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The effects the Earth , Sun and Moon attraction on satellite's orbital motion

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Abstract:

Computer simulation of orbit estimation satellites has been developed. The combined effects of the J_2 zonal harmonics of the Earth gravitational field and the sun and moon attraction effects on the orbital elements , the right ascension of ascending nodes and the argument of perigee ,were investigated. Orbit estimation is based on the iterative solution of Kepler's equation by using Newton method.

1. Introduction

The estimation of future position and orbital velocity of satellites can be predicted if we know the orbit elements . The Keplerian elements can be obtained from the two line elements , TLE, ephemeris data . Orbit perturbations due to the combined effects of the J_2 zonal harmonics of the Earth , sun and moon attractions are dominant on the right ascension of ascending nodes and argument of perigee angles. Orbit estimation can be performed by solving Kepler equation by iterative method, like Newton method. And numerical integration of the time rate of change of these elements using simple Euler method.

General perturbation element sets on all space objects was performed by Felix R. Hoots and Ronald L. Roehrich in 1988 [1]. Orbit estimation of Ncube picosatellite by solving Kepler's equation was introduced by Kritian S.,2003 [2]. The combined effects of orbit perturbation of J_2 spherical zonal harmonic of the gravitational potential and atmospheric drag for low earth orbit satellites was presented by Mohammed J. Al Bermani and Aref S. Baron ,2008 ,[4].Orbit semi-analytical theory using the central method was given by Barlier and Metris , 2006 [5].Orbit estimation of low earth orbit satellites (cubesat) was presented by Hobbs D., and Bohn, P.,2003[10], and Sakamoto e.al.,2003 [11].

In this paper computer simulation for orbit predictions of position and velocity vectors in inertial frame has been developed using Mathlab 2007 a4 . The effects of zonal harmonic of the oblate gravitational potential , sun and moon gravitational attractions on the right ascension of ascending nodes and argument of perigee were taken into account. Kepler's equation was solved by Newton iteration method. Orbit propagation in time was performed by simple Euler iteration method.

The simple orbit estimator was introduced in section 2 ,orbit perturbations due to earth oblateness and the gravitational attractions of the sun and the moon was presented in section 3. Simulation results in section 4 . The conclusion s and future work in section 5.

2. Simple orbit estimator

When the Keplerian elements for a single point in time are known, the estimation of the future position and velocity becomes relatively straight forward. The future prediction of the mean anomaly in time is given by

$$M(to+t)=M(to)+n(t-tp) \qquad (1)$$

Where n is the mean motion given by

$$n=\sqrt{\frac{\mu}{a^3}} \qquad (2)$$

Where tp is the time of perigee passage.
In order to transfer the estimate into earth centered inertial coordinate frame the Kepler's equation should be solved by iterative method

$$M(t)= E-e \sin(E) \qquad (3)$$

Which relates the eccentric anomaly to the mean anomaly M.

Kepler's equation is a transcendental equation which cannot be solved directly for E given arbitrary M. A simple iterative method like Newton method can be used as follows

$$E_{i+1}=M+e\sin E_i \qquad (4)$$

With E=Mo, gives a good estimate

$$E_{i+1}=E_i+\frac{M+e\sin E_i-E_i}{1-e\cos E_i} \qquad (5)$$

Given the eccentric anomaly, the radius vector from the center of the Earth to the satellite in orbit plane is given by[8]

$$r=a\begin{bmatrix}\cos E-e\\ \sqrt{1-e^2}\sin E\\ 0\end{bmatrix} \qquad (6)$$

And the corresponding orbital velocity vector is given by[8]

$$v=\sqrt{\frac{\mu_e}{a}}\begin{bmatrix}\dfrac{\sin E}{1-e\cos E}\\ \dfrac{\sqrt{1-e^2}\cos E}{1-e\cos E}\\ 0\\ \end{bmatrix} \qquad (7)$$

