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Theoretical Investigation of Starved Porous Journal Bearings Anisotropic Permeability with

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Abstract:

The present study is a theoretical analysis to evaluate the static characteristics of oil – filled porous journal bearing with non – uniform permeability using an improved boundary condition. The analysis was carried out for a porous bearing with a permeability of the bearing ends is lower than that of the middle. The circumferential boundary condition for the oil film pressure is obtained by an integral momentum equation to the oil film region in the bearing clearance. It was found that a porous bearing with anisotropic permeability enhances the load carrying capacity of the bearing, also the bearing oil film is extended along specified extent.

Keywords: Design, Boundary layer

1. Introduction:

Bearings made of powder metallurgy products impregnated with lubricate have been widely used for office equipment, audio equipment, car electric equipment, etc. and are increasingly taking the place of rolling – contact bearing in such equipment to meet higher performance and lower cost requirements. During service operation of porous oil bearing, oil would come out from pores lubricate frictioning surface (self – lubrication) and on shut down of operation oil would penetrate back to the pores. Theoretical analysis of porous bearings operating under hydrodynamic lubrication condition have been performed by many authors since Morgan and Cameron (1957); Reason and Dyer (1973); Prakash.J and Vijs (1974); Cusano (1979). Kaneko and Obara (1990) shows experimentally that even if hydrodynamic lubrication condition existed initially the oil film extent in the bearing clearance will decrease with running time as a result of oil loss, eventually resulting in mixed or boundary lubrication conditions. The effect of different boundary conditions on the mechanism of lubrication for the porous bearing was investigated by many workers; Kaneko, Ohkawa and Hashimoto (1994) and Kaneko and Hashimoto (1995).

Recently Kaneko , Hashimoto and Hiroki (1997) used an improved boundary condition rather than Reynolds' boundary condition to analyze the oil film pressure distribution in porous journal bearings. The circumferential condition for oil film pressure is obtained by applying an integral momentum equation to the oil film pressure distribution.

The effect of using non uniform permeability matrix on the performance of porous bearings were studied by Yong – Xi et. al., (1985) and Kaneko and Doi (1989). They are used the classical Reynlods' boundary condition in their works.

Saha and Majumdar (2004) has been analyzed the steady state characteristics of hydrostatic two – layered porous oil journal bearings. The effect of anisotropy of permeability on load carrying capacity, attitude angle and friction factor have been investigated. In present work the effect of an isotropy in journal bearing permeability was studied for a porous bearing working under improved boundary conditions suggested by Kaneko et. al.(1997).

2. Numerical Analysis:
Pressure Distribution In Oil Film:

A schematic diagram of a porous journal bearing with the coordinate system used in the analysis is shown in figure(1).

The journal rotates with rotational speed (ω_j) about the journal center (O_j). the governing equation for the pressure distribution in the oil film is given by the modified Reynolds' equation including the slip velocity effect Kaneko (1989);

$$\frac{\partial}{\partial\theta}\left(h^{\wedge 3}(1+\zeta_{1\theta})\frac{\partial P^{\wedge}}{\partial\theta}\right)+\left(\frac{D_i}{L}\right)^2\frac{\partial}{\partial Z^{\wedge}}\left(h^{\wedge 3}(1+\zeta_{1Z})\frac{\partial P^{\wedge}}{\partial Z^{\wedge}}\right)$$
$$=6\frac{\partial}{\partial\theta}\left(h^{\wedge 3}(1+\zeta_{0\theta})\right)-12\Phi_r\left(\frac{\partial P^{\wedge *}}{\partial r^{\wedge}}\Big|_{r^{\wedge}=1}\right) \quad (1)$$

where

$$\left.\begin{aligned}(\zeta_{0\theta})&=\left(\frac{s_\theta}{(h^{\wedge}+s_\theta)}\right)\\(\zeta_{1\theta})&=\left(3\left(h^{\wedge}s_\theta+2\alpha^2 s_\theta^2\right)/\{h^{\wedge}(h^{\wedge}+s_\theta)\}\right)\\(\zeta_{1Z})&=\left(3\left(h^{\wedge}s_Z+2\alpha^2 s_Z^2\right)/\{h^{\wedge}(h^{\wedge}+s_Z)\}\right)\end{aligned}\right\} \quad (2)$$

The terms $(\zeta_{0\theta})$, $(\zeta_{1\theta})$ and (ζ_{1z}) represent terms which result from the tangential velocity slip.

$$\left.\begin{aligned}(s_\theta)&=(\Phi_\theta c/r_i)^{1/2}/\alpha\\(s_Z)&=(\Phi_Z c/r_i)^{1/2}/\alpha\\(\Phi_r)&=(k_r r_i/c^3)\\(\Phi_\theta)&=(k_\theta r_i/c^3)\\(\Phi_z)&=(k_z r_i/c^3)\end{aligned}\right\} \quad (3)$$

The slip coefficient (α) is dimensionless parameter depending on the material parameter which characterizes the structure of a permeable material within the boundary regions and its value for laminar channel flow has been estimated by Beavers and Joseph (1967) to be 0.1. The oil film thickness can be expressed as;

$$(h^{\wedge})=h/c=(1+\varepsilon\cos(\theta)) \quad (4)$$

Pressure Distribution In The Porous Matrix:

The governing equation for the pressure distribution in the porous matrix with non uniform permeability can be expressed as: Kaneko (1989);

$$\frac{1}{r^{\wedge}}\frac{\partial}{\partial r^{\wedge}}\left(r^{\wedge}\Phi_r\frac{\partial P^{\wedge *}}{\partial r^{\wedge}}\right)+\frac{1}{r^{\wedge 2}}\frac{\partial}{\partial\theta}\left(\Phi_\theta\frac{\partial P^{\wedge *}}{\partial\theta}\right)$$
$$+\left(\frac{D_i}{L}\right)^2\frac{\partial}{\partial Z^{\wedge}}\left(\Phi_Z\frac{\partial P^{\wedge *}}{\partial Z}\right)=0 \quad (5)$$

Boundary Conditions:

