Digital Holography via electrical devices (CCD camera) And liquid crystal display (SLM)

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Abstract

Conventional holography has been widely useful in trade measurement, combining a high resolution charge coupled device (CCD) with a phase modulating, holographic interferometer recording and display could be realized by digital holographic technique. Digital holographic technique has a great number of important and practical applications. In the conventional holography hologram was detected and reconstructed by the photoconductive film. But in the digital holography the numerical version of the conventional holographic technique, hologram is digitally detected by CCD and then displayed on the computer monitor. By reconstruction, the hologram is written into spatial light modulator (SLM), then illuminating the SLM with the reference wave, the object may be reconstructed optically. In this research the application technique of CCD/SLM in holography is theoretically studied and the experimental have been developed.

Keywords : Digital Holography, CCD camera, liquid crystal display

الهولوغرافيا الرقمية بواسطة الكاميرا الالكترونية وشاشة البلورة السائلة

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الهولوغرافيا التقليدية مفيدة في القياسات بصورة واسعة و الهولوغرافيا الرقمية مهمة في كثير من التطبيقات العملية الناتج من الدمج بين الكاميرا الرقمية و معدلة الطور فالإمكانيات التقنية توضح التسجيل الهولوغرافي الرقمي. الهولوغرام التقليدي يُسجل بواسطة الفلم الفوتوغرافي ذو التوصيلية الضوئية، إما النسخة الرقمية للهولوغرام فنسجلها بالكاميرا ومن ثم نظهرها على شاشة الكمبيوتر. ولاسترجاع الصورة الأصلية للجسم بصرياً ، نطبع الهولوغرام على الشاشة (معدلة الضوء المكانية) ونضيئها بموجة الضوء المرجعية. دراسة نظرية حول تطبيقات (CCD/SLM) في الهولوغرافيا الرقمية مع تجربة عملية لتسجيل هولوغرام لصورة تمثل عبارة " Kufa University" مطبوعة على شفاف باستخدام كاميرا CCD واسترجاعها باستخدام SLM و كاميرا CCD آخرا وفي نفس وقت التسجيل.

الكلمات المفتاحية: الهولوكرافيا الرقمي، CCD كاميرا، شاشة البلورة السائلة

1- Introduction

D. Gabor invented holography in 1948 as a method for recording and reconstructing amplitude and phase of a wave field. He created the word holography from the Greek words '*holos*' meaning whole or entire and '*graphein*' meaning to write.

A holographically stored image or hologram is the photographically or otherwise recorded interference pattern between wave fields scattered from the object and a coherent background named reference wave. It is usually recorded on a flat surface (special film), but contains the information about the entire three-dimensional wave field. This information is coded in the form of interference stripes, usually not visible for the human eye due to the high spatial frequencies. The object wave can be reconstructed by illuminating the hologram with the reference wave again; this reconstructed wave is by passive means indistinguishable from the original object wave. An observer recognizes a three-dimensional image with all effects of perspective and depth of focus [1, 2].

U. Schnars applied Digital Holography (DH) to interferometer and demonstrated that digital hologram reconstruction offers much more possibilities than conventional (optical) processing. The phases of the stored light waves can be calculated directly from the digital holograms. The use of electronic devices such as the charge coupled device (CCD) for recording in different states of the object under investigation. The resulting fringe pattern has some similarities to that of conventional or digital holography. Main differences are the speckle appearance of the fringes and the loss of phase in the correlation process [3].

Digital holographic technique has a large number of important and practical applications. Hologram is digitally detected by CCD camera and then displayed on the computer monitor. To reconstruction, the hologram is writing into LCD, then illuminating the LCD with the reference wave, the tested object may be reconstructed optically. The angle between the two interfering waves should be less than (3°) for recording the interference fringes with high spatial frequency [4, 5].

