Journal of AL-Rafidain University College for Sciences (2024); Issue 56; pp. 307-321

AL- Rafidain University College

PISSN: (1681-6870); EISSN: (2790-2293)

Journal of AL-Rafidain University College for Sciences

Available online at: https://www.jrucs.iq

JRUCS

Journal of AL-Rafidain University College for Sciences

Survival Function Estimation for COVID-19 Patients Using TPLD and PTPLD

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Article Information

Article History:
Received: February, 23, 2024
Accepted: April, 12, 2024
Available Online: 31, December, 2024

Keywords:
Three-parameter Lindley distribution, proposed three-parameter Lindley distribution, Survival functions, Method of Moments, MLH.

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Abstract

This paper aims to study some properties of the three-parameter Lindley distribution TPLD and the proposed new formula for the TPLD in studying the life-time distribution, survival function, and hazard function. We proposed a new formula for the TPLD by considering the new mixing weights of the mixing function that were used in constructing the TPLD and by replacing the mixing weights in the mixing equation to get a new formula for the TPLD that is called the proposed three-parameter Lindley distribution, and we denoted it by PTPLD. We study some properties of PTPLD in the lifetime distribution. We use three methods of estimation, the method of moments , the MLH method and Standard Bayesian method, to estimate the parameters and the survival functions for the two distributions (TPLD and PTPLD) and compare the survival functions estimates for both distributions using the above three methods of estimation, and we select the best estimate based on some criteria of error. In addition, we used real data from the AL-Hussain Teaching Hospital in Nasiriya City to estimate the survival time of infected patients with COVID-19 using TPLD and PTPLD. It was concluded that the proposed formula for the distribution is the best for all selected sample sizes.

DOI: https://doi.org/10.55562/jrucs.v56i1.28

1. Introduction

Survival analysis has become a very important statistical technique since the beginning of the 20th century; this theory is based on non-negative probability distribution (right-skewed) like Weibull, Rayleigh, Lognormal distributions, etc. Because of the most phenomena in real life is a mixture of distributions, so mixture distributions raised, that is a part of the population follows a certain distribution and the other follows the mixed distributions such as t-Student, Laplace, Lomax and Lindley distribution. The Lindley distribution was named by Adrian M. Lindley introduced it in 1958 in his paper titled “Fiducial distributions and Bayes' theorem" [1]. But real interest began in 2008, when Ghitany, Atieh and Nadarajah examined its properties and application [2], and this distribution be developed as a generalized Lindley distribution Proposed by Zakerzadeh and Dolati (2009)[3].Shanker and Mishra (2013a, 2013b) [4, 5] developed a two parameters Lindley distribution and studies a quasi-Lindley distribution, Lindley distribution with location parameters. Three-parameter distribution was studied by Abd El-Monsef (2015) [6]. Menezes and Chakraborty (2018) with stand-in Lindley Kumar and Jose Sales (2019).Bayesian survival function estimation of three-parameter Lindley distribution and its practical application was studied by Al-Abadi Karma N. Hussein(2021) [7].In (2023) both (Mathil K. Thamer & Raoudha Zine) studied the properties of Lindley distribution with three-parameters [8].Because emergence of the COVID-19 pandemic, which spread rapidly across continents causing hundreds of millions of infections and thousands of deaths within a short period. This led to fear and panic among human societies and hindered the global economy, necessitating research, study, and interpretation to understand the behavior of this pandemic. The purpose of this paper is to estimate the survival function for individuals infected with the corona virus (COVID-19) by using Lindley distribution with three parameters (TPLD) and proposing a new formula of (TPLD) distribution through suggesting different values of mixing parameter. We studied some properties of (PTPLD) in the life time distribution. We use three methods of estimation (moments, MLH, Bayes estimation) and compare between the results of survival estimated for (TPLD) and (PTPLD) by using some criterions of errors for the various studied distributions to get the better one. And finally, we applied real data from AL-Hussain Teaching Hospital in Nasiriya City to estimate the survival time of infected patients with COVID-19 using TPLD and PTPLD.

2. Theoretical Aspect

2.1 The Three–Parameter Lindley Distribution (TPLD) and Survival measures

The probability density function of the three–parameter Lindley distribution (TPLD) introduced by Shanker et al. (2017) can be expressed as a mixture of two distributions exponential distribution with parameter (η) and gamma distribution with parameters (2, η) with mixing proportion: [9]

$$U = \frac{\eta\zeta}{\eta\zeta + \theta} \tag{1}$$

Using the mixing formula:

$$f(t;\eta,\zeta,\theta) = Uf1(t) + (1-U)\,f2(t) \tag{2}$$

$$f_{(\text{TPLD})}(t;\eta,\zeta,\theta) = \left(\frac{\eta^2}{\eta\zeta+\theta}\right)(\zeta+\theta t)e^{-\eta t}\ ; t,\eta,\zeta > 0\ ; \eta\zeta+\theta > 0 \tag{3}$$

η: $scale\ parameter$

ζ,θ: $shape\ parameters$

The Cumulative distribution function is:

$$\text{F}(\text{t};\ \eta,\zeta,\theta) = 1 - \left[\frac{\eta\zeta+\theta+\eta\theta t}{(\eta\zeta+\theta)}\right]e^{-\eta t}\ \ ; \text{t}, \eta, \theta > 0\ ,\ \ \eta\zeta+\theta > 0 \tag{4}$$

The survival function S(t) is:

$$\text{S}(\text{t};\ \eta,\zeta,\theta) = \left[\frac{\eta\zeta+\theta+\eta\zeta\theta t}{(\eta\zeta+\theta)}\right];\ \text{t},\eta,\theta > 0\ ,\ \eta\zeta+\theta > 0 \tag{5}$$

And the hazard function is:

$$\text{h}(\text{t};\ \eta,\zeta,\theta) = \frac{\eta^2(\zeta+\theta t)}{\eta\zeta+\theta+\eta\theta t}\ \ ;\ \text{t},\eta,\theta > 0\ ,\ \eta\zeta+\theta > 0 \tag{6}$$

The MTTF or mean time to failure for (TPLD) is defined as follows:

$$\text{MTTF} = \int_0^\infty \text{S}(\text{t};\eta,\zeta,\theta)\,dt = \frac{\zeta\eta+2\theta}{\eta(\zeta\eta+\theta)} \tag{7}$$

Non-central and central moments of the TPLD

Non-central moments about zero can be defined as follows: [9]

$$\mu'_r = E(T)^r = \frac{r!(\eta\zeta+(r+1)\theta)}{\eta^r(\eta\zeta+\theta)} \qquad r=1, 2,\ldots \tag{8}$$

So the variance of (TPLD) is defined as:

$$\text{Var}(t) = \frac{\eta^2\zeta^2+4\eta\zeta\theta+2\theta^2}{\eta^2(\eta\zeta+\theta)^2} \tag{9}$$

