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A New Truncated Family of Log Logistic Distribution

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Abstract

Compound probability models have important roles in modeling real life events, in this paper, a new class of continuous distributions was presented named Truncated [0,1] Log logistic -Log logistic distribution ([0,1] TLL-LLD). And we presented the important properties of the new distribution model, which are the r-th moment function, reliability function, the function of hazard, the function stress strength function , and the function of Shannon entropy.

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Introduction

A number of authors have in recent time introduced several compound distributions, has been studied by many researchers. In (2019) Ibrahim [7]. Presented a new model which is named Frechet lengthy distribution and he introduce it is properties. (Jamal, et al. 2019) [8]. Intrudes a new family of distribution which is named the Burr-X family with som continuous distribution the origin probability of G, they describe the corresponding Burr- X created (BXG) distribution with one added non negative parameters. In (2019) Mohammad [5]. Present two ways to generate probability distribution by composition some generalized distributions, which are generalized Gamma distribution, exponentiated weibull distribution generalized inverse weibull distribution, and generalized Gompertz distribution. In(2020) Hassan et al [6] introduced [0,1] truncated Lomax-G family of distributions based on the arguments of Abid and Abdulrazak (2017) [2,3]. Statistical properties like; moments, moment generating function. In (2021) [4] Abid and Jani intrudes certain som properties, the reliability stress strength, Shannon entropy for the generalized Inverse Weibull accidental variable. In (2021) Kadhim [9] discussed twice as truncated Inverted Gamma distribution (DTIGD). Also he derived the distributions of extreme order statistics from

DTIGD.

The log-logistic L-L distribution,(Shomrani et al,. 2016)[10] and (Warsono et al,. 2014)[11] if x is a random variable having L-L distribution, the presence of a shape parameter $\mu > 0$, positive scale parameter $\sigma > 0$, denoted by $X \sim LL(\mu, \sigma)$. The pdf of the L-L distribution is of the form:

$$f(x;\mu,\sigma)_{L-L} = \left(\frac{\mu}{\sigma}\right)\left(\frac{x}{\sigma}\right)^{\mu-1} \left(1+\left(\frac{x}{\sigma}\right)^{\mu}\right)^{-2} \quad (1)$$

$$\mu > 0, \qquad \sigma > 0, \qquad x \geq 0$$

And the CDF of L-L model with two parameters takes the form:

$$F(x;\mu,\sigma)_{L-L} = \frac{1}{1+\left(\frac{x}{\sigma}\right)^{-\mu}} \quad (2)$$

$$\mu > 0, \qquad \sigma > 0, \qquad x \geq 0$$

Truncated [0,1] L-Q Distributions

We will generated new continuous type, based on time period truncated [0,1] cdf L-Q, called [0,1] truncated L-Q ([0,1] TL-QD), has been discussed.

Let $Q(x) and\ q(x)$ any continuous pdf of random variable x. Assume that $L(.)\ and\ l(.)$, the Pdf and Cdf respectively on interval $[0, \infty)$. he suggested generic cdf syntax for this class depends on syntax L with Q is(Boshi,. 2019)[5], (Abid. et al,.2022)[1] .

$$F(x)_{TL-Q} = \frac{L[Q(x)] - L[0]}{L[1] - L[0]} \quad (3)$$

Now, by letting that L(0)=0 Then cdf (3) can be express by,

$$F(x)_{TL-Q} = \frac{L[Q(x)]}{L[1]} \quad (4)$$

And its associated pdf, $f(x)\frac{d}{dx}[F(x)]$ will be,

$$f(x)_{TL-Q} = \frac{l\,[Q(x)]q(x)}{L[1]} \quad (5)$$

[0,1] Truncated Log Logistic- Q Distribution

Here we present a new born family of [0,1] truncated Log logistic (LL) distribution.

Assume that L(.) $and\ l(.)$ that Referred in (4) and (5), are cdf and pdf of distribution (L-L), remember that (1) and (2) with non-negative parameters $(\mu = 0, \sigma > 0)$.that is mean L(0)= 0 So, (Abid. et al,.2022)[1]

$$L[Q(x)] = \frac{1}{1+\left(\frac{Q(x)}{\sigma}\right)^{-\mu}} \quad , \quad L[1] = \frac{1}{1+\left(\frac{1}{\sigma}\right)^{-\mu}}$$

$$And \quad l[Q(x)] = \frac{\left(\frac{\mu}{\sigma}\right)\left(\frac{Q(x)}{\sigma}\right)^{\mu-1}}{\left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)^{2}}$$