Digital holography, also known as electronic holography, is the technology of recording and processing holographic information electronically. Recording of holographic information is conventionally done by using CCD camera, thereby bypassing the use of non-real-time films for recording. Processing or reconstructing of holograms is subsequently performed by digital methods or optical methods such as the use of spatial light modulators (SLM) for real-time applications [6].

2- Optical Computing Hardware

The three most basic hardware components of an optical information processing system are a source, a modulator, and a detector. A source generates the light, a modulator multiplies the light by a (usually, spatially varying) function, and a detector senses the resulting light [7].

2-1 Sources

Lasers are a common source of illumination because at some levels they are mathematically simpler to understand. Usually, the source is monochromatic to avoid the problem of colour dispersion as the light passes through refracting optical components, unless this dispersion is itself the basis for the computation.

2-2 Spatial Light Modulator (SLM)

It is possible to encode a spatial function (a 2D image) in an optical wavefront. This would be called an amplitude-modulating reflective. so an image encoded in the incoming wavefront will be point sensible multiplied by the image on the SLM. Modulators can also act on phase and polarization, and can be transmissive rather than reflective [8].

One class of note is the optically-addressed SLMs, in which, typically, a 2D light pattern falling on a photosensitive layer on one side of the SLM spatially varies (with an identical pattern) the reflective properties of the other side of the SLM.

2-3 Charged Coupled Device (CCD)

The CCD's were invented in the sixties of the last century by researchers at Bell Labs. A CCD is an electrical device that is used to create images of objects, store information or transfer electrical charge. The most popular application today is image recording. CCD's are used as imaging devices in electronic cameras. They are available as line scanning devices, consisting of a single line of light detectors, and as area scanning devices, consisting of a rectangular matrix of light detectors. For Digital Holography only the latter architecture is of interest. CCD imaging is performed in a three step process [9].

1. Light exposure: the incident light separates charges by the internal photo effect. This effect converts light into an electronic charge at the individual detectors called pixels.
2. Charge transfer: the charge transfer function moves the packets of charge within the semiconductor (silicon) substrate to memory cells.
3. Charge to voltage conversion and output amplification.

3- Fresnel Diffraction Formula

If we assume that all operations on $\psi(z,t)$ are linear, we can use the complex representation is that the spatial and temporal parts factorize [11]

$$\psi(z,t) = a exp(i\omega t)\exp(-i\phi) = \psi_A exp(i\omega t) \quad ……(1)$$

Where $\psi(z,t)$ the time-varying electric field at any point due to a linearly polarized monochromatic light wave propagating in a vacuum in the z direction, $\psi_A = a\exp(-i\phi)$ is known as the complex amplitude, a the amplitude, $i = (-1)^{1/2}$, $\phi = kz$ the phase, $\exp(-i\phi)$ the constant phase shift and (ω) (k) the angular and spatial frequency [12]. We can model as the field distribution immediately after an aperture located at (z) as:

$$\psi(x,y,z,t) = \psi_A(x,y;z)\exp(i\omega t) \quad ……(2)$$

Where $\psi_A=(x,y;z)$ the complex amplitude of $\psi(x,y,z)$ and x,y is a spatial variable of a plane wave.

Since to find ψ_A we substituted Eq. (2) into the 3-D scalar wave equation, i.e., the light field must satisfy the wave equation, we have the Helmholtz equation of ψ_A as [13]:

$$\nabla^2\psi_A + k^2\psi_A = 0 \quad …… (3)$$

By taking the 2-D Fourier transform for Eq. (3) we can obtain:

$$\frac{\partial^2\Psi_A}{\partial z^2} + (k^2 - k_x^2 - k_y^2)\Psi_A = 0 \quad ……(4)$$

Where the 2-D Fourier transform is [10]:

$$\mathcal{F}\{f(x,y)\}=F(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, expi(k_x x, k_y y)\, dxdy \quad ...(5)$$