Now the r^{th} central moments is define as follows: [9]

$$\mu_r = E[(t-\mu)^r] = \sum_{i=0}^{r} \binom{r}{i}(-\mu)^{r-i}\mu'_i \tag{10}$$

$$\mu_1 = 0 \tag{11}$$

$$\mu_2 = \frac{\zeta^2\eta^2+4\zeta\eta\theta+2\theta^2}{\eta^2(\eta\zeta+\theta)^2} \tag{12}$$

$$\mu_3 = \frac{2(\zeta^3\eta^3+6\zeta^2\eta^2\theta+6\zeta\eta\theta^2+2\theta^3)}{\eta^3(\eta\zeta+\theta)^3} \tag{13}$$

$$\mu_4 = \frac{3(3\zeta^4\eta^4+24\zeta^3\eta^3\theta+44\zeta^2\eta^2\theta+32\zeta\eta\theta^3+8\theta^4)}{\eta^4(\eta\zeta+\theta)^4} \tag{14}$$

The measures of central tendency for TPLD

The mean, median, and mode for the TPLD are defined as follows: [9]

$$mean = \frac{(\eta\zeta+2\theta)}{\eta(\eta\zeta+\theta)} \tag{15}$$

$$Median = \left[\frac{\eta\zeta+\theta+\eta\theta t_{med}}{(\eta\zeta+\theta)}\right]e^{-\eta t_{med}} = 0.5 \tag{26}$$

$$Mode = \begin{cases} \dfrac{\theta-\eta\zeta}{\theta\eta}, & |\zeta\eta| < \theta \\ zero, & otherwise \end{cases} \tag{27}$$

The standard deviation (σ)and coefficient of variation (C.V) as follows: [9]

$$\sigma = \frac{\sqrt{\zeta^2\eta^2+4\zeta\eta\theta+2\theta^2}}{\eta(\eta\zeta+\theta)} \tag{18}$$

$$C.V = \frac{\sqrt{\zeta^2\eta^2+4\zeta\eta\theta+2\theta^2}}{(\eta\zeta+2\theta)} \tag{19}$$

The Moment Generating Function for TPLD

It can be found according to the following formula: [9]

$$M_x(t) = \sum_{k=0}^{\infty}\left(\frac{\eta\zeta+\theta+k}{\eta\zeta+\theta}\right)\left(\frac{t}{\eta}\right)^k \tag{20}$$

2.2 Proposed Three-Parameter Lindley Distribution (PTPLD)

We suggest a new formula for the (TPLD) by the following assumptions:

1. **The exponential distribution as follows:**

$$f_1(t,\eta) = \eta\, e^{-\eta t} \qquad , t > 0 \tag{21}$$

And gamma distribution as follows:

$$f_2(t,2,\eta) = \eta^2 t\, e^{-\eta t} \qquad , t > 0 \tag{22}$$

2. **The mixing parameter is defined as follows:**

$$U = \frac{\eta\zeta\theta}{\eta\zeta\theta+1} \tag{23}$$

3. **Assuming the mixing formula as follows:**

$$f(t;\eta,\zeta,\theta) = (1-U)f1(t) + Uf2(t) \tag{24}$$

The proposed three-parameter Lindley distribution (PTPLD) can be shown as follows:

$$f_{(\text{ PTPLD})}(t,\eta,\zeta,\theta)=\frac{\eta}{(\eta\zeta\theta+1)}(1+\ \eta^2\zeta\theta t\ )\,e^{-\ \eta t} \qquad ;\ t,\eta,\zeta,\theta>0 \tag{25}$$

ζ,θ : Shape parameters

η : Rate parameter

$$\int_0^\infty \frac{\eta}{(\eta\zeta\theta+1)}(1+\ \eta^2\zeta\theta t\ )\,e^{-\ \eta t}\ dt=1 \tag{26}$$

The cumulative distribution (c.d.f.) for the PTPLD

$$F_{(t)}=1-\left[\frac{1+\ \eta^2\zeta\theta t+\eta\zeta\theta}{(\eta\zeta\theta+1)}\right]e^{-\eta t} \tag{27}$$

And the survival function S(t) for PTPLD is given by:

$$S(t,\eta,\zeta,\theta)=P(T\ >t)=\left[\frac{1+\eta^2\zeta\theta t+\eta\zeta\theta}{(\eta\zeta\theta+1)}\right]e^{-\eta t};\ t,\eta,\theta\ >\ 0\ ,\ \ \eta\zeta+\theta>0 \tag{28}$$

So, the hazard function can be shown as follows:

$$h(t,\eta,\zeta,\theta)=\frac{\eta+\eta^3\zeta\theta t}{1+\ \eta^2\zeta\theta t+\eta\zeta\theta} \qquad ;t,\zeta,\eta,\theta\ >0 \tag{29}$$

The non-central moments about origin of (PTPLD) can be obtained as follows:

$$\mu_r^{'}=\int_0^\infty t^r\,\frac{\eta}{(\eta\zeta\theta+1)}(1+\ \eta^2\zeta\theta t\ )\,e^{-\ \eta t}\,dt \tag{30}$$

$$\mu_r^{'}=\frac{r![1+\eta\zeta\theta(r+1)]}{\eta^r(\eta\zeta\theta+1)} \qquad ;r=1,2,\ldots \tag{31}$$

The first four non-central moments about the origin of PTPLD as follows:

$$\mu_1^{'}=\frac{1+2\eta\zeta\theta}{\eta(\eta\zeta\theta+1)} \tag{32}$$

$$\mu_2^{'}=\frac{2(1+3\eta\zeta\theta)}{\eta^2(\eta\zeta\theta+1)} \tag{33}$$

$$\mu_3^{'}=\frac{6(1+4\eta\zeta\theta)}{\eta^3(\eta\zeta\theta+1)} \tag{34}$$

$$\mu_4^{'}=\frac{24(1+5\eta\zeta\theta)}{\eta^4(\eta\zeta\theta+1)} \tag{35}$$

The central moments about the mean of (PTPLD) are obtained as follows:

$$\mu_r=E(t-\mu)^r=\int_0^\infty (t-\mu)^r\,\frac{\eta}{(\eta\zeta\theta+1)}(1+\ \eta^2\zeta\theta t\ )\,e^{-\ \eta t}\,dt \tag{36}$$

$$\therefore E(t-\mu)^r=\frac{\eta}{\eta\zeta\theta+1}\left[\sum_{i=0}^{r}\binom{r}{i}\left(-\frac{1+2\eta\zeta\theta(r-i)!}{\eta^{r-i+2}(\eta\zeta\theta+1)}\right)+\zeta\theta\sum_{i=0}^{r}\binom{r}{i}\left(-\frac{1+2\eta\zeta\theta(r-i+1)!}{\eta^{r-i+1}(\eta\zeta\theta+1)}\right)\right] \tag{37}$$

The three most common measures of tendency are the mean, median, and mode for new formula of the three parameter Lindley distribution (PTPLD) can be obtained as follows:

$$mean=\frac{1+2\eta\zeta\theta}{\eta(\eta\zeta\theta+1)} \tag{38}$$

$$Median=\left[\frac{\eta\zeta\theta+\eta t_{med}+1}{\eta\zeta\theta+1}\right]e^{-\eta t_{med}}=0.5 \tag{40}$$

$$Mode=\begin{cases}\dfrac{\eta\zeta\theta-1}{\eta^2\zeta\theta}, & |\,\eta\zeta\theta|>1\\ zero, & otherwise\end{cases} \tag{41}$$

