The Usage of Dispersion Measure in Bayesian Selection Approach

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Abstract

In many instances, researchers and or specialists face certain problems in their researches which make them need the range for the purpose of calculating and analyzing. Therefore in this research, we have shed light on selecting range for a calculation of varieties, category, populations, and degrees...etc. using Bayesian approach.

Key word: the range, ranking and selection approach, prior and posterior dis., N-nomial distribution

الخلاصة

في الكثير من الحالات يواجة الباحثون أو ذوي الاختصاص مشكلات معينة في عملهم تجعلهم بحاجة الى أيجاد المدى لانة يساعدهم في بعض الحسابات والتحاليل. في هذا البحث حاولنا تسليط الضوء على اختيار المدى لمجموعة من (أنواع , أصناف ,مجتمعات و درجات...الخ) مستخدمين الاجراء البيزيني

الكلمات المفتاحية: المدى,اجراء الاختبار والترتيب , التوزيع الاحق والسابق ,توزيع متعدد الحدود

1. Introduction

Al-Hassan.K.F addressed in previous research the selection and ranking problem and found formulas to extract the best, least and median (in the case of odd community and even) using the procedure Bayesian and derived the Bayesian risk. To mention them briefly, Bayesian procedure for multinomial selection problem in (2006). AL- Hassan K.F., constructed the fully optimal Bayesian sequential best selection procedure (BOS) using the dynamic programming technique in conjunction with Bayesian decision-theoretic formulation, she compared (BOS) with Bayesian optimal fixed sample size procedure (BOF) and found the simulation for this work.

In 2007, Saad and Kawther concluded Bayesian fixed sample size for selected least (worst) in multinomial distribution. Later in 2011, Al Hassan complemented her work in 2007 by using functional analysis for the purposes of approximation .

In 2010 Al Hassan, developed the Bayesian selection method to find a new approach: median selection where the sample size is odd and she found median selection when sample size is even. Al Hassan also found the approximate of odd median by functional analysis.

The problem of selecting and ranking procedure is very important due to their wide application to find the best and worst from a set of samples directly. In this paper, we worked on another measure of dispersion which is the range. We used this measure since it is needed in medical, technology, air and climate, such as temperature, humidity, atmospheric pressure, weather forecasting and used in quality control…etc

2. Measure of Dispersion (range)

2.1 Defining the range

The range is defined as the difference between the maximum and minimum value in a variable value which is frequently used by geographers in their descriptions and it is also used in natural phenomena and human analysis, such as: continental measurement and climatic extremes, like measuring the slope degree of the surface. In addition, it is used in explaining the wind speed depending on atmospheric pressure between the two areas and the distance between them. Its also used in the studies related to dams and banks design .So another application to the identification of the range of popular television among programs is to show the unity of spatial variation and the impact of local factors. The range is one of the most important measures of spatial variation which reflects extremes statisticians and spatial distance, and helps explain the results.

2.2 Application of the range

- In computer science, the range may refer to the following:
    1. The likely values that may be kept in a variable: The range of a variable is given as the set of likely values that a variable can have. In the instance of a number, the variable is limited to the entire numbers only, and the range will cover all numbers within its range including the maximum and minimum. For example, the range of a signed 16-bit integer variable is all the integers from −32,768 to +32,767.
    2. The upper and lower bounds of an array: When an array is numerically indexed, Its range is the upper and lower bound of the array.
- In the music field, the range of a musical instrument could be the distance from the lowest to the highest pitch it can produce. For a singing voice, the vocal range is equivalent.
- In the health field, a reference range or a reference interval is the range of values for a physiologic measurement in healthy persons. For example, the amount of creatinine in the blood, or partial oxygen pressure).
- In mathematics and statistics, the range is the difference between the largest and smallest values.

