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Applying Nuclear Shell Model to Study the Energy Levels for ^{18}F nucleus

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Abstract:

In this work, we applied the nuclear shell model to study the energy levels for ^{18}F nucleus which contain two nucleons (proton and neutron) outside ^{16}O core and occupy the ($d_{5/2}$ $s_{1/2}$ $d_{3/2}$) shell . In this calculations used the Surface Delta Interaction (SDI) is employed as a residual inter- action. Comparison with experimental values and found to be rather good agreement , and determined some of the total angular momentum values that were undetermined experimentally .

Key Words : Shell Model , Surface Delta Interaction .

1. Introduction

The nuclear shell model is one of the important and useful models of nuclear structure and the progress in the study of nuclear structure has been made by the development of the nuclear shell model [1] .

In this work, we applied the shell model to calculate the energy levels for Fluorine nucleus (^{18}F), we treat this nucleus on the assumption that $^{16}O_8$ is an inert closed core and one neutron and one proton outside the core will be confined to the ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$) orbits .

The surface delta interaction is employed as a residual interaction to calculate the matrix elements .

2. Theory

In shell model calculation one tries to calculate the energy relative to a closed shell rather than the total energy of the system , and for a single nucleon outside a doubly magic core this energy is taken to be eigenvalue of the Hamiltonian H_0 . The Hamiltonian, when there is more than one nucleon outside the core, is given by [2,3] :

$$H = \sum_{k=l}^{A} (H_0)_k + \sum_{k \subset l} V_{\mathrm{kl}} \qquad \text{………………(1)}$$

where V_{kl} is the residual two-body interaction which exists in addition to the average shell-model potential .

If the two particles occupy the same levels, the energy relative to the closed shell is :

$$< H > = 2\varepsilon_j + E_{JT}(j\, j\, ; j\, j\,) \qquad \text{..…………….(2)}$$

where ε_j is the single-particle energy and $E_{JT}(j\, j\, ; j\, j\,)$ is the matrix element of the residual two-body interaction in same orbit j . If we want the description of low- lying states, we consider the several single-particle levels are founded, for example , we assume there are two states denoted by $| j_1 j_2 J M >$ and $| j_3 j_4 J M >$, then the energies with respect to the core are given by :

$$< H >_{11} = \varepsilon_{j1} + \varepsilon_{j2} + E_{JT}(j_1 j_2 ; j_1 j_2) \qquad \ldots\ldots\ldots\ldots..(3)$$

$$< H >_{22} = \varepsilon_{j3} + \varepsilon_{j4} + E_{JT}(j_3 j_4 ; j_3 j_4) \qquad \ldots\ldots\ldots\ldots..(4)$$

These above equations calculate the energies by only one pure configuration .

We assume the residual nucleon - nucleon interaction is the SDI to calculate the matrix element , this interaction can be considered as a δ- interaction for which one make the further assumption that all the radial integrals are equal . We assume that the residual interaction is attraction and of strength V_0 , it can be written as a delta function in the angular co- ordinates of the interacting particles [2,4,5,6] :

$$V_{ik}^{SDI} = -4 \Pi V_0 \; \delta(\Omega_{ik}) \qquad \ldots\ldots\ldots\ldots\ldots(5)$$

where $\delta(\Omega_{ik}) = \delta(\mathbf{r_i} - \mathbf{r_k})$ and $\mathbf{r_i}$, $\mathbf{r_k}$ are the position vectors of the interacting particles . Two-body matrix elements are given by :

$$E_{JT}(j_1 j_2 ; j_3 j_4) = < j_1 j_2 | -4 \Pi V_0 \, \delta(\mathbf{r_1} - \mathbf{r_2}) | j_3 j_4 >_{JT} \qquad \ldots\ldots\ldots\ldots(6)$$

For brevity of notation j_1 stands for the complete set of single-particle quantum numbers n_1 (radial nodes), l_1 (orbital angular momentum) and j_1 (total angular momentum) . The two-particle states are each coupled to total J and T (isospin) which can use to label the strength parameter .

