The effects of external magnetic field on laser beam self focusing through plasma

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Abstract

In this work, the nonlinear dielectric constant due to the relativistic electron motion by the high intense laser beam has been derived. From this, one can evaluate the effect of longitudinal external magnetic field in comparison with transverse external magnetic field on relativistic self focusing of laser beam inside collisionless plasma. Theoretical and numerical calculations show that the increase in the values of external magnetic field, in both cases, lead to the laser beam self focusing is faster and stronger. Furthermore, for fixed values of magnetic field, the self focusing of laser beam in existence of transverse external magnetic field is greater comparing with longitudinal external magnetic field case.

Keywords: Relativistic nonlinearity , Self focusing , Magnetoplasma , Laser beam , Nonlinear dielectric constant.

تأثير المجال المغناطيسي الخارجي على البؤرة الذاتية لحزمة الليزر خلال البلازما

منذر باقر حسن احمد عبيد سواري

الخلاصة

في هذا العمل تم إشتقاق ثابت العزل اللاخطي والناتج بسبب حركة الالكترونات في المدى النسبي داخل حزمة ليزر ذات شدة عالية. من ذلك فإنه يمكن حساب تأثير المجال المغناطيسي الخارجي الطولي بالمقارنة مع تأثير المجال المغناطيسي الخارجي المستعرض على التبؤر الذاتي النسببي لحزمة الليزر داخل بلازما لاتصادمية. الحسابات النظرية والعددية تبين أن زيادة قيم المجال المغناطيسي الخارجي في كلا الحالتين تؤدي الى أن التبؤر الذاتي لحزمة الليزر يكون أسرع وأقوى. بالإضافة لذلك ولقيم مجال مغناصيسي خارجي ثابتة فأن التبؤر الذاتي لحزمة الليزر يكون أعظم في حالة وجود مجال مغناطيسي خارجي طولي بالمقارنة مع وجود مجال مغناطيسي خارجي مستعرض.

كلمات مفتاحية: اللاخطية النسبية ، البؤرة الذاتية ، حزمة الليزر ، ثابت العزل اللاخطي

1. Introduction

Currently the propagation of an intense laser beam through plasmas due to the nonlinear interaction has a wide research interest both theoretically [1, 2] and practically [3, 4] as a result of its relevance with wide-ranging applications such as laser-driven fusion, laser-driven accelerators, x-ray lasers [5-10]. In these applications for efficient interaction between laser beam and plasmas, one needs the laser beam to propagate several Rayleigh lengths without divergence which can be achieved by using the nonlinear self focusing of laser beam. In plasma, there are three main mechanisms of self-focusing: relativistic, poderomotive and thermal. At laser pulse duration τ shorter than $\left(\omega_{pe}\right)^{-1}$, where ω_{pe} is the electron plasma frequency, the relativistic nonlinearity due to the electron mass variation relativistically will predominate by comparing with ponderomotive and thermal nonlinearities. Max [11] studied the nonlinear self-focusing of laser beam propagation in plasma due to ponderomotive nonlinearity. Sodha et al. [12].considered two different situations of nonlinearities arising through the thermal and the ponderomotive force on steady state self-focusing in magnetoplasma. Fedosejevs et al. [13] observed a relativistic self-focusing for hydrogen gas. They employed a *0.3 TW, 250 fs* laser pulse, which gave an axial intensity of $3x10^{17}\ W\ cm^{-2}$. Osman et al. [14] presented numerical calculations on the concept of relativistic self-focusing of a high-power laser beam. Hafizi et al. [15] studied the propagation of an intense laser beam in plasma as a result of the relativistic and ponderomotive effects. Liu and Tripathi [16] reported the effect of a self-generated azimuthal magnetic field on the relativistic self-focusing of an intense laser in plasma. K. I. Hassoon et al. [17] showed the role of perpendicular external magnetic field on relativistic self focusing on other hand M. B. Hassan et al. [18] presented a theoretical study to explain the longitudinal external magnetic field effect on the relativistic self focusing leading to enhance THz radiation production. In the present article, we investigate the nonlinear self focusing of an intense laser beam through plasma in presence of a static magnetic field in longitudinal direction once and perpendicular direction in another with respect to laser beam propagation direction. In section 2 and 3 we derive appropriated expressions to calculate the nonlinear dielectric constant of plasma and beamwidth parameter of laser beam self focuses in longitudinal case and perpendicular case respectively. The typical parameters of the laser beam, plasma and applied magnetic fields have been characterized in Section 4 with discussion of the numerical results and introducing the conclusions of the present work briefly.