Then according to (4) and (5), the cdf and pdf associated with a new family of distribution which is called the function of truncated [0,1] Log logistic- Q ([0,1] TLL- QD) which given by,

$$F(x)_{TLL-Q} = \frac{1+\left(\frac{1}{\sigma}\right)^{-\mu}}{1+\left(\frac{Q(x)}{\sigma}\right)^{-\mu}} \ , \quad \sigma > 0, \quad \mu > 0, x \geq 0 \quad (6)$$

and,

$$f(x)_{TLL-Q}=\frac{\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma}\right)\left(\frac{Q(x)}{\sigma}\right)^{\mu-1}q(x)}{\left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)^{2}}\ ,\qquad \sigma>0,\qquad \mu>0, x\geq 0$$

$$=\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\left(Q(x)\right)^{\mu-1}\left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)^{-2}q(x) \tag{7}$$

by using some Equations, (Boshi,. 2019)[5],

$$(a+u)^{-n}=\sum_{i=0}^{\infty}C_{I}^{-n}\,a^{-n-i}u^{i} \tag{8}$$

$$ln(x)=2\sum_{k=0}^{\infty}\frac{(x-1)^{2k+1}}{(x+1)^{2k+1}}\quad ,x>0 \tag{9}$$

$$e^{-u}=\sum_{i=0}^{\infty}\frac{(-1)^{i}}{i!}u^{i} \tag{10}$$

$$ln(u)=\sum_{i=0}^{\infty}\frac{(-1)^{i}(u-1)^{i-1}}{i+1}\ ;\qquad 0<u\leq 2 \tag{11}$$

$$ln(1-u)=-\sum_{i=0}^{\infty}\frac{u^{i}}{i}\ ;\quad |u|<1 \tag{12}$$

$$(-ln\,u)^{\alpha}=\sum_{k,l=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^{j+k+l}}{\alpha-j}C_{k}^{k-\alpha}C_{j}^{k}C_{l}^{\alpha+k}p_{j,k}u^{l} \tag{13}$$

Where $p_{j,o}=1$ *for* $j\geq 0$ *and*

$$p_{j,k}=k^{-1}\sum_{m=1}^{k}\frac{(-1)^{m}\,(m(j+1)\,-k)}{m+1}\,p_{j,k-m}\quad for\ k=1,2,\ldots$$

$$(1-u)^{-b}=\sum_{i=0}^{\infty}\frac{1}{i!}\frac{\Gamma(b+i)}{\Gamma(b)}u^{i};\ |u|<1, b>0 \tag{14}$$

$$(a+b)^{n}=\sum_{k=0}^{n}C_{k}^{n}a^{n-k}b^{k}=\sum_{k=0}^{n}C_{k}^{n}a^{k}b^{n-k} \tag{15}$$

According to (8), $f(x)_{TLL-Q}$ will be,

$$\begin{aligned}f(x)_{TLL-Q}&=\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\left(Q(x)\right)^{\mu-1}\sum_{i=0}^{\infty}C_{i}^{-2}\left(\frac{Q(x)}{\sigma}\right)^{i\mu}q(x)\\&=\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu(i+1)}}\right)\sum_{i=0}^{\infty}C_{i}^{-2}\left(Q(x)\right)^{\mu(i+1)-1}q(x)\end{aligned} \tag{16}$$

Truncated [0,1] Log Logistic- Log Logistic family

Assume that $Q(x)$ *and* $q(x)$, are Log logistic pdf with non-negative parameters λ *and* β, then cdf and pdf are given by ,(Shomrani et al,. 2016)[10] and (Warsono et al,. 2014)[11]

$$Q(x;\ \lambda,\beta)=\frac{1}{1+\left(\frac{x}{\beta}\right)^{-\lambda}} \tag{17}$$

$$q(x;\ \lambda,\beta)=\left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2} \tag{18}$$

Now, Based on (6), the cdf of our distribution is called Truncated [0,1] Log logistic- Log logistic (denoted by [0,1] TLL-LL D) given by the following formula,

$$F(x)_{TLL-LL}=\frac{1+\left(\frac{1}{\sigma}\right)^{-\mu}}{1+\left(\frac{1}{\sigma\left(1+\left(\frac{x}{\beta}\right)^{-\lambda}\right)}\right)^{-\mu}} \tag{19}$$

The pdf [0,1] TLL-LL distribution can be obtained, according to (6) as,

$$f(x)_{TLL-LL}=\left\{\begin{array}{c}\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\left(\frac{\mathbf{1}}{\mathbf{1}+\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\lambda}}\right)^{\mu-1}\\ \left(1+\sigma^{-\mu}\left(\frac{\mathbf{1}}{\mathbf{1}+\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\lambda}}\right)^{\mu}\right)^{-2}\left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2}\end{array}\right\} \tag{20}$$