3. Original the problem

In many situations, the range is selected from multi (parameters, values, categories, degrees, temperatures….etc.). The order of the data or parameters is important and considered as a requirement. The data or parameters should be in an ascending or a descending order, then range should be selected from finding the difference between the highest and the lowest degrees (or data) .For examples: - Comparing temperature degree of two readings for the same person, pulse, blood pressure, temperature degree in two cities, and brightness of two pictures.

Now Consider a multinomial distribution which is characterized by *N* events (cells) with probability p_i associated with the i^{th} cell (*i*=1, 2,…,*N*). Let $p_{[1]} \le p_{[2]} \le ... \le p_{[N]}$ denote the ordered values of $p_1,..., p_N$, the "range of cells " event, is defined as that cell which has the probability $p_{[R]}$ associated with it. Nothing is known about the value of $p_i's$, that is we do not know which cell associated with $p_{[R]}$, Such that $p_{[R]} = p_{[N]} - p_{[1]}$.

4. Bayesian decision- theoretic formulation

For the multinomial distribution with N cells, let the unknown probability of an observation in the i^{th} cell be p_i, (*i*= 1, 2, …, *N*). Let $\Omega_N = \{\underline{p} = (p_1, p_2,..., p_N), p_i \ge 0\}$ be the parameter space and $D = \{d_1, d_2,..., d_N\}$ be the decision space where in the following terminal *N*-decision rule:

$d_i : p_i$ is the range of cells probability (i=1, 2, …, N).

That is, d_i denote the decision to select the event associated with the i^{th} cell as the $p_{[R]}$ event, after the sampling is terminated.

Suppose the loss function in making decisions d_i defined on $\Omega_N \times D$ which is given as follows:

$$L(d_i, \underline{p}^*) = \begin{Bmatrix} k^*(p_{[R]} - p_i) & \text{if } (p_{[R]} \neq p_i) \\ 0 & \text{if } (p_{[R]} = p_i) \end{Bmatrix}$$

That is the loss if decision d_i is made when the true value of $\underline{p} = \underline{p}^*$, where k^* is, the loss constant, giving losses in terms of cost.

The Bayesian approach requires that we specify a prior probability density function $\pi(\underline{p})$, expressing our beliefs about $\underline{p}$ before we obtain the data. From a mathematical point of view, it would be convenient if $\underline{p}$ be the assigned a prior distribution which is a member of a family of distributions closed under multinomial sampling or as a member of the conjugate family. The conjugate family in this case is the family of Dirichlet distribution. Accordingly, let $\underline{p}$ is assigned Dirichlet prior distribution with parameters $m', n'_1, n'_2, ..., n'_N$. The marginal distribution for p_i is Beta density given by $f(p_i) = \dfrac{(m'-1)!}{(n'_i-1)!(m'-n'_i-1)!} p_i^{n'_i-1}(1-p_i)^{m'-n'_i-1}$. The normalized density function is and $\pi(\underline{p}) = \dfrac{\Gamma m'}{\prod_{i=1}^{N} \Gamma(n'_i)} \prod_{i=1}^{N} p_i^{n'_i-1}$, where $m' = \sum_{i=1}^{N} n'_i$

Here $\underline{n'} = (n'_1, n'_2, ..., n'_N)$, are regarded as hyper parameters specifying the prior distribution. They can be thought of as "imaginary counts" from prior experience. The equivalent sample size is $(n'_1 + n'_2 + ... + n'_N = m')$. In addition to the prior information, we obtain some sample information from the multinomial population. If there are N categories, p_i is the probability that the i^{th} category is selected in a single trail, and the trails are independent, then the number of times each category is selected has a multinomial distribution. More precisely, let K_i be the number of times that category i is chosen in m independent trials. Then $P_r(K_1 = n_1, K_2 = n_2, ..., K_N = n_N \mid p_1, ..., p_N) = P(\underline{n} \mid \underline{p}) = \dfrac{m!}{n_1! n_2! ... n_N!} \prod_{i=1}^{N} p_i^{n_i}$, where

$$\sum_{i=1}^{N} n_i = m \quad \text{and} \quad \sum_{i=1}^{N} p_i = 1.$$

The posterior distribution is derived from the prior probability function and the multinomial distribution by means of Bayes theorem as follows.