Transforming these vectors into inertial coordinates frame ECI yields[2]

$$r_I=R_z(-\Omega)R_x(-i)R_z(-w) \qquad (8)$$

Where the rotation matrices for inclination angle , right ascension of ascending nodes and argument of perigee are given by[2]

$$R_x=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(-i) & \sin(-i)\\ 0 & -\sin(-i) & \cos(-i)\end{bmatrix} \qquad (9a)$$

$$R_{z\Omega} = \begin{bmatrix} \cos(-\Omega) & \sin(-\Omega) & 0 \\ -\sin(-\Omega) & \cos(-\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (9b)$$

$$R_{z\omega} = \begin{bmatrix} \cos(-w) & \sin(-w) & 0 \\ -\sin(-w) & \cos(-w) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (9c)$$

There is another reference frame frequently used in estimation of the earth magnetic field, the so-called Earth-Centered Earth fixed frame (ECEF). In this frame , the x-axis intersects the earth surface at latitude 0 deg and longitude 0 deg , the z-axis in the north direction and the y-axis completes the right hand system. The ECEF rotates with the earth at constant angular velocity .
Transforming r into ECEF frame can be utilized by the following direction cosines

$$r_E = R_{z\Omega\theta}(-\Omega+\theta)R_x(-i)R_{zw}(-w) \qquad (10)$$

Where

$$R_{Z\Omega\theta} = \begin{bmatrix} \cos(-\Omega+\theta) & \sin(-\Omega+\theta) & 0 \\ -\sin(-\Omega+\theta) & \cos(-\Omega+\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And the angle θ is the angle between the two frames ECI and ECEF which is given by

$$\theta = \theta_{GO} + w_E.t \qquad (11)$$

Where w_E is the angular rotational velocity of the earth and θ_{GO} refers to the right ascension of Greenwich at epoch.

3. Orbit perturbations

There are several sources of errors facing Keplerian orbits ,such as the oblateness of the earth, sun and moon gravitational attractions ,tidal earth and oceans , electromagnetic radiations and atmospheric drag forces. In this paper , the zonal spherical harmonic J_2 of the geopotential function , the sun and moon gravitational attractions are taken in consideration.

3-1 Perturbation due to the nonspherical earth

The earth is not spherical , in fact it has a bulge at the equator , is flattened at poles and is slightly pear – shaped. This leads to perturbations in all Keplerian elements. According to Lagrange planetary equations , the time derivatives of the right ascension of ascending nodes and the argument of perigee are given by[5]

$$\dot{\Omega}_{J2} = -\frac{3}{2}na_e{}^2 \frac{\cos i}{a^2(1-e^2)^2} J_2 \qquad (12a)$$

$$\dot{\omega}_{J2} = \frac{3}{4}na_e{}^2 \frac{5\cos^2 i - 1}{a^2(1-e^2)^2} J_2 \qquad (12b)$$

Where a_e is the earth radius and J_2 is the second zonal spherical harmonic coefficient of the earth ($J_2 = 1.08284.10^{-3}$)

3.2 Perturbations due to sun and moon attractions

The sun and moon causes periodic variations in all Keplerian elements, but secular perturbations only to the right ascension of ascending node and argument of perigee. For nearly circular orbits, an approximation suggested by Wertz and Larson 1999 [9] yields

$$\dot{\Omega}_{moon} = -0.00338\frac{\cos i}{n} \quad (13a)$$

$$\dot{\Omega}_{sun} = -0.00154\frac{\cos i}{n} \quad (13b)$$

And

$$\dot{\omega}_{moon} = 0.00169\frac{5\cos^2 i - 1}{n} \quad (13c)$$

$$\dot{\omega}_{sun} = 0.00077\frac{5\cos^2 i - 1}{n} \quad (13d)$$

Where n is the number of revolutions per day and $\dot{\Omega}, \dot{\omega}$ are given in degree/day

