

Comparative Study on the Required Tension Reinforcement for Singly Reinforced Concrete Rectangular Beams According to Different Codes

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Abstract

This paper aims to compare the required, maximum and minimum tension reinforcement of singly reinforced concrete (RC) rectangular beams according to three common Codes. These adopted Codes are the ACI 318M-11, BS 8110-1:1997 and IRAQI Code 1/1987.

The results of analysis and design of (36) various beams cases indicate that the BS Code and the IRAQI Code have given an increase in the required area of tension reinforcement to about (15%) and (30%) respectively with respect to the ACI Code.

It is clearly shown that adopted ACI Code gives the minimum required area of tension reinforcement in the same beam, which leads to good economy and satisfy the ductility provisions in order to prevent sudden and brittle collapse of members subject to flexural.

Finally it is recommended to adopt the provisions of ACI Code to obtain the required area of tension reinforcement in members subjected to flexure in the next edition of IRAQI Code.

الخلاصة

يهدف هذا البحث مقارنة تسليح الشد المطلوب، الحد الأقصى والحد الأدنى في العتبات الخرسانية المسلحة ذو التسليح المنفرد والمقطع المستطيل وفق ثلاثة مدونات مشهورة ومعتمدة عالمياً. المدونات هي المواصفة الأمريكية ACI 318M-11، المواصفة القياسية البريطانية BS 8110-1:1997 والمواصفة العراقية 1/1987.

أظهرت نتائج تحليل وتصميم (٣٦) حالة مختلفة للعتبات أن المواصفة البريطانية والمواصفة العراقية تعطي زيادة في كمية تسليح الشد المطلوب تصل إلى (١٥%) و (٣٠%) مقارنة مع المواصفة الأمريكية على الترتيب.

لقد لوحظ ان اعتماد المواصفة الأمريكية يعطي الكمية المثلى لتسليح الشد المطلوب للعتبة ذاتها، وهذا يؤدي إلى اقتصاد جيد بالتكاليف وتلبية متطلبات المستمطلية لمنع حدوث انهيار مفاجئ وهش في الاعضاء المعرضة للانثناء.

أخيراً يوصى باعتماد فرضيات المواصفة الأمريكية للحصول على كمية تسليح الشد في الاعضاء المعرضة للانثناء في الاصدار القادم للمواصفة العراقية.

1. Introduction

1.1. Previous Studies

The structure should satisfy four major criteria which are the appropriateness, economy, structural adequacy and maintainability [Wight *et al.*, 2012].

Economy is one major criteria of the structure, and *efficient structural designs can reduce the total quantity of concrete and reinforcing steel required for different building components* [Wight *et al.*, 2012]. Steel is much more expensive than concrete [Arya, 2009], and reduce the amount of reinforcement will lead to save the cost and good economy.

Another major criteria of the structure is adequacy which involves the analysis and design. Design deals with amount of required tension reinforcement to satisfy the basic requirements.

The amount of tension reinforcement deals with type of the beam failure. The failure of an under-reinforced beam is termed as tension failure or ductile failure, while the failure of over-reinforced beam is termed as compression failure or brittle failure. The ductile failure is gradual, giving ample prior warning of the impending collapse by way of increased curvatures, deflections and cracking, while brittle failure occurs explosively without warning [(Nawy, 2009) and (Menon *et al.*, 2009)].

Different researches emphasize that the ductility of singly (RC) beams decreases when the percentage of reinforcement increases [(Hasan *et al.*, 2011), (Kheyroddin *et al.*, 2007) and (Siddique *et al.*, 2000)].

Another reason for using smaller percentages of steel is given in ACI where a plastic redistribution of moments is permitted in continuous members whose net tensile strain of steel reinforcement are (0.0075) or greater [McCormac *et al.*, 2006].

Previous comparative study on strength design requirements of ACI-318-02 Code, BS8110 and EuroCode2 was found that the ACI Code gives higher moment capacity for lower steel ratios. The results was based on the analysis of rectangular beam with ($f'_c = 30 \text{ MPa}$) and ($f_y = 420 \text{ MPa}$) [Jawad, 2006].

1.2. Objective of the Research

This paper is devoted to compare the required, maximum and minimum tension reinforcement of singly (RC)rectangular beams according to different Codes, and then adopt the Code which gives the minimum required area of tension reinforcement to save the cost. Three common Codes are selected in this study. These Codes are the ACI 318M-11, BS 8110-1:1997 and IRAQI Code 1/1987. The ACI Code adopts the compressive strength of concrete (f'_c) of the standard cylinder while the BS Code and IRAQI Code adopt the characteristic compressive strength of concrete (f_{cu}) of the standard cube.

The ACI Code adopts the elastic analysis and strength design method, while the BS Code and IRAQI Code adopt the elastic analysis and the ultimate limit state design method [(ACI Committee 318, 2011), (British Standard Committees, 1997) and (IRAQI Code Committee, 1987)].

2. Theoretical Background

2.1. ACI CODE (ACI 318M, 2011)

The strength design method is adopted in this Code. Two factors of safety, one is called the load factors and equal to (1.2 and 1.6) for unfactored dead and unfactored live load respectively and the other is called the strength reduction factor (ϕ) which is equal to (0.9).

