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Optimal LQR Controller Design for Wing Rock Motion Control in Delta wing Aircraft

Abstract: *In this work, an Optimal Linear Quadratic (LQR) and optimal Minimax controller is proposed for Wing Rock Motion Control in Delta wing Aircraft model. The nonlinear Wing Rock Motion dynamics were considered. The LQR and Minimax controllers are designed in order to reduce the Wing Rock Motion. Simulations are performed in order to verify the ability of both controllers to reduce the wing rock motion in delta wing aircraft. Results were plotted together in order to show the difference of performance for both controllers for a comparative point of view.*

Keywords: *Wing Rock Motion, Optimal Linear Quadratic control, optimal Minimax control.*

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1. Introduction

Wing rock is mainly defined as a rolling motion of aircraft often presented when the aircraft maneuvers at high angles of attack, leading to aircraft instability or entering a limit cycle oscillation [1]. The basic sketch of delta wing aircraft is shown in Figure 1.

The wing rock motion is of interest since wing rock may have negative effects on maneuverability, tracking accuracy, and safety of operation. Such vibrations lead to a noticeable loss in lift and may cause severe safety drawbacks throughout maneuvers such as landing or takeoff. However, the underlying mechanism of the wing rock motion is still not very clear because with modern aircraft it is not easy to analyze the aerodynamic flow created by the fore-body, wing and strake, or their relationship together with the wing rock motion due to the complex geometry of high-performance aircraft [3].

Many researches have shown interest for the control of the wing rock motion based on the theory of output feedback linearization and methods of adaptive control [4,5]. When feedback linearization approaches are applied, the gain of feedback control should be calculated by trial and error to attain the desired performance. Anyway, applying the trial methods could consume time, adding to that; the detailed model of system may be needed [4].

When applying the adaptive control methods, the wing rock aerodynamic mechanism should be known; anyway, the aerodynamic model may not be easy to achieve [5].

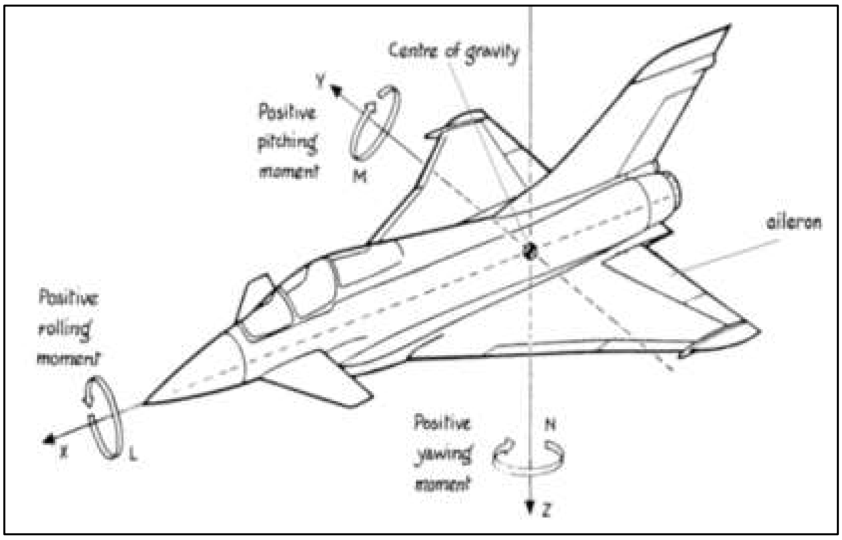

Figure 1: Basic sketch of delta wing aircraft [2]

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Another approach in controlling dynamical systems is attempting to make the system performance to be optimal, in that way, researchers seek to design a controller, which satisfies the desired performance consuming the shortest time or less energy or both [6]. [7] Applied phase plane analysis and existence theorems to describe the overall system behavior in order to design an optimal controller to stabilize the delta wing aircraft. Another approach was adopted by [8] where an optimal controller is designed to meet specific performance requirements such as rise time, settling time and maximum peak overshoot. A Lyapunov stability based optimal LQR controller is presented in [9] where the controller is designed to optimize the aircraft performance by minimizing a specified Lyapunov function.

