

A Study for the Energy levels and Potential Energy Surface of $^{184}_{74}\text{W}_{110}$ Isotope

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Abstract

In the present work , The $^{184}_{74}\text{W}_{110}$ isotope in SU(3) – O(6) transition region were investigated . The Hamiltonian equation for this dynamical symmetry is used in the calculations of energy levels for $^{184}_{74}\text{W}_{110}$ isotope according to the interacting boson model version one (IBM-1) . The potential energy surface was also studied . As spin and parity for some energy levels, which were not exactly determined experimentally have been determined. It was assuring that spin and parity for energy levels number (5) . The results are compared with the most recent experimental data . Good agreement was obtained between our theoretical calculations for this under study isotope .

Key words: IBM-1 , nucleons tructure , potential energy surface

دراسة مستويات الطاقة وطاقة جهد السطح للنظير $^{184}_{74}\text{W}_{110}$

الخلاصة

في البحث الحالي ، النظير $^{184}_{74}\text{W}_{110}$ يقع في منطقة الانتقال SU(3) – O(6) . استخدمت معادلة الهاملتونين لهذا التناظر الديناميكي في حساب مستويات الطاقة للنظير $^{184}_{74}\text{W}_{110}$ وفقاً لأنموذج البوزونات المتفاعلة الأول (IBM-1) . تم أيضاً حساب طاقة جهد السطح . كما تم تحديد البرم والتماثل لبعض مستويات الطاقة غير المحددة عملياً بصورة أكيدة ، حيث تم تأكيد برم وتماثل لمستويات طاقة عدد (5) . وبمقارنة النتائج مع القيم العملية المتوفرة حصلنا على توافق جيد مع حساباتنا النظرية للنظير تحت الدراسة .

الكلمات المفتاحية : نموذج البوزونات المتفاعلة ، التركيب النووي ، طاقة الجهد

1- Introduction :

In 1974 a nuclear model was proposed by Arima and Iachello[1,2], and was called Interacting Boson Model (IBM) in an attempt to describe in a unified way collective properties of nuclei (collective motion).

The interacting boson model (IBM) was based on the well-known shell model and on geometrical collective model of the atomic nucleus. Which is suitable for describing the structure of intermediate and heavy nuclei. In addition, it is of a considerable theoretical interest since it shows the dynamical symmetries of the nuclei, which are made visible through using Lie algebra.

The basic idea of the interacting boson model is to assume that, the low-lying collective states in medium and heavy even-even nuclei away from closed shells, are dominated by excitations of the valence protons and the valence neutrons only, while the closed-shell core is inert[3,4,5].

The (IBM) model, assumed that, the collective behavior arises from the coupling, through the nucleon-nucleon interaction of the separate low-lying systems of valence protons and neutrons defined with respect to a major shell closure. It can be able to describe nuclear properties such as spins and energies of the levels, decay probabilities for the emission of gamma quanta, probabilities of electromagnetic transitions and their reduced matrix elements for different transitions, multipole moments, and mixing ratios[3,6].

Furthermore, it was assumed that the particle configurations which were very important in shaping the properties of the low-lying states were these in which identical particles were coupled together forming pairs of angular momentum L = 0 or 2, In addition, these proton (neutron) pairs are treated as bosons. The bosons with angular momentum L=0 are denoted by s_π (s_ν) and are called s-boson, while proton (neutron) bosons with angular momentum L=2 are denoted by d_π(d_ν) and where they are called d-boson[7].

The number of bosons depends on the number of active nuclear particle (or hole) pairs outside a closed shell, while the total boson number (N) is calculated by adding the partial numbers i.e. N=N_π + N_ν, where N_π and N_ν are the number of proton and neutron bosons respectively[3,6].

2 –Theoretical Basics :

2 –1: The Hamiltonian operator of the (IBM-1)

In the IBM-1, the Hamiltonian operator can be written in terms of (one-body and two-body interaction) as [8].

$$\hat{H} = \sum_{i=1}^N \epsilon_i + \sum_{i<j}^N V_{ij} \quad \dots (1)$$

where

ε_i :- The Boson energy .

V_{ij}:- Boson–Boson interacting energy .