The following boundary conditions have been used with the above governing equation to solve the problem;

$$\left.\begin{aligned} &P^{\wedge}\left(\theta_1, z^{\wedge}\right) = P^{\wedge^*}\left(r^{\wedge}, \theta_1, z^{\wedge}\right) = 0 \\ &P^{\wedge}\left(\theta_2, z^{\wedge}\right) = P^{\wedge^*}\left(r^{\wedge}, \theta_2, z^{\wedge}\right) = 0 \\ &P^{\wedge}\left(\theta, \pm 1\right) = P^{\wedge^*}\left(r^{\wedge}, \theta, \pm 1\right) = 0 \\ &P^{\wedge}\left(\theta, z\right) = P^{\wedge^*}\left(r^{\wedge}, \theta, z\right) \; at \left(r^{\wedge}\right) = 1 \\ &\frac{\partial P^{\wedge}\left(\theta, 0\right)}{\partial Z^{\wedge}} = \frac{\partial P^{\wedge^*}\left(\theta, r^{\wedge}, 0\right)}{\partial Z^{\wedge}} = 0 \end{aligned}\right\} \quad (6)$$

The outer surface of porous matrix consist of two parts as shown in figure(1). The first is the part press – fitted inside the solid housing where the pressure is evaluated from the condition that the permeability of the housing adjacent to the porous matrix is zero i.e.

$$\begin{aligned} &\Phi_{\theta}, \Phi_Z = 0 \; or \frac{\partial P^{\wedge^*}}{\partial r^{\wedge}} \; at \left(r^{\wedge}\right) \geq \left(r_o / r_i\right) \\ &and \; 0.5 \leq \left|Z^{\wedge}\right| \leq 1 \end{aligned} \quad (7)$$

The second is that exposed to the circumferential groove in the housing, where the pressure is given by;

$$\left(P^{\wedge^*}\right) = P_s^{\wedge} = \frac{c_2^2 P_s}{\left(r_i^2 \eta \omega_j\right)} \quad at \left(r^{\wedge}\right) = r_o$$

$$and \quad \left|Z^{\wedge}\right| \leq 0.5$$

The following boundary conditions are used to determine the oil film extent. The leading edge (θ_1) of the oil film can be determine by the boundary condition used by Kaneko et. al. (1997); see figure(2);

$$\left(M_{\theta_1} - M_{\theta 2} - M_{\theta c} - M_{\theta b}\right) = 0 \quad (8)$$

where;

$M_{\theta 1}, M_{\theta 2}, M_{\theta c}$ and $M_{\theta b}$ are the circumferential momentum flow rates across the control surfaces of the oil films, as shown in figure (2). The momentum flow rates are given as follows;

$$\left.\begin{aligned} &\left(M_{\theta_1}\right) = 2\int_0^{L/2}\int_0^{\left(h_{\theta_1}\right)} \rho\left[\left(u_{\theta}\big|_{\theta_1}\right)^2\right] dydz \\ &\left(M_{\theta_2}\right) = 2\int_0^{L/2}\int_0^{\left(h_{\theta_1}\right)} \rho\left[\left(u_{\theta}\big|_{\theta_2}\right)^2\right] dydz \\ &\left(M_{\theta_c}\right) = 2\left(r\right)\int_{\theta_1}^{\theta_2}\int_0^{\left(h\right)} \rho\left[\left(u_{\theta} * u_z\right)\big|_{z=L/2}\right] dyd\theta \\ &\left(M_{\theta_b}\right) = 2\left(r\right)\int_0^{L/2}\int_{\left(\theta_1\right)}^{\left(\theta_2\right)} \rho\left[\left(u_{\theta_m}\right) * \left(u_r^{*}\right)\right]_{r=\left(r\right)} d\theta dz \end{aligned}\right\} \quad (9)$$

The values of the above circumferential momentum flow rates are calculated as shown in Appendix A.

The velocity components $\left(u_{\theta}\right)$ and $\left(u_z\right)$ represent the components of the oil velocity in circumferential and axial directions in the oil film, while $\left(u_r^*\right)$ represents the radial velocity component of the oil inside the porous bearing. The values of (θ_1) and (θ_2) are assumed to be constant in z – direction.

On the other hand the oil film extent at the trailing edge (θ_2) can be obtained by ensuring the continuity of the bulk flow a cross the boundary line at (θ_2).

$$\left(q_{\theta_p} / q_{\theta_c}\right) = 0 \quad (10)$$

where q_{θ_p} and q_{θ_c} are the flow rates a cross the trailing boundary line due to the Poiseuilles' and Couettes' flows respectively. Equation **(10)** can be rewritten as;

$$\left(\frac{q_{\theta_p}}{q_{\theta_c}}\right) = \left(\left(\frac{\left(1+\zeta_{1\theta}\right)}{6\left(1+\zeta_{0\theta}\right)} h^{\wedge 2} \int_0^1 \frac{\partial P^{\wedge}}{\partial \theta} d\hat{z}\right)_{\theta=\theta_2}\right) \quad (11)$$

Knowing the values of $\left(\theta_1\right)$ and $\left(\theta_2\right)$ the angular extent of the oil film $\left(\beta\right)$ is expressed in the form;

$$\left(\beta\right) = \left(\theta_2\right) - \left(\theta_1\right) \quad (12)$$

Bearing Parameters:

Knowing the pressure distribution the dimensionless film force components along and perpendicular to the line of centers can be obtained, respectively as;

$$\left(\hat{W}_R\right) = -\int_0^1\int_{\theta_1}^{\theta_2}\left(P^\wedge(\theta,z)\cos\theta\right) d\theta dz^\wedge \quad (13)$$

$$\left(\hat{W}_T\right) = \int_0^1\int_{\theta_1}^{\theta_2}\left(P^\wedge(\theta,z)\sin\theta\right) d\theta dz^\wedge \quad (14)$$

the total dimensionless load can be expressed as;

$$\left(\hat{W}\right) = \sqrt{\left(\hat{W}_R\right)^2 + \left(\hat{W}_T\right)^2} \quad (15)$$

The attitude angle (Ψ) can be evaluated as;

$$(\Psi) = \tan^{-1}\left(W_T^\wedge / W_R^\wedge\right) \quad (16)$$

The friction force on the inner and outer surfaces of the ring can be evaluated as;

$$F^\wedge = \int_0^1\int_{\theta_1}^{\theta_2}\left(\pm\frac{h^\wedge}{2}\frac{\partial P^\wedge}{\partial\theta} + \frac{h^\wedge}{2}\frac{\zeta_{1\theta}}{3}\frac{\partial P^\wedge}{\partial\theta} + \frac{(1+\zeta_{0\theta})}{h^\wedge}\right) d\theta dz^\wedge \quad (17)$$

Hence the coefficient of friction can be evaluated as ;

$$\left(\mu^\wedge\right) = \frac{\left(F_r^\wedge\right)}{\left(\hat{W}\right)} \quad (18)$$

Method Of Solution:

Pressure distribution in the oil film can be obtained by solving the modified Reynolds' equation which includes the slip velocity effect (equation1). The pressure distribution through the porous matrix can be obtained by solving Darcy's equation including the effect of non uniform permeability (equation5). The permeability of the porous matrix was assumed to be non uniform along the length of the bearing. The permeability of the bearing ends was assumed to be lower than that of middle. The above equations are discretized and solved simultaneously with an appropriate boundary conditions. The interested domain was divided into (180) divisions in circumferential direction, (100 divisions for rupture zone and 80 divisions for the effective zone). Twelve divisions in the axial direction and eight divisions in the radial direction.