$Let\ V_1=\left(\frac{\mathbf{1}}{\mathbf{1}+\left(\frac{x}{\beta}\right)^{-\lambda}}\right)^{\mu-1}$, $V_2=\left(1+\sigma^{-\mu}\left(\frac{\mathbf{1}}{\mathbf{1}+\left(\frac{x}{\beta}\right)^{-\lambda}}\right)^{\mu}\right)^{-2}$

$$V_1=\left(1+\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\lambda}\right)^{-(\mu-1)}$$

According to (8) V_1 will be,

$$V_1=\sum_{i=0}^{\infty}\mathrm{C}_i^{-(\mu-1)}\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\boldsymbol{i\lambda}}$$

$$=\sum_{i=0}^{\infty}\mathrm{C}_i^{-(\mu-1)}(\beta)^{i\lambda}x^{-i\lambda}$$

$$=\sum_{i=0}^{\infty}\mathrm{C}_i^{-(\mu-1)}(\beta)^{i\lambda}x^{-i\lambda}$$

$$V_2=\left(1+\sigma^{-\mu}\left(\frac{\mathbf{1}}{\mathbf{1}+\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\lambda}}\right)^{\mu}\right)^{-2}$$

According to (8) V_2 will be,

$$V_2=\sum_{k=0}^{\infty}C_k^{-2}\sigma^{-k\mu}\left(\frac{\mathbf{1}}{\mathbf{1}+\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\lambda}}\right)^{k\mu}$$
$$=\sum_{k=0}^{\infty}C_k^{-2}\sigma^{-k\mu}\left(1+\left(\frac{\boldsymbol{x}}{\boldsymbol{\beta}}\right)^{-\lambda}\right)^{-k\mu}$$

Also by using (1.24) V_2 will be,

$$V_2=\sum_{\mathrm{k}=0}^{\infty}C_{\mathrm{k}}^{-2}\sigma^{-\mathrm{k}\mu}\sum_{s=0}^{\infty}C_s^{-k\mu}(\beta)^{s\lambda}x^{-s\lambda}$$

Substitute V_1 and (V_2) in (20),by the expansion formula for the pdf of $f(x)_{TLL-LL}$ given by,

$$f(x)_{TLL-LL}=\left\{\begin{matrix}\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty}C_i^{-(\mu-1)}(\beta)^{i\lambda}x^{-i\lambda}\\ \sum_{k=0}^{\infty}C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty}C_s^{-k\mu}(\beta)^{s\lambda}x^{-s\lambda}\\ \left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2}\end{matrix}\right\} \tag{21}$$

The Reliability function, $R(x)=1-F(x)$, of [0,1]TLL-LL distribution it can be obtained as,

$$R(x)_{TLL-LL}=1-\left[\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(1+\left(1+\left(\frac{x}{\beta}\right)^{-\lambda}\right)^{\mu}\right)^{-\mathbf{1}}\right] \tag{22}$$

The function for the hazard rate, $H(x)=\frac{f(x)}{R(x)}$,of [0,1]TLL-LL D, are given the following,

$$H(x)_{TLL-LL}=\frac{\left\{\begin{matrix}\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty}C_i^{-(\mu-1)}(\beta)^{i\lambda}x^{-i\lambda}\\ \sum_{k=0}^{\infty}C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty}C_s^{-k\mu}(\beta)^{s\lambda}x^{-s\lambda}\\ \left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2}\end{matrix}\right\}}{1-\left[\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(1+\left(1+\left(\frac{x}{\beta}\right)^{-\lambda}\right)^{\mu}\right)^{-\mathbf{1}}\right]} \tag{23}$$

properties of the [0,1] TLL-LL distribution respectively given as

Furthermore, the most essential statistical properties of [0,1] TLL-LL D are given following,

i- ***The r-th moment of [0,1] TLL-D, it can be obtained from $\int_0^{\infty}x^r f(x)_{TLL-LL}dx$. According to (21), can be obtained as follows,***

$$E(x^r)_{TLL-LL}=\int_0^{\infty}x^r\left\{\begin{matrix}\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty}C_i^{-(\mu-1)}(\beta)^{i\lambda}x^{-i\lambda}\\ \sum_{k=0}^{\infty}C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty}C_s^{-k\mu}(\beta)^{s\lambda}x^{-s\lambda}\\ \left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2}\end{matrix}\right\}dx$$

$$E(x^r)_{TLL-LL} = \left\{ \begin{array}{c} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^\mu}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu}\beta^{(i+s)\lambda} \\ \int_0^\infty x^{r-(i+s)\lambda}\left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2} dx \end{array} \right\} \tag{24}$$

Now, let $I = \int_0^\infty x^{r-(i+s)\lambda} f(x)_{TLL-LL}\, dx$
since it's ,

$$\int_0^\infty x\, f(x)_{TLL-LL}\, dx = \int_0^\infty x \left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2} dx$$