2. Relativistic self focusing of the laser beam with longitudinal magnetic field

2.1 Nonlinear dielectric constant in relativistic case and $\vec{B}_0 \parallel \vec{k}$

We consider the propagation of Gaussian laser beam in a uniform magnetoplasma of equilibrium electron density n_0 along the direction of a static magnetic field $\vec{B}_0 = \hat{z}B_0$.

The electric field vector $\vec{E}_{0+}$ of laser beam propagating along z-direction via the magnetoplasma can be written as [19]

$$\vec{E}_{0+} = \vec{A}_{0+} \exp i\,(\omega_0 t - k_{0+} z), \quad (1)$$

where $\vec{A}_{0+} = \vec{E}_x + i\vec{E}_y$ is the electric field amplitude of a right circular polarized electromagnetic wave , ω_0 and k_{0+} are the angular frequency and wave vector respectively , and k_{0+} is related with dielectric constant ε_{0+} as $k_{0+}^2 = \frac{\varepsilon_{0+}\omega_0^2}{c^2}$,

where c is the light velocity in the vacuum .

The electron relativistic motion equation in presence of high intense laser is

$$m_0\gamma \frac{\partial}{\partial t}\vec{\upsilon}_j = -e\vec{E}_j - \frac{e}{c}\left(\vec{\upsilon}_j \times \vec{B}_0\right), \quad (2)$$

Where γ, $\vec{\upsilon}$ and $\vec{B}_0$ are the relativistic factor, the oscillation velocity imparted by laser beam and external magnetic field respectively.

Using Eq. (2), we calculate the electron oscillating velocity$\left(\upsilon_{0+}\right)$ for the right circular polarized mode of laser beam as

$$\vec{\upsilon}_{0+} = \vec{\upsilon}_x + i\vec{\upsilon}_y = \frac{ie\vec{E}_{0+}}{m_0\gamma\omega_0(1 - \frac{\omega_{ce}}{\gamma\omega_0})}, \quad (3)$$

where $-e, m_0$ and $\omega_{ce} = \frac{eB_0}{m_0 c}$ are the electronic charge , rest mass of electron and the electron cyclotron frequency respectively and $\gamma = (1 - \frac{\upsilon_{0+}^2}{c^2})^{-\frac{1}{2}}$.

Proposing (γ -1<1) [17] the relativistic factor γ will be

$$\gamma \cong 1 + \frac{1}{2}\cdot\left(\frac{e}{m_0 c\omega_0}\right)^2 \cdot \frac{A_{0+}\cdot A_{0+}^*}{(1 - \frac{\omega_{ce}}{\omega_0})^2} = 1 + \alpha_+ A_{0+}\cdot A_{0+}^*, \quad (4)$$

The relativistic nonlinearity factor $\alpha_+ = \frac{e^2}{2m_0^2c^2\omega_0^2}\cdot\frac{1}{(1 - \frac{\omega_{ce}}{\omega_0})^2}$ will become zero at non-relativistic regime (i.e. $\gamma = 1$).