A series of papers have considered the control of the wing rock motion based on output feedback linearization theory and adaptive control technique. In the feedback linearization, design approaches the feedback control gain should be preselected by trial and error to achieve the desired performance; however, this trial procedure is time consuming and the system model is required as shown in [10]. In the adaptive techniques, the knowledge of the structure of the aerodynamic functions is required; however, the aerodynamic structure of the wing rock is difficult to obtain as shown in [11].

The neural-network-based control technique was presented in [12] as an alternative design method for control of the wing rock dynamics to compensate for the effects of nonlinearities and system uncertainties, so that the stability, convergence and robustness of the control system can be improved. A neural-network-identification based adaptive control (NNIAC) system have been developed in [13] to attenuate the effects of the wing rock motion and tracking error. In addition, The fuzzy neural networks (FNN) proved to have advantages over both of fuzzy systems and neural networks in controlling wing rock motion which was demonstrated in [14] by improving dynamic response and information storing ability.

In this paper, two different controllers (optimal LQR, optimal minimax) are addressed and designed for the wing rock motion control in delta wing aircraft.

This paper is organized as follows: the wing rock motion mathematical model is derived in section two. In section three, the model was used in design two different controllers in order to control the wing rock motion. Simulations were performed in section four in order to investigate the proposed controllers performance and some notes about the results were mentioned in section five.

2. Mathematical Model of Wing Rock

The one–dimensional differential equation which describes the roll angle ϕ is [15]:

$$\ddot{\phi} = \frac{pU_\infty^2 Sb}{2I_x} C_l + Du \quad (1)$$

Where ϕ is the roll angle (rad), ρ is the air density ($kg/m3$), U_∞ is the free stream velocity (m/sec), S is the wing reference area ($m2$), b is the chord (m), D is the effectiveness of the differential ailerons rolling, u is the aileron input angle (deg) and C_l is given by [16];

$$C_l = c_0\phi + c_1\dot{\phi} + c_2|\phi|\dot{\phi} + c_3|\dot{\phi}|\dot{\phi} + c_4\phi^3 \quad (2)$$

Substituting (2) into (1) yields

$$\ddot{\phi} = f(\phi, \dot{\phi}) + Du \quad (3)$$

Where,

$$f(\phi, \dot{\phi}) = b_0\phi + b_1\dot{\phi} + b_2|\phi|\dot{\phi} + b_3|\dot{\phi}|\dot{\phi} + b_4\phi^3 \quad (4)$$

and the parameters b_i, $i = 0,1, \dots, 4$ are given by [16]

$$b_i = \left(\frac{pU_\infty^2 Sb}{2I_x} C_l\right) c_i \quad (5)$$

In order to design an optimal controller for the wing rock motion, the mathematical model should be approximated to linear model. Since the roll angle and its rate of change are considered too small, then the result of their multiplication is considered too small. Thus, the wing rock model will be linearized as in Eq. (6) below,

$$\ddot{\phi} = b_0\phi + b_1\dot{\phi} + Du \quad (6)$$

Let $x_1 = \phi$ and$\dot{x}_1 = x_2 = \dot{\phi}$, then $\dot{x}_2$ can be written as below

$$\dot{x}_2 = \ddot{\phi} = b_0\phi + b_1\dot{\phi} + Du$$

Defining the state vector as$x^T = [x_1 \quad x_2]^T = [\phi \quad \dot{\phi}]^T$, Eq. (6) can be written in state space form,

$$\dot{x} = A\,x + B\,u \quad (7)$$

$$y = C\,x$$

Where,

$$A = \begin{bmatrix} 0 & 1 \\ b_0 & b_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ D \end{bmatrix}, C = [1 \quad 0]$$

The linear model of Eq. (7) will be used to design the linear quadratic controller for the wing rock motion.