N :- Total number of bosons .

Also the Hamiltonian can be written in terms of Creation and Annihilation operators as [5, 8] :-

$$\begin{aligned} \hat{H} = & \epsilon_s (\hat{S}^\dagger, \hat{S}) + \epsilon_d (\hat{d}^\dagger, \hat{d}) + \sum_{L=0,2,4} \frac{1}{2} \\ & \sqrt{2L+1} C_L [[\hat{d}^\dagger \times \hat{d}^\dagger]^L \times [\hat{d} \times \hat{d}]^L]^0 + \\ & \frac{1}{\sqrt{2}} V_2 [[\hat{d}^\dagger \times \hat{d}^\dagger]^2 \times [\hat{d} \times \hat{S}]^2 + [\hat{d}^\dagger \times \hat{S}^\dagger]^2 \times [\hat{d} \times \hat{d}]^2]^0 + \frac{1}{2} V_0 [[\hat{d}^\dagger \times \hat{d}^\dagger]^0 \times \\ & [\hat{S} \times \hat{S}]^0 + [\hat{S}^\dagger \times \hat{S}^\dagger]^0 \times [\hat{d} \times \hat{d}]^0]^0 + \\ & U_2 [[\hat{d}^\dagger \times \hat{S}^\dagger]^2 \times [\hat{d} \times \hat{S}]^2]^0 + \\ & \frac{1}{2} U_0 [[\hat{S}^\dagger \times \hat{S}^\dagger]^0 \times [\hat{S} \times \hat{S}]^0]^0 \quad \dots (2) \end{aligned}$$

Where

ε_L(L = 0, 2), C_L(L = 0, 2, 4), V_L(L = 0, 2), U_L(L = 0, 2) represents the boson energies and interactions .

The creation operators (d[†], S[†]), and the annihilation operator (s, d) can obey the commutation relations [4,8].

$$[\hat{S}, \hat{S}^\dagger] = 1, [\hat{S}, \hat{S}] = 0, [\hat{S}^\dagger, \hat{S}^\dagger] = 0$$

$$[\hat{d}_M, \hat{d}_M] = 0, [\hat{d}_M, \hat{d}_M^\dagger] = \delta_{MM} ,$$

$$[\hat{d}_M^\dagger, \hat{d}_M^\dagger] = 0, [\hat{S}, \hat{d}_M] = 0, [\hat{S}, \hat{d}_M^\dagger] = 0, \\ [\hat{S}^\dagger, \hat{d}_M] = 0, [\hat{S}^\dagger, \hat{d}_M^\dagger] = 0 \quad \dots (3)$$

The general formula of the IBM-1 Hamiltonian, which contains one- body and two-body terms , can be written explicitly in terms of s and d bosons [8] .

$$\hat{H} = \epsilon_s (\hat{S}^\dagger \cdot \hat{S}) + \epsilon_d \sum_m (\hat{d}_m^\dagger \cdot \hat{d}_m) + \hat{V} \quad \dots (4)$$

where :

ϵ_s , ϵ_d are the s- and d- boson energies .
The index $m = 0, \pm 1, \pm 2$.

\hat{V} :- boson-boson interaction , which can be written as :-

$$\hat{V} = \sum_{L=0,2,4} \frac{1}{2} (2L + 1)^{\frac{1}{2}} C_L \left[(\hat{d}^\dagger \times \hat{d}^\dagger)^{(L)} \times (\hat{d} \times \hat{d})^{(L)} \right]_0^{(0)} + \frac{V_2}{\sqrt{2}} \left[(\hat{d}^\dagger \times \hat{d}^\dagger)^{(2)} \times (\hat{d} \times \hat{S})^{(2)} + (\hat{d}^\dagger \times \hat{S}^\dagger)^{(2)} \times (\hat{d} \times \hat{d})^{(2)} \right]_0^{(0)} + \frac{V_0}{2} \left[(\hat{d}^\dagger \times \hat{d}^\dagger)^{(0)} \times (\hat{S} \times \hat{S})^{(0)} + (\hat{S}^\dagger \times \hat{S}^\dagger)^{(0)} \times (\hat{d} \times \hat{d})^{(0)} \right]_0^{(0)} + U_2 \left[(\hat{d}^\dagger \times \hat{S}^\dagger)^{(2)} \times (\hat{d} \times \hat{S})^{(2)} \right]_0^{(0)} + \frac{1}{2} U_0 \left[(\hat{S}^\dagger \times \hat{S}^\dagger)^{(0)} \times (\hat{S} \times \hat{S})^{(0)} \right]_0^{(0)} \quad \dots (5)$$