$$\pi(\underline{p} \mid \underline{n}) = \frac{P(\underline{n} \mid \underline{p})\pi(\underline{p})}{\int P(\underline{n} \mid \underline{p})\pi(\underline{p}) d\underline{p}} = \frac{P(\underline{n} \mid \underline{p})\pi(\underline{p})}{P(\underline{n})} .$$

Now,

$P(\underline{n} \mid \underline{p}$ Proportional to $p_1^{n_1} \ldots . \; p_N^{n_N}$ and $\pi(\underline{p})$ Proportional to $p_1^{n_1'-1} \ldots . \; p_N^{n_N'-1}$.

Then the posterior distribution is $\pi(\underline{p} \mid \underline{n})$ Proportional to $p_1^{n_1+n_1'-1} \ldots . \; p_N^{n_N+n_N'-1}$.

This is a member of the Dirichlet family with parameters $n_i'' = n_i' + n_i$ and $m'' = m' + m$ (i=1, …, N).

Hence, the posterior distribution has density function $\pi(\underline{p} \mid \underline{n}) = \dfrac{(m''-1)!}{(n_1''-1)! \ldots .(n_N''-1)!} p_1^{n_1''-1} \ldots . p_N^{n_N''-1}$, with posterior mean $\hat{p}_i = \dfrac{n_i''}{m''}$ (i= 1, 2, …, N), n_i'' will be termed the posterior frequency in the i^{th} cell. The marginal posterior distribution for p_i is the beta distribution with probability density function

$$f(p_i \mid n_i'') = \frac{\Gamma(m'')}{\Gamma(n_i'')\Gamma(m''-n_i'')} p_i^{n_i''-1}(1-p_i)^{m''-n_i''-1}.$$

5. Deriving Stopping Risk

In this section, we derive the stopping risks (Bayes risk) of making decision d_i for linear loss function. The stopping risk (the posterior expected loss) of the terminal decision d_i when the posterior distribution for $\underline{p}$ has parameters $(n_1'', n_2'', n_3'' \ldots, n_N''; m'')$, denoted by $S_i(n_1'', n_2'', n_3'' \ldots, n_N''; m'')$ can be found as follows. Let the ordered values of $n_1'', n_2'', n_3'' \ldots, n_N''$ is $n_{[1]}'' \le n_{[2]}'', \le n_{[3]}'' \le, \ldots, \le n_{[N]}''$.

$$S_i(n_1'', n_2'', n_3'' \ldots, n_N''; m'') = \underset{\pi(\underline{p}|\underline{n})}{E}[L(d_i, \underline{p}^*)] \quad = k \bullet \left[\underset{\pi(\underline{p}|\underline{n})}{E}(\mathrm{þ}_{[R]}) - \frac{n_i''}{m''} \right]$$

Now, we drive the expected value of $\mathrm{þ}_{[R]}$, such that $\mathrm{þ}_{[R]} = \left(\mathrm{p}_{[N]} - \mathrm{p}_{[1]}\right)$ as follows.

$$\underset{\pi(\underline{p}|\underline{n})}{E}(\mathrm{þ}_{[N]}) = \int_0^1 \int_0^1 f\left(\mathrm{p}_{[N]} - \mathrm{p}_{[1]}\right) g(\mathrm{p}_{[1]}, p_{[N]}) dp_{[1]} dp_{[N]} ,$$

Where $g(\mathrm{p}_{[R]})$ be the joint probability density function of the $p_{[N]} - p_{[1]}$,

$$g(\mathrm{p}_{[1]}, p_{[N]}) = \mathrm{N}(N-1)\left[F(p_{[N]}) - F(p_{[1]})\right]^{N-2} f(p_{[1]}) f(p_{[N]})$$

