ON α-Syndetic Sets

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Abstract
In this paper, we introduce and investigate the notion of α-syndetic sets, using the notion of α-open sets which was introduce by Njastad. And the work of W. Gottschalk , and S. Al-Kutaibi .
Keywords: α-open, $\alpha - closed$, α −closure, $\alpha - compact$, $\alpha - syndetic$

الخلاصة
في هذا البحث قدمنا مفهوم المجموعات الرابطة من النوع الفا مع عدد من النظريات باستخدام تعريف المجموعات المفتوحة من النوع الفا التي قدمها نجاستد ومعتمدين بذلك على المفاهيم التي قدمها كل من جوتسجوك والكتبي في هذا الموضوع.
الكلمات الافتتاحية: المجموعة المفتوحة من النوع الفا،المجموعة المغلقة من النوع الفا، الانغلاق من النوع الفا، المجموعة المرصوصة من النوع الفا، المجموعة المترابطة من النوع الفا.

1.Introduction
Gottschalk in [Gottschalk, 1955] introduced the notions of left (right) syndetic set in topological group. He defined a subset A of topological group G, is called left (right) syndetic if there exists a compact subset M of G, such that $AM = G$ $(MA) = G$. In 1998 Al-kutaibi [Al-kutaibi, 1998] introduced paper semi-syndetic and feebly syndetic subsets.

The aim of this paper is to introduce and investigate a new class of syndetic called $\alpha - syndetic$ which is a subset A of topological group G, is called left (right) $\alpha - syndetic$ if there exists a $\alpha - compact$ set M of G, such that $AM = G$ $(MA) = G$. And we obtain several Theorem abut $\alpha - syndetic$.

2. Preliminaries
In this section we recall some of the basic Definitions.

Throughout this paper X will denote a topological space. If A is a subset of the space X, $Cl(A), Int(A), \alpha - Cl(A)$ denote the closure, interior and the α −closure of A respectively. A subset A of X is said to be $\alpha - open$ set [Njastad, 1965] if $A \subset \text{Int}(C1(\text{Int}(A)))$. The complement of a $\alpha - open$ set is said to be $\alpha - closed$. Let $A \subseteq X$ and $\mathcal{H}$ be a family of $\alpha - open$ subsets of X, then $\mathcal{H}$ is called a α −open cover of A, if $A \subseteq \bigcup_{H \in \mathcal{H}} H$, A is said to be $\alpha - compact$ if and only if any $\alpha - open$ cover has a finite sub cover.

Clear that, every open set is $\alpha - open$ and then every $\alpha - compact$ set is compact also, the union of two $\alpha - compact$ subsets of X is $\alpha - compact$. A topological group is a set G which carries group stricture and a topology and satisfied the two axioms: (i) The map $(x, y) \longrightarrow xy$ of $G \times G$ into G is continuous.(That is, the operation of G is continuous). (ii) The map $x \longrightarrow x^{-1}$ (The inversion map) of G into G is continuous.[Higgins, 1979]

3. The main results
3.1 Theorem:
If M is $\alpha - compact$ set in a topological group G , then M^{-1} is $\alpha - compact$.

Proof:

Let $f: G \longrightarrow G$ be the invers map, that is, $f(x) = x^{-1}$ for all x in G, let $\mathcal{H}$ be a $\alpha - open$ cover of M^{-1}, then $f(\mathcal{H})$ is $\alpha - open$ cover of $f(M^{-1}) = (M^{-1})^{-1} = M$,but M is $\alpha - compact$, which implies $f(\mathcal{H})$ has a finite sub cover $\mathcal{H}^*$, then $f(\mathcal{H}^*)$ covers $f(M) = M^{-1}$. Hence, M^{-1} is $\alpha - compact$ set.

3.2 Definition:

Let A be a subset of a topological group G, then A is called left (right) $\alpha - syndetic$ if there exists a $\alpha - compact$ subset M of G such that $AM = G$ $(MA = G)$.

3.3 Note:

In the following results we will prove the case of left $\alpha - syndetic$ and the case of right $\alpha - syndetic$ will be similar.