3. Controller Design

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In what follows, the design of suggested controller will be developed for the plane under consideration.

I. Optimal LQR Controller Design

In this section, an optimal LQR controller is designed for the wing rock system. If one assumes that all the system states are measurable, then the state variable feedback control function can be written as

$$u = -K\,x \qquad (8)$$

where K is the gain matrix which is designed to minimize the following performance index

$$J = \int_0^{\infty} (x^T Q\, x + u^T R\, u)\, dt \qquad (9)$$

where the performance index J is a quantitative value used to describe the system performance, Q and R are positive semi-definite and positive definite symmetric constant matrices, respectively. Substitute for u from Eq. (8) into Eq. (9)

$$J = \int_0^{\infty} x^T (Q + K^T R\, K)\, x\, dt \qquad (10)$$

The main goal in optimal control design is to select the state variable feedback gain matrix K that minimize the performance index J described by Eq. (10). Figure 2 shows the block diagram of the optimal control configuration.

The optimal feedback matrix K can be calculated using the following procedure. Assuming that there is a positive definite matrix P existed such that

$$x^T (Q\, x + K^T\, R\, K)\, x = -\frac{d}{dt} x^T P x$$

or,

$$x^T (Q\, x + K^T\, R\, K)\, x = -\dot{x}^T P\, x - x^T P\, \dot{x} \qquad (11)$$

To get an expression for $\dot{x}$, substitute Eq. (8) into Eq. (7)

$$\dot{x} = (A - BK)\, x \qquad (12)$$

Substitute for $\dot{x}$ from the above equation into Eq. (11) then we get

$$x^T (Q\, x + K^T R\, K)\, x = -x^T (A - BK)^T P x - x^T P\, (A - BK)\, x \qquad (13)$$

Comparing both sides of the above equation we notice the following

$$(A - BK)^T P + P\, (A - B\, K) + (Q\, x + K^T R\, K) = [0] \qquad (13)$$

If the matrix $(A - BK)$ is proved to be stable [8], then there is a positive-definite matrix P that satisfies Equation (13). Continuing our work we have

$$A^T P - K^T B^T P + PA - PBK + Q + K^T R\, K = [0] \qquad (14)$$

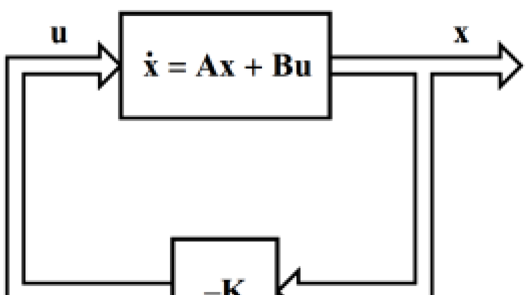

Figure 2: Block diagram for optimal control configuration [8]

This is a matrix quadratic equation. Exactly as for the scalar case, one may complete the squares. However, this procedure is a bit complicated for matrices [8], suppose we select

$$K = R^{-1} B^T P \qquad (15)$$

Substituting for K in Eq. (14), we obtain

$$A^T P + PA + Q - P\, B\, R^{-1} B^T P = [0] \qquad (16)$$

The above equation is called the algebraic Riccati equation. P matrix can be calculated by solving the above algebraic Riccati equation assuming that other matrices are given. Finally, optimal feedback gain matrix K is given by Eq. (15).