And

$$f\left(\mathrm{p}_{[R]}\right) = \mathrm{p}_{[N]} - \mathrm{p}_{[1]} \quad ,$$

The marginal posterior probability density functions of $p_{[1]}$ & $p_{[N]}$ are respectively

$$f(p_{[1]}) = \frac{(m''-1)!}{(n_{[1]}''-1)!(m''-n_{[1]}''-1)!} p_{[1]}^{n_{[1]}''-1}(1-p_{[1]})^{m''-n_{[1]}''-1} \quad ,$$

$$f(p_{[N]}) = \frac{(m''-1)!}{(n_{[N]}''-1)!(m''-n_{[N]}''-1)!} p_{[N]}^{n_{[N]}''-1}(1-p_{[N]})^{m''-n_{[N]}''-1} .$$

And the cumulative density function of $p_{[1]}$ & $p_{[N]}$ are respectively

$$F(p_{[1]})=\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}$$ &

$$F(p_{[N]})=\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}.p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}$$

Then,

$$g(\mathrm{p}_{[R]})=\mathrm{N}(N-1)\left[\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}.p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}-\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}\right]^{N-2}$$

$$\left\langle\frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!}p_{[N]}^{n''_{[N]}-1}(1-p_{[N]})^{m''-n''_{[N]}-1}\right\rangle\left\langle\frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!}p_{[1]}^{n''_{[1]}-1}(1-p_{[1]})^{m''-n''_{[1]}-1}\right\rangle$$

$$=\mathrm{N}(N-1)\left[\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}.p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}-\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}\right]^{N-2}$$

$$\left\langle\frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!}\frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!}\right\rangle\left\langle p_{[N]}^{n''_{[N]}-1}(1-p_{[N]})^{m''-n''_{[N]}-1}p_{[1]}^{n''_{[1]}-1}(1-p_{[1]})^{m''-n''_{[1]}-1}\right\rangle$$

Now,

$$\underset{\pi(\underline{p}|\underline{n})}{E}(\mathrm{p}_{[R]})=\int_0^1\!\!\int_0^1\mathrm{N}(N-1)\frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!}\left\langle\frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!}\right\rangle p_{[N]}^{n''_{[N]}-1}(1-p_{[N]})^{m''-n''_{[N]}-1}$$

$$p_{[1]}^{n''_{[1]}-1}(1-p_{[1]})^{m''-n''_{[1]}-1}\left(\mathrm{p}_{[N]}-\mathrm{p}_{[1]}\right)\left[\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}-\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.\right.$$

$$\left.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}\right]^{N-2}dp_{[1]}dp_{[N]}$$

So,

$$\left[\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}-\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}\right]^{N-2}$$

$$=\left[\sum_{q=0}^{t}(-1)^q\binom{t}{q}\left[\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}\right]^{t-q}\left[\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}\right]^q\right]$$

$$Let\ \ R_1=\left[\sum_{j_2=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{j_2!(m''-1-j_2)!}\ \ p_{[N]}{}^{j_2}(1-p_{[N]})^{m''-1-j_2}\right]^{t-q}$$

$$and\ \ let\ \ R_2=\left[\sum_{j_1=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{j_1!(m''-1-j_1)!}.p_{[1]}{}^{j_1}(1-p_{[1]})^{m''-1-j_1}\right]^q$$

Then,

$$R_1=\left[\left((1-p_{[N]})^{m''-1}\right)^{t-q}\sum_{b_1=n''_{[N]}}^{m''-1}\ldots\ldots\sum_{b_{t-q}=n''_{[N]}}^{m''-1}\left[\frac{p_{[N]}}{(1-p_{[N]})}\right]^{b_1+b_2+\ldots+b_{t-q}}\frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!}\right]$$

$$R_2=\left[\sum_{h=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{h!(m''-1-h)!}.p_{[1]}h(1-p_{[1]})^{m''-1-h}\right]^q$$