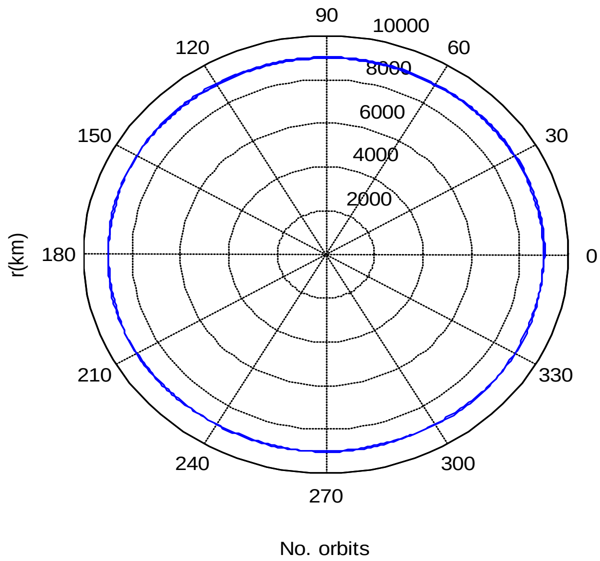
4. Computer simulation and results

Computer simulation of orbit estimation has been developed in Mathlab 2007a4 . The orbit elements were entered in the form of the NORAD two-line Element Sets , TLE ,as proposed by NORAD and NASA to describe satellites and their position in orbit [4] . The propagation of orbit elements in time were performed by simple Euler integration method. Kepler's equation was solved by Newton iteration method. The input data are the keplerian element of sunsynchronous quasi-circular orbit .
a=9000 km,e=0.0045,i=98 deg,Ω=155 deg ,w=85 deg , M=55 deg .
The outputs are shown in the figures below . Figure(1) is a polar plot of the geocentric radial distance (km) against true anomaly (deg) . Transformation of coordinates from orbit plane into inertial frame ECI and earth centered earth fixed frame ECEF are shown in figures (2) and (3) for 15 orbits .The time history of the angle between the two frames is shown in figure(4) which depends upon the earth spinning about it's axis. It can be seen from this figure that the angle varies linearly with time.
The orbital velocity components (vx,vy,vz) defined in inertial frame are shown in figure(5) for a time of one day (i.e. 15 orbits). The solution of Kepler's equation for eccentricity is shown in figure(6) , where the eccentric anomaly coincides with the mean anomaly . The result for eccentric orbit e=0.5 is shown in figure(7). It can be seen from this figure that the eccentric and mean anomalies are equal at perigee then the eccentric anomaly grows faster than the mean anomaly until the orbit passes through apogee θ=π, where the two angles are equal at the first half of the orbit . and finally at the end of the orbit they are equal θ=2π . The variations of argument of perigee and the right ascension of ascending node longitude are shown in figures(8) and (9) respectively .

5. Conclusion and future work

Orbit estimation under the effects of J_2 zonal harmonics of the earth gravitational field and the attraction of moon and sun for sunsynchronour quasi- circular orbit has been simulated .Kepler's equation was solved using Newton iteration method. And orbit propagation by simple Euler's integration method. The simulation was tested for quasi-circular sunsynchronour orbit (e=0.0045) and for eccentric orbit (e=0.5) for one day (15 orbits) .
It can be concluded from the results that the J_2 zonal harmonic of the gravitational field was dominant over the sun and moon attractions for this type of orbit. and there is appreciable changes of position vectors defined in inertial frame ECI and ECEF frame during the day .

The future work will include the estimation of the earth magnetic field on orbit defined in earth rotating frame ECEF.



Fig(1). The geocentric radial distance (km) against polar angle (deg) in orbit plane