Let $y = \left(\frac{x}{\beta}\right)^{\lambda}$, we find $x = y^{\frac{1}{\lambda}}\beta$ With $dx = \frac{\beta}{\lambda} y^{\frac{1}{\lambda}-1} dy$

It means that if boundary of $x = 0$ then boundary of $y = 0$ and for $x = \infty$ then $y = \infty$. We have the result as:

$$\int_0^\infty x\, f(x)_{TLL-LL}\, dx = \beta \int_0^\infty \frac{y^{\frac{1}{\lambda}}}{(1+y)^2} dy$$

By using Beta function, we get,

$$\int_0^\infty x\, f(x)_{TLL-LL} = \beta B\left(\frac{\lambda+1}{\lambda}, \frac{\lambda-1}{\lambda}\right) \tag{25}$$

By similar Processes in (4.47) then we have the follows:

$$\int_0^\infty x^2\, f(x)_{TLL-LL} = \beta^2 B\left(\frac{\lambda+2}{\lambda}, \frac{\lambda-2}{\lambda}\right)$$

$$\int_0^\infty x^3\, f(x)_{TLL-LL} = \beta^3 B\left(\frac{\lambda+3}{\lambda}, \frac{\lambda-3}{\lambda}\right)$$

And $\int_0^\infty x^4\, f(x)_{TLL-LL} = \beta^4 B\left(\frac{\lambda+4}{\lambda}, \frac{\lambda-4}{\lambda}\right)$

Consequently by mathematical inductor we get (I) as follows:

$$I = \int_0^\infty x^{r-(i+s)\lambda} f(x)_{TLL-LL}\, dx$$

$$= \beta^{r-(i+s)\lambda} B\left(\frac{\lambda+r-(i+s)\lambda}{\lambda}, \frac{\lambda-r-(i+s)\lambda}{\lambda}\right)$$

Substitute (I) in (4.46),then the $E(x^r)_{TLL-LL}$ can be obtained as,

$$E(x^r)_{TLL-LL} = \left\{ \begin{array}{c} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^\mu}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu}\beta^{(i+s)\lambda} \\ \beta^{r-(i+s)\lambda} B\left(\frac{\lambda+r-(i+s)\lambda}{\lambda}, \frac{\lambda-r-(i+s)\lambda}{\lambda}\right) \end{array} \right\}$$

$$E(x^r)_{TLL-LL} = \left\{ \begin{matrix} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu} \\ \beta^r B\left(\frac{\lambda+r-(i+s)\lambda}{\lambda},\frac{\lambda-r-(i+s)\lambda}{\lambda}\right) \end{matrix} \right\} \tag{26}$$

Depending on the particular $E(X^r)_{TLL-LL}$; $(r = 1,2,3,4)$, some of our distribution such as the Mean, Variance, Coefficient of skewness and coefficient of kurtosis can be given as,

$$E(x)_{TLL-LL} = \left\{ \begin{matrix} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu} \\ \beta B\left(\frac{\lambda+1-(i+s)\lambda}{\lambda},\frac{\lambda-1-(i+s)\lambda}{\lambda}\right) \end{matrix} \right\}$$

$$E(x^2)_{TLL-LL} = \left\{ \begin{matrix} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu} \\ \beta^2 B\left(\frac{\lambda+2-(i+s)\lambda}{\lambda},\frac{\lambda-2-(i+s)\lambda}{\lambda}\right) \end{matrix} \right\}$$

$$E(x^3)_{TLL-LL} = \left\{ \begin{matrix} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu} \\ \beta^3 B\left(\frac{\lambda+3-(i+s)\lambda}{\lambda},\frac{\lambda-3-(i+s)\lambda}{\lambda}\right) \end{matrix} \right\}$$

$$E(x^4)_{TLL-LL} = \left\{ \begin{matrix} \left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu} \\ \beta^4 B\left(\frac{\lambda+4-(i+s)\lambda}{\lambda},\frac{\lambda-4-(i+s)\lambda}{\lambda}\right) \end{matrix} \right\}$$

ii- ***The characteristic function of the TLL-LL D can be can be given as,***

$$E(e^{itx}) = \sum_{r=0}^{\infty}\frac{(it)^r}{r!}E(X^r)_{TLL-LL}$$

Therefor the characteristic function of the TLL-LL distribution is given by,

$$\emptyset_X(t)_{TLL-LL} = \left\{ \begin{matrix} \sum_{r=0}^{\infty}\frac{(it)^r}{r!}\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)\sum_{i=0}^{\infty} C_i^{-(\mu-1)} \\ \sum_{k=0}^{\infty} C_k^{-2}\sigma^{-k\mu}\sum_{s=0}^{\infty} C_s^{-k\mu} \\ \beta^r B\left(\frac{\lambda+r-(i+s)\lambda}{\lambda},\frac{\lambda-r-(i+s)\lambda}{\lambda}\right) \end{matrix} \right\} \tag{27}$$

iii- ***The function of Shannon entropy for the TLL-LL D can be get by the following formula***

$$-\int_0^\infty \ln(f(x)_{TLL-LL}) f(x)_{TLL-LL}\, dx$$

By applying the natural logarithm of the pdf in (7) and according (17) and (18), we get.