Referring to **Fig. (1)** and applying the conditions of equilibrium and compatibility of strains with maximum concrete compressive strain at crushing of the concrete equal to (0.003) and other hypotheticals sanctioned by this Code,

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad (1)$$

$$M_u = 0.9 A_s f_y \left(d - \frac{a}{2} \right) \quad (2.a)$$

Substituting Eq. (1) in Eq. (2.a) gives:

$$M_u = 0.9 A_s f_y \left(d - \frac{A_s f_y}{2 \times 0.85 f'_c b} \right) \quad (2.b)$$

To ensure under-reinforced behavior, ACI Code encourages the use of lower reinforcement ratios. The Code defines a tension – controlled member as one with a net tensile strain greater than or equal to (0.005) [(Nawy, 2009), (Nilson *et al.*, 2010) and (Subramanian, 2010)]. Such a tension-controlled section will give ample warning of failure with excessive deflection and cracking) [Subramanian, 2010]. Based on that limitation, the maximum reinforcement ratio is given by:

$$\rho_{max} (\epsilon_s=0.005) = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_c}{\epsilon_c + \epsilon_s} \quad (3.a)$$

where (β_1) is equal to (0.85) for (f'_c) up to and including (28 MPa) and (0.05) less for each (7 MPa) of strength in excess of (28 MPa), but (β_1) shall not be taken less than (0.65).

Substituting ($\epsilon_c = 0.003$) and ($\epsilon_s = 0.005$), Eq. (3.a) can be expressed as:

$$\rho_{max} (\epsilon_s = 0.005) = 0.31875 \beta_1 \frac{f'_c}{f_y} \quad (3.b)$$

The minimum reinforcement ratio of a flexural member where tensile reinforcement is required by analysis should not be less than that given by:

$$\rho_{min} = \frac{0.25 \sqrt{f'_c}}{f_y} \geq \frac{1.4}{f_y} \quad (4)$$

2.2. Bs code (BS 8110-1, 1997)

The ultimate limit state design method is adopted in this Code for bending through the using of two partial safety factors, one is related with loads and equal to (1.4 and 1.6) for characteristic dead and characteristic live loads respectively and other for materials (γ_m). The partial safety factors for reinforcement and concrete are (1.05) and (1.5) respectively.

Referring to Fig. (1) and applying the conditions of equilibrium and compatibility of strains with maximum strain in concrete at failure equal to (0.0035) and other hypotheticals sanctioned by this Code,

$$x = \frac{0.95 A_s f_y}{0.4 f_{cu} b} \quad (5)$$

$$M_u = 0.95 A_s f_y \left(d - \frac{0.9x}{2} \right) \quad (6.a)$$

Substituting Eq. (5) in Eq. (6.a) gives [Bayagoob et al, 2013]:

$$M_u = 0.95 A_s f_y \left(d - \frac{0.9 \times 0.95 A_s f_y}{0.8 f_{cu} b} \right) \quad (6.b)$$

To ensure under-reinforced behavior, BS Code encourages the use of lower reinforcement ratios also. If the section is under-reinforced, the steel yields and failure will again occur due to crushing of the concrete. However, the beam will show considerable deflection which will be accompanied by severe cracking and spalling from the tension face thus providing ample warning signs of failure [(Arya, 2009) and (Bhatt et al., 2006)]. In order to ensure that the section is under-reinforced, BS 8110 limits the ultimate moment of resistance by [Arya, 2009]:

$$M_u = 0.156 f_{cu} b d^2 \quad (7)$$

Based on the above limitation, the maximum reinforcement ratio can be calculated.

The minimum areas of reinforcement in a beam section to control cracking as well as to resist tension or compression due to bending in different types of beam section are given in BS 8110: Part 1 [Bhatt et al., 2006].

For rectangular beams with overall dimensions b and h , the area of tension reinforcement, A_s , should lie within the following limits [Arya, 2009], the minimum tension reinforcement ratio for mild steel and high yield steel are:

$$A_{s,min} = 0.0024 b h, \text{ when } f_y = 250 \text{ MPa} \quad (8.a)$$

$$A_{s,min} = 0.0013 b h, \text{ when } f_y = 460 \text{ MPa} \quad (8.b)$$

2.3. Iraqi Code (Iraqi Code 1, 1987)

The ultimate limit state design method is adopted in this Code for bending through the using of two partial safety factors, one is related with loads and equal to (1.4 and

1.7) for unfactored dead and unfactored live loads respectively and other for materials (γ_m). The partial safety factors for reinforcement and concrete are (1.15) and (1.5) respectively.

Referring to Fig. (1) and applying the conditions of equilibrium and compatibility of strains with maximum strain in concrete at failure equal to (0.0035) and other hypotheticals sanctioned by this Code,

$$x = \frac{0.87 A_s f_y}{0.4 f_{cu} b} \quad (9)$$

$$M_u = 0.87 A_s f_y \left(d - \frac{0.9x}{2} \right) \quad (10.a)$$

Substituting Eq. (9) in Eq. (10.a) gives:

$$M_u = 0.87 A_s f_y \left(d - \frac{0.9 \times 0.87 A_s f_y}{0.8 f_{cu} b} \right) \quad (10.b)$$

The IRAQI Code is similar to BS Code, which limits the ultimate moment of resistance by above Eq. (7). Also, based on that limitation, the maximum reinforcement ratio can be calculated.