We can now readily solve the above equation to get:

$$\Psi_A(k_x,k_y;z) = \Psi_{Ao}(k_x,k_y) exp\left[-ikz\sqrt{(k^2 - k_x^2 - k_y^2)}\right] \quad …(6)$$

Where $\Psi_A(k_x,k_y;z)$ is the Fourier transform of $\psi_A(x,y;z)$, k_x , k_y is spatial radian frequency (radian/meter), and $\mathcal{F}\left\{ \frac{\partial^2\psi(x,y)}{\partial x\, \partial y}\right\}= k^2\Psi(k_x,k_y)$ $\Psi_{Ao}(k_x,k_y)$ the Fourier transform $\psi_{Ao}(x,y)$ at z=0.

To find the field distribution at z in the spatial domain, we take the inverse Fourier transform of Eq. (6) to obtain the Fresnel diffraction formula which describes the Fresnel diffraction of a beam during propagation and has an arbitrary initial complex profile $\psi_{A_o}(x,y)$ as [14]:

$$\psi_A(x,y;z) = \psi_{Ao}(x,y)\otimes h(x,y;z) = \left(\frac{ik}{2\pi z}\right)\exp(-ikz) \iint \psi_o(x^\backslash,y^\backslash) \times \exp\left\{\left(\frac{-ik}{2z}\right)[(x - x^\backslash)^2 + (y - y^\backslash)^2]\right\} dx^\backslash dy^\backslash ...(7)$$

Where $\otimes$ is a symbol denoting the convolution of f and h; the expression $(f\otimes g)$ reads as f convolves g and $h(x,y;z)$ is the Point Spread Function (PSF) of the linear space invariant system (LSI), aside from a constant phase shift, $\exp(-ikz)$ define as:

$$h(x,y;z) = \left(\frac{ik}{2\pi z}\right)\exp\left\{\left(\frac{-ik}{2z}\right)[x^2 + y^2]\right\} \exp(-ikz) …(8)$$

4- Digital Holography

The concept of digital hologram recording is a plane reference wave and the wave reflected from the object interfering at the surface of a Charged Coupled Device [9]. The resulting hologram is electronically recorded and stored.

The object is in general a three-dimensional body with diffusely reflecting surface, located at a certain distance from the CCD. In optical reconstruction the virtual image appears at the position of the original object and the real image is formed at a same distance as well, but in the opposite direction from the CCD. The diffraction of a light wave at an aperture (in this case a hologram) which is perpendicular to the incoming beam, then accordant to Fresnel diffraction formula Eq. (7), the reconstruction of object image is equal to:

$$H(x,y) \otimes h(x,y;\ z_o) \quad ….(9)$$

Where $h(x,y;z_o=\mathrm{d})$ is the hologram function or (PSF) and d is the distance between a point in the hologram plane and a point in the reconstruction plane and $H(x,y)$ is a hologram. The geometrical quantities are explained in Fig. (1), for various values of

$z = z_1, z_2$, etc., we can reconstruct different planes normal to the hologram.

The whole 3-D volume of the object is then constructs a plane by plane. The diffraction pattern is calculated at a distance *d* behind the CCD plane, which means that it reconstructs the complex amplitude in the plane of the real image. Because the reconstructed wave field is a complex function, both the intensity as well as the phase can be calculated [15].

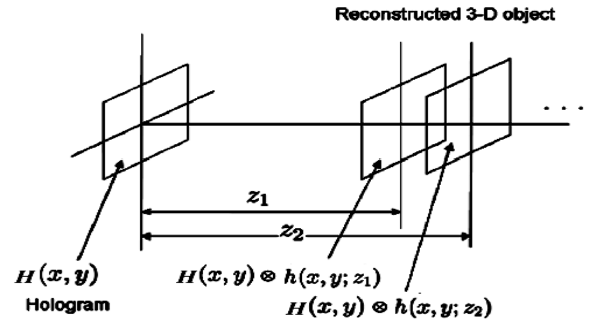


Fig. (2-3) The Digital Holographic Reconstruction. [50]