In relativistic regime the components of the dielectric constant tensor $\underline{\underline{\varepsilon}}$ will be as following

$$\varepsilon_{xx} = \varepsilon_{yy} = 1 - \frac{\omega_{pe}^2}{\omega_0^2\gamma\left(1 - \frac{\omega_{ce}^2}{\omega_0^2\gamma^2}\right)},$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = \frac{-i\left(\frac{\omega^2{}_{pe}}{\omega_0^2\gamma}\right)\left(\frac{\omega_{ce}}{\omega_0\gamma}\right)}{\left(1 - \frac{\omega_{ce}^2}{\omega_0^2\gamma^2}\right)},$$

$$\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zx} = \varepsilon_{zy} = 0,$$

$$\varepsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega_0^2\gamma},$$

where the effective dielectric constant corresponding to right circular polarized laser beam ε_+ will take the following formula

$$\varepsilon_+ = \varepsilon_{xx} - i\,\varepsilon_{xy} = 1 - \frac{\dfrac{\omega_{pe}^2}{\omega_0^2\gamma}}{\left(1-\dfrac{\omega_{ce}}{\omega_0\gamma}\right)},$$

where $\omega_{pe} = \left(\dfrac{4\pi n_0 e^2}{m_0}\right)^{\frac{1}{2}}$ is the electron plasma frequency.

Using Eq. (4) the effective dielectric constant ε_+ can be written as following

$$\varepsilon_+ = 1 - \frac{\left(\dfrac{\omega_{pe}}{\omega_0}\right)^2}{\left(1-\dfrac{\omega_{ce}}{\omega_0}\right)} + \frac{\left(\dfrac{\omega_{pe}}{\omega_0}\right)^2}{\left(1-\dfrac{\omega_{ce}}{\omega_0}\right)^2}\alpha_+ A_{0+}A_{0+}^*, \quad (5)$$

Eq. (5) shows that the effective dielectric constant ε_+ consists of a linear part ε_{0+} and a nonlinear part $\phi_+\left(A_{0+}A_{0+}^*\right)$, where the latter is appearing as a result of relativistic electron mass increasing. Both parts of the effective dielectric constant ε_+ may be written as

$$\varepsilon_{0+} = 1 - \frac{\left(\dfrac{\omega_{pe}}{\omega_0}\right)^2}{\left(1-\dfrac{\omega_{ce}}{\omega_0}\right)}, \quad (6)$$

$$\phi_+ = \varepsilon_{2+}A_{0+}A_{0+}^*, \quad (7)$$

where ε_{2+} is given by

$$\varepsilon_{2+} = \frac{1}{2}\left(\frac{e}{m_0 c\omega_0}\right)^2 \frac{\left(\dfrac{\omega_{pe}}{\omega_0}\right)^2}{\left(1-\dfrac{\omega_{ce}}{\omega_0}\right)^4}. \quad (8)$$

2.2 Relativistic self focusing with $\vec{B}_0 \parallel \vec{k}$

The general wave equation of electromagnetic wave propagating through magnetized plasma can be given as

$$\nabla^2\vec{E} - \nabla\left(\vec{\nabla}\cdot\vec{E}\right) + \frac{\omega_0^2}{c^2}\underline{\underline{\varepsilon}}\cdot\vec{E} = 0, \quad (9)$$

One can consider that the electromagnetic wave inside magnetoplasma is transverse wave since its field vary along external magnetic field(i.e. z-direction) larger than its variation via wave front plane(i.e. $x-y$ plane)[20]so no space charge occur and thus

$$\vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot\left(\underline{\underline{\varepsilon}}\vec{E}\right) = 0, \quad (10)$$

Using Eq. (10) with components of dielectric tensor, one can obtain

$$\frac{\partial E_z}{\partial z} \cong -\frac{1}{\varepsilon_{zz}}\left[\varepsilon_{xx}\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}\right)+\varepsilon_{xy}\left(\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}\right)\right], \quad (11)$$

Putting Eq.(11) in Eq.(8) and using zero-order approximation, thus we obtained the differential equation for the circular polarized electric field amplitude A_{0+} as

$$\frac{\partial^2 A_{0+}}{\partial z^2} + \frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)A_{0+} + \frac{\omega_0^2}{c^2}\left(\varepsilon_{0+}+\varepsilon_{2+}A_{0+}A_{0+}^*\right)A_{0+} = 0, \quad (12)$$

where the product of nonlinear part with $\dfrac{\partial^2 A_{0+}}{\partial x^2}$ or $\dfrac{\partial^2 A_{0+}}{\partial y^2}$ have been neglected[21].

