Periodic changes in orbital elements of GLONASS satellite due to the solar radiation pressure

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Abstract:

This paper presents the effect of the sun's radiation pressure on the orbital elements of Russian GLONASS satellites eclipses of the satellite as it passes through the Earth's shadow will be taken into considerations when calculating the pressure effect of solar radiation. First solar radiation effects in the calculation the variation in orbital elements of GLONASS satellites at different values of argument of perigee. Second the effect of solar radiation pressure of the GLONASS satellites was obtained in different three orbital planes (i.e. three different values of right ascension of ascending nodes). The amplitude of oscillations in the semi major axis is large due to the solar pressure radiation and the variation depends on the location of the satellite in the plane (depends on arguments of perigee). The effect of solar radiation pressure on the GLONASS satellite shows that amplitudes of oscillations also depends on the direction of plane(right ascension of node).

Keywords: solar radiation pressure, periodic changes, orbital elements, perturbations, Earth shadow.

التغيرات الدورية للعناصر المدارية للأقمار الصناعية (غلوناس) الناتجة من تأثير ضغط الإشعاع الشمسي

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الخلاصة:

يتناول هذا البحث ضغط الإشعاع الشمسي على العناصر المدارية لمنظومة أقمار الملاحة الروسي غلوناس وقد اخذ في الاعتبار عندما يكون القمر الاصطناعي في ظل الأرض أثناء إجراء الحسابات. في البداية حسب التغير في العناصر المدارية لمنظومة غلوناس عند قيم مختلفة (الإزاحة الزاوية لنقطة الحضيض)، وتبعها دراسة تأثير ضغط الرياح الشمسية على أقمار غلوناس في ثلاث مستويات مختلفة (لقيم مختلفة من المطلع المستقيم للعقدة الصاعدة). وتبين من النتائج أن سعة التذبذب بالمحور الكبير تكون عالية وتعتمد على موقع القمر الصناعي في المدار ، وان تأثير الرياح الشمسية على سعة التذبذب لهذه الأقمار تعتمد أيضا على اتجاه المستوي الذي يقع فيه.

الكلمات المفتاحية: ضغط الإشعاع الشمسي، تغيرات دورية، العناصر المدارية، الاضطرابات، ظل الأرض.

1. Introduction

GLONASS "Global Navigation Satellite System", is a space-based satellite navigation system operated by the Russian Aerospace Defense Forces describing GLONASS as a system " The purpose of the GLONASS is to provide real time position and velocity determination for military and civilian users; provide air, marine, and any other type of users with positioning, velocity measuring and timing data . It provides an alternative to Global Positioning System (GPS) and is the second alternative navigational system in operation with global coverage and of comparable precision[1,2].

Development of GLONASS began in the Soviet Union in 1976 to determine positioning of civil aviation aircraft, navy transport and fishing-boats of the Soviet Union at the beginning on the first generation GLONASS satellites were 7.8 m tall, had a width of 7.2 m, measured across their solar panels, and a mass of 1260 kg The second generation of satellites, known as GLONASS-M, were development was beginning in 1990 and first launched in 2003 [3]. These satellites possess a substantially increased lifetime of seven years and weigh slightly more at 1480 kg. They are approximately 2.4 m diameter and 3.7 m high, with a solar array span of 7.2 m . The new satellite also had better accuracy and ability to broadcast two extra civilian signals[4]. The first GLONASS-K satellite launched in 2011[5,6]. GLONASS-K is a substantial improvement of the previous generation: it is the first unpressurised GLONASS satellite with a much reduced mass (750 kg versus 1480 kg of GLONASS-M). It has an operational lifetime of 10 years, compared to the 7-year lifetime of the second generation GLONASS-M[7]. It would be able to promptly fix the receiving station's position based on signals from 4 satellites, and also reveal the object's speed and direction[8]. The force due to solar radiation is greater than air drag for satellites above 1000 km; the effects of solar radiation pressure, however, are mainly important for high orbit e.g. GPS and GLONASS satellites. Solar radiation perturbations on the orbital elements of artificial satellites have been treated by a number of authers. Kozai(1963)[9], Cook(1962)[10] discussed solar radiation pressure on the motion of spherical Earth satellite.