The minimum reinforcement ratio of flexural members is given by:

$$\rho_{min} = \frac{1.4}{f_y} \quad (11)$$

3. Theretical Approach and Discussion

3.1. Comarision Between Different Codes

For ordinary applications of RC, the strength of concrete of 21 and 28 MPa concretes are used [McCormac et al, 2006]. Using the following relation between (f'_c) which is used in the ACI Code and (f_{cu}) which is used in other Codes [(IRAQI Code Committee, 1987), (Subramanian, 2010) and (McCormac et al, 2006)].

$$f_{cu} = 1.25 f'_c \quad (12)$$

the three f'_c (f_{cu}) values of [21(26.25), 24(30) and 28(35)] MPa are used.

The strength of steel reinforcement is selected based on (f_y) value of 280 and 420 MPa for ordinary grade and high-strength grade respectively [Hassoun et al, 2008].

Before selecting miscellaneous cases of beams, the values of factored flexural resistance (R_u) is investigated, where (R_u) is given by [Hassoun et al, 2008]:

$$R_u = \frac{M_u}{bd^2} \quad (13)$$

so that the required area of tension reinforcement satisfies the limitations of minimum and maximum ratios for all Codes.

The area A_s of tension reinforcement in a reinforced-concrete flexural member can be expressed as the ratio [Merritt, 2001]:

$$\rho = \frac{A_s}{bd} \quad (14)$$

For ACI Code, rearranging Eq. (2.b) by using Eqs. (13) and (14), the factored flexural resistance (R_u) of this Code can be written as [Hassoun et al, 2008].

$$R_u = 0.9 \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c} \right) \quad (15)$$

The value of (ρ) can be determined by solving the quadratic Eq. (15) [Wang et al., 2006]. Letting the ratio (m_l) for strength of concrete and reinforcement for ACI Code,

$$m_1 = 0.85 \frac{f'_c}{f_y} \quad (16)$$

and the ratio (m_2) for strength of concrete for ACI Code,

$$m_2 = \frac{2}{0.765} \frac{1}{f'_c} \quad (17)$$

then (ρ) for ACI Code can be determined by the following:

$$\rho = m_1 (1 - \sqrt{1 - m_2 R_u}) \quad (18)$$

For BS Code, rearranging Eq. (6.b) by using Eqs. (13) and (14), the factored flexural resistance (R_u) for this Code can be written as:

$$R_u = 0.95 \rho f_y \left(1 - \frac{0.9 \times 0.95 \rho f_y}{0.8 f_{cu}} \right) \quad (19)$$

Also, the value of (ρ) can be determined by solving the quadratic Eq. (19). Letting the ratio (m_1) for strength of concrete and reinforcement for BS Code,

$$m_1 = \frac{0.4}{0.90 \times 0.95} \frac{f_{cu}}{f_y} \quad (20)$$

and the ratio (m_2) for strength of concrete for BS Code,

$$m_2 = 4.5 \frac{1}{f_{cu}} \quad (21)$$

then, also (ρ) for BS Code can be determined by the same above Eq. (18), but with ratios (m_1) and (m_2) for BS Code.

Also, for BS Code, rearranging Eq.(7) by using Eqs. (13), the ultimate factored flexural resistance (R_u) for this Code which is related with the maximum reinforcement ratio can be written as:

$$R_u = 0.156 f_{cu} \quad (22)$$

Substituting Eqs. (20), (21) and (22) in Eq. (18), the maximum reinforcement ratio for BS Code can be written as:

$$\rho_{max} = 0.2124 \frac{f_{cu}}{f_y} \quad (23)$$

For BS Code and based on ($d/h = 0.9$) [Concrete Centre, 2009], the minimum reinforcement ratio in Eqs. (8.a) and (8.b) can be calculated by interpolation as:

$$\rho_{min} = 0.0025, \text{ when } f_y = 280 \text{ MPa} \quad (24.a)$$

$$\rho_{min} = 0.0016, \text{ when } f_y = 420 \text{ MPa} \quad (24.b)$$

For IRAQ Code, rearranging Eq. (10.b) by using Eqs. (13) and (14), the factored flexural resistance (R_u) for this Code can be written as:

$$R_u = 0.87 \rho f_y \left(1 - \frac{0.9 \times 0.87 \rho f_y}{0.8 f_{cu}} \right) \quad (25)$$

Also, the value of (ρ) can be determined by solving the quadratic Eq. (25). Letting the ratio (m_1) for strength of concrete and reinforcement for IRAQ Code,

$$m_1 = \frac{0.4}{0.90 \times 0.87} \frac{f_{cu}}{f_y}$$

$$m_1 = 0.5109 \frac{f_{cu}}{f_y} \quad (26)$$

and the ratio (m_2) for strength of concrete for IRAQ Code is equal to the same ratio (m_2) for strength of concrete for BS Code in Eq. (21).

then, also (ρ) for IRAQI Code can be determined by the same above Eq. (18), but with ratios (m_1) and (m_2) for IRAQI Code.

Also, for IRAQI Code, rearranging Eq. (7) by using Eqs. (13), the ultimate factored flexural resistance (R_u) for this Code which is related with the maximum reinforcement ratio can be written in a similar way of Eq. (22) for BS Code.

Substituting Eqs. (21), (22) and (26) in Eq. (18), the maximum reinforcement ratio for IRAQI Code can be written as:

$$\rho_{max} = 0.2320 \frac{f_{cu}}{f_y} \quad (27)$$

For all Codes, the limitations of reinforcement ratios are shown in **Table (1)**, based on Eqs. (3.b and 4), (23 , 24.a and 24.b) and (11 and 27) for ACI, BS Code and IRAQI Code respectively.