We assume $A_{0+} = A'_{0+}\exp i\left(\omega_0 t - k_{0+}z\right)$ and substituting its value in Eq. (12), one can get

$$-2ik_{0+}\frac{\partial A'_{0+}}{\partial z}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)A'_{0+}$$
$$+\frac{\omega_0^2}{c^2}(\varepsilon_{2+}A'_{0+}A'^{*}_{0+})A'_{0+}=0, \qquad (13)$$

where A'_{0+} is the complex amplitude.

Proposing a two dimensional Gaussian beam (i.e. $\frac{\partial}{\partial y}=0$) and introducing an eikonal $A'_{0+}=A^0_{0+}\exp\left(ik_{0+}S_+\right)$, where A^0_{0+} and S_+ are a real function and the phase of the laser beam inside magnetoplasma respectively, hence Eq.(13) after separating real and imaginary parts can be written as following [20]

$$2\frac{\partial S_+}{\partial z}+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\partial S_+}{\partial x}\right)^2-\frac{1}{2k_{0+}^2A^0_{0+}}$$
$$\times\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial^2 A^0_{0+}}{\partial x^2}=\frac{\varepsilon_{2+}}{\varepsilon_{0+}}\left(A^0_{0+}\right)^2, \qquad (14)$$

$$\frac{\partial\left(A^0_{0+}\right)^2}{\partial z}+\frac{1}{2}\left(A^0_{0+}\right)^2\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial^2 S_+}{\partial x^2}$$
$$+\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\frac{\partial S_+}{\partial x}\frac{\partial\left(A^0_{0+}\right)^2}{\partial x}=0. \qquad (15)$$

In the paraxial ray approximation S_+ can be expanded to $S_+=\frac{1}{2}x^2\beta_+\left(z\right)+\varphi_+\left(z\right)$

where β_+^{-1} may be explained as the curvature radius of laser beam and φ_+ is a constant

independent of x.

Introducing initially Gaussian beam with initial beam radius x_0 as

$$\left(A^0_{0+}\right)^2=\frac{E_{00}^2}{f_+}\exp\left(-\frac{x^2}{r_0^2f_+^2}\right), \qquad (16)$$

and substituting S_+ in Eq.(15), $\beta\left(z\right)$ will take the following formula [22]

$$\beta_+\left(z\right)=2\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)^{-1}\frac{1}{f_+}\frac{df_+}{dz},$$

where f_+ represents the beam width parameter.

Using A^0_{0+} and $\beta_+\left(z\right)$ values in Eq. (19) and assuming initially plane wavefront condition ($f_+=1$ and $\frac{df_+}{dz}=0$ at $z=0$) we obtain

$$\frac{d^2f_+}{dz^2}=\frac{1}{4}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)^2\frac{1}{R_{d+}^2f_+^3}$$
$$-\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\varepsilon_{2+}E_{00}^2}{\varepsilon_{0+}}\right)\frac{1}{r_0^2f_+^2}. \qquad (17)$$

where $R_{d+}=k_0r_0^2$ represents diffraction length and r_0 is the initial radius beam.