Paper by Lala and Sehnal 1969 [11] investigated the short periodic perturbations during one revolution of the satellite around the earth caused by solar radiation, Lala 1971[12] presented the general futures of analytical theory of the direct solar radiation perturbation of the satellite orbits . Aksnes 1975[13] calculate the solar radiation perturbation on orbital elements by semi analytic algorithm. Casotto1991[14] developed the transformation of classical orbit elements perturbations to perturbations in position and velocity in the radial, transverse and normal directions of the orbital elements. Hubaux2012[15] presented in his paper a symplectic integration scheme to numerically compute space debris motion takes into account the Earth’s gravitational potential, luni-solar and planetary gravitational perturbations and direct solar radiation pressure. Saleh2013[16] studied some perturbations of a satellite orbit with heights more than 10000km. Alrofiae 2012[17] investigated the effects of the solar radiation pressure and the attraction of the sun on medium Earth orbit satellite. Vokronhlicky [18] developed an approximate method for perturbation force due to solar radiation and applied it to study the long term perturbations associated with the penumbra effect on the orbital element of semi major axis of LAGEOS.

In this paper, perturbations due to solar radiation pressure on orbital elements of GLONASS satellites in the three different planes at eight different values of argument of perigee are determined from Gaussian form of variation of parameter using numerical integration; the effects of Earth's shadow were also considered to evaluate the eccentric anomalies of shadow exit and entry.

2. Solar Radiation Perturbation Analysis

Perturbation on orbital motion result in secular and periodic changes, secular changes in a particular element vary linearly over time, or proportional to some power of time and error in secular terms produce unbounded error growth. Period changes are either short or long periodic, depending on the length of time required for an effect to repeat. Short period effects repeat on the order of the satellite's period or less, this mean for satellite at altitude 400km could vary with period about 100 min, whereas for geosynchronous satellite would be about 24 hours. Long period effects have cycles longer than one orbital period; these long period effects are seen in the motion of the node and perigee.

The orbital elements are distinguished as either fast or slow variables depending on their relative rate of change. Fast variables, like mean, true, and eccentric anomalies change a lot during one orbital revolution, even in the absence of perturbations. Slow variables change very little during one orbital revolution. Perturbation cause these changes; the orbital elements changes as slow variable are semi major axis, eccentricity, inclination, right ascension of ascending node and argument of perigee. All slow elements would remain constant in the absence of perturbations while the fast variables would continue to change.

Solar radiation pressure is a nonconservative perturbation it become more important at higher altitude, during solar storms the effect of this type of perturbation be much larger than other perturbation therefore solar radiation pressure leads to determine the effect of shadowing on the satellite. The incoming radiation from the sun cause a force on the satellite; using the reflectivity, the solar radiation pressure and the exposed area to the sun the effect force can be expressed as[19]:

$$F_{SR} = P_{SR}\, C_R\, A$$

Where

P_{SR} is the solar pressure (force per area) $P_{SR} = 4.5 * 10^{-6} \frac{N}{m^2}$

C_R is the reflectivity (how the satellite reflects incoming radiation) its value between 0.0 to 2.0.

A exposed area to the sun.

Newton's second law helps us to express the acceleration of the radiation force as:

$$\vec{a}_{radiation} = -\frac{P_{SR} C_R A}{m} \frac{\vec{r}_{\odot sat}}{|\vec{r}_{\odot sat}|}$$

Most satellites including GLONASS undergo periodic eclipses behind the Earth hence the satellite does not exposed to solar radiation pressure, then modeling to turn the solar radiation pressure calculation on when the satellite exit penumbra and brings it off when entry the Earth's shadow.

The shadow function in a forth order equation in the cosine of the true anomaly is obtained by Escobal 1965 [19]:

$$S = \alpha_1 cos^4 \upsilon + \alpha_2 cos^3 \upsilon + \alpha_3 cos^2 \upsilon + \alpha_4 cos\, \upsilon + \alpha_5$$

Solving the quartic analytically yields the value of true anomaly for entry and exit; if its entry, the value of shadow function changed from negative to positive. The values of $\alpha_1, \alpha_2, \alpha_3$, are given in appendix A.