Where the parameters C_L ($L = 0, 2, 4$) , V_L ($L = 0, 2$) , U_L ($L = 0, 2$) describe the boson interaction and the parenthesis denotes angular momentum couplings .

The most commonly used form of the IBM-1 Hamiltonian is [4, 8] .

$$\hat{H} = \epsilon \hat{n}_d + a_0(\hat{p} \cdot \hat{p}) + a_1(\hat{L} \cdot \hat{L}) + a_2(\hat{Q} \cdot \hat{Q}) + a_3(\hat{T}_3 \cdot \hat{T}_3) + a_4(\hat{T}_4 \cdot \hat{T}_4) \quad \dots (6)$$

Where $\epsilon = \epsilon_d - \epsilon_s$ is the boson energy .

The operators :

$$\left. \begin{aligned} \hat{n}_d &= (\hat{d}^\dagger \cdot \hat{d}) && \text{the boson number operator} \\ \hat{P} &= \frac{1}{2} (\hat{d} \cdot \hat{d}) - \frac{1}{2} (\hat{S} \cdot \hat{S}) && \text{the pairing bosons operator} \\ \hat{L} &= \sqrt{10} [\hat{d}^\dagger \times \hat{d}]^{(1)} && \text{the angular momentum operator} \\ \hat{Q} &= \left[(\hat{d}^\dagger \times \hat{S}) + (\hat{S}^\dagger \times \hat{d}) \right]^{(2)} - \frac{1}{2} \sqrt{7} [\hat{d}^\dagger \times \hat{d}]^{(2)} && \text{the quadrupole operator} \\ \hat{T}_3 &= [\hat{d}^\dagger \times \hat{d}]^{(3)} && \text{the octupole operator} \\ \hat{T}_4 &= [\hat{d}^\dagger \times \hat{d}]^{(4)} && \text{the hexadecapole operator} \end{aligned} \right\} \dots (7)$$

And a_0, a_1, a_2, a_3, a_4 are the phenomenological parameters .

$CHI = -\sqrt{\frac{7}{2}}$ for rotational dynamical symmetry and $CHI = \text{Zero}$ for vibrational and γ - soft dynamical symmetry .

2-2: Potential Energy Surface (P.E.S.)

The general formula for the potential energy surface as a function of geometrical variables β and γ is given by[3, 8] :-

$$V(N, \beta, \gamma) = \frac{N(\epsilon_s + \epsilon_d \beta^2)}{1 + \beta^2} + \frac{N(N-1)}{(1 + \beta^2)^2} (\alpha_1 \beta^4 + \alpha_2 \beta^3 \cos 3\gamma + \alpha_3 \beta^2 + \alpha_4) \quad \dots (8)$$

Where

$N =$ is the total boson number .

$\hat{\beta}^2 =$ is the quadrupole deformation parameter operator from $0 \rightarrow 2.4$.

$\hat{\gamma} =$ is the distortion parameter operator or (asymmetry angle) for $0^\circ \rightarrow 60^\circ$.

The variables ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$) are related to the parameters C_L, V_L and U_L which is given in equation (3) .

The relationships between the variables (α 's) and these parameters have been expressed by Van Isacker and Chen (1981) as[9] :-

$$\alpha_1 = \frac{C_0}{10} + \frac{C_2}{7} + \frac{9}{35} * C_4 \quad \dots (9)$$

$$\alpha_2 = -SQRT\left(\frac{8}{35}\right) * V_2 \quad \dots (10)$$

$$\alpha_3 = (V_0 + U_2)/SQRT(5) \quad \dots (11)$$

$$\alpha_4 = U_0 \quad \dots (12)$$

One must take into account that the asymmetry angle which occurs only in the term $\cos 3\gamma$.

