Performance Algorithem of Processing Stochastic Signals in Receiving System with Antenna Array in Real Arithmetic

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Abstract

We consider four problems of processing stochastic signals at the antenna array of receiving, which are: adaptive spatial filtering, detected, the estimation and direction finding of source signals. The solution of which is generally performed in a complex calculation. For the central symmetric structure of antenna array it is verified that the solution of these problems can be performed in the real calculation. The results of real and complex calculation are actually the same, but in complex calculation the quantity of calculations is four times larger than the real calculation.

Keywords: stochastic signals, algorithms of signal processing, detection signals, estimating the number of sources, direction finding, adaptive spatial filtering, real arithmetic.

الخلاصة

أربعة مسائل لمعالجة الإشارات العشوائية في نظام الاستقبال مع مجموعة من الهوائيات وتتمثل هذه المسائل بعمليات الكشف، تحديد الاتجاه، تقدير عدد مصادر الإشارات والمرشحات المكانية التكيفية والتعامل مع هذه المسائل وإيجاد الحلول لها يتم تنفيذه بصورة عامة بإجراء العمليات الحسابية على المركبات المعقدة . ثبت إن جميع هذه المسائل يمكن إيجاد حلول لها بإجراء العمليات الحسابية على المركبات الحقيقية عندما تكون مجموعة الهوائيات لديها بنية متماثلة مركزية . نتائج الحلول المعقدة و الحقيقية متطابقة لجميع الحالات ولكن في الحالة الثانية باستخدام المركبات الحقيقية إن كمية الحسابات هي اقل بأربعة مرات بالمقارنة مع استخدام العمليات الحسابية على المركبات المعقدة

الكلمات المفتاحية: خوارزمية معالجة الإشارات ، الإشارات العشوائية ، كشف الإشارات،تقدير عدد المصادر، إيجاد الاتجاه ، المرشحات المكانية التكيفية ، العمليات الحسابية المنطقية

Introduction

The Problem of processing random or quasi-random signals with the different nature of the receiving system with antenna arrays occur in various applications - in radar, sonar, radio astronomy, communications, seismology, biomedicine (Van, 2002). The further statement for definiteness, we shall bear in mind the radar application, with the processing narrowband stochastic signals in the radar station with a digital phased array, for which applies the concept of the complex envelope, although the final results may have wider application. Consider the solution of the following four problems of signal processing

- Detection signals.
- Estimate of the number of source signals.
- Direction finding of source signals.
- Adaptive spatial filtering.

Rectangular matrix A of the size $N \times M$ is used as an input information

$$A = (A_1, \dots, A_M), \qquad (1)$$

Columns which are N-dimensional complex vectors $A_m, m = 1, \dots. M$, signals (complex envelope) from the outputs of phased array elements in M of sequential time points of space from one to another by the sampling interval selected according to the Nyquist theorem, so that serial sampling signals can be considered practically independent. By the package (A) can be calculated maximum likelihood estimation sample of the correlation matrix of the input signals.

$$R = \frac{1}{2M} A.A^{\mathrm{H}}, \qquad (2)$$

Where $(\cdot)^H -$ Sign of Hermitian conjugation (transposition and complex conjugation). The matrix R - Hermitian, positive definite and with a Gaussian of external statistics signals and the intrinsic noise of the antenna array contains all information about the received signals. In general, the algorithms of signal processing implemented in the complex arithmetic. However, there is an important exception to this general rule is the case where the antenna array has a centrally symmetrical structure. To this private, other than often occur in practice, the case include, for example: uniform, linear, regular rectangular and hexagonal, uniform ring (for an even number of radiators) array and a number of other (Van , 2002). It is known that the correct processing of information in antenna arrays has a centrally symmetric structure allows to solve many problems practically double the number of independent samples of the input signal (For example (Zaritskii , Kokin, Lekhovitskii , Salamatin ,1985 ; Van , 2002)), and this station which at first glance may seem almost obvious, actually in need of far from a trivial proof. Well, as for the implementation algorithms of signal processing in real arithmetic, the individual aspects of this problem touched on the available publications (Linebarger , DeGroat ,1994; Van , 2002), but the description of a unified approach to the solutions are listed at the beginning of this section problems not known to us, and its consideration is dedicated to the remaining part of the paper.