$$ln(f(x)_{TLL-LL}) = ln\left\{\begin{matrix}\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)\left(\frac{\mu}{\sigma^{\mu}}\right)(Q(x))^{\mu-1} \\ \left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)^{-2} q(x)\end{matrix}\right\}$$

$$ln(f(x)_{TLL-LL})\left\{\begin{matrix} ln\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)+ln\left(\frac{\mu}{\sigma^{\mu}}\right)+(\mu-1)ln\big(Q(x)\big) \\ -2ln\left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)+\ln q(x)\end{matrix}\right\} \tag{28}$$

Let, $I = ln\big(Q(x)\big)$, $II = -2ln\left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)$, $III = \ln(q(x))$

$$I = ln\big(Q(x)\big)$$

$$= ln\left(\frac{1}{1+\left(\frac{x}{\beta}\right)^{-\lambda}}\right)$$

$$= ln\left(1+\left(\frac{x}{\beta}\right)^{-\lambda}\right)^{-1}$$

$$= -ln\left(1+\left(\frac{x}{\beta}\right)^{-\lambda}\right)$$

According to (11) will be,

$$I = -\sum_{i=0}^{\infty}\frac{(-1)^i\left(\left(\frac{x}{\beta}\right)^{-\lambda}\right)^{i-1}}{i+1}$$

$$= -\sum_{i=0}^{\infty}\frac{(-1)^i\left(\frac{x}{\beta}\right)^{-\lambda(i-1)}}{i+1}$$

$$= -\sum_{i=0}^{\infty}\frac{(-1)^i(\beta)^{\lambda(i-1)}}{i+1}x^{-\lambda(i-1)}$$

$$II = -2ln\left(1+\left(\frac{Q(x)}{\sigma}\right)^{\mu}\right)$$

According to (11) will be,

$$II = -2\sum_{t=0}^{\infty}\frac{(-1)^t}{(t+1)}\left(\frac{Q(x)}{\sigma}\right)^{\mu(t-1)}$$

$$II = -2\sum_{t=0}^{\infty}\frac{(-1)^t}{(t+1)}\left(\frac{1}{\sigma}\right)^{\mu(t-1)}\big(Q(x)\big)^{\mu(t-1)} \tag{29}$$

Now,

$$\big(Q(x)\big)^{\mu(t-1)} = \left(\frac{1}{1+\left(\frac{x}{\beta}\right)^{-\lambda}}\right)^{\mu(t-1)}$$

$$=\left(1+\left(\frac{x}{\beta}\right)^{-\lambda}\right)^{-\mu(t-1)}$$

According to (8) will be,

$$Q(x)^{\mu(t-1)}=\sum_{k=0}^{\infty}\mathrm{C}_k^{-\mu(t-1)}\left(\frac{x}{\beta}\right)^{-k\lambda}$$

$$=\sum_{k=0}^{\infty}\mathrm{C}_k^{-\mu(t-1)}\beta^{k\lambda}x^{-k\lambda} \tag{30}$$

Substitute (29) in (30), II will be,

$$II=\left\{-2\sum_{t=0}^{\infty}\frac{(-1)^t}{(t+1)}\left(\frac{1}{\sigma}\right)^{\mu(t-1)}\sum_{k=0}^{\infty}\mathrm{C}_k^{-\mu(t-1)}\beta^{k\lambda}x^{-k\lambda}\right\}$$

And,

$$III=\ln(q(x))$$

$$=ln\left(\left(\frac{\lambda}{\beta}\right)\left(\frac{x}{\beta}\right)^{\lambda-1}\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)^{-2}\right)$$

$$=\left\{\begin{matrix} ln\left(\frac{\lambda}{\beta}\right)+(\lambda-1)ln(x) \\ -(\lambda-1)\ln(\beta)-2ln\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)\end{matrix}\right\} \tag{31}$$

According (9) we obtain,

$$ln(x)=2\sum_{s=0}^{\infty}\frac{(x-1)^{2s+1}}{(x+1)^{2s+1}}$$

According to (15) and (8) we get ,

$$ln(x)=2\sum_{s=0}^{\infty}\sum_{j=0}^{2s+1}\mathrm{C}_j^{2s+1}(-1)^{(2s+1)-j}x^j\sum_{n=0}^{\infty}\mathrm{C}_n^{-(2s+1)}x^n \tag{32}$$