II. Optimal Minimax Controller Design

If the disturbance model is included in the wing rock model, then Eq. (7) becomes

$$\dot{x} = A\, x + B\, u + L\, d \qquad (17)$$

where, L is the gain matrix and given by $L^T = [0 \quad 1]^T$. If the disturbance d was considered as a scalar function of the state variables x, then Eq. (17) can be rewritten as

$$\dot{x} = A\, x + B\, u + \gamma\, L\, x \qquad (18)$$

where γ is a scalar representing the worst case disturbance. In this case the performance index J will be given by

$$J = \int_0^{\infty} (x^T Q\, x + u^T R\, u + d^T H d)\ dt$$

$$= \int_0^{\infty} x^T (Q + K^T R\, K + \gamma^2 L^T H\, L)\, x\, dt \qquad (19)$$

where H is a positive definite symmetric constant matrix. Repeating the same procedure used in the regular LQR controller we get the following Riccati equation

$$A^T P + PA + Q - P(BR^{-1}B^T - \gamma^2 L^T H^{-1} L)\, P = [0] \qquad (20)$$

Depending on the desired performance, the parameters of the matrices H, R and Q are chosen by the designer and generally they are taken as unity matrices [6]. A summary for the calculation of the optimal LQR feedback matrix K can be addressed by the steps below:

- ❑ Selection of design parameter matrices Q and R according to above argument,
- ❑ Solution of the algebraic Riccati equation for matrix P,
- ❑ Find the optimal state variable feedback gain matrix K using Eq. (15).

4. Simulation Results

The aerodynamic parameters of the delta wing used for simulation are given by [7];

b_0= -0.01859521, b_1 = -0.015162375, b_2= -0.06245153;

b_3= -0.00954708, b_4= -0.0214529 and D= 0.75.

The scaling factor γ is set at unity, $\gamma = 1$, and the matrices R, Q and H are taken as unity matrices;

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Considering the above values and solving the Reccati equation in Eq. (16) for the P matrix then substitute for matrix P matrix in Eq. (14) to find the optimal gain matrix K results in

$$P = \begin{bmatrix} 104.22 & 63.47 \\ 63.47 & 43.93 \end{bmatrix}, \ K = [16.667 \quad 4.177]$$

The above matrix represents the optimal LQR controller gain.

In order to find the optimal Minimax gain the procedure adopted in section 3.2 was adopted and the following P and gain matrix were obtained

$$P = \begin{bmatrix} 226.04 & 137.68 \\ 137.68 & 61.67 \end{bmatrix}, \ K = [63.33 \quad 47.02]$$

The matrix above represents the optimal Minimax controller gain.

To determine the stability of the system one can easily substitute the values of gain matrix K into the closed loop equation represented by Eq. (7) and find the location of the closed loop as in the following manner:

$$\dot{x} = A\,x - B\,Kx = (A - BK)x$$

Then, the eigenvalues of the matrix *(A-BK)* are the roots of the characteristic equation, which calculated as follows

$$|\lambda I - A + BK| = 0$$

Substitute values for matrices *A, B* and *K* the characteristic equation is

$$|\lambda I - A + BK| = \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -0.0186 & -0.015 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.75 \end{bmatrix} [16.667 \quad 4.177] \right|$$

$$= \left| \begin{bmatrix} \lambda & -1 \\ 0.0186 & \lambda + 0.015 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 12.5 & 3.133 \end{bmatrix} \right| = \left| \begin{bmatrix} \lambda & -1 \\ 12.5186 & \lambda + 3.148 \end{bmatrix} \right|$$

Solving for λ yields

$\lambda_1 = -0.7474, \quad \lambda_2 = -2.677$

Repeating the same procedure for the optimal Minimax controller we get

$\lambda_1 = -1.4033, \quad \lambda_2 = -3.8617$

Initially, both controllers have the roots in the left hand of the S-plane meaning that the system under the proposed controllers is stable. However, it can be noted that with the optimal Minimax controller the roots lie further from the origin resulting in more stable performance and a faster response.

Both of the optimal LQR and Minimax optimal controller was applied for the delta wing aircraft and the simulation based on MATLAB package is performed. It is important to mention that the control signal is considered as unbounded function and the control signal obtained is theoretical but practically it cannot exceed the physical aileron limitations, which were not included in the model.