Thus, the energy surfaces has minima only at $\gamma = 0^\circ$ and 60° . The energy expressions in their limits, can display the essential dependence on β and γ , which are been given as [3] :-

$$E(N, \beta, \gamma) = \varepsilon_d N \frac{\beta^2}{1+\beta^2}$$

in SU(5) limit $\dots (13)$

$$E(N, \beta, \gamma) = KN(N-1) \frac{1+\frac{3}{4}\beta^4 - \sqrt{2}\beta^2 \cos 3\gamma}{(1+\beta^2)}$$

in SU(3) limit $\dots (14)$

$$E(N, \beta, \gamma) = K'N(N-1) \left[\frac{1-\beta^2}{1+\beta^2} \right]$$

in O(6) limit $\dots (15)$

3- Results and Discussion :

3-1: Energy levels and energy ratios

To introduce a comprehensive description for a nuclear structure of the $^{184}_{74}W_{110}$ isotope, we have to emphasize first of all on finding the dynamical symmetry of this isotope, by comparing the energy ratios with their identical and experimental values as shown in table (1).

Table (1) : The energy ratios of corresponding limits[3,4,8].

Limit	R4 = $\frac{E(4_1^+)}{E(2_1^+)}$	R6 = $\frac{E(6_1^+)}{E(2_1^+)}$	R8 = $\frac{E(8_1^+)}{E(2_1^+)}$
SU(5)	2	3	4
SU(3)	3.33	7	12
O(6)	2.5	4.5	7

For each dynamical symmetry there is an equivalent equation of the Hamiltonian operator function, which is used to calculate the energy levels, their ratios and the (g, β, γ) –energy bands.

Show that, the $^{184}_{74}W_{110}$ isotope is belonging to the dynamical symmetry SU(3) – O(6) and the equation of the Hamiltonian operator function as shown in table (2).

Table (2) : Hamiltonian representation for the dynamical symmetry corresponding to the $^{184}_{74}W_{110}$ isotope used in the present work .

Isotope	Dynamical Symmetry	Hamiltonian
$^{184}_{74}W_{110}$	SU(3) – O(6)	$\hat{H} = a_0 \hat{P}^\dagger \cdot \hat{P} + a_1 \hat{L}^2 + a_2 \hat{Q}^2 + a_3 \hat{T}_3^2$

Table (3) shows the corresponding parameters obtained with the best fitting from the Hamiltonian operator with the convenient dynamical symmetry .

Table (3) : The parameters values of Hamiltonian operator for the $^{184}_{74}W_{110}$ isotope by using (IBSS1. For) program .

Isotope	$^{184}_{74}W_{110}$
N_π	4
N_ν	8
N	12
EPS (Mev)	0.0000
$\hat{P}^\dagger \cdot \hat{P}$ (Mev)	0.0127
$\hat{L} \cdot \hat{L}$ (Mev)	0.0100
$\hat{Q} \cdot \hat{Q}$ (Mev)	-0.0144
$\hat{T}_3 \cdot \hat{T}_3$ (Mev)	0.0280
$\hat{T}_4 \cdot \hat{T}_4$ (Mev)	0.0000
CHI (Mev)	-0.9000

Table (4) shows the energy levels and their transitions were obtained from the program (IBSS1. For) for the $^{184}_{74}W_{110}$ isotope . The results have been used for calculating the energy ratios and compared them with the identical and available experimental values as shown in table (5).

Table (4) : Theoretical energy levels and energy transitions compared with the experimental data for the $^{184}_{74}\text{W}_{110}$ isotope by using (IBSS1. For) program .