The field resulting from the superposition of the reference wave ψ_R and object wave ψ_O is simply given by ($\psi_R + \psi_O$) if both fields are linearly polarized and collinear. The hologram *H*(*x*,*y*) recorded intensity is given by the simple mathematical expression [15].

$$H(x,y) = I_{int}(x,y) = (\psi_R + \psi_O)^*(\psi_R + \psi_O)$$
$$= |\psi_R|^2 + |\psi_O|^2 + \psi_R^*\psi_O + \psi_R\psi_O^* \quad \text{....(10)}$$

The last two terms yield the desired holographic image while the two first terms of this development are called "zero order terms", are slowly varying in time or space and do not depend on the phase of ψ_R and ψ_O. They can be easily eliminated by high pass spatial filters. But the last two terms, on the contrary are called "interference terms. They are sensitive to the phase difference between ψ_R and ψ_O, and will be processed to retrieve the phase information. To reconstruction of the object wave field can be achieved by forming the product:

$$\psi_R H(x,y) = \psi_R(\psi_R + \psi_O)^*(\psi_R + \psi_O)$$
$$= \psi_R(|\psi_R|^2 + |\psi_O|^2) + |\psi_R|^2\psi_O + \psi_R^2\psi_O^* \quad \text{...(11)}$$

The first term on the right part of Eq. (11) corresponds to the reconstruction of the "zero order term" and should be eliminated as completely as possible. The second term corresponds to the "virtual image" and restores exactly in principle the object wave after computation of the object wave field propagated over the exact distance *d* separating the object from the hologram. The third term corresponds to a wave outgoing from the hologram and converging to the "real image" of the object, which appears as a mirror image of the "virtual image" (with respect to the hologram) [9].

According to Fresnel diffraction formula, the object wave arises from the point-object-offset $\delta(x - x_o, y - y_o)$, on the hologram plane and is given by describing an offset delta function [15]:

$$\psi_o(x,y;z) = \delta(x - x_o, y - y_o) \otimes h(x,y;\, z_o) \ldots \text{(12)}$$

The off-axis holography is a method that was devised by Leith and Upatnieks to separate the "twin-image" and the "zero-order beam" from the desired image [16]. for off-axis recording, the reference plane wave, ψ_R, is now an off-axis plane wave given as:

$$\psi_R(z_o) = a exp[ikx\, sin(\theta)] \quad \text{..... (13)}$$

This can be done by simply, for example, rotating the beamsplitter (BS) between the pinhole aperture and the hologram plane in a clockwise direction or rotate the mirror in the clockwise direction such, so that the reference plane wave is incident on the hologram plane at an angle θ and a is a constant (the amplitude). Hence, the intensity distribution that is being recorded on the hologram plane, or the CCD camera, is given by substituting (13) and (12) into (10), we get:

$$H(x,y) = A + (\tfrac{B}{2i}) exp\left\{\left(\tfrac{ik}{2z_o}\right)[(x - x_o)^2 + (y - y_o)^2] + kxsin(\theta)\right\} - (\tfrac{B}{2i}) exp\left[\left(\tfrac{-ik}{2z_o}\right)[(x - x_o)^2 + (y - y_o)^2] + kxsin(\theta)\right] \quad \text{......(14)}$$

Where $A = a^2 + \left(\frac{k}{2\pi z_o}\right)^2$, $B = \frac{k}{\pi z_o}$

The holographic field distribution defined by (14) is called the Fresnel Zone Plate FZP(*x*,*y*;*z*), with its center displaced at (x_o, y_o) from the center (0, 0) of the holographic plate (*x*, *y*), and the spatial variation of the zone plate is governed by a sine function with a quadratic spatial dependence.