For all Codes and depending on the limitations of reinforcement ratios of **Table (1)**, the limitations of (R_u) are listed in **Table (2)**, based on Eqs. (15), (19) and (25) for ACI Code, BS Code and IRAQI Code respectively.

It is obvious from **Table (2)** that the limitations of greatest lower bound and least upper bound of (R_u) are (1.21 and 4.10 MPa), (1.22 and 4.68 MPa) and (1.22 and 5.46 MPa) for the f'_c (f_{cu}) of [21(26.25), 24(30) and 28(35)] MPa respectively for all Codes. *The lower and upper bounds designate the range of permissible values for [Guerra,2006] the limitations of (R_u) which are (1.22 and 4.10 MPa) of greatest lower and least upper bound for all Codes.* Six miscellaneous realistic beams which have values of (R_u) ranging between (1.30 and 3.98 MPa) are chosen to investigate the reinforcement. The details of all beams are shown in **Table(3)**. Hence (36) various beams which have different strength of concrete, yield strength of reinforcement and factored flexural resistance (R_u) are studied in this comparison between the ACI Code, BS Code and IRAQI Code.

3.2. Optimum (Minimum) Required Tension Reinforcement

The optimized Code is the Code which gives minimum required area of tension reinforcement in the same beam, satisfy the basic requirements and most economical. The same beam is the beam with the same dimensions, unfactored or characteristic loads, strength of concrete, strength of reinforcement and same support conditions. The loads having same values, positions and concentrations.

Reducing the quantity of reinforcement, will reduce the complex or congested reinforcement and will lead to economic design [**Wight et al., 2012**], and also satisfy the ductility provisions for cracks and deflections in order to ensure happening the ductile failure and avoiding the brittle failure.

The required area of tension reinforcement in all selected beams is calculated based on Eq. (18) and Eq. (14), using the values of ratios (m_1) and (m_2) in **Table (4)**. These values are computed by using Eqs. (16 and 17), (20 and 21) and (26 and 21) for ACI Code, BS Code and IRAQI Code respectively.

Table (5) and **Table (6)** signify that the required area of reinforcement for ACI Code is less than that for BS Code and IRAQI Code. **Table (5)** and **Table (6)** also illustrate the percentage increase in the required area of reinforcement for BS Code and IRAQI Code with respect to ACI Code. It is clearly appeared that percentage increase in the required area of reinforcement is not affected with the strength of

reinforcement (f_y). Hence, **Table (7)** is established depending on values of related (R_u) and the percentage increase in the required area of reinforcement of **Table (5)** and **Table (6)**.

Figs. (2, 3 and 4) are drawn with regression analysis to illustrate the relation between (R_u) and the percentage increase in the required area of reinforcement of **Table (7)**. These figures clearly show that there is an increase in required area of reinforcement for BS Code and IRAQI Code with respect to ACI Code. To avoid exaggeration and to attain rational equations for the best fitted curves, polynomial equations of second degree with correlation coefficient are calculated and shown in these figures. Based on the equations of these best fitted curves, the percentage increase in the required area of reinforcement for least upper bound of (R_u) of table (2), are calculated and presented in **Table (8)**. It is clearly appear from **Table (8)** that the percentage increase in the required area of reinforcement may reach to about (15%) and (30%) for BS Code and IRAQI Code respectively.

For the above several reasons mentioned, it is obvious that the ACI Code gives the minimum required area of tension reinforcement compared with BS Code and IRAQI Code.

4. Conclusions and Recommendation

The main results of this study can be indicated as following:

1. ACI Code gives the minimum required area of tension reinforcement in the same beam compared with BS Code and IRAQI Code, which leads to good economy and satisfy the basic requirements.
2. The results indicate that the BS Code have given an increase in the required area of tension reinforcement up to (15 %) with respect to ACI Code.
3. The results indicate that the IRAQI Code have given an increase in the required area of tension reinforcement up to (30 %) with respect to ACI Code.
4. The paper provides swift, simple and accurate computations to find the percentage of required tension reinforcement for the ACI Code, through using the ratios (m_1) and (m_2) in **Table (4)** and the formula of Eq. (18).

It is recommended to adopt the provisions of ACI code to obtain the required area of tension reinforcement in members subjected to flexure in the next edition of IRAQI code.

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Notation

A_s	= area of tension reinforcement
a	= depth of equivalent rectangular stress block for ACI Code
b	= width of beam
C	= concrete compression force
c	= distance from extreme compression fiber to neutral axis for ACI Code
d	= distance from extreme compression fiber to centroid of tension reinforcement
f'_c	= cylinder compressive strength of concrete for ACI Code
f_{cu}	= cube characteristic strength of concrete for BS Code and IRAQI Code
f_y	= specified yield strength of reinforcement
h	= overall depth of beam
L	= span length of beam
M_D	= dead load bending moment at section
M_L	= live load bending moment at section
M_u	= factored bending moment at section

m_1	= ratio for strength of concrete and reinforcement (see equations 16, 20 and 26 for ACI Code, BS Code and IRAQI Code respectively or Table 4)
m_2	= ratio for strength of concrete (see equations 17 for ACI Code, 21 for BS Code and IRAQI Code or Table 4)
RC	= Reinforced Concrete
R_u	= factored flexural resistance
T	=steel reinforcement tension force
x	=distance from extreme compression fiber to neutral axis for BS Code and IRAQI Code
W_D	=uniformly distributed dead load over the span length of beam
W_L	=uniformly distributed live load over the span length of beam
β_1	= factor relating depth of equivalent rectangular compressive stress block to neutral axis depth for ACI Code
ϕ	= strength reduction factor for ACI Code
ρ	= ratio of A_s to bd
ϵ_c	=maximum usable strain at extreme concrete compression fiber
ϵ_s	=tension steel strain
γ_m	= BS Code and IRAQI Code partial safety factors for materials