The variance, standard deviation and coefficient of variation of (PTPLD) can be shown as follows:

$$var(t)=\frac{2\eta^2\zeta^2\theta^2+4\eta\zeta\theta+1}{\eta^2(\eta\zeta\theta+1)^2} \tag{42}$$

$$\sigma = \sqrt{\frac{2\eta^2\zeta^2\theta^2+4\eta\zeta\theta+1}{\eta^2(\eta\zeta\theta+1)^2}} \tag{43}$$

$$C.V = \sqrt{\frac{2\eta^2\zeta^2\theta^2+4\eta\zeta\theta+1}{(1+2\eta\zeta\theta)^2}} \tag{44}$$

And the coefficient of Skewness (C.S.) of PTPLD is:

$$C.S = \frac{3\eta^2(1+4\eta\zeta\theta)(\eta\zeta\theta+1)}{2(1+3\eta\zeta\theta)^2} \tag{45}$$

The Moment Generating Function of (PTPLD)

It can be found according to the following formula:

$$M_x(t) = \frac{\eta}{\eta\zeta\theta+1}\left[\frac{1}{\eta}\sum_{k=0}^{\infty}\left(\frac{t}{\eta}\right)^k + \zeta\theta\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\eta}\right)^k\right] \tag{46}$$

2.3 Moments Method (MME) for (TPLD) [4]

$$\frac{(\eta\zeta+2\theta)}{\eta(\eta\zeta+\theta)} = \frac{1}{n}\sum_{i=1}^{n} t_i \tag{47}$$

$$\frac{2(\eta\zeta+3\theta)}{\eta^2(\eta\zeta+\theta)} = \frac{1}{n}\sum_{i=1}^{n} t_i^2 \tag{48}$$

$$\frac{6(\eta\zeta+4\theta)}{\eta^3(\eta\zeta+\theta)} = \frac{1}{n}\sum_{i=1}^{n} t_i^3 \tag{49}$$

We see that they $(47), (48)$and(49) are non- linear equation and do not seem to be solved directly to obtain estimator $\hat{\eta}_{mom}, \hat{\zeta}_{mom}$ and $\hat{\theta}_{mom}$. Thus, we must use the Lindley approximation to solve these equations for the survival function of TPLD as follows:

$$\hat{R}_{mom}(t_i) = \left[\frac{\hat{\eta}_{mom}\hat{\zeta}_{mom}+\hat{\theta}_{mom}+\hat{\eta}_{mom}\hat{\zeta}_{mom}\hat{\theta}_{mom}t_i}{(\hat{\eta}_{mom}\hat{\zeta}_{mom}+\hat{\theta}_{mom})}\right]e^{-\hat{\eta}_{mom}t_i} \tag{50}$$

2.4 Similarly the MME to PTPLD

$$\frac{\eta\zeta\theta+2}{\eta(\eta\zeta\theta+1)} = \frac{1}{n}\sum_{i=1}^{n} t_i \tag{51}$$

$$\frac{2\eta\zeta\theta+6}{\eta^2(\eta\zeta\theta+1)} = \frac{1}{n}\sum_{i=1}^{n} t_i^2 \tag{52}$$

$$\frac{6\eta\zeta\theta+24}{\eta^3(\eta\zeta\theta+1)} = \frac{1}{n}\sum_{i=1}^{n} t_i^3 \tag{53}$$

We see that they $(51), (52)$and(53) are non- linear equation and do not seem to be solved directly to obtain estimator$\hat{\eta}_{mom}, \hat{\zeta}_{mom}$ and $\hat{\theta}_{mom}$. Thus, we must use the Lindley approximation to solve these equations for the survival function of PTPLD as follows:

$$\hat{R}_{mom}(t_i) = \left[\frac{\hat{\eta}_{mom}\hat{\zeta}_{mom}\hat{\theta}_{mom}+\hat{\eta}_{mom}t_i+1}{(\hat{\eta}_{mom}\hat{\zeta}_{mom}\hat{\theta}_{mom}+1)}\right]e^{-\hat{\eta}_{mom}t_i} \tag{54}$$

2.5 Maximum Likelihood Estimate (MLH)

Let $t_1, t_2, t_3, \dots . t_n$ be a random sample of size n from (TPLD) (3).The likelihood function of (3) is given by: [9, 12]

$$L = \left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \prod_{i=1}^{n}(\zeta+\theta t)e^{-n\eta\bar{t}} \tag{55}$$

$$\therefore \frac{\partial LnL}{\partial\hat{\eta}} = \frac{2n}{\hat{\eta}} - \frac{-n\hat{\zeta}}{(\hat{\eta}\hat{\zeta}+\hat{\theta})} - n\bar{t} = 0 \tag{56}$$

$$\therefore \frac{\partial LnL}{\partial\hat{\zeta}} = \sum_{i=1}^{n}\left[\frac{1}{\hat{\zeta}+\hat{\theta}ti}\right] - \frac{n\,\hat{\eta}}{\hat{\eta}\hat{\zeta}+\hat{\theta}} = 0 \tag{57}$$

$$\therefore \frac{\partial LnL}{\partial\hat{\theta}} = \sum_{i=1}^{n}\left[\frac{ti}{\hat{\zeta}+\hat{\theta}ti}\right] - \frac{n}{\hat{\eta}\hat{\zeta}+\hat{\theta}} = 0 \tag{58}$$

Since the first derivatives of likelihood function with respect to $\eta, \zeta \; and \; \theta$ are non-linear equations and do not seem to be solved directly .so, we will use the Fisher's scoring method to solve these equations as follows:

$$\frac{\partial^2 LnL}{\partial \eta^2} = \frac{-2\mathrm{n}}{\eta^2} + \frac{\mathrm{n}\zeta^2}{(\eta\zeta+\theta)^2} \tag{59}$$

$$\frac{\partial^2 LnL}{\partial \eta \partial \zeta} = \frac{-\mathrm{n}\theta}{(\eta\zeta+\theta)^2} = \frac{\partial^2 LnL}{\partial \zeta \partial \eta} \tag{60}$$

$$\frac{\partial^2 LnL}{\partial \eta \partial \theta} = \frac{-\mathrm{n}\zeta}{(\eta\zeta+\theta)^2} = \frac{\partial^2 LnL}{\partial \theta \partial \eta} \tag{61}$$

$$\frac{\partial^2 LnL}{\partial \zeta^2} = \sum_{i=1}^{n} \frac{-1}{(\zeta+\theta ti)^2} - \frac{n\,\eta^2}{(\eta\zeta+\theta)^2} \tag{62}$$

$$\frac{\partial^2 LnL}{\partial \zeta \partial \theta} = \sum_{i=1}^{n} \frac{-ti}{(\zeta+\theta ti)^2} + \frac{n\,\eta}{(\eta\zeta+\theta)^2} = \frac{\partial^2 LnL}{\partial \theta \partial \zeta} \tag{63}$$

$$\frac{\partial^2 LnL}{\partial \theta^2} = \sum_{i=1}^{n} \frac{-ti^2}{(\zeta+\theta ti)^2} + \frac{\mathrm{n}}{(\eta\zeta+\theta)^2} \tag{64}$$