By illuminating the off-axis hologram with a reconstruction wave identical to the reference wave $(\psi_{Rc} = \psi_R)$ we have immediately after the hologram, and by performing Fresnel diffraction Fig. (2); i.e. $FZP(x,y;z) = H(x,y)$.

$$FZP(x,y;z) \otimes h(x,y;z) \quad \text{......(15)}$$

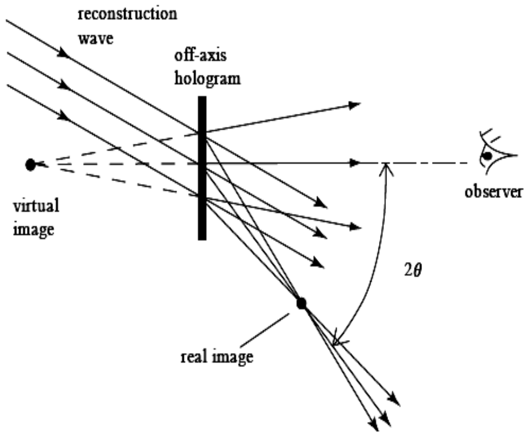


Fig.(2-8)Holographic reconstruction 851of off-axis hologram

Thereby creating three waves as follows:

The Zero-order beam:

$Aaexp[ikxsin(\theta)] \otimes h(x,y;z=z_o)$
$\sim \exp(ikxsin\theta)$. ……(16)

The Real-image (or the twin image):

$aexp[ikxsin(\theta)]exp\left\{\left(\frac{ik}{2z_o}\right)[(x-x_o)^2+(y-y_o)^2]+kxsin(\theta)\right\} \otimes h(x,y;z=z_o)$
$\sim \delta(x+2z_o sin\theta\ , y-y_o)$ …(17)

The Virtual-image:

$aexp[ikxsin(\theta)]exp\left\{\left(\frac{-ik}{2z_o}\right)[(x-x_o)^2+(y-y_o)^2]+kxsin(\theta)\right\} \otimes h(x,y;z=-z_o)$
$\sim \delta(x-x_o, y-y_o)$ ……(18)

The point $\delta(x-x_o, y-y_o)$can be captured in CCD camera; it would be directly correlated as the intensity of the pixel in the image.

This technique of off-axis recording is also known as carrier frequency holography [4].We can re-write Eq. (13) as

$$\psi_{R}(z_o) = aexp[i2\pi\nu_c\, x] \quad ...\ (19)$$

Where $\nu_c = \frac{k}{2\pi}(sin\theta) = \frac{sin\theta}{\lambda}$ is the spatial carrier.

By use electronic devices such as CCD cameras we can have holographic recording. We can perform the electronic recording of holographic information by using Spatial Light Modulator (SLM) electrical addressable liquid crystal display (EALCD), has been coming into the holography as a light wave front transmission device. SLM are electro-optical devices used to modulate optical waves. An SLM is an array of cells from liquid crystal material. An individual SLM cell changes its transmittance or its refractive index depending on the applied voltage. It is therefore possible to modulate the brightness or phase of light, which passes the device, i.e. an SLM can be applied to the reconstruction of an optical hologram. At first a digital hologram is recorded with a CCD camera. The hologram is stored and transmitted to the measurement site and optically reconstructed by the SLM, which modulates the optical wave corresponding to the reference wave with the hologram function and generates the original object wave. The virtual image can be observed at the position of the original object.

Classic optical interference theory gives us the following equation, which exposes the relationship of interference fringes frequency (ν) to the angle θ between reference wave and object wave:

$$\nu = 2\frac{\sin\left(\frac{\theta}{2}\right)}{\lambda} \quad …….(20)$$

So, the fringe frequency must not be higher than the CCD cut$_{off}$ frequency 1/(2Δx), that means $\nu_{max} = 1/(2\Delta x)$ in which Δx is CCD pixel dimension. Thus, the maximum angle θ_{max} between reference wave and object wave is