We continues to simplify III by using (13) we get,

$$-2ln\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)=2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w}\mathrm{C}_m^{m-1}\mathrm{C}_w^{m}\mathrm{C}_l^{m+1}P_{w,m}\left(1+\left(\frac{x^{\lambda}}{\beta^{\lambda}}\right)\right)^{l}$$

According to (15) will be,

$$-2ln\left(1+\left(\frac{x}{\beta}\right)^{\lambda}\right)=\left\{\begin{matrix} 2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w}\mathrm{C}_m^{m-1}\mathrm{C}_w^{m}\mathrm{C}_l^{m+1}P_{w,m} \\ \sum_{p=0}^{l}\mathrm{C}_p^{l}\frac{1}{\beta^{\lambda p}}x^{\lambda p}\end{matrix}\right\} \tag{33}$$

Substitute (23) and (33) in (31), $we\ get,$

$$III=\left\{\begin{matrix} ln\left(\frac{\lambda}{\beta}\right)+(\lambda-1)2\sum_{s=0}^{\infty}\sum_{j=0}^{2s+1}\mathrm{C}_j^{2s+1}(-1)^{(2s+1)-j}x^j \\ \sum_{n=0}^{\infty}\mathrm{C}_n^{-(2s+1)}x^n-(\lambda-1)\ln(\beta)+2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w} \\ \mathrm{C}_m^{m-1}\mathrm{C}_w^{m}\mathrm{C}_l^{m+1}P_{w,m}\sum_{p=0}^{l}\mathrm{C}_p^{l}\frac{1}{\beta^{\lambda p}}x^{\lambda p}\end{matrix}\right\}$$

Substitute (I), (II) and (III) in (28) we get,

$$ln(f(x)_{TLL-LL}) = \left\{\begin{array}{c} ln\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right) + ln\left(\frac{\mu}{\sigma^{\mu}}\right) - (\mu-1) - \sum_{i=0}^{\infty}\frac{(-1)^{i}(\beta)^{\lambda(i-1)}}{i+1}x^{-\lambda(i-1)} \\ -2\sum_{t=0}^{\infty}\frac{(-1)^{t}}{(t+1)}\left(\frac{1}{\sigma}\right)^{\mu(t-1)}\sum_{k=0}^{\infty}C_{k}^{-\mu(t-1)}\beta^{k\lambda}x^{-k\lambda} \\ +ln\left(\frac{\lambda}{\beta}\right) + (\lambda-1)2\sum_{s=0}^{\infty}\sum_{j=0}^{2s+1}C_{j}^{2s+1}(-1)^{(2s+1)-j}x^{j} \\ +\sum_{n=0}^{\infty}C_{n}^{-(2s+1)}x^{n} - (\lambda-1)\,ln(\beta) + 2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w} \\ C_{m}^{m-1}C_{w}^{m}C_{l}^{m+1}P_{w,m}\sum_{p=0}^{l}C_{p}^{l}\frac{1}{\beta^{\lambda p}}x^{\lambda p} \end{array}\right\}$$

The function of Shannon entropy of the TLL-LL D given by:

$$SH_{TLL-LL} = -\int_{0}^{\infty}\left\{\begin{array}{c} ln\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right) + ln\left(\frac{\mu}{\sigma^{\mu}}\right) - (\mu-1) - \sum_{i=0}^{\infty}\frac{(-1)^{i}(\beta)^{\lambda(i-1)}}{i+1}x^{-\lambda(i-1)} \\ -2\sum_{t=0}^{\infty}\frac{(-1)^{t}}{(t+1)}\left(\frac{1}{\sigma}\right)^{\mu(t-1)}\sum_{k=0}^{\infty}C_{k}^{-\mu(t-1)}\beta^{k\lambda}x^{-k\lambda} \\ +ln\left(\frac{\lambda}{\beta}\right) + (\lambda-1)2\sum_{s=0}^{\infty}\sum_{j=0}^{2s+1}C_{j}^{2s+1}(-1)^{(2s+1)-j}x^{j} \\ +\sum_{n=0}^{\infty}C_{n}^{-(2s+1)}x^{n} - (\lambda-1)\,ln(\beta) + 2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w} \\ C_{m}^{m-1}C_{w}^{m}C_{l}^{m+1}P_{w,m}\sum_{p=0}^{l}C_{p}^{l}\frac{1}{\beta^{\lambda p}}x^{\lambda p} \end{array}\right\} f(x)_{TLL-LL}$$