Initial condition

The vector X amplitude-phase of the field distribution generated by a point source in the aperture phased array centrally symmetric structure is a "conjugate symmetric":

$$X = J.X^* \ , \tag{3}$$

Here $(\cdot)^* -$ A sign of complex conjugation; J - permutation matrix, that has of unity on the secondary diagonal and zeros in all other positions.

According to maximum likelihood estimation sample of the of correlation matrix in this case has the form (Van , 2002)

$$R_{cs} = \frac{1}{4M}\left(A.A^{\mathrm{H}} + JA^*.A^{\mathrm{T}}\mathrm{J}\right),$$

Where $(\cdot)^{\mathrm{T}} -$A sign of transposition. The last equation can be written also in the form

$$R_{cs} = \frac{1}{4M}\,\mathrm{A_{cs}}.\mathrm{A_{cs}^{H}}, \tag{4}$$

Where the block matrix $\mathrm{A_{cs}}$ Is a $N \times 2M$ which consisting of two blocks with size $N \times M$

$$\mathrm{A_{cs}} = [A \ \ JA^*]. \tag{5}$$

Use of estimates (4), (5) instead of (2) (1) precisely gives the effect, equivalent to the mentioned in the introduction of an increase in the number of independent samples of the input signals.

If matrix A present in the form of two blocks with sizing $\frac{N}{2} \times M$,

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix},$$

The complex package $\mathrm{A_{cs}}$ Is possible to put in an equivalent real package $\mathrm{A_r}$ of the same size, an equal [Van Trees H.L ,2002]

$$\mathrm{A_r} = \begin{bmatrix} Re(A_1 + JA_2) & -Im(A_1 + JA_2) \\ Im(A_1 - JA_2) & Re(A_1 - JA_2) \end{bmatrix}, \tag{6}$$

Where Re (), Im () $-$ real and imaginary parts of the corresponding quantities.

Matrices $\mathrm{A_{cs}}$ and $\mathrm{A_r}$ are related

$$A_r = Q^{\mathrm{H}}.\mathrm{A_{cs}}.\mathrm{L}, \tag{7}$$

Where Q and L - unitary matrices of sizes $N \times N$ and $2M \times 2M$ respectively, equal to

$$Q = \frac{1}{\sqrt{2}}\begin{bmatrix} I & jI \\ J & -jJ \end{bmatrix}, \; L = \frac{1}{\sqrt{2}}\begin{bmatrix} I & jI \\ I & -jI \end{bmatrix}. \tag{8}$$

Where I - the identity matrix, j - imaginary unit. Since the singular values of the matrices are not changed by multiplying its from left or right to any unitary matrices (Gene , Charles , 1996), the singular values of the matrices A_{cs} (Complex) and A_r (Real) are the same.

Detection and estimation of the number of source signals

The computing part of problems of detection and estimates of the number of source stochastic signals add up to finding a (maximum) or more (the largest) eigenvalues correlation matrix R_{cs} by (4) or equivalently, to finding the suitable singular numbers of package A_{cs} by (5). Since the singular numbers of packages A_{cs} by (5) and A_r by (6) are the same, All calculations can be performed in the real matrix A_r with appropriate reducing the amount of computation.

Direction finding of source signals

The solved problems of direction finding of source stochastic signals consider by the example of the Capon's method of direction finding with superresolution. The computing part problems of direction finding in this case consist of finding the values of direction finding pattern P according to the expression

$$P = (V^{H}.R^{-1}.V)^{-\mathbf{1}} \tag{9}$$

Where V - normalized vector hypothesis on structure coinciding with a vector of signal of a taken the bearings of a point source receivable with the direction under consideration. Obviously, in the case of phased array centrally symmetric structures the vector hypothesis V also is "involves symmetrical", i.e. conform the relation (3).