Table (1) - The limitations of reinforcement ratios

f_y , MPa	Code	$f_c(f_{cu})$, MPa					
		21(26.25)		24(30)		28(35)	
		Min.	max.	min.	max.	min.	max.
280	ACI	0.0050	0.0203	0.0050	0.0232	0.0050	0.0271
	BS	0.0025	0.0199	0.0025	0.0228	0.0025	0.0266
	IRAQI	0.0050	0.0218	0.0050	0.0249	0.0050	0.0290
420	ACI	0.0033	0.0135	0.0033	0.0155	0.0033	0.0181
	BS	0.0016	0.0133	0.0016	0.0152	0.0016	0.0177
	IRAQI	0.0033	0.0145	0.0033	0.0166	0.0033	0.0193

Table (2) - The limitations of factored flexural resistance (R_u)

f_y , MPa	Code	R_u (MPa) for different $f_c(f_{cu})$, MPa								
		21(26.25)			24(30)			28(35)		
		Min.	Max.	Diff.	Min.	Max.	Diff.	Min.	Max.	Diff.
280	ACI	1.21	4.30	3.09	1.22	4.92	3.70	1.22	5.74	4.52
	BS	0.65	4.10	3.45	0.65	4.68	4.03	0.65	5.46	4.81
	IRAQI	1.15	4.10	2.95	1.16	4.68	3.52	1.17	5.46	4.29
420	ACI	1.20	4.29	3.09	1.21	4.92	3.71	1.21	5.75	4.54
	BS	0.62	4.10	3.48	0.62	4.68	4.06	0.62	5.46	4.84
	IRAQI	1.14	4.10	2.96	1.15	4.68	3.53	1.16	5.46	4.30

Table (3) - Beams details

Beam Information	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Notes
b , mm	250	300	350	400	450	500	
d , mm	340	390	490	540	740	840	Assuming effective cover=60 mm [McCormac <i>et al</i> , 2006]
h , mm	400	450	550	600	800	900	Satisfy deflection for all Codes
h / b ratio	1.60	1.50	1.57	1.50	1.78	1.80	Ratio between 1.5 to 2 [(McCormac <i>et al</i> , 2006) and (Wang <i>et al</i> , 2006)]
d / h ratio	0.85	0.87	0.89	0.90	0.93	0.93	
L , mm	3500	4000	6000	6500	7000	8000	All beams are simply supported
L / d ratio	10.29	10.26	12.24	12.04	9.46	9.52	
w _L , kN/m	6	10	10	15	35	45	Unfactored load
w _D , kN/m	10	16	16	25	50	60	Unfactored load
Beam dead weight , kN/m	2.4	3.24	4.62	5.76	8.64	10.8	Unfactored load, concrete density=24 kN/m ³ [(Arya, 2009) and (McCormac <i>et al</i> , 2006)]
M _L , kN.m	9.19	20.00	45.00	79.22	214.38	360.00	Unfactored moment at center [(Arya, 2009) and (McCormac <i>et al</i> , 2006)]
M _D , kN.m	18.99	38.48	92.79	162.45	359.17	566.40	
M _u , kN.m	37.49	78.18	183.35	321.69	774.00	1255.68	Load factors - ACI Code =1.2, 1.6
	41.28	85.87	201.91	354.18	845.84	1368.96	Load factors - BS Code=1.4, 1.6
	42.20	87.87	206.41	362.10	867.28	1404.96	Load factors - IRAQI Code=1.4, 1.7
R _u , MPa	1.30	1.71	2.18	2.76	3.14	3.56	ACI Code
	1.43	1.88	2.40	3.04	3.43	3.88	BS Code
	1.46	1.93	2.46	3.10	3.52	3.98	IRAQI Code

Table (4) - Values of (m_1) and (m_2) ratios

f _y , MPa	Code	f _c '(f _{cu}) , (MPa)					
		21(26.25)		24(30)		28(35)	
		m ₁	m ₂ , 1/MPa	m ₁	m ₂ , 1/MPa	m ₁	m ₂ , 1/MPa
280	ACI	0.0638	0.1245	0.0729	0.1089	0.0850	0.0934
	BS	0.0439	0.1714	0.0501	0.1500	0.0585	0.1286
	IRAQI	0.0479	0.1714	0.0547	0.1500	0.0639	0.1286

Table (4) - Values of (m_1) and (m_2) ratios (Continued)

f_y , MPa	Code	$f_c'(f_{cu})$, (MPa)					
		21(26.25)		24(30)		28(35)	
		m_1	m_2 , 1/MPa	m_1	m_2 , 1/MPa	m_1	m_2 , 1/MPa
420	ACI	0.0425	0.1245	0.0486	0.1089	0.0567	0.0934
	BS	0.0292	0.1714	0.0334	0.1500	0.0390	0.1286
	IRAQI	0.0319	0.1714	0.0365	0.1500	0.0426	0.1286