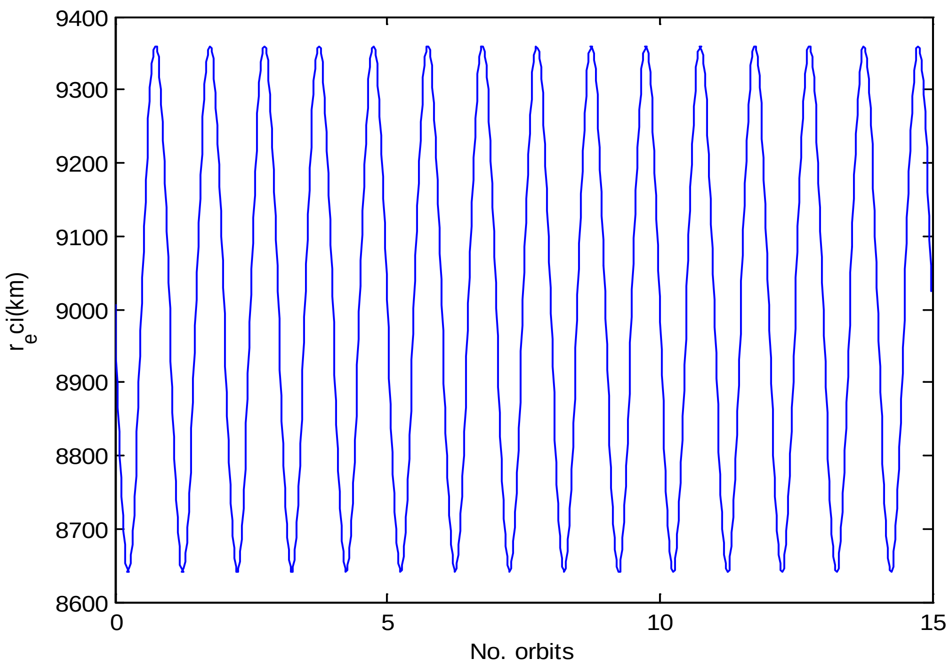
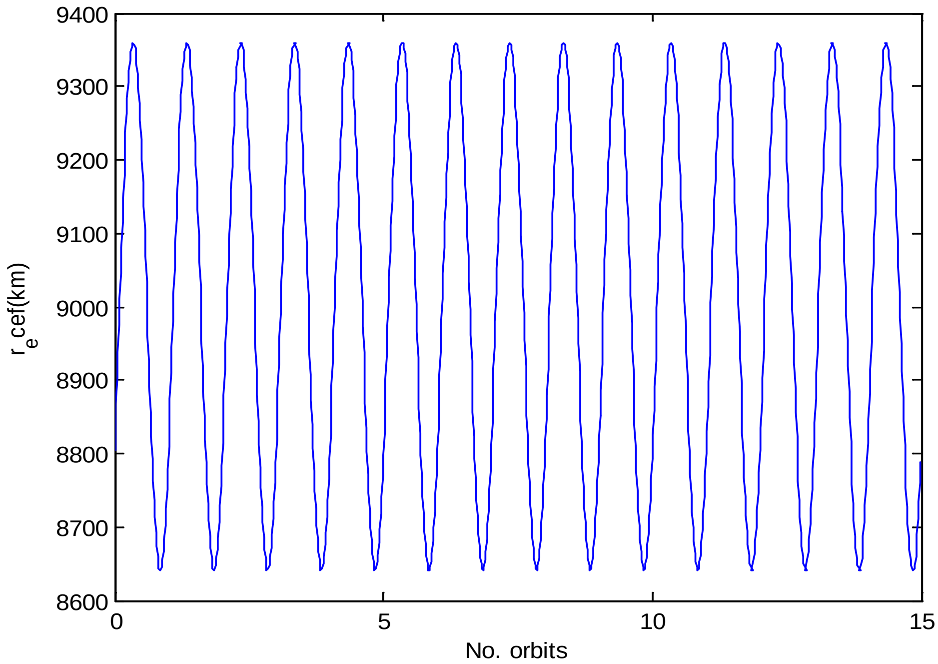
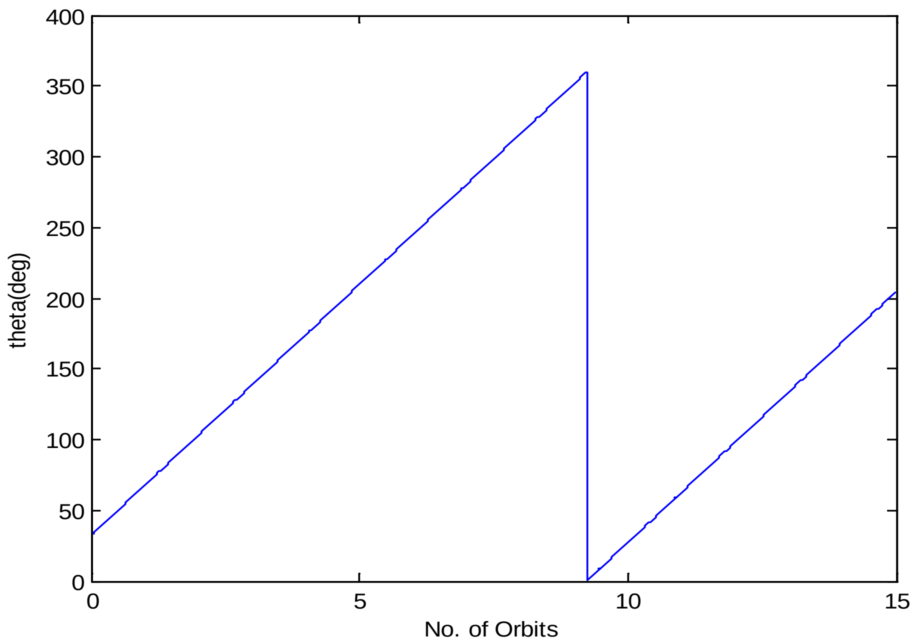