3.4 Theorem:

Let G be a topological group and let $A \subseteq G$, then A is left (right) $\alpha - syndetic$ set in G if and only if there exists a $\alpha - compact$ subset M of G such that every left (right) translation of M intersects A.

Proof:

Sufficiency, suppose A is a left $\alpha - syndetic$ set then, there exists a $\alpha - compact$ subset M of G such that $AM = G$, let $g \in G$ then, there exists $a \in A, m \in M$ such that $g = am$ which implies $a = gm^{-1}$ and then $a \in gm^{-1}$but m^{-1} is $\alpha - compact$. Hence $gm^{-1} \cap A \neq \emptyset$, (i.e m^{-1} is the $\alpha - compact$ set we need).

Efficiency, let $g \in G$, there exists $\alpha - compact$ subset M of G such that, $gM \cap A \neq \emptyset$, for each g in G, there exists $a \in A, m \in M$ such that $gm = a$, so $g = am^{-1}$, which implies $G = AM^{-1}$, and since M^{-1} is $\alpha - compact$ then A is a left $\alpha - syndetic$.

3.5 Theorem:

Let A be a subset of a topological group G, then A is left (right) $\alpha - syndetic$ in G, if and only if A^{-1} right (left) $\alpha - syndetic$.

Proof:

Let A is a left $\alpha - syndetic$, then there exists $\alpha - syndetic$ subset M of G such that $AM = G$. Since $G = G^{-1} = (AM)^{-1} = M^{-1}A^{-1}$ and since M^{-1} is $\alpha - syndetic$, then A^{-1} is right $\alpha - syndetic$.

3.6 Theorem:

Let G be a topological group, let A, B be two subsets of G such that $A \subseteq B$. If A is left (right) $\alpha - syndetic$ set, then so is B.

Proof:

Let A be a left $\alpha - syndetic$ set, then there exists a $\alpha - compact$ subset M of G such that $AM = G$, since $A \subseteq B$, then $BM = G$, which implies that B is left $\alpha - syndetic$.

3.7 Theorem:

Let G be a topological group, then

1- If A is a left$\alpha - syndetic$ set in G,then $cl(A)\ and\ \alpha - cl(A)$ are left
2- The union of any family of left (right) $\alpha - syndetic$ sets is left (right) $\alpha - syndetic$.

Proof:

Directly from (Theorem 3.6).

3.8 Theorem:

Let G be a topological group, let A, B are two left (right) $\alpha - syndetic$ sub sets of G, then $A \cap B$ is a left (right) $\alpha - syndetic$.

Proof:

Since both $A\ and\ B$ are left $\alpha - syndetic$, then there exist $\alpha - compact$ sets $M\ and\ N$ such that $AM = G\ and\ BN = G$ and then $(A \cap B)(M \cup N) = A(M \cup N) \cap B(M \cup N) = G \cap G = G$ and since $(M \cup N)$ is $\alpha - compact$, then $(A \cap B)$ is left $\alpha - syndetic$.

3.9 Theorem:

Let A be a subset of topological group G.If A is a subgroup of G or if G is an abelian group, then A is left $\alpha - syndetic$ in G if and only if A is a right $\alpha - syndetic$ in G.

Proof:

Let A is a left $\alpha - syndetic$ subgroup of G, then $G = AM$, that means $G^{-1} = M^{-1}A^{-1}$ and hence $G = M^{-1}A$, but M^{-1} is $\alpha - compact$, which implies A is a right $\alpha - syndetic$ in G.

3.10 Theorem:

Let A be a $\alpha - syndetic$ subgroup of a topological group G, then the quotient space $G\ /\ A$ is compact.

Proof:

Let A be a $\alpha - syndetic$ subgroup of a topological group G, then there exists a $\alpha - compact$ set M such that $MA = G$. Let $f: G \longrightarrow G\ /\ A$ is the quotient map. Clear that $f(M) \subset G/A$. Let $gA \in G\ /\ A$, then $ga \subset G$ which implies $gA \subset MA$, that is $gA \in f(M)$ and then, $G/A \subset f(M)$. Hence $G/A = f(M)$. Since f is continuous and M is $\alpha - compact$ and hence is compact, then $f(M) = G/A$ is compact set.

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