As mentioned the solar radiation effects depend on various parameters. Assuming the disturbing acceleration F_{SR} is constant while the satellite in the sun light and zero if it is in Earth shadow; Cook 1962 [10] expressed the perturbations in terms of radial, transverse, and normal components of the disturbing force as

$$\begin{Bmatrix} S(\upsilon) \\ T(\upsilon) \end{Bmatrix} = F_{SR}\{-cos^2\frac{i}{2}cos^2\frac{\epsilon}{2}\begin{Bmatrix} cos \\ sin \end{Bmatrix}(\lambda_\odot - \upsilon - \omega - \Omega) - sin^2\frac{i}{2}sin^2\frac{\epsilon}{2}\begin{Bmatrix} cos \\ sin \end{Bmatrix}(\lambda_\odot - \upsilon - \omega + \Omega) - \frac{1}{2}\sin i \sin\epsilon\left[\begin{Bmatrix} cos \\ sin \end{Bmatrix}((\lambda_\odot - \upsilon - \omega) - \begin{Bmatrix} cos \\ sin \end{Bmatrix}(-\lambda_\odot - \upsilon - \omega)\right] - sin^2\frac{i}{2}cos^2\frac{\epsilon}{2}\begin{Bmatrix} cos \\ sin \end{Bmatrix}(-\lambda_\odot - \upsilon - \omega + \Omega) - cos^2\frac{i}{2}sin^2\frac{\epsilon}{2}\begin{Bmatrix} cos \\ sin \end{Bmatrix}(-\lambda_\odot - \upsilon - \omega - \Omega)\}$$

$$W = -F_{SR}\{\sin i\, cos^2\frac{\epsilon}{2} sin(\lambda_\odot - \Omega) - \sin i\, sin^2\frac{\epsilon}{2}\sin(\lambda_\odot + \Omega) - \sin i\, sin^2\frac{\epsilon}{2} sin(\lambda_\odot + \Omega) - \cos i \sin\epsilon\ sin\lambda_\odot\}$$

Notice that the obliquity of the ecliptic $\epsilon = 23.5^\circ$, and $\lambda_\odot$ is the ecliptic longitude of the Sun.

Gauss's form is advantageous for neoconservative force, therefore it is convenient to express the rates of change of the elements in terms of disturbing forces. Many sources present Gauss's variation of parameter such as Brouwer 1961[20]. Vallado [21] followed Bate [22] to obtain a general form of Gaussian variation of parameter using the disturbing force with specific force components resolved in the RSW system and is given as below:

$$\frac{da}{dt} = \frac{2na^3}{\sqrt{1-e^2}} F\left[eS(\upsilon)\sin\upsilon + T(\upsilon)\frac{p}{r}\right]$$

$$\frac{de}{dt} = na^2\sqrt{1-e^2}\, F\left[S(\upsilon)\sin\upsilon + T(\upsilon)\left\{cos\upsilon + \frac{1}{e}\left(1 - \frac{r}{a}\right)\right\}\right]$$

$$\frac{di}{dt} = \frac{na^2}{\sqrt{1-e^2}} FW\frac{r}{a}\cos(\upsilon + \omega)$$

$$\sin i\ \frac{d\Omega}{dt} = \frac{na^2}{\sqrt{1-e^2}} FW\frac{r}{a}\sin(\upsilon + \omega)$$

$$\frac{d\omega}{dt} = -\cos i\ \frac{d\Omega}{dt} + \frac{na^2\sqrt{(1-e^2}}{e} F\left[-S(\upsilon)\cos\upsilon + T(\upsilon)\left(1 + \frac{r}{p}\right)\sin\upsilon\right]$$

$$\frac{dM}{dt} = n - 2na^2 FS(\upsilon)\frac{r}{a} - \sqrt{1-e^2}\left[\frac{d\omega}{dt} + \cos i\ \frac{d\Omega}{dt}\right]$$

Here $= a(1-e^2)$, υ is the true anomaly; $n^2a^3FS(\upsilon), n^2a^3FT(\upsilon), and\ n^2a^3FW$ are three components of the disturbing force due to solar radiation pressure along the satellite's radius vector, perpendicular to it in the orbit plane in the direction of satellite motion, and W normal to the orbit plane respectively.