Table (5) - The required area of reinforcement; $f_y=280$ MPa

Beam Case	Code	$f_c'(f_{cu})$, MPa					
		21(26.25)		24(30)		28(35)	
		A_s , mm ²	% Increase	A_s , mm ²	% Increase	A_s , mm ²	% Increase
1	ACI	457	0.0	454	0.0	452	0.0
	BS	488	6.9	484	6.6	480	6.2
	IRAQI	546	19.6	541	19.1	536	18.7
2	ACI	843	0.0	836	0.0	830	0.0
	BS	908	7.7	896	7.1	885	6.6
	IRAQI	1017	20.7	1004	20.0	991	19.3
3	ACI	1602	0.0	1585	0.0	1569	0.0
	BS	1753	9.4	1721	8.6	1692	7.8
	IRAQI	1964	22.6	1927	21.6	1893	20.6
4	ACI	2612	0.0	2575	0.0	2540	0.0
	BS	2914	11.6	2837	10.2	2769	9.0
	IRAQI	3270	25.2	3181	23.5	3102	22.1
5	ACI	4663	0.0	4584	0.0	4510	0.0
	BS	5235	12.3	5065	10.5	4918	9.0
	IRAQI	5904	26.6	5704	24.4	5531	22.6
6	ACI	6794	0.0	6656	0.0	6529	0.0
	BS	7762	14.2	7442	11.8	7174	9.9
	IRAQI	8785	29.3	8402	26.2	8085	23.8

Table (6) - The required area of reinforcement; $f_y=420$ MPa

Beam Case	Code	$f_c'(f_{cu})$, MPa					
		21(26.25)		24(30)		28(35)	
		A_s , mm ²	% Increase	A_s , mm ²	% Increase	A_s , mm ²	% Increase
1	ACI	304	0.0	303	0.0	301	0.0
	BS	326	6.9	323	6.6	320	6.2
	IRAQI	364	19.6	361	19.1	357	18.7
2	ACI	562	0.0	558	0.0	553	0.0
	BS	605	7.7	597	7.1	590	6.6
	IRAQI	678	20.7	669	20.0	660	19.3
3	ACI	1068	0.0	1057	0.0	1046	0.0
	BS	1169	9.4	1148	8.6	1128	7.8
	IRAQI	1309	22.6	1285	21.6	1262	20.6

Table (6) - The required area of reinforcement; $f_y=420$ MPa (Continued)

Beam Case	Code	$\bar{f}_c(f_{cu})$, MPa					
		21(26.25)		24(30)		28(35)	
		A_s , mm ²	% Increase	A_s , mm ²	% Increase	A_s , mm ²	% Increase
4	ACI	1741	0.0	1716	0.0	1693	0.0
	BS	1942	11.6	1892	10.2	1846	9.0
	IRAQI	2180	25.2	2121	23.5	2068	22.1
5	ACI	3108	0.0	3056	0.0	3007	0.0
	BS	3490	12.3	3377	10.5	3278	9.0
	IRAQI	3936	26.6	3803	24.4	3687	22.6
6	ACI	4529	0.0	4437	0.0	4353	0.0
	BS	5174	14.2	4961	11.8	4783	9.9
	IRAQI	5857	29.3	5601	26.2	5390	23.8

Table (7) - The relation between factored flexural resistance (R_u) and the percentage increase in the required area of reinforcement with respect to ACI Code provisions

Beam Case	Code	$\bar{f}_c(f_{cu})$, MPa					
		21(26.25)		24(30)		28(35)	
		R_u , MPa	% Increase	R_u , MPa	% Increase	R_u , MPa	% Increase
1	ACI	1.30	0.0	1.30	0.0	1.30	0.0
	BS	1.43	6.9	1.43	6.6	1.43	6.2
	IRAQI	1.46	19.6	1.46	19.1	1.46	18.7
2	ACI	1.71	0.0	1.71	0.0	1.71	0.0
	BS	1.88	7.7	1.88	7.1	1.88	6.6
	IRAQI	1.93	20.7	1.93	20.0	1.93	19.3
3	ACI	2.18	0.0	2.18	0.0	2.18	0.0
	BS	2.40	9.4	2.40	8.6	2.40	7.8
	IRAQI	2.46	22.6	2.46	21.6	2.46	20.6
4	ACI	2.76	0.0	2.76	0.0	2.76	0.0
	BS	3.04	11.6	3.04	10.2	3.04	9.0
	IRAQI	3.10	25.2	3.10	23.5	3.10	22.1
5	ACI	3.14	0.0	3.14	0.0	3.14	0.0
	BS	3.43	12.3	3.43	10.5	3.43	9.0
	IRAQI	3.52	26.6	3.52	24.4	3.52	22.6
6	ACI	3.56	0.0	3.56	0.0	3.56	0.0
	BS	3.88	14.2	3.88	11.8	3.88	9.9
	IRAQI	3.98	29.3	3.98	26.2	3.98	23.8

Table (8) - The relation between least upper bound of factored flexural resistance (R_u) and the percentage increase in the required area of reinforcement with respect to ACI Code provisions

Code	$\tilde{f}_c(f_{cu})$, MPa					
	21(26.25)		24(30)		28(35)	
	R_u , MPa	% Increase	R_u , MPa	% Increase	R_u , MPa	% Increase
BS	4.10	14.98	4.68	13.53	5.46	11.40
IRAQI	4.10	29.83	4.68	28.69	5.46	27.11