$$=\sum_{h=n''_{[1]}}^{m''-1}\frac{(m''-1)!}{h!(m''-1-h)!}.\left(\left(\frac{p_{[1]}}{1-p_{[1]}}\right)^h\right)^q\left((1-p_{[1]})^{m''-1}\right)^q$$

$$=\left((1-p_{[1]})^{m''-1}\right)^q\sum_{h_1=n''_{[1]}}^{m''-1}\sum_{h_2=n''_{[1]}}^{m''-1}\ldots\ldots\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!..h_q!(m''-1-h_1)!\ldots(m''-1-h_q)!}..\left(\frac{p_{[1]}}{1-p_{[1]}}\right)^{h_1.h_2\ldots.h_q}$$

$$\underset{\pi(\underline{p}|\underline{n})}{E}(\text{þ}_{[R]})=\int_0^1\int_0^1 \mathrm{N}(N-1)\left\langle\frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!}.\frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!}\right\rangle\Big\{p_{[N]}^{n''_{[N]}-1}(1-p_{[N]})^{m''-n''_{[N]}-1}$$

$$p_{[1]}^{n''_{[1]}-1}(1-p_{[1]})^{m''-n''_{[1]}-1}\mathrm{p}_{[N]}\sum_{q=0}^{t}(-1)^q\binom{t}{q}(1-p_{[N]})^{(m''-1)(t-q)}\left[\sum_{b_1=n''_{[N]}}^{m''-1}\ldots\ldots\sum_{b_{t-q}=n''_{[N]}}^{m''-1}\left[\frac{p_{[N]}}{(1-p_{[N]})}\right]^{b_1+b_2+\ldots+b_{t-q}}\right.$$

$$\left.\frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!}\right]\left[(1-p_{[1]})^{m''-1}\right]^q\sum_{h_1=n''_{[1]}}^{m''-1}\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!\ldots h_q!(m''-1-h_1)!\ldots(m''-1-h_q)!}..$$

$$\left.\left(\frac{p_{[1]}}{1-p_{[1]}}\right)^{h_1.h_2\ldots.h_q}\right\}-\Big\{p_{[N]}{}^{n''_{[N]}-1}(1-p_{[N]})^{m''-1-n''_{[N]}}\,p_{[1]}{}^{n''_{[1]}-1}(1-p_{[1]})^{m''-1-n''_{[N]}}\,p_{[1]}\sum_{q=0}^{t}(-1)^q\binom{t}{q}\left[\left[(1-p_{[N]})^{m''-1}\right]^{(t-q)}\right.$$

$$\left[\sum_{b_1=n''_{[N]}}^{m''-1}\ldots\ldots\sum_{b_{t-q}=n''_{[N]}}^{m''-1}\left[\frac{p_{[N]}}{(1-p_{[N]})}\right]^{b_1+b_2+\ldots+b_{t-q}}\frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!}\right]\left[(1-p_{[1]})^{m''-1}\right]^q\sum_{h_1=n''_{[1]}}^{m''-1}\ldots..$$

$$\left.\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!\ldots h_q!(m''-1-h_1)!\ldots(m''-1-h_q)!}\left[\frac{p_{[1]}}{(1-p_{[1]})}\right]^{h_1+h_2+\ldots+h_q}\right\}.dp_{[1]}dp_{[N]}$$

$$= \int_0^1 \int_0^1 N(N-1) \left\langle \frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!} \cdot \frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!} \right\rangle \left\{ \sum_{q=0}^{t} (-1)^q \binom{t}{q} \left[\sum_{b_1=n''_{[N]}}^{m''-1} \ldots\ldots \sum_{b_{t-q}=n''_{[N]}}^{m''-1} \right. \right.$$

$$\frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!} \sum_{h_1=n''_{[1]}}^{m''-1} \ldots \sum_{h=n''_{[q]}}^{m''-1} \frac{(m''-1)!}{h_1!\ldots h_q!(m''-1-h_1)!\ldots(m''-1-h_q)!} \ldots \left[\frac{p_{[N]}}{(1-p_{[N]})} \right]^{b_1+b_2+\ldots+b_{t-q}}$$