Vallado 1997 [21] use the true anomaly when the satellite exits the Earth's shadow,υ_{exit} and the true anomaly on entering the shadow, υ_{ent} , to express the rate change in orbital elements as

$$\dot{a} = \frac{1}{\pi n a}\left[r_{eev}S + a\{cos(E_{ent}) - cos(E_{exit})\}T\right]$$

$$\dot{e} = \frac{nS}{2\pi a}\left[3a^2\sqrt{1-e^2}\tau_{ee} - \frac{1}{2e}r_{eev2} + \frac{a}{2e}(1-4e^2)r_{eev}\right] - \frac{nR}{4\pi\mu}\left[r_{ee2} + \frac{a(1-e^2)}{e^2}r_{ee} + \frac{1}{e}\{r_{ent}^2 cos(\upsilon_{ent}) - r_{exit}^2 cos(\upsilon_{ent})\}\right]$$

$$\dot{i} = \frac{nW}{2\pi\mu}\left\{\left[\frac{r_{eev}}{2(1-e^2)} + \frac{1+2e^2}{2(1-e^2)}ar_{eev} - \frac{3a^2e}{\sqrt{1-e^2}}\tau_{ee}\right]cos(\omega) - \frac{r_{ee2}}{2e}sin(\omega)\right\}$$

$$\dot{\Omega} = \frac{nW}{2\pi\mu sin(i)}\left\{cos(\omega) + \left[\frac{r_{eev2}}{2(1-e^2)} + \frac{1+2e^2}{2(1-e^2)}ar_{eev} - \frac{3a^2e}{\sqrt{1-e^2}}\tau_{ee}\right]sin(\omega)\right\}$$

$$\dot{\omega} = \frac{nR}{2\pi\mu e}\left[3a^2\sqrt{1-e^2}\tau_{ee} + \frac{1}{2e}r_{eev2} - \frac{a}{2e}(1+2e^2)r_{eev}\right] + \frac{nS}{4\pi\mu e^3}\left[e\{r_{ent}^2 cos(\upsilon_{ent}) - r_{exit}^2 cos(\upsilon_{exit})\} - a(1-e^2)r_{ee}\right] - \frac{\dot{\Omega}n}{2\pi}cos(i)$$

Where

$$\tau_{ee} = tan^{-1}\left[\frac{\sqrt{1-e^2}tan\left(\frac{\upsilon_{ent}}{2}\right)}{1+e}\right] - tan^{-1}\left[\frac{\sqrt{1-e^2}tan\left(\frac{\upsilon_{exit}}{2}\right)}{1+e}\right]$$

$$r_{ee} = r_{ent} - r_{exit} \quad and \quad r_{ee2} = r_{ent}^2 - r_{exit}^2$$

$$r_{eev} = r_{ent}sin(\upsilon_{ent}) - r_{exit}sin(\upsilon_{exit})$$

$$r_{eev2} = r_{ent}^2 sin(\upsilon_{ent}) - r_{exit}^2 sin(\upsilon_{exit})$$

Determination of true anomaly on entering and exiting the shadow, as well as the geocentric distance and times are very important in the calculation of the variation in orbital elements due to the solar radiation pressure. The method for determining these quantities in terms of satellite's position and the sun geocentric coordinates are given in appendix B.

3. Result and Discussion:

GLONASS constellation is composed of 24 satellites in three orbital planes whose ascending nodes are 120^0 apart. 8 satellites are equally spaced in each plane with argument of latitude displacement 45^0. The satellites in adjoining planes are shifted in argument of latitude by 15 degrees. The satellites operate in circular orbits with semi-major axis of approximately 25510 km at an inclination 64.8, and each satellite completes the orbit in approximately 11 hours 15 minutes [23,24].

In order to illustrate the solar radiation effect on the behavior of orbital element, a computer simulation has been developed using the Matlab R2010a to determine the perturbation effect on orbital elements of four GLONASS satellites (cosmos 2485, cosmos 2494, cosmos 2500, cosmos 2501), its osculating orbital elements is given in table (1).

Figures (1- 4) show the development of orbital elements of GLONASS satellites under the influence of radiation pressure and remarkably different perturbation characteristics at different values of initial elements "argument of perigee ".The perturbations are over the time interval of four days and amplitudes ranging between Δa = (6 – 13), (5 –10) and (5 – 9) in semi major axis for cosmos 2501, 2500 and 2495 respectively.

The changes in the semi major axis cause changes in the mean motion of the satellites, the offset in the mean motion in degrees per day can be given by [15] as

$$\Delta n \left[\frac{0}{day}\right] = -\frac{3n}{2a}\Delta a$$

The oscillations perturbation do not occur in the semi major axis.

Also figures (1 - 4) show the amplitudes eccentricity ranged between (0.002 – 0.004), (0.0001 – 0.0003), (0.002 – 0.0045) and (0.0015– 0.006) for cosmos 2501, 2885, 2500. and 2495 respectively. The change in inclinations is nothing due to radiation perturbation.