If the roll angle of the aircraft is assumed to be initially at 30 degrees and allowed to settle at zero angle, the response of the plane roll response based on both controllers are shown in Figures 3, 4 and 5. It is evident from the figures that the roll response base on MiniMax controller is faster than its classical controller. Moreover, the response based on MiniMax controller shows an oscillation at steady state of lower amplitude than that would be shown from other controller. However, the Minimax controller gives higher control action than its counterpart; of course this is price for better response behavior given by this controller.

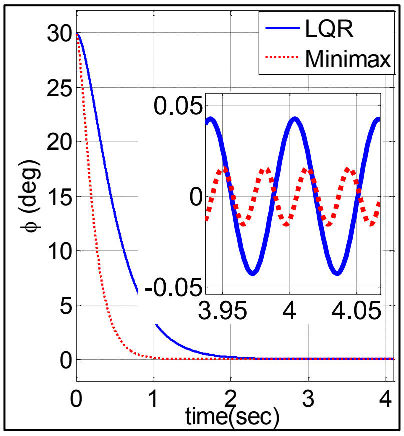

Figure 3: Roll angle response

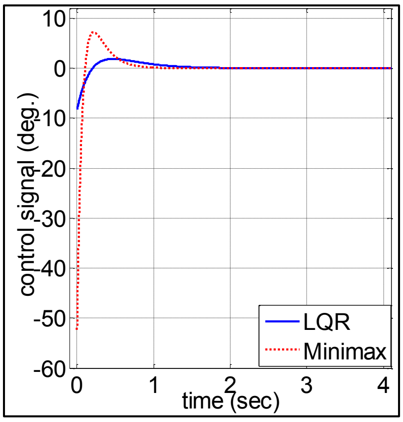

Figure 4: Control signal

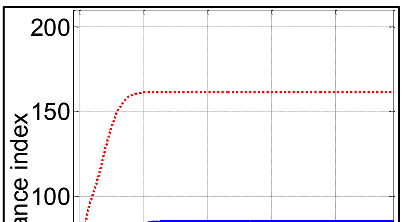

Figure 5: Performance index

To examine the robustness of both controllers, an external disturbance torque of height 1000 N.m has been exerted beyond the steady state of the response, i.e., at time t=1 sec., as shown in Figures 6, 7, 8 and 9. It is clear from the figures that system based on Minimax controller shows higher robustness characteristics as compared to other. Certainly, the structure of classical optimal controller lacks the ability to compensate the disturbing torque.