$$p_{[N]}^{n''_{[N]}+b_1+b_2+\ldots+b_{t-q}} (1-p_{[N]})^{m''-n''_{[N]}-1+(m''-1)(t-q)-(b_1+b_2+..+b_{t-q})+q(m''-1)} \left[\frac{p_{[1]}}{(1-p_{[1]})} \right]^{h_1+h_2+\ldots+h_{t-q}} p_{[1]}^{n''_{[1]}-1+(h_1+h_2+\ldots+h_q)}$$

$$(1-p_{[1]})^{m''-n''_{[1]}-1-(h_1+h_2+\ldots+h_q)} \Big\} - \left\{ \sum_{q=0}^{t} (-1)^q \binom{t}{q} \left[\sum_{b_1=n''_{[N]}}^{m''-1} \ldots\ldots \sum_{b_{t-q}=n''_{[N]}}^{m''-1} \frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!} \right] \right.$$

$$\sum_{h_1=n''_{[1]}}^{m''-1} \ldots \sum_{h=n''_{[q]}}^{m''-1} \frac{(m''-1)!}{h_1!\ldots h_q!(m''-1-h_1)!\ldots(m''-1-h_q)!} \left[\frac{p_{[N]}}{(1-p_{[N]})} \right]^{b_1+b_2+\ldots+b_{t-q}} . p_{[N]}^{n''_{[N]}+b_1+b_2+\ldots+b_{t-q}}$$

$$\left. (1-p_{[N]})^{m''-n''_{[N]}-1+(m''-1)(t-q)-(b_1+b_2+..+b_{t-q})+(t-q)} \left[\frac{p_{[1]}}{(1-p_{[1]})} \right]^{h_1+h_2+\ldots+h_{t-q}} p_{[1]}^{n''_{[1]}+(h_1+h_2+\ldots+h_q)} (1-p_{[1]})^{m''-n''_{[1]}-1+m''+1-(h_1+h_2+\ldots+h_q)} \right\} dp_{[1]} dp_{[N]}$$

$$= N(N-1) \left\langle \frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!} \cdot \frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!} \right\rangle \left\{ \sum_{q=0}^{t} (-1)^q \binom{t}{q} \left[\sum_{b_1=n''_{[N]}}^{m''-1} \ldots\ldots \sum_{b_{t-q}=n''_{[N]}}^{m''-1} \right. \right.$$

$$\frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!} \sum_{h_1=n''_{[1]}}^{m''-1} \ldots \sum_{h=n''_{[q]}}^{m''-1} \frac{(m''-1)!}{h_1!\ldots h_q!(m''-1-h_1)!\ldots(m''-1-h_q)!} \ldots$$

$$\frac{\Gamma(n''_{[N]}+b_1+\ldots+b_{t-q}+1)\Gamma((m''+n''_{[N]}+(m''-1)(t-q)-(b_1+b_2+..+b_{t-q})+q(m''-1))}{\Gamma((n''_{[N]}+1)m''+n''_{[N]}+(m''-1)(t-q)+q(m''-1))}$$

$$\frac{\Gamma(n''_{[N]}+h_1+\ldots+h_q+1)\Gamma((m''+n''_{[N]}-(h_1+h_2+..+h_q)}{\Gamma(m'')}$$

$$- \left\{ \sum_{q=0}^{t} (-1)^q \binom{t}{q} \left[\sum_{b_1=n''_{[N]}}^{m''-1} \ldots\ldots \sum_{b_{t-q}=n''_{[N]}}^{m''-1} \frac{(m''-1)!}{b_1!\ldots.b_{t-q}(m''-1-b_1)!\ldots.(m''-1-b_{t-q})!} \right] \right.$$