The following equations can be solved for getting MLEs $\hat{\eta}$, $\hat{\zeta}$ and $\hat{\theta}$ of (TPLD)

$$\begin{bmatrix} \frac{\partial^2 LnL}{\partial \eta^2} & \frac{\partial^2 LnL}{\partial \eta \partial \zeta} & \frac{\partial^2 LnL}{\partial \eta \partial \theta} \\ \frac{\partial^2 LnL}{\partial \zeta \partial \eta} & \frac{\partial^2 LnL}{\partial \zeta^2} & \frac{\partial^2 LnL}{\partial \zeta \partial \theta} \\ \frac{\partial^2 LnL}{\partial \theta \partial \eta} & \frac{\partial^2 LnL}{\partial \theta \partial \zeta} & \frac{\partial^2 LnL}{\partial \theta^2} \end{bmatrix}_{\substack{\hat{\eta}=\eta_0 \\ \hat{\zeta}=\zeta_0 \\ \hat{\theta}=\theta_0}} \begin{bmatrix} \hat{\eta}-\eta_0 \\ \hat{\zeta}-\zeta_0 \\ \hat{\theta}-\theta_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial LnL}{\partial \eta} \\ \frac{\partial LnL}{\partial \zeta} \\ \frac{\partial LnL}{\partial \theta} \end{bmatrix}_{\substack{\hat{\eta}=\eta_0 \\ \hat{\zeta}=\zeta_0 \\ \hat{\theta}=\theta_0}} \tag{65}$$

Where,η_0, ζ_0 and θ_0 are the initial values of η , ζ and θ respectively. These equations are solved iteratively till sufficiently close values of $\hat{\eta}$, $\hat{\zeta}$ and $\hat{\theta}$ are obtained.

2.6 Similarly for MLE to (PTPLD)

$$L = \left(\frac{\eta}{\eta\zeta\theta+1}\right)^n \prod_{i=1}^{n}(1 + \eta^2\zeta\theta t\ )\, e^{-\eta t} \tag{66}$$

$$\therefore \frac{\partial LnL}{\partial \hat{\eta}} = \frac{\mathrm{n}}{\hat{\eta}} - \frac{\mathrm{n}\hat{\zeta}\hat{\theta}}{(\hat{\eta}\hat{\zeta}\hat{\theta}+1)} + \sum_{i=1}^{n} \frac{2\hat{\eta}\hat{\zeta}\hat{\theta} ti}{1+\hat{\eta}^2\hat{\zeta}\hat{\theta} t_i} - n\bar{t} = 0 \tag{67}$$

$$\therefore \frac{\partial LnL}{\partial \hat{\zeta}} = \sum_{i=1}^{n}\left[\frac{\widehat{\eta^2}\hat{\theta} ti}{1+\hat{\eta}^2\hat{\zeta}\hat{\theta} t_i}\right] - \frac{n\,\hat{\eta}\hat{\theta}}{\hat{\eta}\hat{\zeta}\hat{\theta}+1} = 0 \tag{68}$$

$$\therefore \frac{\partial LnL}{\partial \hat{\theta}} = \sum_{i=1}^{n}\left[\frac{\widehat{\eta^2}\hat{\zeta} ti}{1+\hat{\eta}^2\hat{\zeta}\hat{\theta} t_i}\right] - \frac{n\,\hat{\eta}\hat{\zeta}}{\hat{\eta}\hat{\zeta}\hat{\theta}+1} = 0 \tag{69}$$

We see from (67), (68) and (69) they are non-linear equations and do not seem to be solved directly. Thus we will use the Fisher's scoring method to solve these equations as follows:

$$\frac{\partial^2 \mathrm{LnL}}{\partial \eta^2} = \frac{-\mathrm{n}}{\eta^2} + \frac{\mathrm{n}\zeta^2\theta^2}{(\eta\zeta\theta+1)^2} + \sum_{i=1}^{n} \frac{\eta\zeta\theta t_i}{(1+\eta^2\zeta\theta t\zeta)^2} \tag{70}$$

$$\frac{\partial^2 \mathrm{LnL}}{\partial \eta\, \partial \zeta} = \frac{-\mathrm{n}\theta}{(\eta\zeta\theta+1)^2} + \sum_{i=1}^{n} \frac{2\eta\theta ti}{(1+\eta^2\zeta\theta t)^2} = \frac{\partial^2 \mathrm{LnL}}{\partial \zeta\, \partial \eta} \tag{71}$$

$$\frac{\partial^2 \mathrm{LnL}}{\partial \eta\, \partial \theta} = \frac{-\mathrm{n}\zeta}{(\eta\zeta\theta+1)^2} - \sum_{i=1}^{n} \frac{2\eta\zeta ti}{(1+\eta^2\zeta\theta t)^2} = \frac{\partial^2 \mathrm{LnL}}{\partial \theta\, \partial \eta} \tag{72}$$

$$\frac{\partial^2 \mathrm{LnL}}{\partial \zeta^2} = \frac{\mathrm{n}\,\eta^2\theta^2}{(\eta\zeta\theta+1)^2} - \sum_{i=1}^{n} \frac{\eta^4\theta^2 ti^2}{(1+\eta^2\zeta\theta t)^2} \tag{73}$$

$$\frac{\partial^2 \mathrm{LnL}}{\partial \theta^2} = \frac{n\eta^2\zeta^2}{(\eta\zeta\theta+1)^2} - \sum_{i=1}^{n} \frac{\eta^4\zeta^2 ti^2}{(1+\eta^2\zeta\theta t)^2} \tag{74}$$

$$\frac{\partial^2 \mathrm{LnL}}{\partial \zeta\, \partial \theta} = \sum_{i=1}^{n} \frac{-\eta^2 ti}{(1+\eta^2\zeta\theta t)^2} - \frac{n\,\eta}{(\eta\zeta\theta+1)^2} = \frac{\partial^2 \mathrm{LnL}}{\partial \theta\, \partial \zeta} \tag{75}$$

The following equations can be solved for getting MLEs $\hat{\eta}$, $\hat{\zeta}$ and $\hat{\theta}$ of (PTPLD) where η_0, ζ_0 and

$$\begin{bmatrix} \frac{\partial^2 \mathrm{LnL}}{\partial \eta^2} & \frac{\partial^2 \mathrm{LnL}}{\partial \eta\, \partial \zeta} & \frac{\partial^2 \mathrm{LnL}}{\partial \eta\, \partial \theta} \\ \frac{\partial^2 \mathrm{LnL}}{\partial \zeta\, \partial \eta} & \frac{\partial^2 \mathrm{LnL}}{\partial \zeta^2} & \frac{\partial^2 \mathrm{LnL}}{\partial \zeta\, \partial \theta} \\ \frac{\partial^2 \mathrm{LnL}}{\partial \theta\, \partial \eta} & \frac{\partial^2 \mathrm{LnL}}{\partial \theta\, \partial \zeta} & \frac{\partial^2 \mathrm{LnL}}{\partial \theta^2} \end{bmatrix}_{\substack{\hat{\eta}=\eta_0 \\ \hat{\zeta}=\zeta_0 \\ \hat{\theta}=\theta_0}} \begin{bmatrix} \hat{\eta}-\eta_0 \\ \hat{\zeta}-\zeta_0 \\ \hat{\theta}-\theta_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathrm{LnL}}{\partial \eta} \\ \frac{\partial \mathrm{LnL}}{\partial \zeta} \\ \frac{\partial \mathrm{LnL}}{\partial \theta} \end{bmatrix}_{\substack{\hat{\eta}=\eta_0 \\ \hat{\zeta}=\zeta_0 \\ \hat{\theta}=\theta_0}} \tag{76}$$

θ_0 are the initial values of, ζ and θ respectively.These equations are solved iteratively till Sufficiently close values of $\hat{\eta}$, $\hat{\zeta}$ and $\hat{\theta}$ are obtained.