Figure (5) Represents the radiation pressure on GLONASS (2501) satellite in three different orbital planes, the change in orbital elements is given in table (2), as seen from table the change in amplitude of orbital elements is much effected by the solar radiation pressure at high value of ascending node, this means the change in orbital elements due to solar radiation perturbation depends on the orbital plane of the satellite.

4. Conclusions

The equation given in section 2 enable us to determine the solar radiation pressure of GLONASS satellites. This equation under consideration of Earth's shadow is applied to GLONASS satellites to determine the corresponding change in orbital elements over four days, eight revolutions the variations in the orbital elements of GLONASS satellites shows the change in the amplitude of semi major axis depends on the position of the satellite in the orbital plane, also we conclude that the change in the amplitudes of orbital elements will be varying due to the change in the direction of the orbital plane.

Table (1) osculating orbital elements for GLONASS satellites

Elements	Cosmos 2501	Cosmos 2500	Cosmos 2494	Cosmos 2485
Lunch	Nov. 30, 2014	June 14, 2014	March 21, 2014	26 April 2013
Perigee (km)	19090	19112.1	19084	19087.8
Apogee (km)	19169	19147.5	19175.5	19172
Inclination (deg)	64.8	64.9	64.9	64.7
Period (min)	675.7	675.7	675.7	675.7
Major axis (km)	25508	25507	25507	25508
Right ascension (deg)	332.6665	91.337	91.276	210.58
Eccentricity	0.001547	0.0006951	0.001794	0.00164
Argument of perigee (deg)	239.9929	228.896	328.36	243.99
Mean anomaly(deg)	295.5758	167.582	244.7972	8.564

Table (2) change in orbital elements of cosmos (2501)

$\Delta \boldsymbol{a}\ (\boldsymbol{km})$	$\Delta e\ \ 10^{-3}$	$\Delta\omega\ (deg)$	$\Delta M\ (deg)$	$\Omega\ (deg)$
8	0.25	6	5	91.27
9	0.35	8	7	228.8
14	0.4	12	25	332.6

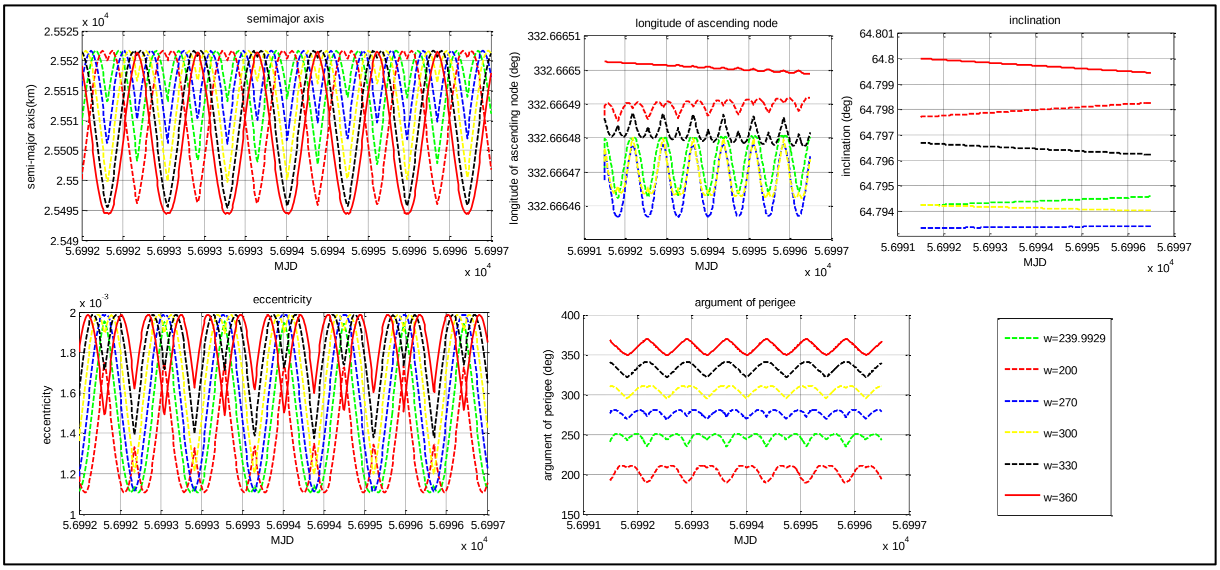

figure (1) solar radiation pressure perturbation on six GLOSNASS satellites (cosmos 2501) in same orbital plane