Isotope	I_i^+	Energy level (MeV)		Spin sequences $I_i^+ - I_f^+$	Transition Energy (MeV)	
		Exp.[10,11]	IBM-1		Exp.[10,11]	IBM-1
$^{184}_{74}\text{W}_{110}$	$0_1^+(g)$	0.0000	0.0000	–	–	–
	$2_1^+(g)$	0.1112	0.1090	$2_1^+ - 0_1^+$	0.1112	0.1090
	$4_1^+(g)$	0.3641	0.3606	$4_1^+ - 2_1^+$	0.2528	0.2516
	$6_1^+(g)$	0.7483	0.7500	$6_1^+ - 4_1^+$	0.3842	0.3894
	$2_2^+(\gamma_1)$	0.9033	0.9602	$2_2^+ - 4_1^+$	0.5392	0.5996
				$2_2^+ - 2_1^+$	0.7921	0.8512
				$2_2^+ - 0_1^+$	0.9033	0.9602
	$0_2^+(\beta_1)$	1.0025	1.0005	$0_2^+ - 2_1^+$	–	0.8915
	$3_1^+(\gamma_1)$	1.0060	1.0859	$3_1^+ - 4_1^+$	0.6419	0.7253
				$3_1^+ - 2_1^+$	0.8948	0.9769
	$2_3^+(\beta_1)$	1.1214	1.1160	$2_3^+ - 4_1^+$	0.7574	0.7554
				$2_3^+ - 2_1^+$	1.0102	1.0070
				$2_3^+ - 0_1^+$	1.1214	1.1160
	$4_2^+(\gamma_1)$	1.1338	1.2382	$4_2^+ - 3_1^+$	0.1277	0.1523
				$4_2^+ - 2_2^+$	0.2305	0.2780
				$4_2^+ - 4_1^+$	0.7698	0.8776
				$4_2^+ - 2_1^+$	1.0226	1.1292
	$8_1^+(g)$	1.2520	1.2721	–	–	–
	$5_1^+(\gamma_1)$	1.2941	1.4434	$5_1^+ - 4_1^+$	0.9309	1.0828
	$0_3^+(\beta_2)$	(1.3221)	1.5910	$0_3^+ - 2_2^+$	–	0.6308
$4_3^+(\beta_1)$	(1.3590)	1.3777	–	–	–	
$2_4^+(\gamma_2)$	1.3863	1.7458	$2_4^+ - 3_1^+$	0.3803	0.6599	
			$2_4^+ - 2_2^+$	0.4830	0.7856	
			$2_4^+ - 2_1^+$	1.2751	1.6368	
			$2_4^+ - 0_1^+$	1.3867	1.7458	
$2_5^+(\beta_2)$	1.4310	1.8147	$2_5^+ - 2_1^+$	–	1.7057	
			$2_5^+ - 0_1^+$	–	1.8147	
$6_2^+(\gamma_1)$	(1.4790)	1.6498	–	–	–	
$3_2^+(\gamma_2)$	(1.5233)	1.9555	$3_2^+ - 2_1^+$	–	1.8465	
$0_4^+(\beta_3)$	1.6149	1.8792	$0_4^+ - 2_2^+$	–	0.9190	
			$0_4^+ - 2_1^+$	–	1.7702	
$4_4^+(\gamma_2)$	(1.6960)	1.8760	–	–	–	
$10_1^+(g)$	1.8610	1.9219	–	–	–	

Table (5): Theoretical energy ratios values $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$, $E(8_1^+)/E(2_1^+)$ and their comparison with the experimental data according to the (IBM-1) for the $^{184}_{74}W_{110}$ isotope .

Isotope	$^{184}_{74}W_{110}$	
$E(4_1^+)/E(2_1^+)$	Exp.[10]	3.2737
	IBM-1	3.3076
$E(6_1^+)/E(2_1^+)$	Exp.[10]	6.7290
	IBM-1	6.8793
$E(8_1^+)/E(2_1^+)$	Exp.[10]	11.2583
	IBM-1	11.6673

3 – 2: (g, β, γ) – Energy bands spectrum

The researcher classifies the energy levels of the $^{184}_{74}W_{110}$ isotope, according to the energy bands (g , β , γ – bands) .

Table (6) shows the energy band values for the $^{184}_{74}W_{110}$ isotope and their comparison with the available experimental values , the results shows that, there are good agreement of the levels

of the present work comparison with the experimental results , as assuring the energy levels of the spins (6_2^+ , 4_3^+ , 0_3^+ , 3_2^+ , 4_4^+) from (1.479, 1.359, 1.3221, 1.52326, 1.696) in experimental results to (1.6498, 1.3777, 1.5910 , 1.9555 , 1.8760) in the present work respectively, as well as the isotope belongs to the dynamical symmetry $SU(3) – O(6)$, that the (β – band) is widely appearance than the (γ – band) .