In this work, the SDI matrix elements (eq.6) take the following form [2]:

$$E_{IT}(j_1 j_2; j_3 j_4) = -(-1)^Q \{V_0 R_0 / 4(2I+1)\} \{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)/(1+\delta_{j_1 j_2}\delta_{l_1 l_2})$$
$$(1+\delta_{j_3 j_4}\delta_{l_3 l_4})\}^{1/2} \{1+(-1)^{l_1+l_2+l_3+l_4}\} [\{1+(-1)^T\}(j_1 j_2 1/2\, 1/2|I1)(j_3 j_4 1/2\, 1/2|I1)$$
$$+(-1)^{l_2+l_4+j_2-j_4} \{1-(-1)^{I+T+l_3+l_4}\}(j_1 j_2 1/2 -1/2|I0)(j_3 j_4 1/2 -1/2|I0)] \qquad \ldots\ldots\ldots\ldots\ldots(7)$$

where $Q = j_1+j_2+j_3+j_4+n_1+n_2+n_3+n_4$, R_0 is a positive number and (|) is the Clebsch – Gordan coefficients [2,4,7].

The allowable angular momentum states for two particles calculate from two theorems[2], first, if two identical particles in the same single particle orbit j (j half integer) can only couple their spins to even values of I :

$$I = 0, 2, 4, \ldots\ldots,(2j-1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

but if two different particles the configuration $(j)^2$ have odd values of I (I = 1, 3, 5, …..) too.

Second theorem, for two particles in the states j_1 and j_2 ($j_1 \neq j_2$)the allowable angular momentum values are

$$I = j_1+j_2 ,\; j_1+j_2-1 ,\; j_1+j_2-2 , \ldots\ldots,| j_1-j_2| \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

3. Calculations and Results

^{18}F contain on one proton and one neutron outside the ^{16}O core occupy the ($0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$)orbits. The energies of this orbits (single - particle energy), when the nucleon is neutron, are -4.143, -3.272 and 0.937 MeV respectively and when the nucleon is proton are -0.09, 0.41 and 5.01 MeV respectively [8,9] .

From equations 8 & 9, we can determined the possible total angular momentum values which are $J^+ = 0 , 1 , 2 , 3 , 4$ and 5 .

In this work, we calculated the spectrum of this nucleus by pure configuration of energy levels .

Matrix elements of the SDI (eq.7) are calculated by using the value of strength of the interaction is 3.0744 MeV and from these elements plus the single - particle energies we obtain the energy matrix elements by applying the equations 3 & 4 are shown w.r.t.(g.s.) in table (1) with the experimental values [8,9] .

Table(1): Theoretical energy levels w.r.t.(g.s.) for ^{18}F by using pure configuration with experimental .

The. Results		Exp. Results	
J^+	Energy(MeV)	Energy(MeV)	J
1	0.0	0.0	1^+
3	0.6796	0.9372	3^+
1	0.7498		
5	1.2938	1.1213	5^+
3	2.6305		
2	3.2247	3.0618	2^+
2	3.427		
0	3.7788	4.7533	0^+
2	3.7986	3.8391	2^+
4	4.2643	4.3981	4^-
1	4.6452	4.36	1^+
1	5.3886	4.8602	1^-
1	5.7796	5.6033	1^+
2,4	6.3738	6.3855	2^+
2,4	6.3938		
1	6.412	6.6331	1
2	6.8441	6.8031	$1^+,2,3^+$
2	7.2351	7.2472	(1^+)
3	8.1561	8.1158	----
3	8.1761		
0	9.9886	10.58	----
3	12.068	12.75	(6^-)
2	14.1473	14.1804	(8^+)

4. Conclusions

From table (1) , we can shown the following conclusions :

1. We calculate 23 energy level for 6 allowable total angular momentum.
2. Many calculated levels with their spin in good agreement with the experimental data .
3. There is uncertainty or undetermined in the spin of some energy levels experimentally , we predict the same energy with a single spin value . These levels with energies (7.2472, 8.1158, 10.58, 12.75 and 14.1804 MeV) with total angular momentum and positive parity states (2, 3, 0, 3 and 2) respectively .
4. The shell model calculation using the SDI is quit successful in introducing the energy spectrum of this nuclei .

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تطبيق أنموذج القشرة النووي لدراسة مستويات الطاقة لنواة ^{18}F

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الخلاصة

في هدا البحث تم اختيار انمودج القشرة النووي لدراسة مستويات الطاقة لنواة ^{18}F والتي توصف بوجود نيوكلونين (بروتون ونيوترون) خارج القلب المغلق ^{16}O يحتلان القشرة $d_{5/2}\ s_{1/2}d_{3/2}$. في هذه الحسابات استخدم تفاعل دلتا السطحي SDIكتفاعل متبقي بين الجسيمين . وبعد مقارنة نتائج البحث بالنتائج العملية وجدنا تقاربا مقبولا بينهما،كما تم تحديد بعض قيم الزخم الزاوي لمستويات طاقة لم تحدد عمليا .