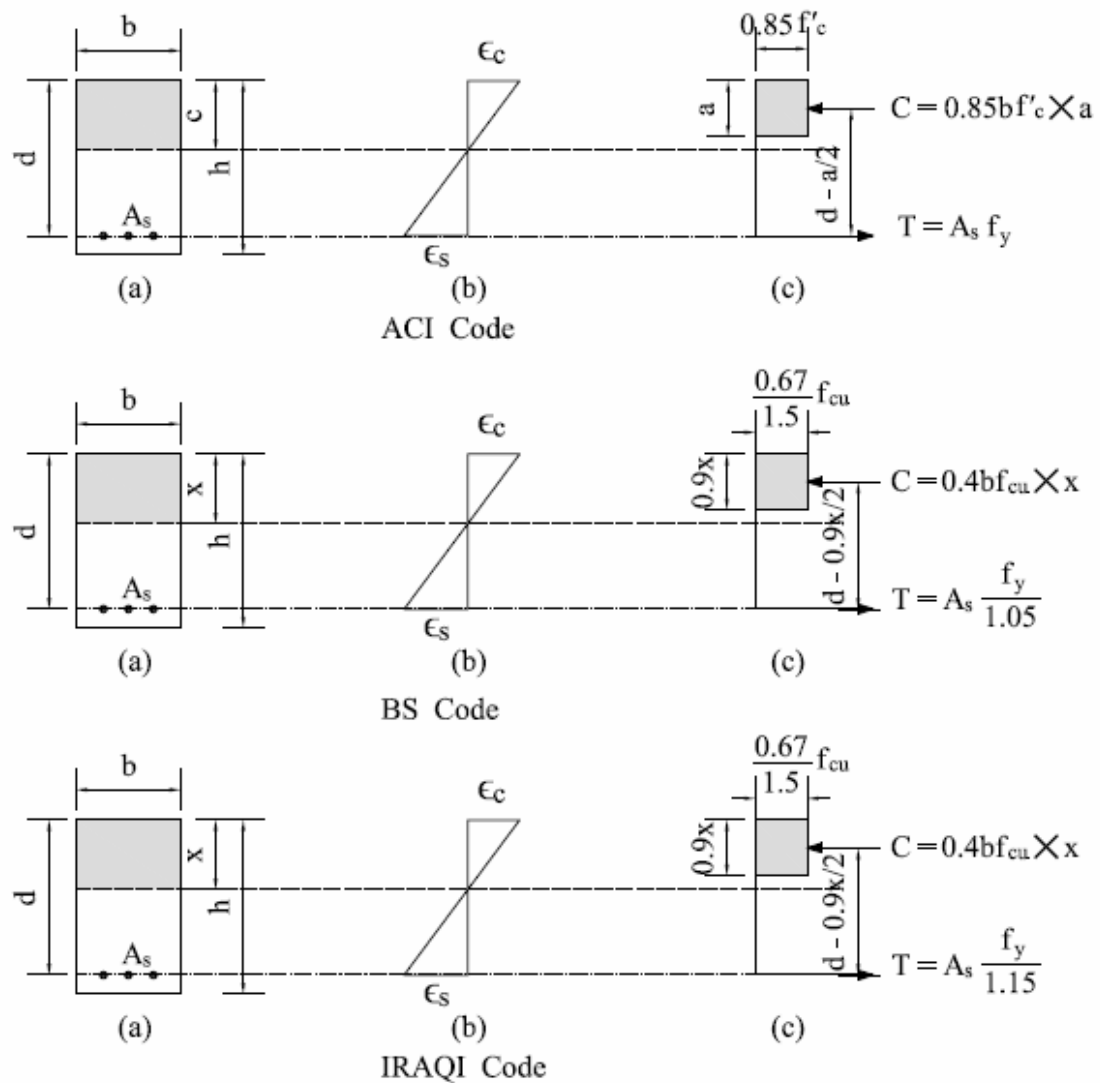


Fig. 1 - Strain distribution and stress block

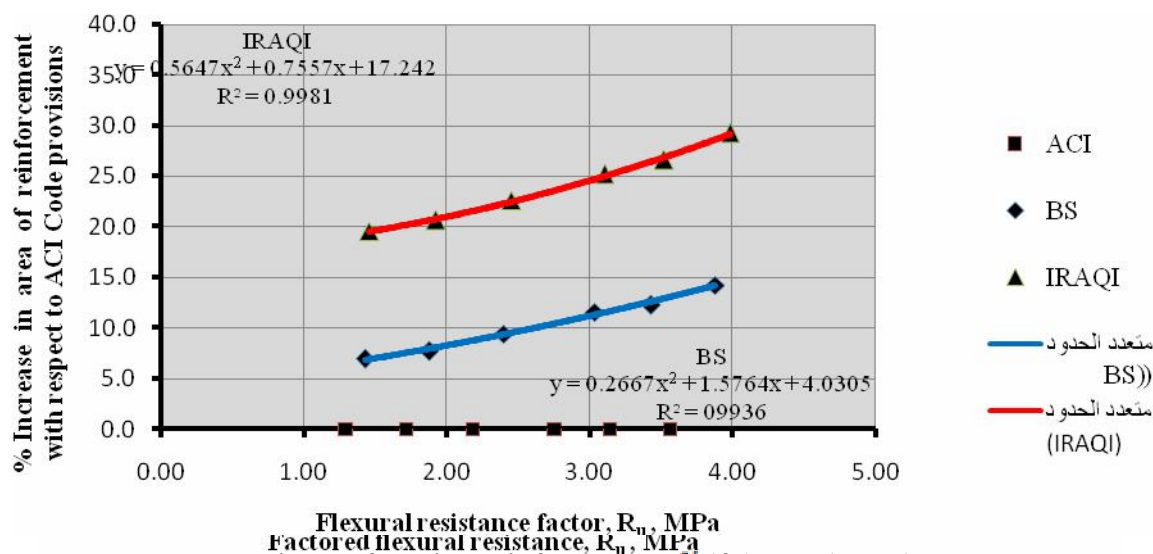


Fig. 2 - Comparison of tension reinforcement, $f'_c (f_{cu}) = 21(26.25)$ MPa