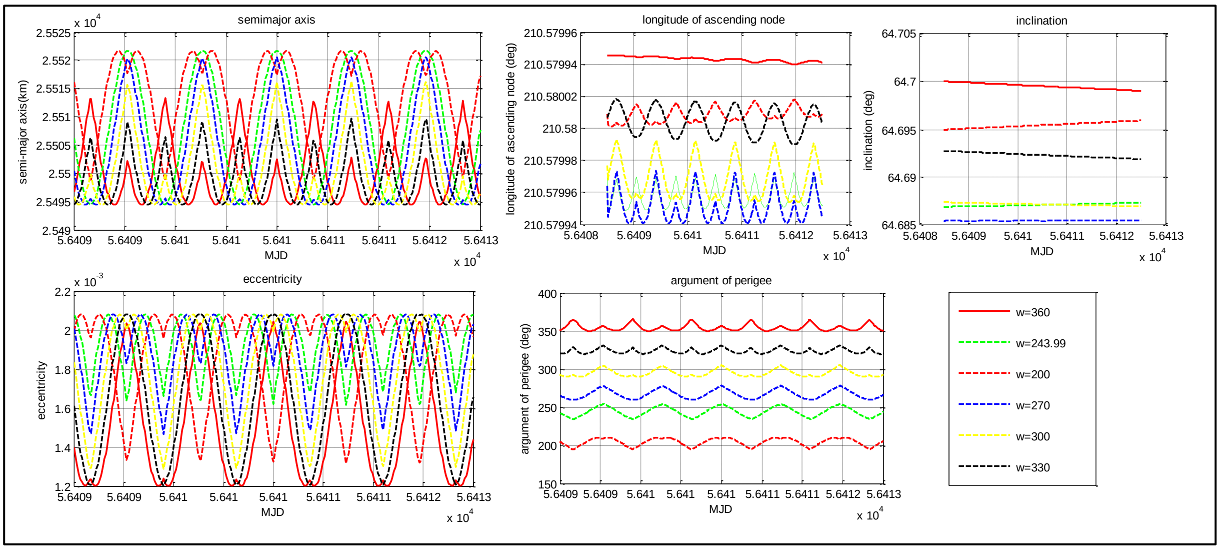

figure (2) solar radiation pressure perturbation on six GLOSNASS satellites (cosmos 2485) in same orbital plane.

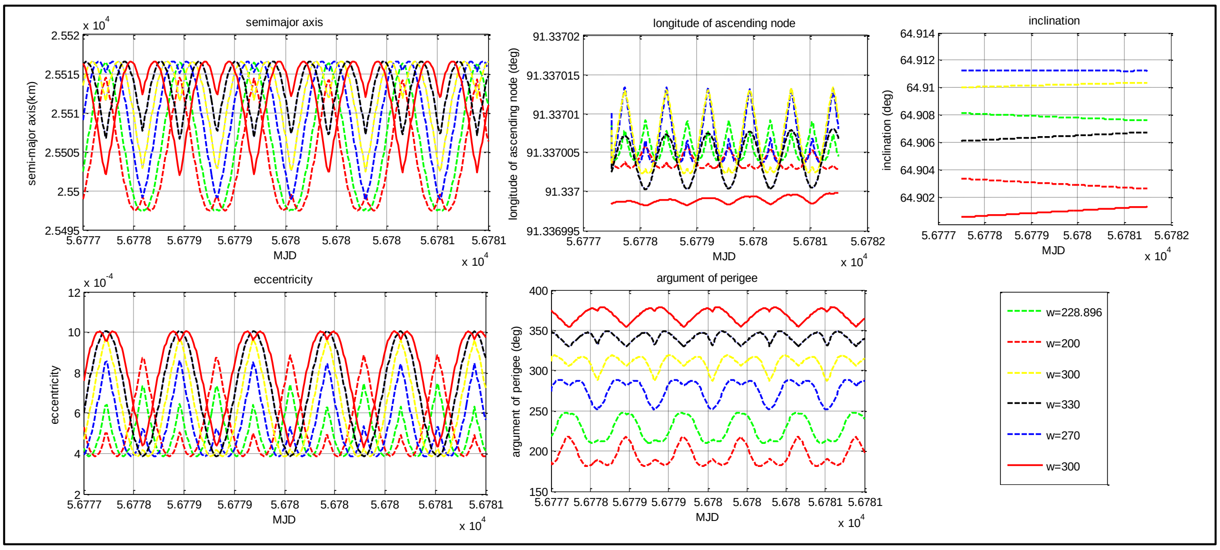

figure (3) solar radiation pressure perturbation on six GLOSNASS satellites (cosmos 2500) in same orbital plane.