Figure (1) shows the energy levels for the $^{184}_{74}W_{110}$ isotope, where that obey the typical energy bands spectrum . Noticed that , there are good agreement of the level sequences of each band with the typical sequence of ground band (0^+ , 2^+ , 4^+ ) , β – band (0^+ , 2^+ , 4^+ ) and γ – band (2^+ , 3^+ , 4^+ , 5^+ ) . There are good agreement between the present results and the experimental values of the energy bands .

Table (6): The comparison between the theoretical (pw) and experimental [10] energy bands (g, β, γ-bands) for the $^{184}_{74}W_{110}$ isotope by using IBM-1 .

Isotopes	Spin	0^+	2^+	4^+	6^+	8^+	10^+
	Band	2^+	3^+	4^+	5^+	6^+	7^+
$^{184}_{74}W_{110}$ SU(3) – O(6)	g- exp.	0.0000	0.1112	0.3641	0.7483	1.2520	1.8610
	g-pw	0.0000	0.1090	0.3606	0.7500	1.2721	1.9219
	γ_1 -exp.	0.9033	1.0060	1.1338	1.2941	(1.4790)	–
	γ_1 -pw	0.9602	1.0859	1.2382	1.4434	1.6498	–
	β_1 -exp.	1.0025	1.1214	(1.3590)	–	–	–
	β_1 -pw	1.0005	1.1160	1.3777	–	–	–
	β_2 -exp.	(1.3221)	1.4310	–	–	–	–
	β_2 -pw	1.5910	1.8147	–	–	–	–
	γ_2 -exp.	1.3863	(1.5233)	(1.6960)	–	–	–
	γ_2 -pw	1.7458	1.9555	1.8760	–	–	–
	β_3 -exp.	1.6149	–	–	–	–	–
	β_3 -pw	1.8792	–	–	–	–	–

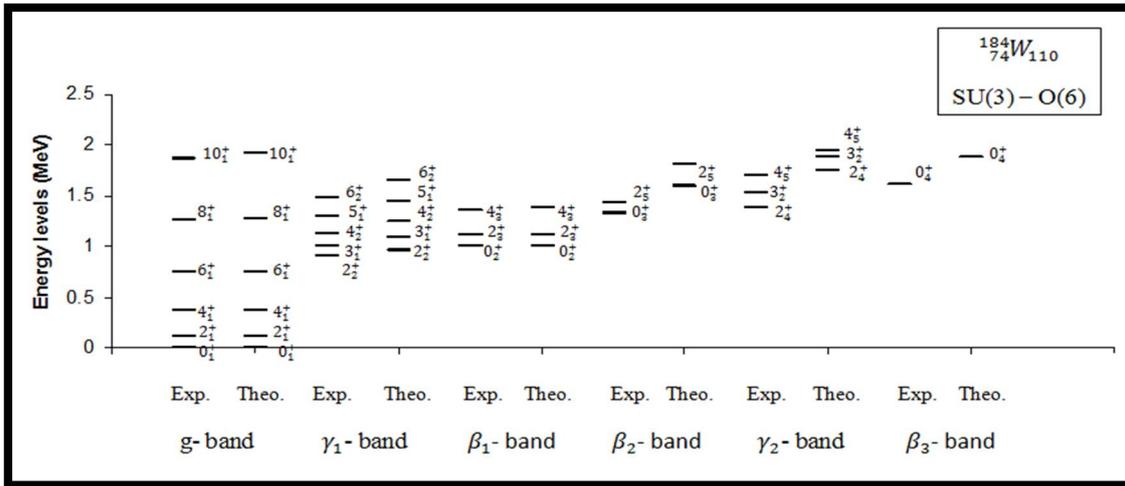


Figure (1) : Comparison between calculated IBM (pw); and experimental[10] energy bands states (g, β , γ - bands) in $^{184}_{74}W_{110}$ isotope of the dynamical symmetry $SU(3) - O(6)$.