The governing equations are solved iteratively with successive under relaxation factor. The iterations are continued until the following inequalities are satisfied simultaneously;

$$\left(\frac{\sum\sum\sum\left|P_{i,j,k}^{\wedge*(n+1)} - P_{i,j,k}^{\wedge*(n)}\right|}{\sum\sum\sum\left|P_{i,j,k}^{\wedge*(n)}\right|} < 10^{-5}\right) \quad (19)$$

$$\left(\frac{\sum\sum\left|P_{j,k}^{\wedge(n+1)} - P_{j,k}^{\wedge(n)}\right|}{\sum\sum\left|P_{j,k}^{\wedge(n)}\right|} < 10^{-5}\right) \quad (20)$$

$$\left(\left|M_{\theta_1}^\wedge - M_{\theta 2}^\wedge - M_{\theta c}^\wedge - M_{\theta_b}^\wedge / M_{\theta_1}^\wedge\right| < 10^{3}\right) \quad (21)$$

$$\left(\left|\frac{q_{\theta_p}}{q_{\theta_c}}\right|\right)_{ii} = \left(\left|\left(\frac{(1+\zeta_{1\theta})}{6(1+\zeta_{0\theta})}h^{\wedge 2}\int_0^1\frac{\partial P^\wedge}{\partial\theta}dz^\wedge\right)_{\theta=\theta_2}\right| < 10^{3}\right) \quad (22)$$

Always (n) and (n+1) used in above equations denote two consecutive iterations. Tthe points i, j, k represent the grid number in radial, circumferential, and axial directions respectively.

3. Results And Discussion:

The results of the present work were checked against the that obtained by Kaneko et. al. (1989) for a porous bearing with conventional Reynolds' boundary condition. Good agreement exists between the two suggesting a good accuracy of the results obtained from the prepared computer program used in this work. The maximum error obtained was seen to be less (5%) as shown in figure(3). The results obtained in figure(4) and figure(5) shows that the bearing with nonuniform permeability has a higher load carrying capacity particularly under condition of higher eccentricity ratio which indicates a high degree of hydrodynamic lubrication. Hence the oil film extent for a porous bearing in this case seems to be extended a long wide range of Sommerefeld number for the bearing with nonuniform permeability as shown in figure(6. It is clear from figure (7) and figure (8) that the oil film extent increases and the coefficient of friction decreases as the oil supply pressure increases. This is true since the bearing become more hydrodynamic in this case. In general the porous bearing with matrix of orthotropic porosity shows as light increase in coefficient of friction and attitude angle as shown in figures (9,10,11). This is indicates that the porous bearing with nonuniform bearing has a higher load carrying capacity in expense of slight decrease of bearing stability.

4. Conclusions

Using an improved boundary condition rather than the conventional Reynolds' boundary condition to analyze the performance of porous bearing with nonuniform permeability shown that the oil film extent was not start from the inlet of bearing. Also it can be deduced from the results obtained that using varying permeability is an effective way to affect the load capacity of the porous bearing. The load capacity of the bearing with nonuniform permeability has been shown to be higher than the load capacity of the bearing with uniform permeability. The oil film extent seen to be higher (in general) for the bearing with nonuniform permeability than that of uniform permeability which was seen to work a long a wide range of load parameter (S). The oil film extent for a porous bearing with nonuniform permeability shown to be higher for higher values of supply pressure. The coefficient of friction for the bearing of a nonuniform permeability shows a slight increase than that of uniform permeability.

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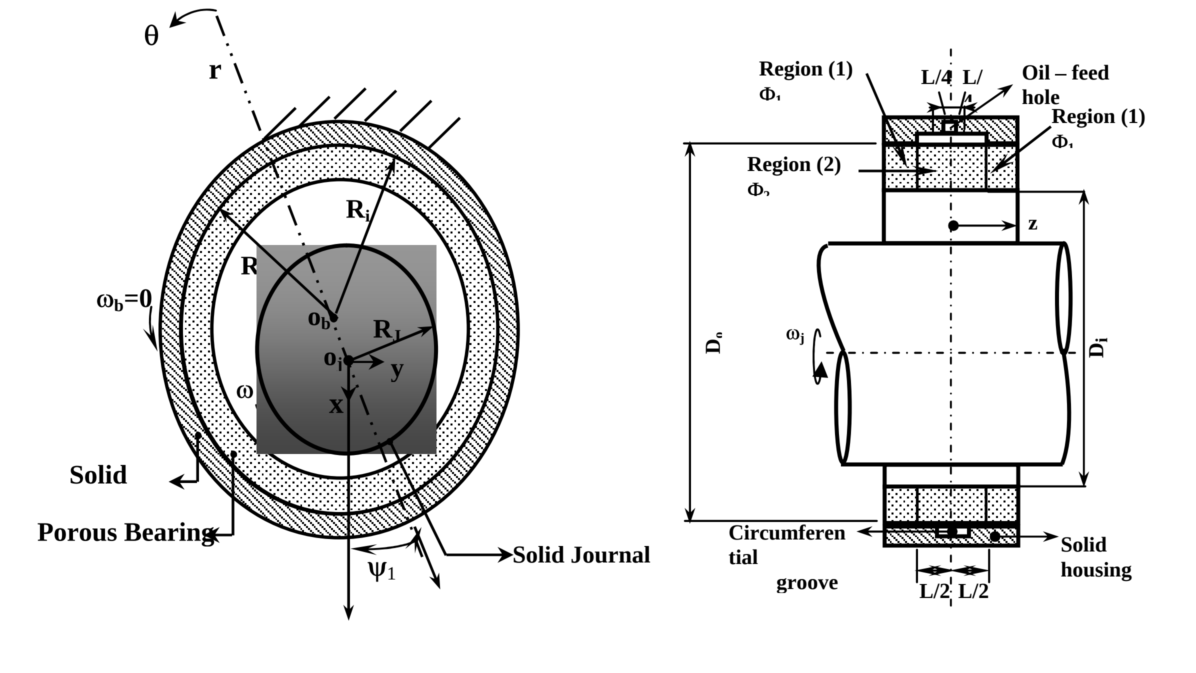