2.7 Standard Bayesian Estimations

By integrating the starting probability, the Bayes theorem may describe the likelihoods function of the observations based on the sample's current knowledge. We obtain the posterior probability distribution by integrating the density function for the parameters with the greatest possibility function for the current observation. We employ a loss function in the Bayes method, which allows us to measure the loss that results from making decisions based on the value of (η), whereas the option to take relies on(η). That is, the parameter and its estimate diverge from one another. In order to estimate b on a, we must provide the initial distribution of the parameters to be estimated η , ζ and θ. Based on the researcher's knowledge of the initial distribution of the parameters, Consider that those parameters' initial distributions will be as follows: [13]

$\eta \sim Gamma\ (a_1, b_1)$, scale parameter
$\zeta \sim Gamma\ (a_2, b_2)$, shape parameter
$\theta \sim Beta\ (c, d)$, shape parameter

Thus, the following defines how each parameter's priority distribution function is formed:

$$\pi_1(\eta) \propto \frac{{b_1}^{a_1}}{\Gamma(a_1)}\ \eta^{a_1-1}\, e^{-b_1\eta} \quad ; \eta > 0 \tag{77}$$

$$\pi_2(\zeta) \propto \frac{{b_2}^{a_2}}{\Gamma(a_2)}\ \zeta^{a_2-1}\, e^{-b_2\zeta} \quad ; \zeta > 0 \tag{78}$$

$$\pi_3(\theta) \propto \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\ \theta^{c-1}\,(1-\theta)^{d-1} \quad ; 0 < \theta < 1 \tag{79}$$

With three parameters (η , ζ and θ), the prior distribution of the Lindley distribution's parameters is represented by equations (77, 78) and (78). Based on prior experiences and the researchers' access to past experiences, parameter b has a gamma distribution with the Hyper-parameters (a_1) and (b_1), parameter has a gamma distribution with the Hyper-parameters (a_2) and (b_2), and parameter has a beta distribution with the Hyper-parameters (c) and (d).

Therefore, will be a joint prior distribution of η, ζ, and θ as follows:

$$\pi_{(\eta,\zeta,\theta)} \propto \frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\, \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\, \eta^{a_1-1}\ \zeta^{a_2-1}\ \theta^{c-1}(1-\theta)^{d-1} e^{-(b_1\eta+b_2\zeta)} \tag{80}$$

$$\log \pi_{(\eta,\zeta,\theta)} = log\left(\frac{{b_1}^{a_1}{b_2}^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\, \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)}\, \eta^{a_1-1}\ \zeta^{a_2-1}\ \theta^{c-1}(1-\theta)^{d-1} e^{-(b_1\eta+b_2\zeta)}\right) \tag{81}$$

The observations probability function $t_1, t_2, \dots \dots, t_n$ can be given as follows:

$$L(t_1, t_2, \dots\dots, t_n|\eta) = \prod_{i=1}^{n} f(x, \eta, \zeta, \theta) \tag{82}$$

$$L(t_1, t_2, \dots\dots, t_n|\eta) = \left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}} \tag{83}$$

The joint posterior distribution at the observed data and for parameters, ζ and θ is as follows:

$$h(\eta, \zeta, \theta|t_1, t_2, \dots\dots, t_n) = \frac{L(t_1, t_2, \dots\dots, t_n|\eta)\pi_{(\eta,\zeta,\theta)}}{\iiint_{\eta,\zeta,\theta} L(t_1, t_2, \dots\dots, t_n|\eta)\pi_{(\eta,\zeta,\theta)} d\eta d\zeta d\theta}$$

$$= \frac{\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}} \frac{b_1{}^{a_1} b_2{}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} \eta^{a_1-1} \zeta^{a_2-1} \theta^{c-1}(1-\theta)^{d-1} e^{-(b_1\eta+b_2\zeta)}}{\iiint_{\eta,\zeta,\theta} \prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}} \frac{b_1{}^{a_1} b_2{}^{a_2}}{\Gamma(a_1)\Gamma(a_2)} \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} \eta^{a_1-1} \zeta^{a_2-1} \theta^{c-1}(1-\theta)^{d-1} e^{-(b_1\eta+b_2\zeta)}\, d\eta d\zeta d\theta}$$

$$\therefore h(\eta, \zeta, \theta|t_1, t_2, \dots\dots, t_n) = \frac{\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\theta^{c-1}(1-\theta)^{d-1}\prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}}{\iiint_{\eta,\zeta,\theta}\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\theta^{c-1}(1-\theta)^{d-1}\prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\, d\eta d\zeta d\theta} \tag{84}$$

By using the squared loss function, the Bayes estimate for the distribution's parameters (PTPLD) as follows:

$$\hat{S}_{SBayes}(t_1, t_2, \dots\dots, t_n) = E(\theta|t_1, t_2, \dots\dots, t_n)$$

$$= \frac{\iiint_{\eta,\zeta,\theta}^{\infty}(\hat{\theta} - \theta)^2\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\theta^{c-1}(1-\theta)^{d-1}\prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\, d\eta d\zeta d\theta}{\iiint_{\eta,\zeta,\theta}^{\infty}\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\theta^{c-1}(1-\theta)^{d-1}\prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\, d\eta d\zeta d\theta} \tag{85}$$

Therefore, the purpose of Bayes is to distribute (PTPLD) the survival function:

$$\hat{R}_{SBayes} = \frac{\iiint_{\eta,\zeta,\theta}^{\infty}\left(\frac{1+\eta^2\zeta\theta t+\eta\zeta\theta}{(\eta\zeta\theta+1)}e^{-\eta t}\right)\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\theta^{c-1}(1-\theta)^{d-1}\prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\, d\eta d\zeta d\theta}{\iiint_{\eta,\zeta,\theta}^{\infty}\left(\frac{\eta^2}{\eta\zeta+\theta}\right)^n \eta^{a_1-1}\zeta^{a_2-1}\theta^{c-1}(1-\theta)^{d-1}\prod_{i=1}^{n}(\zeta + \theta t_i)e^{-n\eta\bar{t}-b_1\eta-b_2\zeta}\, d\eta d\zeta d\theta} \tag{86}$$

The survival function of PTPLD is represented by equation (86) which shows that the survival function is not conceptually difficult and does not have a closed formula. It is necessary to compute these complex integrals numerically using the Lindley Approximation in order to derive the estimate of the dependability function (survival).

3. Empirical Aspect

3.1 Simulation Experiments

A Monte Carlo simulation was performed as follows:

First step: theoretical determination of values

1- Determining the theoretical values of the parameters of the distributions under study: this was done by carrying out several experiments, analyzing and evaluating the outcomes, and drawing conclusions that provided a comprehensive understanding of the capabilities and behavior patterns.

2- Determining theoretical values for the sample size (n =10, 25, 50, 100)

Second step: Data Generation:

At this step, the following formulas are used to create data that corresponds to the Lindley distribution with three parameters and the three distributions we suggested with parameters (η, ζ, θ):

1- TPLD by using the Lambert W Function as follows:

$$x = \frac{\eta}{\theta} - \frac{1}{\zeta} - \frac{1}{\zeta}W_{-1}\left(\frac{(\zeta\eta+\eta)(1-u)e^{-\frac{(\zeta\eta+\theta)}{\theta}}}{\theta}\right) \tag{87}$$

2- PTPLD by suing the Lambert W Function as follows:

Third step: Discuss about the outcomes of the simulation exercises.