$$\theta_{max} = \lambda/(2\Delta x) \quad ……(21)$$

Only when the fringe frequency is less than CCD cut$_{off}$ frequency, the fringe can be recorded. It means that in order to keep all the information of every point in the object, the angle between reference wave and object wave must be less than θ_{max}. For Semiconductor Laser, λ = 650nm, consequently, the maximum angle θ_{max} between reference wave and object wave is θ_{max} = 1.56^o. These data means a small object or a long distance. Thus the object dimensions and the distance from object to CCD are confined. The best CCD camera record off-axis holograms efficiently because the resolution requirement by making the recording angle smaller, but this approach requires a very small recording angle that often makes it impractical. Because of this reason, on-axis holography seems to be prevalent in digital holography [17]. On the other hand, twin-image problems need to be tackled when on-axis holography is employed. Indeed twin-image elimination is an important research topic [18]. We can digitally evaluate Fresnel diffraction by performing the convolution, which is Eq. (15), where some constant amplitude of the reconstruction beam, $FZP(x, y; z)$ is the recorded hologram, and $h(x,y;z)$ is the PSF in Fourier optics. An alternative way to utilize electronically or digitally recorded hologram is to have it displayed on some sort of spatial light modulator (SLM) for real-time coherent reconstruction. A 2-D spatial light modulator is a device with which one can imprint a 2-D pattern on it then passing the laser beam through it. In fact, we can think of a spatial light modulator as a real-time transparency because one can update 2-D images or holograms upon the spatial light modulator in real time without developing films into transparencies.

5-Experimental Setup

The experiment is recording and reconstructing the hologram in mach-zehnder interferometer by implementation the Spatial Light Modulation (SLM).

A transparency object in the form of text "Kufa University" with [0,1] i.e. black line on white background and [1,0] i.e. white line writing on black background. The objectives of this experiment are to show the recorded hologram on one Laptop computer and the reconstructed object on other desktop computer. Optical elements have been arranged on the workbench (1000×800) *mm* with Laptop and desktop and two CCD cameras. The basic of these experiments rest on the Mach-Zehnder Interferometer which is an amplitude splitting device, it consists of two beamsplitters and two totally reflecting mirrors. The two waves within the apparatus travel along separate paths. A difference between the optical paths can be introduced by a slight tilt of one of the beam splitters. Since the two paths are separated, the interferometer is relatively difficult to align.

In this experiment beam expanded was added. This optical element consists of two pieces the beam divergent and collimation lens in order to obtain a plane wave. The expansion beam is outside the mach-zehnder interferometer in order to get wide and collimate beam from the beginning.

The purpose of the beam splitter is to get two beams one for recording stage and the second for the reconstructing stage because the two stages are working at the same time. Then the two beams by beam splitter of the mach-zehnder were obtained, the reference wave and object wave which illuminate the object. There was a beam splitter outside the mach-zehnder interferometer which is used to obtain a simultaneous reconstruction wave which illuminates the SLM that is imprinted upon a 2-D pattern of hologram to reconstruct the object. Semiconductor laser is split by beam splitter (BS) into two paths: (half-reflecting half-transparent) and two total reflection mirrors (M); four reflection planes are close to parallel to each other, and the central light paths form a parallelogram. Fig.(3) illustrates the sketch of experiment used for recording the transparency object image (Kufa University) one with black line font on white background Fig.(4). and the other is white line font on black background Fig.(5). The beam about 2*mm* in diameter from a source of 4-milliwatt, 650*nm* semiconductor laser follow the polarizer in order to obtain a linear polarization laser beam of attenuated intensity which is followed by the first beam splitter which gives two beams, the reflected beam will be used to illuminate (SLM) after widening and collimation and this beam will be utilized in the reconstruction part.

The transmitted beam will be widened and collimated by beam divergent and collimation lens (*f*=10cm), then by the first beam splitter of mach-zehnder interferometer divided into two plane waves. One plane wave is used to illuminate the object (Kufa University), and the other is used to illuminate directly the recording plate CCD camera.