Substituting instead of R in (9) R_{cs} by (4) and going from A_{cs} to A_r by (7) after simple transformations of Hermitian form of expression (9) we obtain

$$T = V^{H}.R^{-1}.V = 4M.V^{H}.Q.(A_r.A_r{}^{T})^{-\mathbf{1}}Q^{H}.V,$$

Where the matrix Q is determined by the first from the equations (8). Introducing the hypothesis vector V as two blocks of size $\frac{N}{2} \times 1$,

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix},$$

And using the obvious relation

$$V_1 = R_eV_1 + jI_mV_1 = JV_2^* = JR_eV_2 - jJI_mV_2 ,$$
$$V_2 = R_eV_2 + jI_mV_2 = JV_1^* = JR_eV_1 - jJI_mV_1 ,$$

After elementary transformations we obtain the expression for the real vector hypothesis V_r:

$$V_r = Q^{H}V = \sqrt{2}\begin{bmatrix} R_eV_1 \\ I_mV_1 \end{bmatrix} = \sqrt{2}\begin{bmatrix} JR_eV_2 \\ -JI_mV_{2\,,} \end{bmatrix}.$$

After elementary transformations we obtain the expression for the real vector hypothesis

$$P = T^{-\mathbf{1}} = \frac{1}{4M}(V_r^T.(V_r.V_r^T)^{-1}.V_r)^{-1}. \tag{10}$$

Of course, instead of forming and explicit inversion matrixes $A_rA_r{}^{T}$ can use the known method of orthogonal of rows package A_r and then using orthogonal filter Φ, so that

$$P = T^{-\mathbf{1}} = \frac{1}{4M}\left(V_r^T.\Phi^T.\Phi V_r\right)^{-1} = \frac{1}{4M}\|\Phi.V_r\|^{-2} ,$$

Where $\|\cdot\|$ – Euclidean norm vector, besides evident that in this case all the calculations are performed in real arithmetic.

Adaptive spatial filtering

The solution of the problem of Adaptive spatial filtering according to the method of direct inversion sample estimate R of the correlation matrix of input signals to calculate a complex weighting vector W

$$W = R^{-1}.\mathrm{S}, \quad (11)$$

Where S - the base vector, all elements that at the presence of an independent system of the phasing antenna array are real and are equal to 1.

Similarly to the above, substituting in (11) R_{cs} by (4) instead of R and going from $\mathrm{A_{cs}}$ to A_r by (7), we obtain:

$$W = 4\mathrm{M}.\mathrm{Q}.(A_r.A_r{}^{\mathrm{T}})^{-1}Q^{\mathrm{H}}.\mathrm{S} \quad . \quad (12)$$

Introducing base vector S in the form of two blocks of size $\frac{N}{2} \times 1$

$$S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix},$$

And in this case $S_1 = S_2$, we obtain the following expression for the intermediate real base vector S_r:

$$S_r = Q^{\mathrm{H}}.\mathrm{S} = \sqrt{2}\begin{bmatrix} S_1 \\ 0 \end{bmatrix}.$$

The second block vector in the last expression – vector of size $\frac{N}{2} \times 1$ with zero elements. Thus, according to (12) can be calculated in real arithmetic value of an intermediate vector W_r,

$$W_r = 4\mathrm{M}.(A_{r.}A_r{}^{\mathrm{T}})^{-1}.S_r, \quad (13)$$

Then the final value of the desired complex vector W is obtained actually repacking elements of a real vector W_r:

$$W = Q.W_r \ .$$

As in the problem of direction finding, use orthogonal filter Φ rows of A_r package instead of (13) we obtain an equivalent expression

$$W_r = 4\mathrm{M}.\Phi^{\mathrm{T}}.\Phi.S_r \ ,$$

According to which all calculations are also performed in real arithmetic.

Conclusion

I have shown that in the receiving system with an antenna array having a centrally symmetrical structure, solving the problems of detection, estimates the number and direction finding of source stochastic signal, as well as adaptive spatial filtering problems can be implemented in real arithmetic.

Since the combination of operations of multiplying two complex numbers and edition two complex numbers is equivalent to eight operations of multiplication or addition of real numbers, the transition from the complex to the real arithmetic leads to reduce the amount of calculation four times.

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