So the function of Shannon entropy for TLL-LL D (7) we get,

$$SH_{TLL-LL} = \left\{\begin{array}{c} -ln\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right) - ln\left(\frac{\mu}{\sigma^{\mu}}\right) + (\mu-1) + \sum_{i=0}^{\infty}\frac{(-1)^{i}(\beta)^{\lambda(i-1)}}{i+1}\int_{0}^{\infty}x^{-\lambda(i-1)}f(x)_{TLL-LL}dx \\ +2\sum_{t=0}^{\infty}\frac{(-1)^{t}}{(t+1)}\left(\frac{1}{\sigma}\right)^{\mu(t-1)}\sum_{k=0}^{\infty}C_{k}^{-\mu(t-1)}\beta^{k\lambda}\int_{0}^{\infty}x^{-k\lambda}f(x)_{TLL-LL}\,dx \\ -ln\left(\frac{\lambda}{\beta}\right) - (\lambda-1)2\sum_{s=0}^{\infty}\sum_{j=0}^{2s+1}C_{j}^{2s+1}(-1)^{(2s+1)-j}\int_{0}^{\infty}x^{j}f(x)_{TLL-LL}dx \\ -\sum_{n=0}^{\infty}C_{n}^{-(2s+1)}\int_{0}^{\infty}x^{n}f(x)_{TLL-LL}dx + (\lambda-1)\,ln(\beta) - 2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w} \\ C_{m}^{m-1}C_{w}^{m}C_{l}^{m+1}P_{w,m}\sum_{p=0}^{l}C_{p}^{l}\frac{1}{\beta^{\lambda p}}\int_{0}^{\infty}x^{\lambda p}f(x)_{TLL-LL}dx \end{array}\right\}$$

$$SH_{TLL-LL}=\left\{\begin{array}{c}-ln\left(1+\left(\frac{1}{\sigma}\right)^{-\mu}\right)-ln\left(\frac{\mu}{\sigma^{\mu}}\right)+(\mu-1)+\sum_{i=0}^{\infty}\frac{(-1)^{i}(\beta)^{\lambda(i-1)}}{i+1}E(x^{-\lambda(i-1)})\\ +2\sum_{t=0}^{\infty}\frac{(-1)^{t}}{(t+1)}\left(\frac{1}{\sigma}\right)^{\mu(t-1)}\sum_{k=0}^{\infty}C_{k}^{-\mu(t-1)}\beta^{k\lambda}E(x^{-k\lambda})\\ -ln\left(\frac{\lambda}{\beta}\right)-(\lambda-1)2\sum_{s=0}^{\infty}\sum_{j=0}^{2s+1}C_{j}^{2s+1}(-1)^{(2s+1)-j}E(x^{j})\\ -\sum_{n=0}^{\infty}C_{n}^{-(2s+1)}E(x^{n})+(\lambda-1)\,ln(\beta)-2\sum_{m,l=0}^{\infty}\sum_{w=0}^{\infty}\frac{(-1)^{m+w+l}}{1-w}\\ C_{m}^{m-1}C_{w}^{m}C_{l}^{m+1}P_{w,m}\sum_{p=0}^{l}C_{p}^{l}\frac{1}{\beta^{\lambda p}}E(x^{\lambda p})\end{array}\right\}$$

Where,

$E\left(X^{-\lambda(i-1)}\right), E\left(X^{-k\lambda}\right), E\left(X^{j}\right), E(X^{n}) and E\left(X^{\lambda p}\right)$ as in (26) with $(r=-\lambda(i-1),-k\lambda,j,n,\lambda p)$.

iv- ***The function of Stress Strength , spouse that X and Y be the independent random variables of Stress Strength respectively with [0,1] TLL-LLD with different Parameters, then the function of Stress -Strength can give by,***

$$SS_{TLL-LL}=\int_{0}^{\infty}f_{X}(x)_{TLL-LL}\,F_{y}(x)dx$$

$$Fy(x)=\frac{1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}}{1+\left(\frac{1}{\sigma_1\left(1+\left(\frac{x}{\beta_1}\right)^{-\lambda_1}\right)}\right)^{-\mu_1}} \tag{34}$$

$$=1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}\left(1+{\sigma_1}^{\mu_1}\left(1+\left(\frac{x}{\beta_1}\right)^{-\lambda_1}\right)^{\mu_1}\right)^{-1}$$

by using (8) can be rewritten as,

$$Fy(x)=1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}\sum_{k=0}^{\infty}C_{k}^{-1}{\sigma_1}^{k\mu_1}\left(1+\left(\frac{x}{\beta_1}\right)^{-\lambda_1}\right)^{k\mu_1}$$