Figure (1): Oil film region and oil flow of porous journal bearing

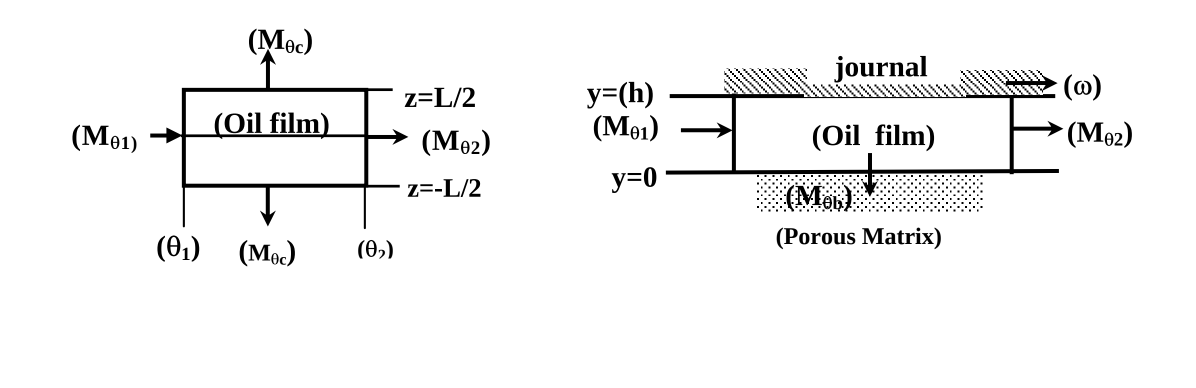

Figure (2): Circumferential momentum flow rates

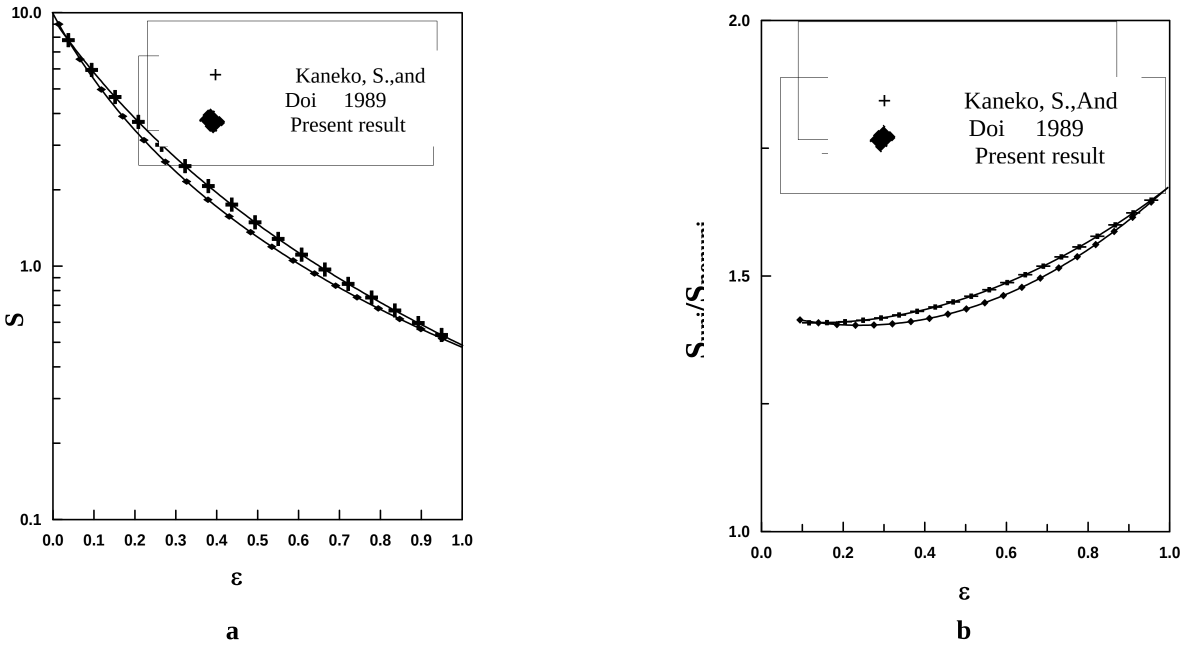

Fig. (1): Comparison between present and published result Kaneko et. al. (1989) at P_s= 0.1, Φ_1=0.01 and Φ_2=0.1
a: Sommerfeld number versus eccentricity ratio at nonuniform permeability,
b: Ratio of the Sommerfeld number for uniform type Φ=0.1 to that nonuniform type Φ_1=0.01 and Φ_2=0.1 versus eccentricity ratio.

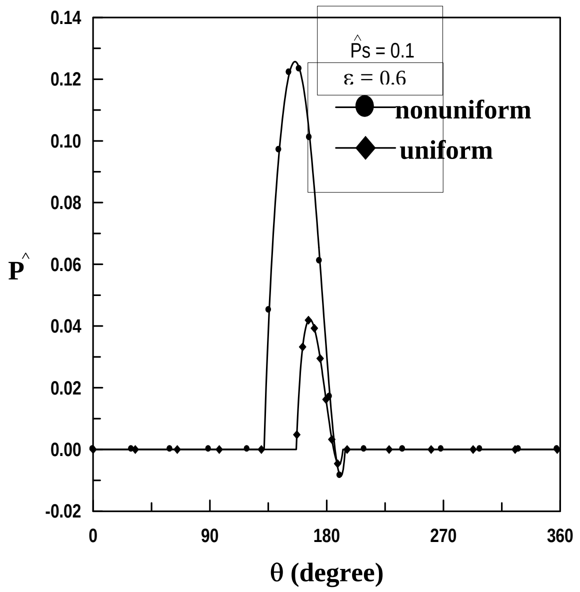

Fig. (4): Comparison pressure distribution between uniform and nonuniform permeability at P_s= 0.1 and ε = 0.6 .