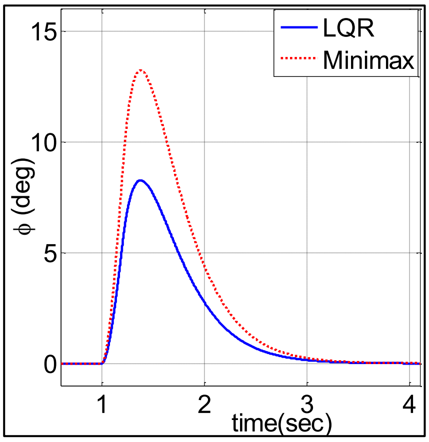

Figure 6: Roll angle

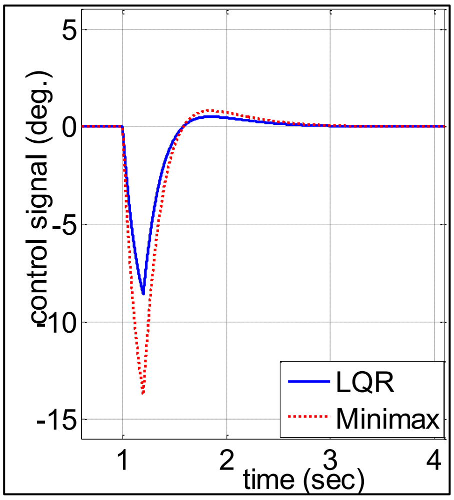

Figure 7: Control signal

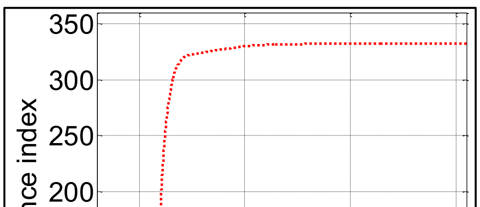

Figure 8: Performance index

The last simulation was intended to test the tracking capability for both controllers. In this case, the aircraft was forced to track a sinusoidal roll angle reference defined by $\phi_d = 15\sin(\pi t)$. The results are illustrated in Figures 10 and 11. The figures show that the response based on Minimax controller gives better tracking performance than the classical one. The Minimax controller-based response is more adjacent to reference than the other one.

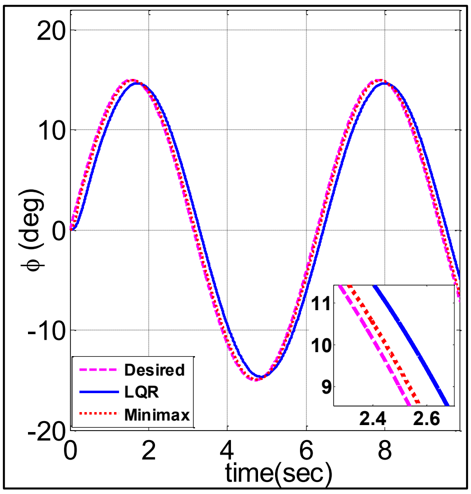

Figure 9: Roll angle

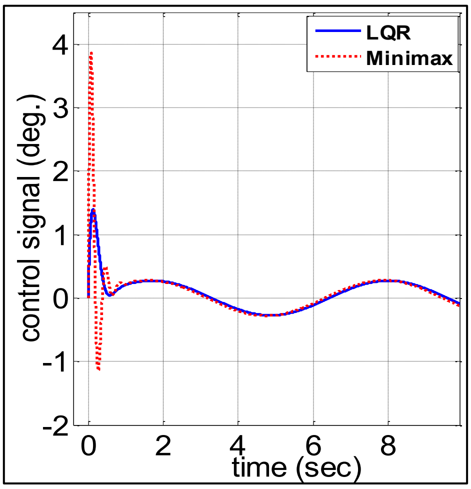

Figure 10: Control signal

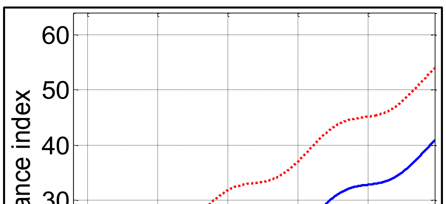

Figure 11: Performance index

5. Conclusion

In this paper, two different controllers were addressed for the problem of the wing rock motion control in delta wing aircraft. The wing rock motion mathematical model was used to represent the dynamic behavior of the aircraft. The mathematical model then was used to design two different controllers which have been addressed to control the wing rock motion. Simulation was performed using MATLAB/Simulink in order to investigate the proposed controllers performance in order to stabilize the aircraft. Initially, both of the designed controllers showed the ability to control the control the wing rock motion and retain the aircraft back to horizontal level. However, the simulation shows a relative difference in the performance of the proposed controllers.

It can be noted that the optimal minimax has relatively a better performance than the optimal LQR by stabilizing the aircraft in shorter time where the optimal minimax controller stabilizes the aircraft within 0.5 second while the optimal LQR takes about 1 second to stabilize the aircraft. Also, the optimal minimax controller has better disturbance rejection by keeping the roll angle below 20^o while it reaches about 30^o when applying the optimal LQR controller. Finally, In terms of reference tracking, both controllers show satisfactory response, though the Minimax controller results in less tracking error. However, in despite that the Minimax optimal controller shows better performance than the optimal LQR controller especially in disturbance rejection, it can be noted that the performance index for the Minimax controller is higher than of the LQR which means that the better performance and disturbance rejection is opposed by more consumption of energy which is the usual price for such better performance.

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