According (15) $Fy(x)$ will be,

$$Fy(x)=1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}\sum_{k=0}^{\infty}C_{k}^{-1}{\sigma_1}^{k\mu_1}\sum_{m=0}^{k\mu_1}C_{m}^{k\mu_1}\left(\frac{x}{\beta_1}\right)^{-k\mu_1\lambda_1}$$

$$=\left\{\begin{array}{l}1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}\sum_{k=0}^{\infty}C_{k}^{-1}{\sigma_1}^{k\mu_1}\\ \sum_{m=0}^{k\mu_1}C_{m}^{k\mu_1}{\beta_1}^{k\mu_1\lambda_1}x^{-k\mu_1\lambda_1}\end{array}\right\} \tag{35}$$

So it is based on (35), the stress strength of the [0,1] TLL-LL distribution can be get as,

$$SS_{TLL-LL}=\left\{\begin{array}{c}1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}\sum_{k=0}^{\infty}C_{k}^{-1}{\sigma_1}^{k\mu_1}\\ \sum_{m=0}^{k\mu_1}C_{m}^{k\mu_1}{\beta_1}^{k\mu_1\lambda_1}\int_{0}^{\infty}x^{-k\mu_1\lambda_1}f(x)_{TLL-LL}dx\end{array}\right\}$$

$$= \left\{ \begin{matrix} 1+\left(\frac{1}{\sigma_1}\right)^{-\mu_1}\sum_{k=0}^{\infty} C_k^{-1}{\sigma_1}^{k\mu_1} \\ \sum_{m=0}^{k\mu_1} C_m^{k\mu_1}{\beta_1}^{k\mu_1\lambda_1}\, E(x^{-k\mu_1\lambda_1}) \end{matrix} \right\}$$

Where,
$E\left(X^{-k\mu_1\lambda_1}\right)$ as in (26) with $(r = -k\mu_1\lambda_1)$.

Empirical study

We simulate a data of random variable from [0,1] TLL-LL for different sample sizes (10, 50, 100) and different parameter values, can be simulated by solving numerically the above nonlinear equations and the MSE,s are calculated a for parameters estimations .By using R program codes, we obtained the results.

Empirical MSE for the parameters estimation of [0,1]TLL-LL.

Table (1): The empirical MSE values of the parameter estimates of the [0,1] TLL-LL

Default parameter value			Sample size	Empirical MSE		
δ	β		n	$\hat{\delta}$	$\hat{\beta}$	$\hat{\lambda}$
0.4	0.4	0.4	10	0.001241	0.000823	0.001416
			50	0.001232	0.000643	0.001387
			100	0.001201	0.000365	0.001275
		1.3	10	0.001573	0.001253	0.016533
			50	0.001492	0.001178	0.001582
			100	0.001322	0.000432	0.001453
	1.3	0.4	10	0.001552	0.001147	0.017864
			50	0.001474	0.001056	0.007342
			100	0.001412	0.000873	0.005439
		1.3	10	0.001725	0.001003	0.001543
			50	0.001432	0.000654	0.001439
			100	0.001238	0.000482	0.001342
1.3	0.4	0.4	10	0.001854	0.001732	0.003279
			50	0.001763	0.001538	0.002992
			100	0.001661	0.000893	0.001729
		1.3	10	0.001834	0.001521	0.003752
			50	0.001752	0.001451	0.002724
			100	0.001619	0.001167	0.002238
	1.3	0.4	10	0.001715	0.001626	0.002983
			50	0.001639	0.001471	0.001875
			100	0.001593	0.001168	0.001762
		1.3	10	0.001743	0.001583	0.001634
			50	0.001694	0.001395	0.001592
			100	0.001489	0.001184	0.001501

In this table you can see:

i- We notice that the lowest MSE for all sample size when the estimated parameters were small (δ = 0.4, β = 0.4, λ = 0.4)

ii- MSE decreases with increasing sample size for all cases. This result is consistent with statistical theory.

Conclusion

We created a new family of distributions with random variable of continues type, named truncated [0,1] LL-LLD, which is based on a beta distribution formula by substitution the [0,1] truncated distributions in the beta- G family. The our proposed family distribution is, truncated [0,1] Log Logistic- Log Logistic distribution.

We provided the formula's characteristic function, the function of the r^{th} moment, the function of reliability , the function of hazard rate, the function of Shannon entropy and stress strength.

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عائلة جديدة من التوزيع اللوغاريتمي اللوجستي المبتور

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المستخلص

لعبت النماذج الاحتمالية المركبة ادوارا مهمة في نمذجة الاحداث الحقيقية في الحياة، في هذا البحث، تم تقديم فئة جديدة من التوزيعات المستمرة تسمى Truncated [0,1] Log logistic -Log logistic distribution ([0,1] TLL-LLD), وتم اشتقاق بعض الخصائص المهمة بالنموذج الجديد، مثل دالة العزوم، الدالة المعولية، دالة المخاطرة، دالة القوة والاجهاد، و دالة شانون انتروبي.

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