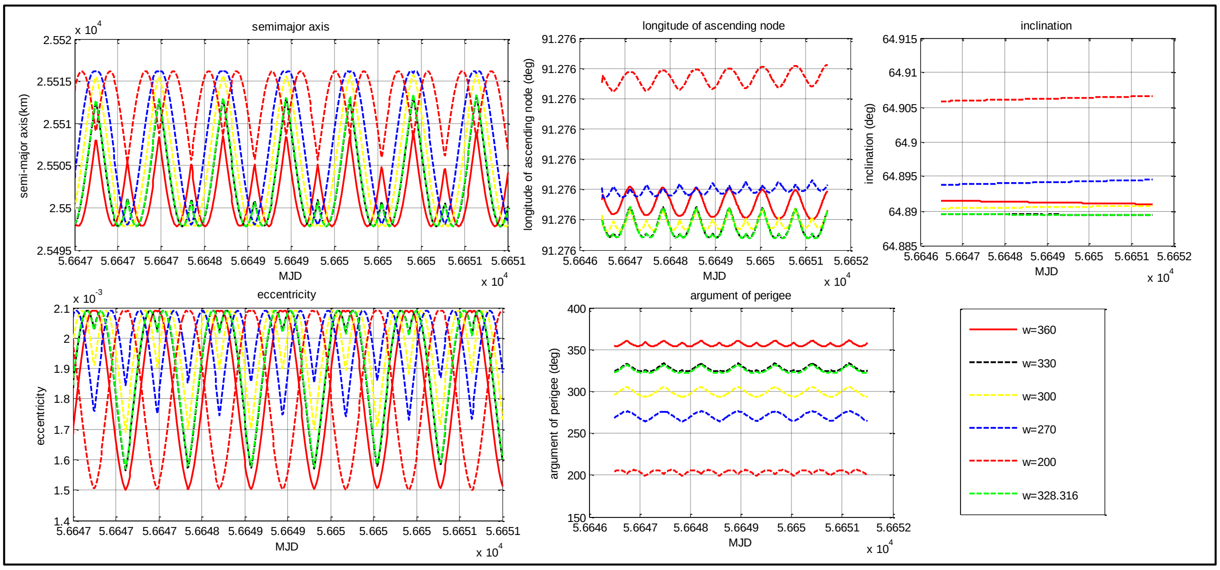

figure (4) solar radiation pressure perturbation on six GLOSNASS satellites (cosmos 2494) in same orbital plane.