Table (1) Simulation outcomes when (η=1.5, ζ= 0.8, θ =0.5)

n	Dist.	TPLD				PTPLD			
	Parameters	$S_{TPLDReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$	$S_{PTPLDReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$
			1.678, 0.997, 0.776 1.643, 0.865, 0.765 1.542, 0.854,0.664				1.635, 0.989, 0.711 1.663, 0.876, 0.656 1.555, 0.855,0.671		
10	Survival		$\hat{S}_{TPLDML}$	$\hat{S}_{TTPLDMom}$	$\hat{S}_{TPLDBayes}$		$\hat{S}_{PTPLDML}$	$\hat{S}_{PTPLDMom}$	$\hat{S}_{TPTPLDBaye}$
		0.99019	0.87022	0.89194	0.99130	0.99787	0.92138	0.99292	0.98680
		0.94939	0.83023	0.87939	0.95193	0.93667	0.86537	0.96351	0.94667
		0.87802	0.68065	0.77802	0.88587	0.76663	0.71757	0.89434	0.77663
		0.78322	0.66837	0.68322	0.79848	0.75435	0.69933	0.79806	0.76446
		0.67387	0.56185	0.58387	0.69748	0.67782	0.63984	0.70603	0.68788
		0.59746	0.53729	0.49746	0.60656	0.63327	0.57759	0.62515	0.64333
		0.52125	0.43277	0.44125	0.55542	0.55875	0.50723	0.56400	0.56873
		0.44749	0.36043	0.37749	0.46601	0.46641	0.43344	0.45160	0.45646
		0.37800	0.27204	0.33800	0.39997	0.36801	0.31398	0.40852	0.37845
		0.34531	0.14975	0.23531	0.32861	0.26534	0.20335	0.33720	0.27573
IMSE			0.18688	0.14716	0.13924		0.15118	0.11146	0.10345
-2lnL	17.98		17.59	17.66	17.52	17.94	17.50	17.50	17.38
AIC	19.89		19.99	19.39	19.55	19.85	19.68	19.30	19.44
CAIC	18.93		18.79	18.49	18.55	18.89	18.55	18.46	18.30
BIC	14.87		14.82	14.85	14.65	14.84	14.78	14.81	14.43
Best Model	2					1			
Best Method	$\hat{S}_{TPLDBayes}$					$\hat{S}_{TPTPLDBayes}$			
n	Dist.	TPLD				PTPLD			
	Parameters	$S_{ThldReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$	$S_{PTPLDReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$
			1.667, 0.987, 0.745 1.578, 0.667, 0.657 1.554, 0.835,0.651				1.590, 0.988, 0.556 1.578, 0.878, 0.566 1.532, 0.827,0.551		
25	Survival		$\hat{S}_{TPLDML}$	$\hat{S}_{TTPLDMom}$	$\hat{S}_{TPLDBayes}$		$\hat{S}_{PTPLDML}$	$\hat{S}_{PTPLDMom}$	$\hat{S}_{TPTPLDBayes}$
		0.89009	0.77012	0.79184	0.89120	0.89170	0.89878	0.85341	0.90170
		0.84927	0.73013	0.77924	0.85193	0.84611	0.86052	0.79522	0.85611
		0.77792	0.58055	0.65792	0.78577	0.69653	0.70436	0.67389	0.70653
		0.68312	0.56828	0.58312	0.69888	0.68426	0.68146	0.64910	0.69426
		0.57377	0.46175	0.48377	0.59738	0.57772	0.60596	0.54974	0.58772
		0.49736	0.43719	0.39736	0.52646	0.45317	0.53505	0.41334	0.46317
		0.42115	0.33267	0.32115	0.45532	0.34865	0.46390	0.38713	0.35865

		0.34739	0.26036	0.27739	0.38591	0.32634	0.39450	0.32337	0.33634
		0.27790	0.17194	0.22390	0.29986	0.28792	0.30845	0.23988	0.29792
		0.24521	0.14965	0.13521	0.22851	0.25563	0.19710	0.22119	0.27563
IMSE			0.16490	0.12518	0.11726		0.12920	0.08948	0.08147
-2lnL	**20.96**		20.57	20.64	20.50	20.80	20.48	20.48	20.36
AIC	**22.87**		22.97	22.37	22.53	22.82	22.66	22.28	22.42
CAIC	**21.91**		21.77	21.47	21.53	21.86	21.53	21.44	21.28
BIC	**17.85**		17.80	17.83	17.63	17.86	17.76	17.79	17.41
Best Model	**2**					1			
Best Method	$\hat{S}_{TPLDBayes}$					$\hat{S}_{TPTPLDBayes}$			
n	**Dist.**	TPLD				PTPLD			
	Parameters	$S_{ThldReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$	$S_{PTPLDReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$
			1.543, 0.867 0.554 1.654, 0.897, 0.675 1.667, 0.888,0.786				1.607, 0.852 0.645 1.554, 0.844, 0.574 1.533, 0.835,0.543		
50	**Survival**		$\hat{S}_{TPLDML}$	$\hat{S}_{TTPLDMom}$	$\hat{S}_{TPLDBayes}$		$\hat{S}_{PTPLDML}$	$\hat{S}_{PTPLDMom}$	$\hat{S}_{TPTPLDBayes}$
		0.66742	0.65739	0.57012	0.55844	0.77896	0.59169	0.56702	0.78896
		0.62650	0.61568	0.55712	0.51674	0.73166	0.57310	0.52532	0.74166
		0.55514	0.46710	0.43580	0.36816	0.63308	0.45177	0.37674	0.64308
		0.46035	0.45483	0.36100	0.35588	0.57081	0.37698	0.36447	0.58081
		0.35099	0.34830	0.26165	0.24935	0.46428	0.27762	0.25794	0.47428
		0.27458	0.32374	0.17524	0.22480	0.43972	0.19122	0.23338	0.44972
		0.19838	0.21922	0.15903	0.21928	0.32520	0.17501	0.12786	0.33520
		0.12462	0.14692	0.14527	0.18797	0.26289	0.16125	0.11655	0.27289
		0.11513	0.13849	0.12378	0.16705	0.23447	0.13976	0.11563	0.24447
		0.11044	0.12221	0.11691	0.15274	0.22019	0.13288	0.01132	0.23019
IMSE			0.00158	0.00389	0.03201		0.02675	0.01887	0.00123
-2lnL	**28.49**		28.32	28.43	28.49	28.44	28.19	28.21	28.34
AIC	**31.52**		30.19	30.32	31.52	30.56	30.06	31.11	30.23
CAIC	**29.42**		29.12	29.18	29.42	29.45	29.11	29.12	29.10
BIC	**25.43**		25.31	25.43	25.43	25.44	25.01	25.22	25.15
Best Model	**2**					1			
Best Method	$\hat{S}_{ThLD3dBayes}$					$\hat{S}_{TPLDBayes}$			