In term of normalization distance of propagation $\xi_+=\frac{z}{R_{d+}}$, the last equation may be rewritten to become more suitable for the computing programs as follows

$$\frac{d^2f_+}{d\xi_+^2}=\frac{1}{4}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)^2\frac{1}{f_+^3}$$
$$-\frac{1}{2}\left(1+\frac{\varepsilon_{0+}}{\varepsilon_{0zz}}\right)\left(\frac{\varepsilon_{2+}E_{00}^2}{\varepsilon_{0+}}\right)\frac{R_{d+}^2}{r_0^2f_+^2}. \qquad (18)$$

Eq. (18) represents the spot size variation of laser beam profile as a result of competition between the diffraction and self-focusing terms (first and second terms on the right hand side of Eq.(18) respectively), it has been solved numerically for several external magnetic fields.

3. Relativistic self focusing of the laser beam with transverse magnetic field

3.1 Nonlinear dielectric constant in relativistic case and $\vec{B}_0 \perp \vec{k}$

Consider the propagation of extraordinary laser beam (X-mode) inside homogeneous magnetoplasma along z-direction and perpendicular on an external magnetic field $\vec{B}_0$ aligned in y-direction. The variation of X-mode electric field $\vec{E}$ may be written as follows

$$\vec{E}_0 = \left(E_x \hat{x} + E_z \hat{z}\right) \exp i\left(\omega_0 t - k_0 z\right), \quad (19)$$

where ω_0 and k_0 are the angular frequency and wave vector respectively which can be related by

$$k_0 = \left(\varepsilon_r\right)^{\frac{1}{2}} \left(\frac{\omega_0}{c}\right), \quad (20)$$

c is the light speed and ε_r is the dielectric constant in relativistic case which may given by

$$\varepsilon_r = 1 - \frac{\frac{\omega_{pe}^2}{\gamma}\left(\omega_0^2 - \frac{\omega_{pe}^2}{\gamma}\right)}{\omega_0^2\left(\omega_0^2 - \frac{(\gamma-1)\omega_{pe}^2 + \omega_u^2}{\gamma^2}\right)}, \quad (21)$$

where $\omega_{pe} = \left(\frac{4\pi n_0 e^2}{m_0}\right)^{\frac{1}{2}}, \omega_u = \left(\omega_{pe}^2 + \omega_{ce}^2\right)^{\frac{1}{2}}$ and $\omega_{ce} = \frac{eB_0}{m_0 c}$ are the angular frequencies of the electron plasma wave , upper hybrid wave and electron cyclotron respectively, also n_0, m_0 and $-e$ represent the plasma density, electron rest mass and electron charge respectively.

The relativistic factor γ in Eq. (21) refers to the variation of the dielectric constant due to the electron mass increasing at relativistic case, which can be written as following [17]

$$\gamma = \left(1 + \frac{\upsilon_0^2}{2c^2}\right), \quad (22)$$

where υ_0 is the electron oscillating velocity imparted by electric field of laser beam

$$\upsilon_0 = \frac{1}{\sqrt{2}}\left(\upsilon_{0x} \cdot \upsilon_{0x}^* + \upsilon_{0z} \cdot \upsilon_{0z}^*\right)^{\frac{1}{2}}, \quad (23)$$

The relativistic motion equation of an electron in electric field of laser beam is given as

$$m_0 \gamma \frac{\partial}{\partial t} \vec{\upsilon}_0 = -e\vec{E}_0 - \frac{e}{c}\left(\vec{\upsilon}_0 \times \vec{B}_0\right), \quad (24)$$

From Eq. (24) , one may get the velocity components υ_{0x} and υ_{0z} of the electron as

$$\upsilon_{0x} = \frac{e\omega_{ce}}{m_0\gamma^2\omega_0^2\left(1 - \frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)} E_z + \frac{ie}{m_0\gamma\omega_0\left(1 - \frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)} E_x, (25)$$

$$\upsilon_{0z} = \frac{ie}{m_0\gamma\omega_0\left(1 - \frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)} E_z - \frac{e\omega_{ce}}{m_0\gamma^2\omega_0^2\left(1 - \frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)} E_x, (26)$$