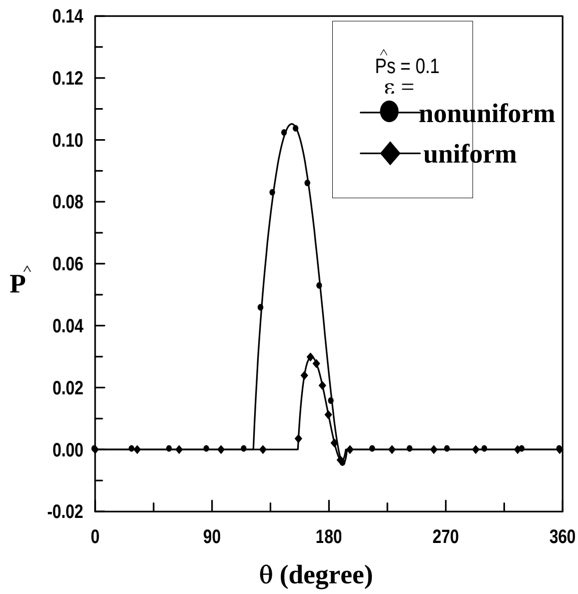

Fig. (5): Comparison pressure distribution between uniform and nonuniform permeability at P_s= 0.1 and ε = 0.55 .

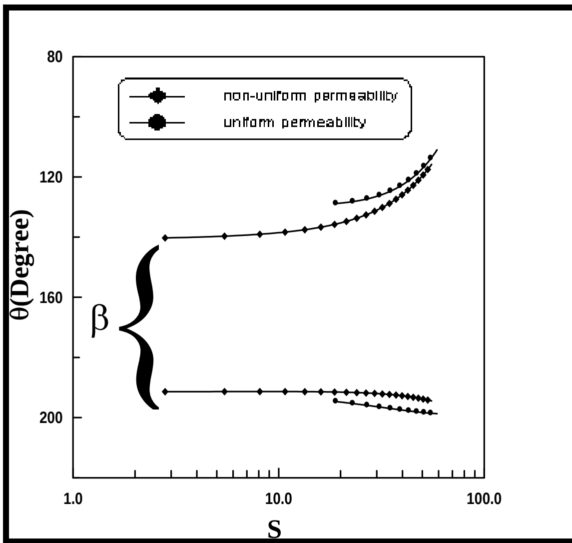

Figure (6): Oil film extent versus Sommerfeld number for uniform and nonuniform permeability at P_s= 0.1.

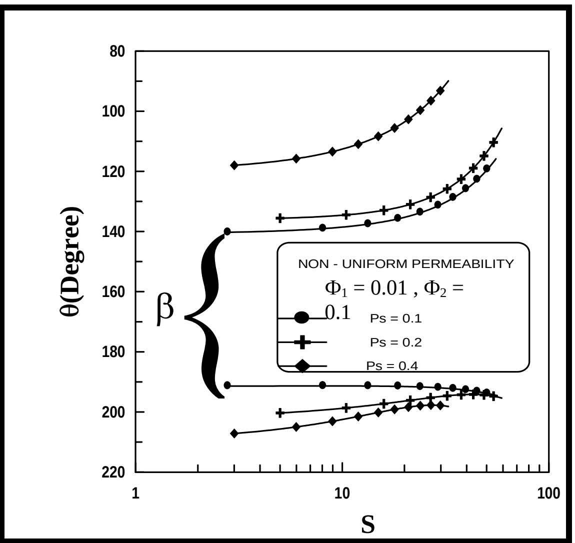

Figure (7): Oil film extent versus Sommerfeld number for various values of dimensionless supply pressure at nonuniform permeability parameter .

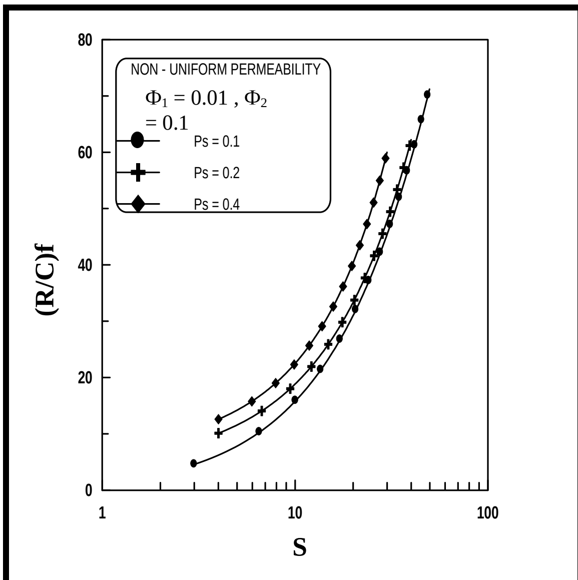

Figure (8): Coefficient of friction versus Sommerfeld number for various values of dimensionless supply pressure at nonuniform permeability.

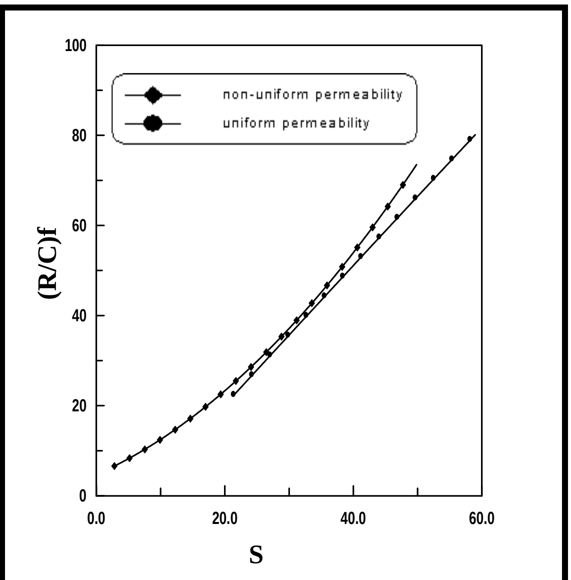

Figure (9): Coefficient of friction versus Sommerfeld number for uniform permeability Φ= 0.1 and nonuniform permeability $Φ_1$= 0.01 and $Φ_2$= 0.1 at P_s= 0.1.