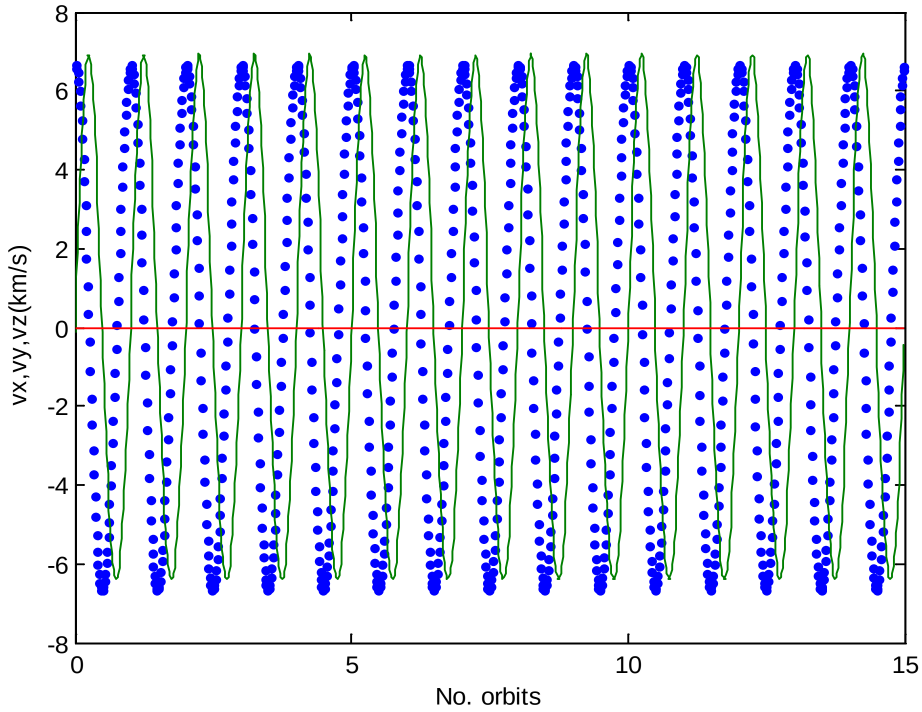
Fig.(2) The position vector (km) in inertial frame ECI versus no. of orbits