N	**Dist.**	TPLD	PTPLD

	Parameters	$S_{ThldReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$	$S_{PTPLDReal}$	$\hat{\eta}$	$\hat{\zeta}$	$\hat{\theta}$
			1.511, 0.812 0.526 1.556, 0.876, 0.667 1.664, 0.998,0.767				1.515, 0.821 0.518 1.567, 0.665, 0.567 1.645, 0.867,0.699		
			$\hat{S}_{TPLDML}$	$\hat{S}_{TTPLDMom}$	$\hat{S}_{TPLDBayes}$		$\hat{S}_{PTPLDML}$	$\hat{S}_{PTPLDMom}$	$\hat{S}_{TPTPLDBayes}$
100	**Survival**	0.55527	0.54224	0.44497	0.44329	0.59019	0.57179	0.40955	0.47422
		0.51435	0.50053	0.43197	0.40159	0.54289	0.55320	0.39785	0.45563
		0.44299	0.45195	0.32065	0.25301	0.46431	0.43187	0.25927	0.33430
		0.34820	0.33968	0.24585	0.24074	0.38204	0.34708	0.24700	0.25950
		0.23884	0.23315	0.14650	0.13420	0.27550	0.25772	0.14046	0.16015
		0.16244	0.17859	0.06009	0.10965	0.21095	0.16132	0.11591	0.07374
		0.08623	0.10407	0.04388	0.00413	0.14643	0.14511	0.01039	0.05754
		0.01247	0.02218	0.00012	0.00282	0.06412	0.06360	0.00092	0.04378
		0.00298	0.00334	0.00064	0.00190	0.06570	0.06186	0.00184	0.02229
		0.00171	0.00306	0.00076	0.00759	0.03941	0.02298	0.00915	0.01541
IMSE			0.00132	0.00128	0.01168		0.00119	0.00112	0.01134
-2lnL	**39.90**		39.12	39.37	39.40	39.85	39.09	39.35	39.39
AIC	**42.92**		41.14	41.35	42.39	42.87	41.07	41.33	42.38
CAIC	**40.89**		40.23	40.38	40.50	40.86	40.09	40.32	40.49
BIC	**36.88**		36.22	36.33	36.51	36.83	36.14	36.31	36.50
Best Model	**2**					1			
Best Method	$\hat{S}_{TPLDML}$					$\hat{S}_{PTPLDML}$			

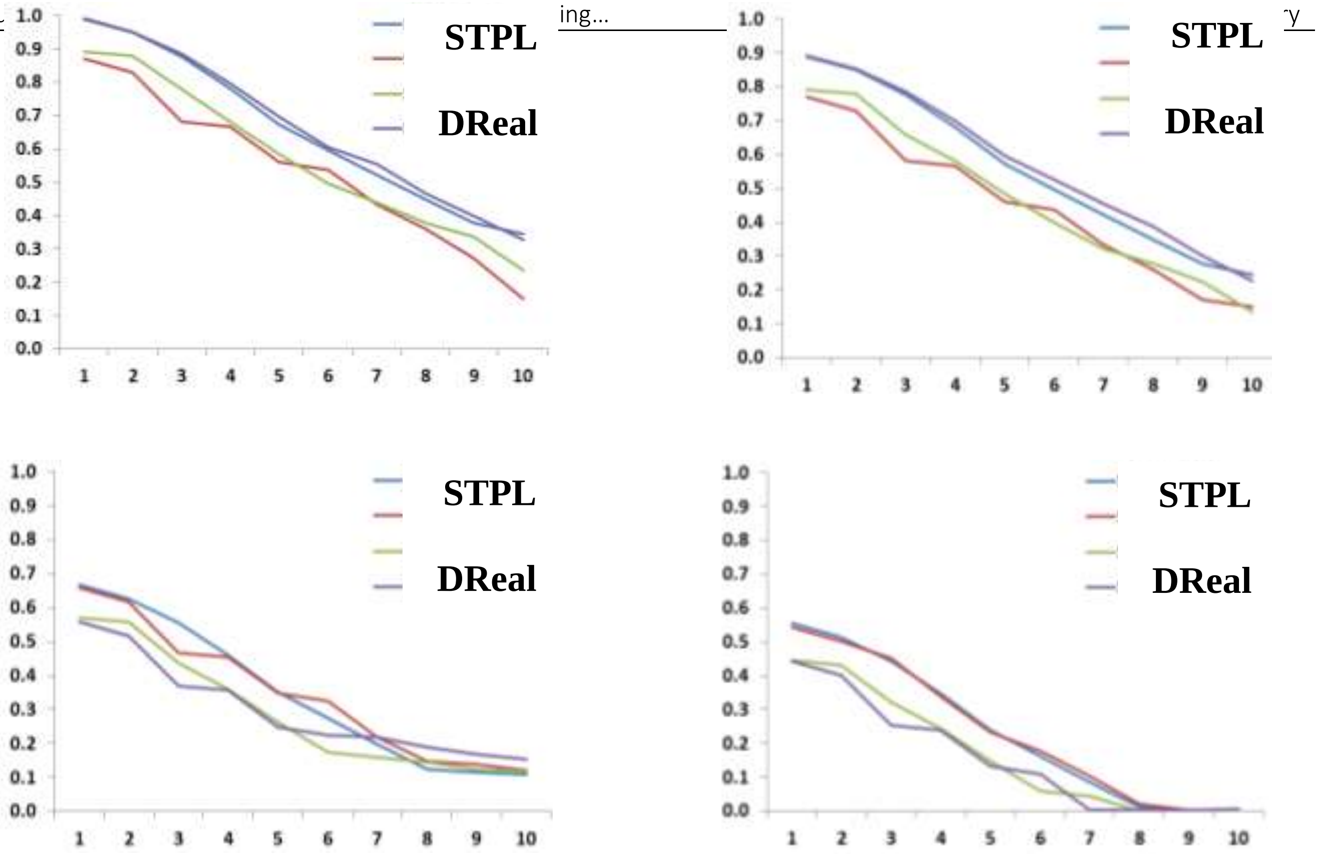

Figure (1) the TPLD Distribution's Reliability Function Curve's (Survival) Behavior for the sample size (n =10, 25, 50, 100)