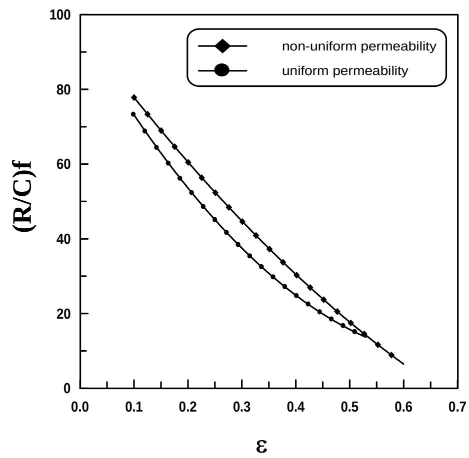

Figure (10): Coefficient of friction versus eccentricity ratio for uniform permeability Φ= 0.1 and nonuniform permeability Φ_1= 0.01 and Φ_2= 0.1 at P_s= 0.1.

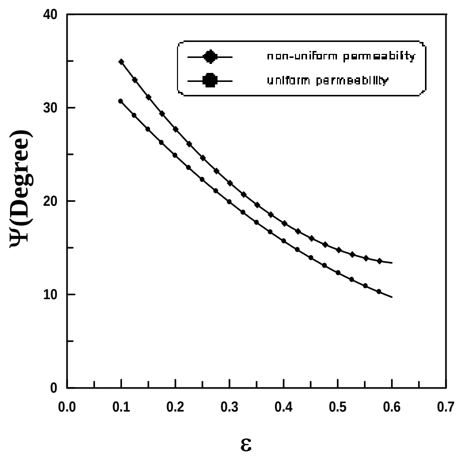

Figure (11): Attitude angle versus eccentricity ratio for nonuniform and uniform permeability parameter at P_s=0.1.

Nomenclature:

c_1	Journal Bearing Clearance (m)
$h^{\wedge}$	Dimensionless Film Thickness,($h^{\wedge}$ =h/c)$_{ii}$
k_θ,r,z	Permeability of the Porous Matrix in Circumferential, Radial and Axial direction respectively (m^2)
L	Length of the bearing (m)
Mθ1	Circumferential Momentum flow rate across oil film surface at inlet end of oil – film region, i.e. at θ = θ1
Mθ2	Circumferential Momentum flow rate across oil – film surface at trailing end of oil –film region, i.e. at θ = θ2
Mθc	Circumferential Momentum flow rate across oil–film surface at both axial ends(z= ±L/2)
Mθb	Circumferential Momentum flow rate across oil – film surface adjacent to Inner Surface of the bearing, i.e. (y=0)
Nj	Journal rotational speed (r.p.m)
P^	Dimensionless Oil-Film Pressure,P^ =c2P/(r2 ηω)
P^*	Dimensionless Oil – Film Pressure Inside the Porous Matrix,P^*=c2P*/(r2ηω)
Ps	Supply Pressure (N/m2)
r^	Normalized radial coordinate, r^= r/ri
Rj	Journal Radius(m)
ri	Inner Radius(m)
ro	Outer Radius(m)
S	Sommerfeld Number , S= (rηωjL / W)*(ri / c)2
s	Slip parameter

T^	Dimensionless Frictional Torque, $T^\wedge = T c / \eta\omega j r_i^3 L$
Uj	Journal Velocity (m/s)
u,v,w	Oil – Film Velocity Components in θ,r,z Directions Respectively(m/s)
u*,v*,w*	Oil Velocity Components inside the Porous Matrix in θ,r,z Directions Respectively (m/s)
W^	Dimensionless Load Carrying Capacity, $W^\wedge = W c2 / \eta\omega j\ r_i^3 L$
(W^r)	Dimensionless Component of Oil – Film Force Along the Line of Centers,
Z^	Normalized axial coordinate, Z^= z/(L/2)

Greek Symbols

ε	Eccentricity ratio
η	Absolute Viscosity of Oil(pa . s)
θ	Angular Coordinate from Maximum Film Thickness Position (Degree)
μ^	Dimensionless Friction Coefficient μ∧ = (R/c)μ
ρ	Density of oil (kg/m3)
(Φ)	Permeability parameter (m2).
ψ	Attitude Angle (degrees)
r, θ, z	Bearing coordinates in radial, circumferential and axial directions.

Subscript and Superscript

B	Referring to Bearing
J	Referring to Journal
^	dimensionless Quantity
*	Porous Parameter

APPENDIX A

To derive the momentum equations and the momentum flow rates figure (2) represents a cross sectional area of the porous matrix. From this figure it can be shown that;

$$dA = dy * dz \quad \text{for } M_{\theta_1} \text{ and } M_{\theta_2} \tag{A1}$$

$$dA = dy * d\theta \quad \text{for } M_{\theta c} \tag{A2}$$

$$dA = d\theta * dz \quad \text{for } M_{\theta b} \tag{A3}$$

$\therefore \; M_{\theta_1}$ can be calculated as **Kaneko (1997)**;

$$\left(M_{\theta_1}\right) = 2\rho\left(\int_0^{L/2}\int_0^{h_{\theta_1}} \left(u_\theta\right)^2\Big|_{\theta_1} dydz\right) \tag{A4}$$

where;

$$\left(u_\theta\right) = \frac{1}{2\eta}\left(\frac{\partial P}{r_i \partial\theta}\right)(y-h)\left(y+\frac{1}{3}h\zeta_{1\theta}\right) + \left(\frac{r_i\omega_j}{h}\right)\left(y\left(1-\zeta_{0\theta}\right)+h\zeta_{0\theta}\right) \tag{A5}$$

Substitute equation **(A5)** into equation **(A4)** to get;

$$\left(M_{\theta_1}\right) = \left[\begin{array}{l} 2\rho\int_0^{L/2}\int_0^{h_{\theta_1}} \frac{1}{4\eta^2 r_i^2}\left(\frac{\partial P}{\partial\theta}\right)^2 (y-h)^2 * \\ \left(y+\frac{1}{3}h\zeta_{1\theta}\right)^2_{ii} + \frac{1}{\eta r_i}\left(\frac{\partial P}{\partial\theta}\frac{r_i\omega_j}{h}(y-h)\right) * \\ \left(y+\frac{1}{3}h\zeta_{1\theta}\right)\left(y\left(1-\zeta_{0\theta}\right)+h\zeta_{0\theta}\right) + \\ \left(\frac{r_i^2\omega_j^2}{h^2}\right)_{ii}\left(\left(y\left(1-\zeta_{0\theta}\right)+h\zeta_{0\theta}\right)^2\right) \end{array}\right] dyd \tag{A6}$$