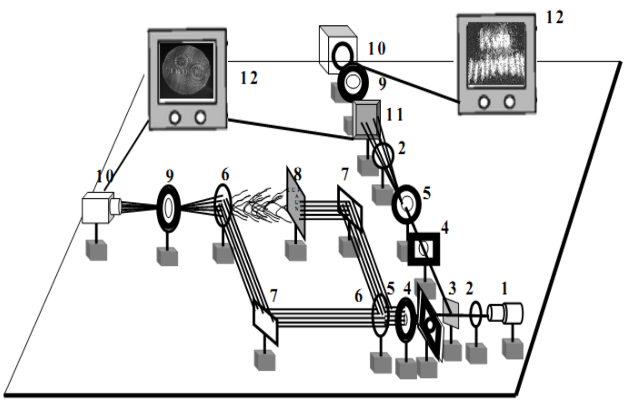


Fig.(3) the Experimental setup Digital Hologram for transparencies
1, semiconductor laser; **2**, polarizer; **3**, first beam splitter; **4**,beam divergent; **5**, collimation lens; **6**, beam splitter **7**, mirror; **8**, object; **9**,lens; **10**,CCD camera; **11**, SLM; **12** computer .

The plane wave that is diffracted by the object generates an object wave. The plane wave that directly illuminates the recording plate is known as a reference wave. The CCD now records the interference of the reference wave and the object wave (Digital hologram of object).

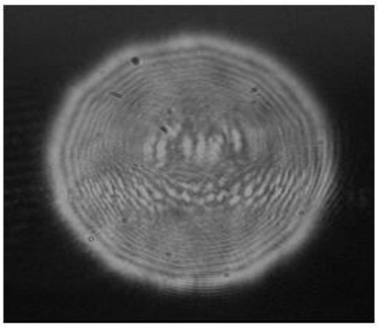
The reflected beam from the first beam splitter will be responsible for reconstructing the object, this beam will become a plane reference wave (reconstruction wave) by beam divergent and collimation lens (f =10cm), and now become ready to illuminate the (SLM), then the CCD camera will capture the same object image.

Because of the polarizing plate component, SLM has a great requirement upon the polarization characteristic of the modulation light. Thus, in order to satisfy the light from laser to the requirement of SLM, a polarizer was inserted into the light path and adjusts it onto a proper position according with the real experiment conditions.

6-Results

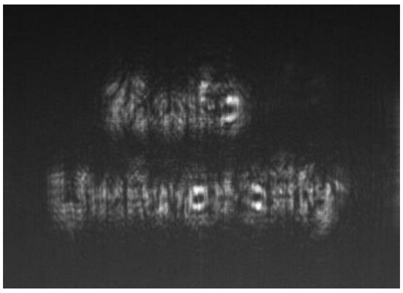
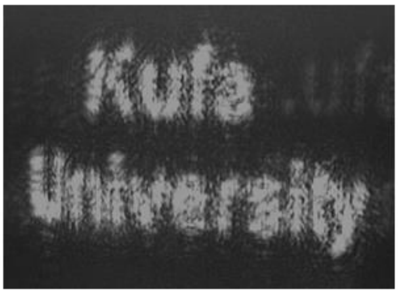
The object was prepared on a transparency (Kufa University) as a bitmap image. The original image is shown in Fig. (4-a). The resulted hologram is shown in Fig. (4-b) and the reconstruction hologram is shown in Fig. (4-c). It can be seen from these figures that the text was written in black font on white background. When the text (Kufa University) was written in white font on black background as shown in Fig. (5-a) the resulted image was distorted by noise, as shown in Fig. (5-c).

We notice that the reconstructed image was more obvious when the text was black font on white background, because the light distribution for diffraction pattern on opaque aperture is less than the transparent aperture.

a

b


c

Fig. (4) the recorded and reconstructed hologram for black font text

a

b


c

Fig. (5) the recorded and reconstructed hologram for white font text

7-Reference

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