For the extraordinary mode, one can introduce the following equations [23]

$$\nabla \cdot \underline{\underline{\varepsilon}} \cdot \vec{E}_0 = 0, \quad (27)$$

$$E_z \simeq \frac{-\varepsilon_{xz}}{\varepsilon_{xx}} E_x, \quad (28)$$

$$\varepsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\gamma\omega_0^2\left(1 - \frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)}, \quad (29)$$

$$\varepsilon_{xz} = \frac{-i\,\omega_{pe}^2\omega_{ce}}{\gamma^2\omega_0^3\left(1 - \frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)}, \quad (30)$$

using Esq.(27-30) , then Eq.(25) and Eq.(26) will take the following formulas

$$\upsilon_{0x} = \frac{ie}{m_0\gamma\omega_0\left(1-\frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)}\left[1+\frac{\left(\frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)\left(\frac{\omega_{pe}^2}{\gamma\omega_0^2}\right)}{\left(1-\frac{(\gamma-1)\omega_{pe}^2+\omega_u^2}{\gamma^2\omega_0^2}\right)}\right]E_x, \quad (31)$$

$$\upsilon_{0z} = \frac{-e}{m_0\gamma\omega_0\left(1-\frac{\omega_{ce}^2}{\gamma^2\omega_0^2}\right)}\left[\frac{\left(\frac{\omega_{ce}}{\gamma\omega_0}\right)\left(\frac{\omega_{pe}^2}{\gamma\omega_0^2}\right)}{\left(1-\frac{(\gamma-1)\omega_{pe}^2+\omega_u^2}{\gamma^2\omega_0^2}\right)}+\frac{\omega_{ce}}{\gamma\omega_0}\right]E_x, \quad (32)$$

Taking the complex conjugates υ_{0x}^* and υ_{0z}^* of υ_{0x} and υ_{0z} then substituting in Eq.(23) and Eq.(22), the relativistic factor γ can be written as the following

$$\begin{aligned}\gamma = 1+\frac{1}{4}\left(\frac{e}{m_0\omega_0 c}\right)^2\Bigg[&1+3\left(\frac{\omega_{ce}}{\omega_0}\right)^2+4\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^2+2\left(\frac{\omega_{ce}}{\omega_0}\right)^4\\ &+5\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^4+2\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^6+13\left(\frac{\omega_{ce}}{\omega_0}\right)^4\left(\frac{\omega_{pe}}{\omega_0}\right)^4\\ &+8\left(\frac{\omega_{ce}}{\omega_0}\right)^6\left(\frac{\omega_{pe}}{\omega_0}\right)^2+6\left(\frac{\omega_{ce}}{\omega_0}\right)^4\left(\frac{\omega_{pe}}{\omega_0}\right)^6+8\left(\frac{\omega_{ce}}{\omega_0}\right)^6\left(\frac{\omega_{pe}}{\omega_0}\right)^4 \quad (33)\\ &+4\left(\frac{\omega_{ce}}{\omega_0}\right)^6\left(\frac{\omega_{pe}}{\omega_0}\right)^6+4\left(\frac{\omega_{ce}}{\omega_0}\right)^8\left(\frac{\omega_{pe}}{\omega_0}\right)^4\Bigg]E_xE_x^*,\end{aligned}$$

It is important to mention that we have neglected the terms of higher orders than $E_xE_x^*$ order.

The relativistic factor γ may be written as following [24]