Integrate equation **(A6)** with respect to (y) and substitute with ($h_{\theta 1}$) as;

$$\left(M_{\theta_1}\right) = \left(\begin{array}{l} 2\rho\int_0^{L/2}\frac{h^5}{4\eta^2 r_i^2}\left(\frac{\partial P}{\partial\theta}\right)^2\left(\frac{9+15\zeta_{1\theta}+10\zeta_{1\theta}^2}{270}\right)_{ii} \\ -\frac{\omega_j}{\eta}\left(\frac{\partial P}{\partial\theta}\right)\frac{h^3}{36}\left(3+3\zeta_{0\theta}+2\zeta_{1\theta}+4\zeta_{1\theta}\zeta_{0\theta}\right) \\ +\left(r_i^2\omega_j^2\frac{h}{3}\left(1+\zeta_{0\theta}+\zeta_{0\theta}^2\right)\right)\Bigg) dz \end{array}\right. \tag{A7}$$

Using dimensionless form for each term of **(A7)**, $\left(M_{\theta_1}\right)$ can be written as dimensionless form as follows;

$$\begin{array}{l} \left(\hat{M}_{\theta_1}\right) = \left(M_{\theta_1}\right)/\left(\rho c r_i^2\omega_j^2 L\right) = \\ \int_0^1\left(\left(\frac{\hat{h}^5}{1080}\left(\frac{\partial\hat{P}}{\partial\theta}\right)^2\left(9+15\zeta_{1\theta}+10\zeta_{1\theta}^2\right)\right)_{\theta=\theta_1}\right)d\hat{Z} \\ -\int_0^1\left(\left(\frac{\hat{h}^3}{36}\frac{\partial\hat{P}}{\partial\theta}\left(3+3\zeta_{0\theta}+2\zeta_{1\theta}+4\zeta_{0\theta}\zeta_{1\theta}\right)\right)_{\theta=\theta_1}\right)d\hat{Z} + \\ \int_0^1\left(\left(\frac{\hat{h}}{3}\left(1+\zeta_{0\theta}+\zeta_{0\theta}^2\right)\right)_{\theta=\theta_1}\right)d\hat{Z} \end{array} \tag{A8}$$

Similarly $\left(M_{\theta_2}\right)$ can be calculated

$$\begin{array}{l} \left(\hat{M}_{\theta 2}\right) = \left(M_{\theta_2}\right)/\left(\rho c\, r_i^2\omega_j^2 L\right) = \\ \int_0^1\left(\left(\frac{\hat{h}^5}{1080}\left(\frac{\partial\hat{P}}{\partial\theta}\right)^2\left(9+15\zeta_{1\theta}+10\zeta_{1\theta}^2\right)\right)_{\theta=\theta 2}\right)d\hat{Z} \\ -\int_0^1\left(\left(\frac{\hat{h}^3}{36}\frac{\partial\hat{P}}{\partial\theta}\left(3+3\zeta_{0\theta}+2\zeta_{1\theta}+4\zeta_{0\theta}\zeta_{1\theta}\right)\right)_{\theta=\theta_2}\right)d\hat{Z} \\ +\int_0^1\left(\left(\frac{\hat{h}}{3}\left(1+\zeta_{0\theta}+\zeta_{0\theta}^2\right)\right)_{\theta=\theta_2}\right)d\hat{Z} \end{array} \tag{A9}$$

From figure (2) $\left(M_{\theta c}\right)$ can be evaluated as **Kaneko (1997)**;

$$\left(M_{\theta_c}\right) = 2\left(r_i\right)\int_{\theta_1}^{\theta_2}\int_0^{(h)}\rho\left[\left(u_\theta * u_z\right)\Big|_{z=L/2}\right] dyd\theta \tag{A10}$$

where;

$$\left(u_Z\right) = \frac{1}{2\eta}\left(\frac{\partial P}{\partial Z}\right)(y-h)\left(y+\frac{1}{3}h\zeta_{1z}\right) \tag{A11}$$

In equation **(A11)** $\left(u_z\right)$ at z=L/2 takes negative value, i.e., $\left(\partial P/\partial z\big|_{z=L/2}\right) > 0$ and (u_θ)

at z=L/2 is assumed to be zero, since (u_θ) would be zero just outside the axial end of the oil film. It yields **Kaneko (1997)**;

$$\left(u_\theta\big|_{z=L/2}\right)=0 \quad \text{if} \quad \left(u_z\big|_{z=L/2}\right)<0$$

$$\left(\partial P/\partial z\big|_{z=L/2}\right)>0 \qquad \textbf{A12}$$

So,

$$\left(M_{\theta c}\right)=0 \quad \text{at} \quad \left(\partial P^\wedge/\partial z\big|_{z^\wedge=1}\right)>0 \qquad \textbf{A13}$$

Substituted equations **(A5)** and **(A11)** into **(A10)**, therefore, $(M_{\theta c})$ cane be evaluated as;

$$\left(M_{\theta c}\right)=2*r_i\int_{\theta_1}^{\theta_2}\int_0^h\left(\begin{array}{l}\frac{r_i\omega_j}{h}\{y-y\zeta_{0\theta}+h\zeta_{0\theta}\}\\ *\frac{1}{2\eta}\frac{\partial P}{\partial Z}(y-h)\left(y+\frac{1}{3}h\zeta_{1Z}\right)\end{array}\right)dy\,d\theta \qquad \textbf{A14}$$

Equation **(A14)** can be integration with respect to (y) and simplified to equal;

$$\left(M_{\theta_c}\right)=\frac{\rho}{\eta}\left(\begin{array}{l}\omega_j r_i^2\int_{\theta_1}^{\theta_2}h^3\frac{\partial P}{\partial z}\\ \frac{-3\{3+3\zeta_{0\theta}+2\zeta_{1Z}+4\zeta_{1Z}\zeta_{0\theta}\}}{108}d\theta\end{array}\right) \qquad \textbf{A15}$$