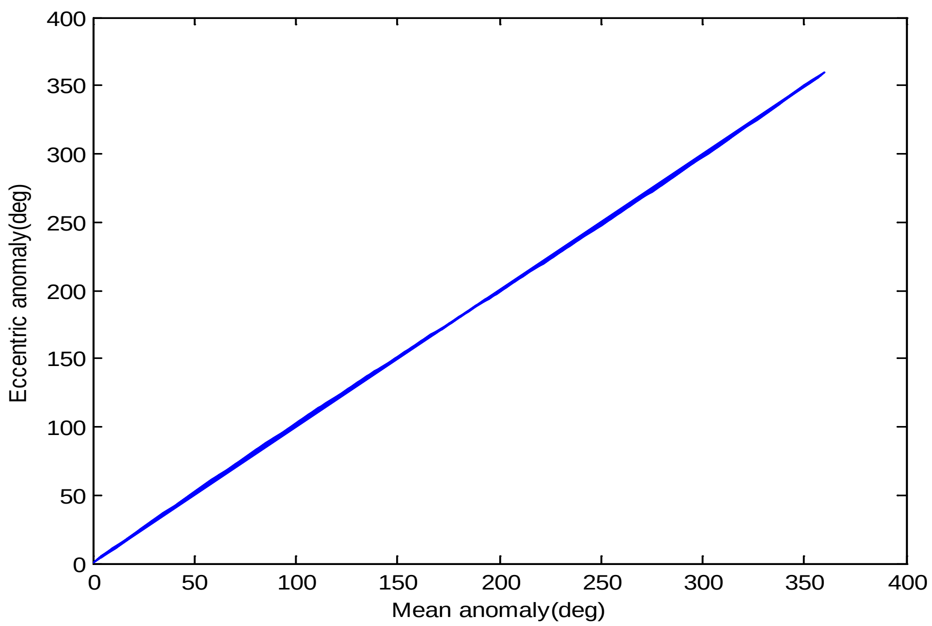
Fig(3). The position vector defined in the ECEF frame (km) versus no. of orbit



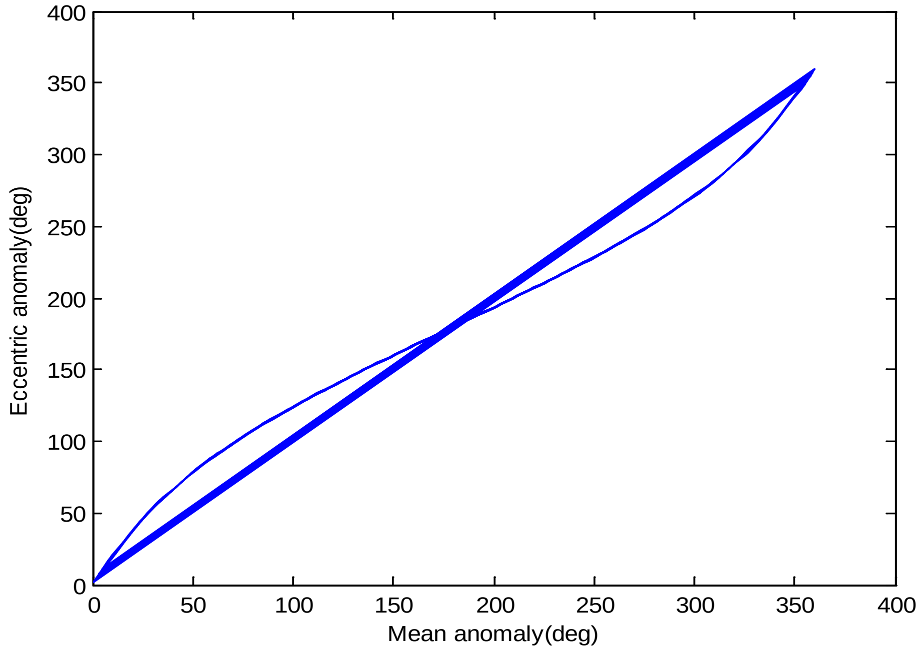
Fig(4). The angle between the two frames ECI and ECEF (deg) against no. of orbits