$$\gamma \approx 1+\alpha E_x E_x^*, \quad (34)$$

Thus by equaling terms of order $E_xE_x^*$ in both Eq.(33) and Eq.(34) we get

$$\begin{aligned}\alpha E_xE_x^* = \frac{1}{4}\left(\frac{e}{m_0\omega_0 c}\right)^2\Bigg[&1+3\left(\frac{\omega_{ce}}{\omega_0}\right)^2+4\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^2+2\left(\frac{\omega_{ce}}{\omega_0}\right)^4\\ &+5\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^4+2\left(\frac{\omega_{ce}}{\omega_0}\right)^2\left(\frac{\omega_{pe}}{\omega_0}\right)^6+13\left(\frac{\omega_{ce}}{\omega_0}\right)^4\left(\frac{\omega_{pe}}{\omega_0}\right)^4\\ &+8\left(\frac{\omega_{ce}}{\omega_0}\right)^6\left(\frac{\omega_{pe}}{\omega_0}\right)^2+6\left(\frac{\omega_{ce}}{\omega_0}\right)^4\left(\frac{\omega_{pe}}{\omega_0}\right)^6+8\left(\frac{\omega_{ce}}{\omega_0}\right)^6\left(\frac{\omega_{pe}}{\omega_0}\right)^4 \quad (35)\\ &+4\left(\frac{\omega_{ce}}{\omega_0}\right)^6\left(\frac{\omega_{pe}}{\omega_0}\right)^6+4\left(\frac{\omega_{ce}}{\omega_0}\right)^8\left(\frac{\omega_{pe}}{\omega_0}\right)^4\Bigg]E_xE_x^*,\end{aligned}$$

where α refers to the relativistic nonlinearity which is occurring due to electron mass increment which oscillating in high intense laser field.

Substituting the value of γ from Eq. (34) into Eq.(22), one obtains

$$\varepsilon_r = 1-\frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)}+\left[\frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)^2+\frac{\omega_{ce}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)^2}\right]\alpha E_xE_x^*, \quad (36)$$

In the above equation ε_r includes the non relativistic part ε_0 and the relativistic part ε_2 where,

$$\varepsilon_0 = 1-\frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)}, \quad (37)$$

$$\varepsilon_2=\left[\frac{\frac{\omega_{pe}^2}{\omega_0^2}\left(\left(1-\frac{\omega_{pe}^2}{\omega_0^2}\right)^2+\frac{\omega_{ce}^2}{\omega_0^2}\right)}{\left(1-\frac{\omega_u^2}{\omega_0^2}\right)^2}\right]\alpha. \quad (38)$$

3.2 Relativistic self focusing with $\vec{B}_0 \perp \vec{k}$

The wave equation governing the laser beam propagation inside plasma may be written as

$$\nabla^2\vec{E}_0=\nabla\left(\nabla\cdot\vec{E}_0\right)-\frac{\omega_0^2}{c^2}\left(\varepsilon_r\cdot\vec{E}_0\right), \quad (39)$$

Using Eq. (27) and Eq.(28) we can write [23]

$$\nabla\cdot\vec{E}_0 \approx \frac{-\varepsilon_{xz}}{\varepsilon_{zz}}\left(\frac{\varepsilon_{xz}}{\varepsilon_{zz}}\frac{\partial E_x}{\partial x}+\frac{\partial E_x}{\partial z}\right), \quad (40)$$

Substituting Eq.(36) and Eq.(40) in Eq.(39)

$$\frac{\partial^2E_x}{\partial z^2}+\frac{\partial^2E_x}{\partial y^2}=\left(k_0^2+\frac{\omega_0^2}{c^2}\varepsilon_2E_xE_x^*\right)E_x, \quad (41)$$

where the products of nonlinear terms with second order space derivatives have been ignored and assuming $\frac{\partial}{\partial x}=0$ in two dimensional beam case.

To solve Eq.(41) we introduce [20]

$$E_x=A_x\exp i\left(\omega_0t-k_0z\right), \quad (42)$$

where A_x is a complex function of space which may be written as [21]

$$A_x=A_0\left(y,z\right)\exp i\left(k_0S\right), \quad (43)$$

A_0 is a real function and S is the phase of the beam laser.