$$\frac{\Gamma(n''_{[N]}+h_1+...+h_q+1)\Gamma((m''+n''_{[N]}-(h_1+h_2+..+h_q)}{\Gamma(m'')}$$

$$-\left\{\sum_{q=0}^{t}(-1)^q\binom{t}{q}\left[\sum_{b_1=n''_{[N]}}^{m''-1}.....\sum_{b_{t-q}=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{b_1!....b_{t-q}(m''-1-b_1)!....(m''-1-b_{t-q})!}\right]\right.$$

$$\sum_{h_1=n''_{[1]}}^{m''-1}....\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!...h_q!(m''-1-h_1)!...(m''-1-h_q)!}.\frac{\Gamma(n''_{[N]}+b_1+...+b_{t-q}+1)}{\Gamma((n''_{[N]}+1)m''+n''_{[N]}+(m''-1)(t-q)+q(m''-1))}$$

$$\frac{\Gamma((m''+n''_{[N]}+(m''-1)(t-q)-(b_1+b_2+..+b_{t-q}))}{\Gamma(m'')}\frac{\Gamma(n''_{[N]}+h_1+...+h_q+1)}{\Gamma(2(m''+1)}$$

$$\frac{\Gamma((m''-n''_{[N]}+(m''+1)-(h_1+...+h_q+1)}{\Gamma(2(m''+1)}$$

Then,

$$S_i(n''_1,n''_2,n''_3...,n''_N;m'')=k^{\bullet}\left[\mathrm{N}(N-1)\left\langle\frac{(m''-1)!}{(n''_{[N]}-1)!(m''-n''_{[N]}-1)!}.\frac{(m''-1)!}{(n''_{[1]}-1)!(m''-n''_{[1]}-1)!}\right\rangle\left\{\sum_{q=0}^{t}(-1)^q\binom{t}{q}\right.\right.$$

$$\left[\sum_{b_1=n''_{[N]}}^{m''-1}......\sum_{b_{t-q}=n''_{[N]}}^{m''-1}\frac{(m''-1)!}{b_1!....b_{t-q}(m''-1-b_1)!....(m''-1-b_{t-q})!}\sum_{h_1=n''_{[1]}}^{m''-1}....\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!...h_q!(m''-1-h_1)!...(m''-1-h_q)!}..\right.$$

$$\frac{\Gamma(n''_{[N]}+b_1+...+b_{t-q}+1)\Gamma((m''+n''_{[N]}+(m''-1)(t-q)-(b_1+b_2+..+b_{t-q})+q(m''-1))}{\Gamma((n''_{[N]}+1)m''+n''_{[N]}+(m''-1)(t-q)+q(m''-1))}$$

$$\sum_{h_1=n''_{[1]}}^{m''-1}...\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!...h_q!(m''-1-h_1)!...(m''-1-h_q)!}.\frac{\Gamma(n''_{[N]}+b_1+...+b_{t-q}+1)}{\Gamma((n''_{[N]}+1)m''+n''_{[N]}+(m''-1)(t-q)+q(m''-1))}$$

$$\frac{\Gamma((m''+n''_{[N]}+(m''-1)(t-q)-(b_1+b_2+..+b_{t-q}))}{\Gamma(m'')}\frac{\Gamma(n''_{[N]}+h_1+...+h_q+1)}{\Gamma(2(m''+1)}$$

$$\frac{\Gamma((m''-n''_{[N]}+(m''+1)-(h_1+...+h_q+1)}{\Gamma(2(m''+1)}\sum_{h_1=n''_{[1]}}^{m''-1}....\sum_{h=n''_{[q]}}^{m''-1}\frac{(m''-1)!}{h_1!...h_q!(m''-1-h_1)!...(m''-1-h_q)!}.$$

$$\left.\frac{\Gamma(n''_{[N]}+b_1+...+b_{t-q}+1)}{\Gamma((n''_{[N]}+1)m''+n''_{[N]}+(m''-1)(t-q)+q(m''-1))}-\frac{n''_i}{m''}\right].$$

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