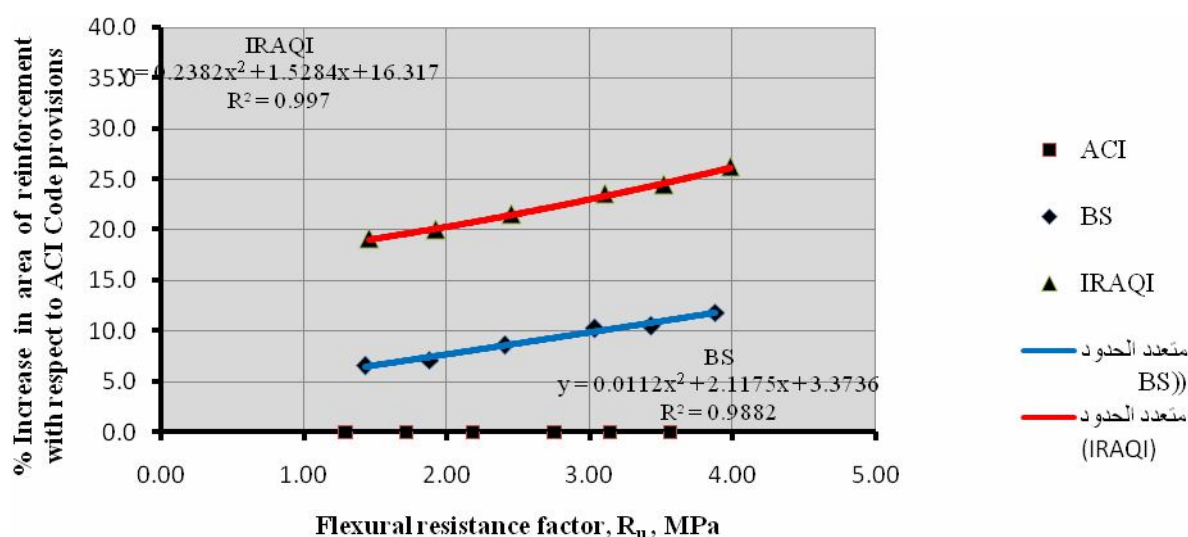


Fig. 3 - Comparison of tension reinforcement, $f'_c (f_{cu}) = 24(30)$ MPa

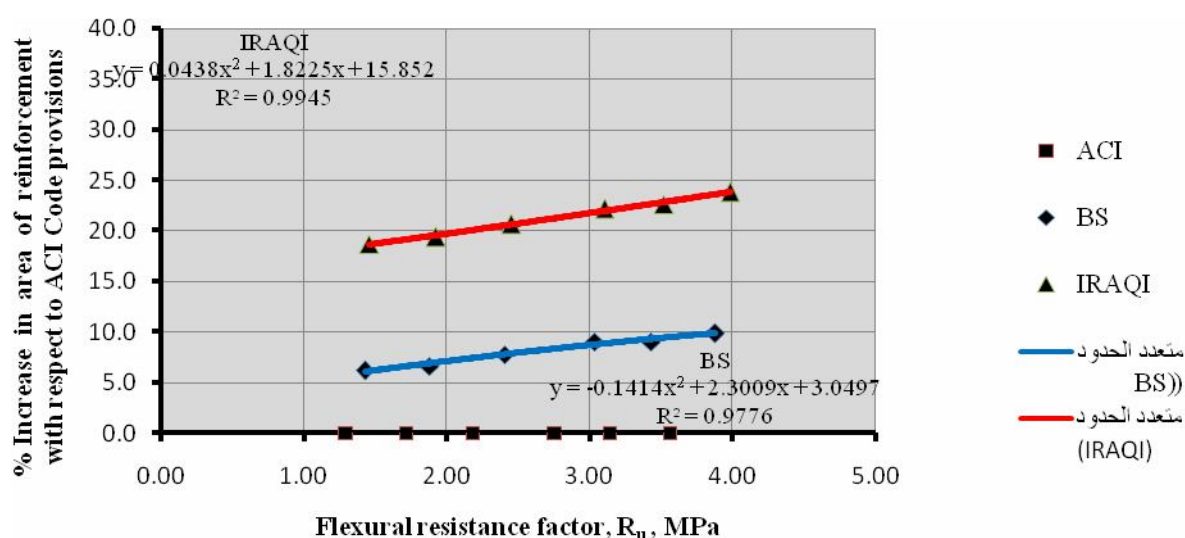


Fig. 4 - Comparison of tension reinforcement, $f'_c (f_{cu}) = 28(35)$ MPa