Multiply and divide equation **(A15)** by (L/2), then use dimensionless form for each term, so, (M_{θ_c}) can be rewritten as;

$$\left(\hat{M}_{\theta_c}\right)=\left(M_{\theta c}\right)/\left(\rho\, c r_i^2\omega_j^2 L\right)$$
$$=-\frac{1}{72}\left(\left(\frac{D}{L}\right)^2\int_{\theta_1}^{\theta_2}A\,d\theta\right) \qquad \textbf{A16}$$

Where:

$$\left(A=h^{\wedge 3}\left(3+3\zeta_{0\theta}+2\zeta_{1Z}+4\zeta_{0\theta}\zeta_{1Z}\right)\frac{\partial P^\wedge}{\partial Z^\wedge}\bigg|_{Z^\wedge=1}\right)\le 0$$

$$\text{if} \quad \left(\frac{\partial P^\wedge}{\partial Z^\wedge}\bigg|_{Z^\wedge=1}\right)\le 0$$

Finally $\left(\hat{M}_{\theta_b}\right)$ can be found as **Kaneko (1997)**;

$$\left(M_{\theta_b}\right)=2(r_i)\int_0^{L/2}\int_{(\theta_1)}^{(\theta_2)}\rho\left[\left(u_{\theta_m}\right)*\left(u_r^*\right)\right]_{r=(r_i)}d\theta\,dz \qquad \textbf{A17}$$

where;

$$\left(u_r^*\right)=\frac{-k_r}{\eta}\left(\frac{\partial P^*}{\partial r}\right) \qquad \textbf{A18}$$

The circumferential velocity component u_{θ_m} across the control surface at (y=0) i.e. (bearing surface) is given for both the case where the oil in the clearance gap flows into the porous matrix and the case where the oil in the porous matrix flows into the clearance gap. It expressed as;

$$\left(u_{\theta_m}\right)=\left(\frac{1}{h}\int_0^h u_\theta\,dy\right)$$

$$=\left(-\frac{h^2}{12r_i}\frac{\partial P}{\partial\theta}\left(1+\zeta_{1\theta}\right)\right)+\frac{\left(r_i\omega_j\right)}{2}\left(1+\zeta_{0\theta}\right)$$
$$\left(\left(u_r^*\right)\Big|_{r=r_i}\right)\ge 0 \qquad \textbf{A19}$$

$$\left(u_{\theta_m}\right)=-\frac{k_\theta}{\eta}\left(\frac{\partial P}{r\partial\theta}\bigg|_{r=r_i}\right)\text{if}\left(u_r^*\Big|_{r=r_i}\right)<0 \qquad \textbf{A20}$$

So; for $\left(\left(u_r^*\right)\Big|_{r=r_i}\right)<0$

$$\left(M_{\theta_b}\right)=\left(2*r_i\int_0^{l/2}\int_{\theta_1}^{\theta_2}\left(\begin{array}{l}\left\{-\frac{k_\theta}{r_i\eta}\frac{\partial P}{\partial\theta}\bigg|_{r=r_i}\right\}*\\ \left\{-\frac{k_r}{\eta}\frac{\partial P^*}{\partial r}\right\}\end{array}\right)d\theta\,dz\right) \qquad \textbf{A21}$$

if dimensionless form using equation **(A21)** can be written as;

$$\left(\hat{M}_{\theta_b}\right) = \left(M_{\theta b}\right) / \left(\rho c r_i^2 \omega_j^2 L\right)$$
$$= \left(\int_0^1 \int_{(\theta_1)}^{(\theta_2)} \left(\left(\frac{c}{r_i}\right) \Phi_\theta * \atop \Phi_Z \left(\frac{\partial P^\wedge}{\partial \theta}\right) \left(\left. \frac{\partial P^{\wedge *}}{\partial r^\wedge} \right|_{r^\wedge = 1} \right) \right) \right) d\theta \, dz^\wedge \qquad \textbf{A22}$$

but where $\left(\left. \left(\boldsymbol{u}_r^* \right) \right|_{r = r_i} \right) \geq 0$

$$\left(M_{\theta_b}\right) = \left(2 r_i \int_0^{L/2} \int_{\theta_1}^{\theta_2} \left(\left\{ -\frac{h^2}{12 \eta r_i} \frac{\partial P}{\partial \theta} \left(1 + \zeta_{1\theta}\right) \atop + \frac{r_i \omega_j}{2} \left(1 + \zeta_{0\theta}\right) \right\} * \left\{ -\frac{k_r}{\eta} \frac{\partial P^*}{\partial r} \right\} \right) \right) d\theta dz \qquad \textbf{A23}$$

Use dimensionless form and simplified equation **(A23)** to get;

$$\left(\hat{M}_{\theta_b}\right) = \left(M_{\theta b}\right) / \left(\rho c r_i^2 \omega_j^2 L\right)$$
$$= \int_0^1 \int_{(\theta_1)}^{(\theta_2)} \left(\Phi_r \left\{ \frac{h^{\wedge 2}}{12} \left(1 + \zeta_{1\theta}\right) \frac{\partial P^\wedge}{\partial \theta} \atop -\frac{1}{2} \left(1 + \zeta_{0\theta}\right) \right\} * \left. \frac{\partial P^{\wedge *}}{\partial r^\wedge} \right|_{r^\wedge = 1} \right) d\theta dz \qquad \textbf{A24}$$

Dividing each momentum flow rate by $\left(\rho c r_i^2 \omega_j^2 L\right)$, equation **(A24)** can be written in dimensionless form as;

$$\left(\hat{M}_{\theta_1} - \hat{M}_{\theta 2} - \hat{M}_{\theta c} - \hat{M}_{\theta b} \right) = 0 \qquad \textbf{A25}$$

دراسة

نظرية لاداء المساند المقعدية ذاتية التزيت تحت ظروف عدم اكتمال غشاء الزيت والنفاذية الغير منتظمة

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الخلاصة:

يتضمن البحث الحالي دراسة نظرية للخصائص الساكنة للمساندالمقعدية المسامية ذات النفاذية الغير منتظمة بأستخدام الشروط الحدية المحسنة. تم أعتماد النموذج بأعتبار النفاذية في نهايات المسند أقل من الوسط. أستخدمت معادلة العزم التكاملي لطبقة الزيت كشرط حدي للحصول على توزيع ضغط الزيت في تلك المنطقة. أظهرت الدراسة الحالية زيادة في قابلية المسند لتحمل الحمل وزيادة في تمدد حدود طبقة الزيت