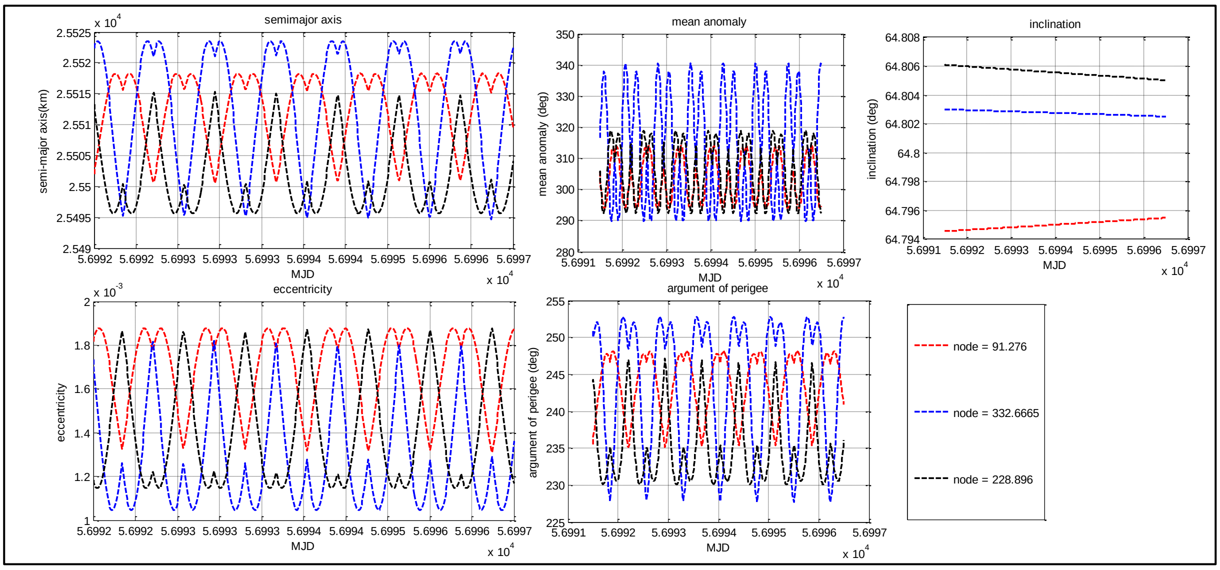

figure (5) solar radiation pressure perturbation on GLOSNASS satellites (cosmos 2501) in three different orbital plane.

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Appendix A: shadow parameter

$$\alpha_1 = \alpha^4 e^4 - 2\alpha^2(\beta_2^2 - \beta_1^2)e^2 + (\beta_1^2 + \beta_2^2)^2$$

$$\alpha_2 = 4\alpha^4 e^3 - 4\alpha^2(\beta_2^2 - \beta_1^2)e$$

$$\alpha_3 = 6\alpha^4 e^2 - 2\alpha^2(\beta_2^2 - \beta_1^2) - 2\alpha^2(1 - \beta_2^2)e^2 + 2(\beta_2^2 - \beta_1^2)(1 - \beta_2^2) - 4\beta_2^2\beta_2^2$$

$$\alpha_4 = 4\alpha^4 e - 4\alpha^2(1 - \beta_2^2)e$$

$$\alpha_5 = \alpha^4 - 2\alpha^2(1 - \beta_2^2) + (1 - \beta_2^2)^2$$

$$\alpha = \frac{R_\oplus}{p}$$

$$\beta_1 = \frac{\vec{r}_\odot \cdot \hat{p}}{r_\odot} \;, \qquad \beta_2 = \frac{\vec{r}_\odot \cdot \hat{Q}}{r_\odot}$$

Appendix B: determination of geocentric position of the sun

Begin to find the number of Julian centuries from:

$T_{UT1} = \frac{JD-2451545.0}{36525}$

Then determine the mean longitude of the sun by

$$\lambda_{M_\odot} = 280.460618° + 36000.77005361 T_{UT1}$$

Find the mean anomaly

$M_\odot = 357.527\,723\,3° + 35{,}999.050\,347 T_{TDB}.$

Ecliptic longitude of the sun

$\lambda_{eclptic} = \lambda_{M_\odot} + 1.914666471° \, sin\left(M_\odot\right) + 0.019\,994\,643 \, sin\left(2 M_\odot\right)$

Find the position magnitude

$r_\odot = 1.000\,140\,612 - 0.016\,708\,617 cos\left(M_\odot\right) - 0.000\,139\,589 cos\left(2M_\odot\right)$

the position vector in geocentric equatorial coordinates

$$\vec{r}_\odot = r_\odot \, cos\left(\lambda_{eclptic}\right)\hat{I} + r_\odot \, cos(\epsilon) sin\left(\lambda_{eclptic}\right)\hat{J} + r_\odot \, sin(\epsilon) sin\left(\lambda_{eclptic}\right)\hat{K}$$

satellite position

begin to find the position vector in perfocal coordinate

$$\vec{r}_{PQW} = \begin{bmatrix} \frac{p\,cos(v)}{1+ecos(v)} \\ \frac{p\,sin(v)}{1+ecos(v)} \\ 0 \end{bmatrix}$$

Rotate these vectors to the geocentric equatorial system using the following rotation matrices:

$$\left[\frac{IJK}{PQW}\right] = \begin{bmatrix} cos(\Omega)cos(\omega) - sin(\Omega)sin(\omega)cos(i) & -cos(\Omega)sin(\omega) - sin(\Omega)cos(\omega)cos(i) & sin(\Omega)sin(i) \\ cos(\Omega)cos(\omega) + sin(\Omega)sin(\omega)cos(i) & -sin(\Omega)sin(\omega) + cos(\Omega)cos(\omega)cos(i) & -cos(\Omega)sin(i) \\ sin(\omega)sin(i) & cos(\omega)sin(i) & cos(i) \end{bmatrix}$$