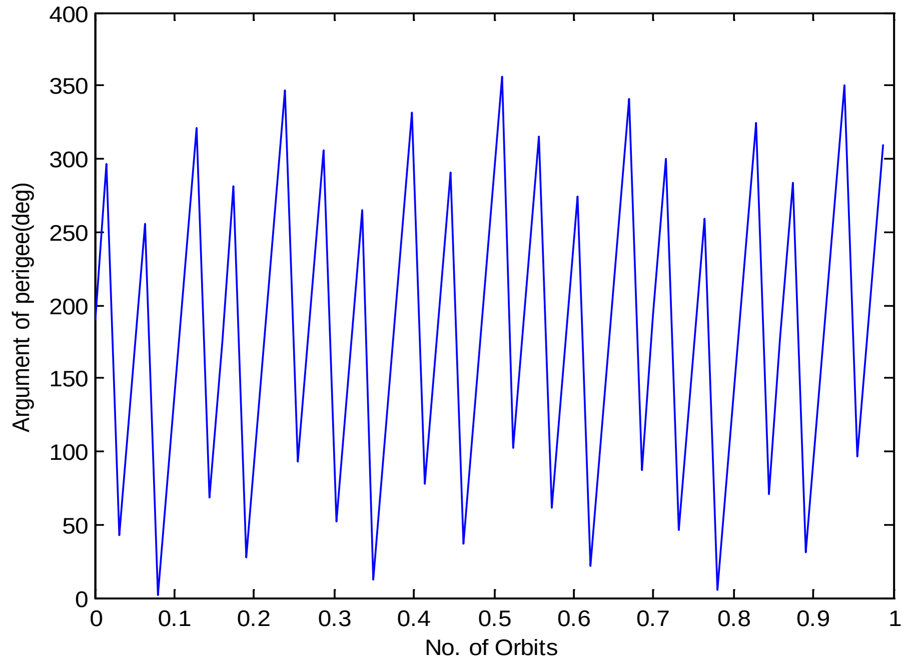
Fig(5). Orbital velocity components in ECI frame (km/s) against no. of orbits



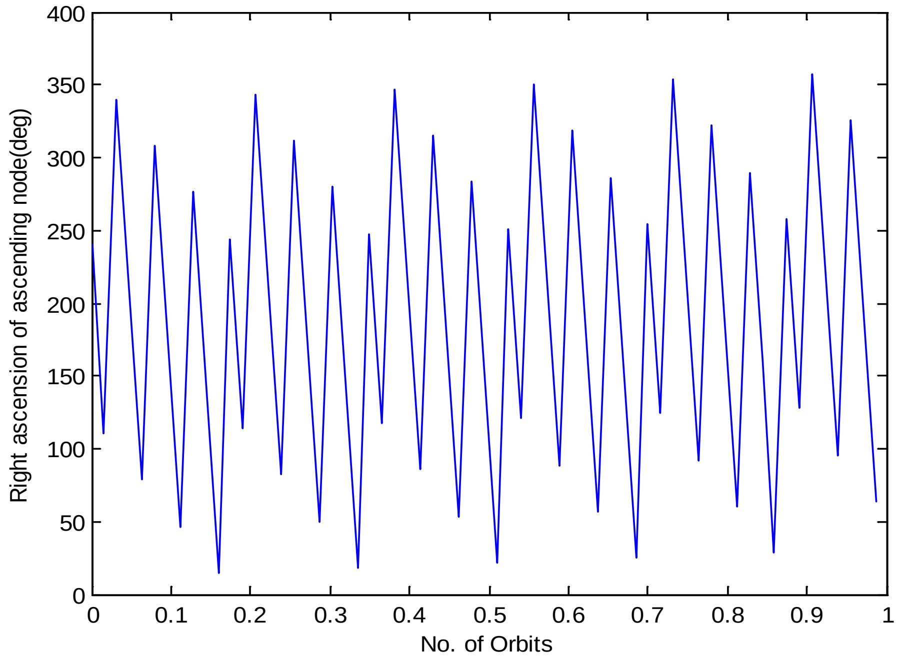
Fig(6). The eccentric anomaly (deg) against mean anomaly (deg) for quasi-circular orbit



Fig(7). The eccentric anomaly (deg) versus mean anomaly (deg), when e=0.5



Fig(8). Argument of perigee (deg) for one orbit



Fig(9). The right ascension longitude of ascending nodes (deg) for one orbit

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تأثيرات جاذبية الأرض والشمس والقمر على مدارات الأقمار الصناعية

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الخلاصة

تم في هذا البحث تطوير محاكاة تخمين مدارات الاقمار الصناعية تحت تاثير جاذبية الارض والشمس والقمر . حيث تم اجراء حسابات المسافة النصف قطرية والسرعة المدارية في الاحداثيات المرجعية الثابتة والمتحركة بسرعة دوران الارض حول محورها . وقد اشتمل البحث تاثيرات تلك الاضطرابات المدارية على كل من زاوية المطلع المستقيم للعقدة الصاعدة وزاوية مثابة الحضيض وذلك من خلال حل معادلة كبلر بطريقة نيوتن .