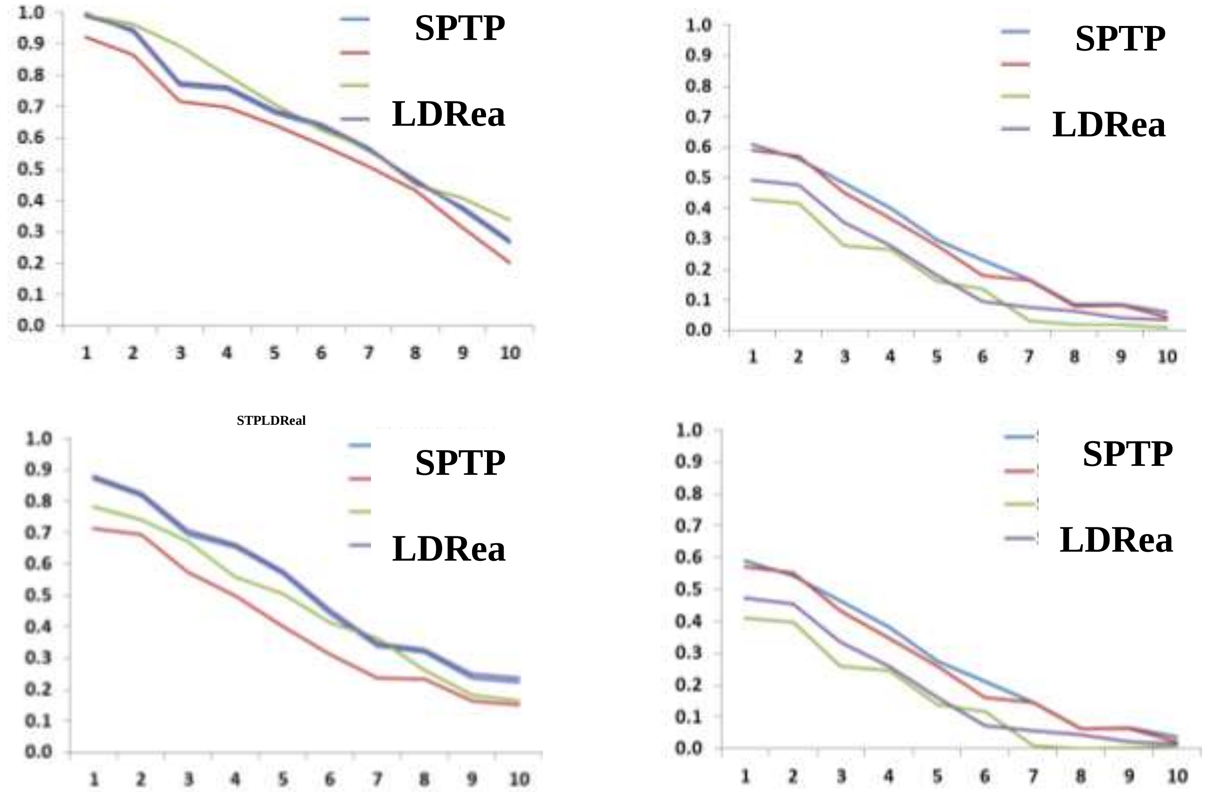

Figure (2) the PTPLD Distribution's Reliability (Survival) Function Curve's Behavior for the sample size (n =10, 25, 50, 100)

3.2 Discussion of Results

a. At all sample sizes; the model (PTPLD) was better than the rest of the models as it recorded the lowest comparison standards, followed by the model (TPLD)

b. At a sample size of (10,25), the Bayes method scored superiority over the rest of the estimation methods for all models because it recorded the lowest comparison standards, followed by the method of moments and finally the method of maximum likelihood.
c. The MLE was the best in the model (TPLD) at a sample size of (50); followed by the method of moments, Bayes, and the Bayes method outperformed the other estimating methods used in the model (PTPLD). The method of moments comes after that, and the method of MLE comes last.
d. The maximum likelihood approach, which matched the lowest comparison criteria, was the most effective model for sample sizes of (100). It was followed by the moments methods and Bayes comes last.

3.3 Real Data Application

Data for patients with the Corona virus are taken from the Al Hussein Educational Hospital in Nasiriyah, Statistics Division, which represents the time of survival to death due to Corona from 100 patients who entered the hospital and died as a result of the Corona virus. After doing the proper work, after applying χ_c^2 statistics to analyze the data for good of fit, the following outcomes were found. Utilizing the Matalap program**.**

Table (2): Results of the data fit test for the PTPLD distribution

Distribution	χ_c^2	χ_t^2	Sig.	Decision
PTPLD	0.14855	123.225	0.46754	Accept H_0

Table (2) shows that the value of Sig = 0.46754 is more than the threshold of significance (0.05) and that the estimated value of χ_c^2 (0.14855) is smaller than the tabulated value of χ_t^2 (123.225). As a result, the null hypothesis cannot be rejected, indicating that the actual data are distributed in accordance with the Proposed Lindley distribution.

In order to determine which distribution performed the best when used with actual data, TPLD Distribution and PTPLD distribution were compared (-2LnL, AIC, CAIC, BIC), were used for model selection; the outcomes are shown in Table (3):

Table (3): Results of comparison and accuracy tests applied to real data

Distribution	-2LnL	AIC	CAIC	BIC
TPLD	46.7784	46.9984	46.2484	46.2784
PTPLD	39.7584	39.0984	39.2084	39.3884

Table (3) shows that the Proposed Lindley distribution had lower criteria for the special tests, suggesting that it is a better fit for the actual data than the previous distributions.

4. Conclusions

Through analyzing the simulation and estimating the survival function for the original and proposed distribution with three estimation methods (moments methods, MLH, Bayes method) we concluded that the proposed formula for the distribution is the best for all selected sample sizes and also superior to the Bayesian method at sample size n = 10.25 for all models because it recorded the lowest. The maximum likelihood approach, which matched the lowest comparison criteria, was the most effective model for sample sizes of (50,100). It was followed by the moments methods and Bayes comes last comparison standards, followed by the method of moments**.**

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AL- Rafidain University College

PISSN: (1681-6870); EISSN: (2790-2293)
مجلة كلية الرافدين الجامعة للعلوم

Available online at: https://www.jrucs.iq

JRUCS
Journal of AL-Rafidain University College for Sciences

تقدير دالة البقاء للمرضى المصابين بفيروس كورونا باستخدام TPLD وPTPLD

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المستخلص

يهدف البحث إلى دراسة بعض خواص توزيع ليندلي ثلاثي المعلمات (TPLD) والصيغة الجديدة المقترحة لتوزيع ليندلي ثلاثي المعلمات في دراسة التوزيع الزمني ووظيفة البقاء ووظيفة الخطر. اقترحنا صيغة جديدة لـ TPLD من خلال النظر في أوزان خلط جديدة لدالة الخلط التي استخدمت في بناء TPLD، واستبدال أوزان الخلط في معادلة الخلط للحصول على صيغة جديدة لـ TPLD تسمى المعلمات الثلاثة المقترحة ليندلي التوزيع ورمزنا له بـ (PTPLD).تم دراسة بعض خصائص PTPLD في التوزيع مدى الحياة. لقد استخدمنا طريقتين في التقدير، طريقة العزوم وطريقة MLH لتقدير المعلمات ودوال البقاء للتوزيعين TPLD وPTPLD ومقارنتها بين تقديرات دوال البقاء لكلا التوزيعين باستخدام طريقتين للتقدير أعلاه، وقد قمنا اختيار أفضل تقدير من خلال بعض معايير الأخطاء.

بالإضافة إلى ذلك، استخدمنا بيانات حقيقية من مستشفى الحسين التعليمي في مدينة الناصرية لتقدير مدة بقاء المرضى المصابين بكوفيد-19 على قيد الحياة باستخدام TPLD .

واستنتج أن الصيغة المقترحة للتوزيع هي الأفضل لجميع أحجام العينات المختارة.

معلومات البحث

تواريخ البحث:

تاريخ تقديم البحث:23/2/2024
تاريخ قبول البحث:12/4/2024
تاريخ رفع البحث على الموقع: 31/12/2024

الكلمات المفتاحية:
توزيع ليندلي ذو ثلاثة معلمات، توزيع ليندلي ثلاثي المعلمات المقترح، دوال البقاء، التقدير بطريقة العزوم ، تقدير MLH.

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DOI: https://doi.org/10.55562/jrucs.v56i1.28