3 – 3 : The potential energy surface (P.E.S)

In the present work , the researcher has applied the geometrical model of the IBM-1 for the $^{184}_{74}W_{110}$ isotope, since the IBM-1 geometrical model of nuclear collective motion is provided as with an alternative description of nuclear collective excitations , which are more sensitive than the phenomenological model .

The IBM-1 analysis of the counter plots of the potential energy function $V(N, \beta, \gamma)$ is calculated by using the parameters (α 's) that was deduced from (IBSS1. For) program , as shown in table (7) .

Figure (2a) elucidates the potential energy surface as a function of deformed parameters (β, γ), the contour lines are in good agreement with the typical plots .

The axially symmetric ($\gamma = 0^\circ, \gamma = 30^\circ, \gamma = 60^\circ$) plots of the potential function is calculated in the present work , for the $^{184}_{74}W_{110}$ isotope show that :

The axially symmetric for the isotope $^{184}_{74}W_{110}$ of the dynamical symmetry $SU(3) - O(6)$, Figure (2b) shows the

behaviors of the potential energy surfaces of (3.109 MeV) on the prolate shape at $\beta=1.2, \gamma=0^\circ$ and for oblate shape at $\beta=0.8, \gamma=60^\circ$ which is (1.302 MeV) . Also it shows good agreement with the typical axially symmetric of $SU(3) - O(6)$ limits .

Table (7) : The parameters values of the potential energy surface (P.E.S) for the $^{184}_{74}W_{110}$ isotope .

Isotope	$^{184}_{74}W_{110}$
N_π	4
N_ν	8
N	12
ϵ_s	-0.0720
ϵ_d	0.0730
α_1	0.0000
α_2	-0.0280
α_3	-0.0640
α_4	0.0000

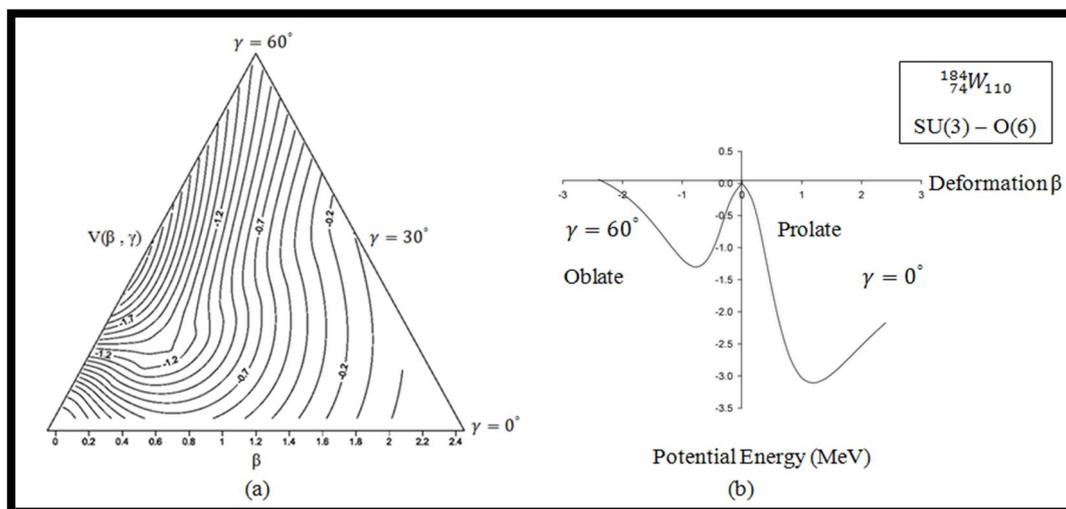


Figure (2 a, b) : The Contour plots and the axial symmetric for the $^{184}_{74}\text{W}_{110}$ isotope of the $\text{SU}(3) - \text{O}(6)$ limit .

3 – 4: Conclusions

The following results are the outcome of this research :

1– The calculating values for energy levels , their ratios , transition energies using IBM-1 for the $^{184}_{74}\text{W}_{110}$ isotope is in a good agreement with the experimental values .

2 – Determining spin and parity for some energy levels , which were not exactly determined experimentally .

3– Determined the nuclei shape by the potential energy surface for the $^{184}_{74}\text{W}_{110}$ isotope .

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