Putting Eq. (42) and Eq.(43) in Eq.(41) , then separating real and imaginary parts we get

$$2\frac{\partial S}{\partial z}+\left(\frac{\partial S}{\partial y}\right)^2=\frac{\varepsilon_2}{\varepsilon_0}A_0^2+\frac{1}{k_0^2A_0}\frac{\partial^2A_0}{\partial y^2}, \quad (44)$$

$$\frac{\partial A_0^2}{\partial z}+\left(\frac{\partial S}{\partial y}\right)\left(\frac{\partial A_0^2}{\partial y}\right)+\frac{\partial^2S}{\partial y^2}A_0^2=0, \quad (45)$$

Assuming $S=\frac{1}{2}y^2\beta\left(z\right)+\phi$ and introducing Gaussian laser beam $A_0^2=\frac{E_{00}^2}{f}\exp\left(\frac{-y^2}{r_0^2f}\right)$ where $\beta\left(z\right)$ is the curvature of the wave front and $f\left(z\right)$ beam width parameter also using paraxial ray approximation then substituting in Eq.(45) we get $\beta\left(z\right)=\frac{1}{f}\frac{df}{dz}$.

Employing A_0^2 and $\beta\left(z\right)$ in Eq. (44) thus for initially plane wavefront conditions (i.e. $f=1$ and $\frac{df}{dz}=0$ at $z=0$) we obtain

$$\frac{d^2f}{dz^2}=\frac{1}{R_d^2f^3}-\frac{\varepsilon_2E_{00}^2}{\varepsilon_0r_0^2f^2}, \quad (46)$$

where $R_d=k_0r_0^2$ represents diffraction length and r_0 is the initial radius beam.

In term of normalization distance of propagation $\xi=\frac{z}{R_d}$, the last equation may be rewritten to become more suitable for the computing programs as follows

$$\frac{d^2f}{d\zeta^2}=\frac{1}{f^3}-\frac{R_d^2\varepsilon_2E_{00}^2}{\varepsilon_0r_0^2f^2}, \quad (47)$$

The first term in RHS of Eq.(47) represents the diffraction term (linear term) while the second term is the converging term (nonlinear term).When the initial laser power is greater than critical power , the nonlinear term will overcome the linear term and hence the laser beam spot size f will decrease along beam propagation direction. One can expect that at the balance of the diffraction and converging terms the laser beam will propagate inside plasma with constant spot size.

4. Result discussion and conclusions

The plasma density will fluctuate as a result of the relativistic nonlinearity by the Gaussian laser beam. The beam width parameter of incident laser beam also will undergo variations leading to vary the laser beam intensity via plasma, this may be understood depending on Eq. (18) in longitudinal case and Eq.(47) in perpendicular case. Numerical simulations have been achieved to Eq. (18) and Eq. (47) with following typical parameters: laser beam intensity $\approx 4\times10^{16} W.cm^{-2}$ (laser strength parameter $\alpha_0 = \frac{eE_{00}}{m_e \omega_0 c} \approx 0.5$), the frequency of pump laser $(\omega_0) = 10^{14}\, rad.\sec^{-1}$, laser beam radius $3\mu m$, plasma density $(n_0) = 2.55\times10^{18} cm^{-3}$ and external magnetic fields $(B_0) = (51, 102, \text{and } 153) MG$.

Figure (1) (related with longitudinal case) and Figure (2) (related with perpendicular case) demonstrate the nonlinear manner of laser beam propagation inside magnetoplasma to several magnetic field values $(B_0) = (51, 102, \text{and } 153) MG$ while relativistic nonlinearity is considered. The decreasing of laser beam spot size is due to the increasing of magnetic field values.

For longitudinal case, Figure (1) shows that the beam width parameter f_+ of laser beam is decreasing after short normalized distance of propagation ξ_+ and leading to increase the laser beam intensity extremely (see Eq. (18)). On the other hand in perpendicular case the beam width parameter f of laser beam is decreasing after shorter normalized distance of propagation ξ than longitudinal case (see Figure 2).

One may conclude that the presence of magnetic field has significant role on enhanced the self-focusing in both cases but in perpendicular case this role will be more affected on self-focusing laser beam. Therefore for confined plasma applications the perpendicular magnetic field will be more candidate than longitudinal magnetic field.

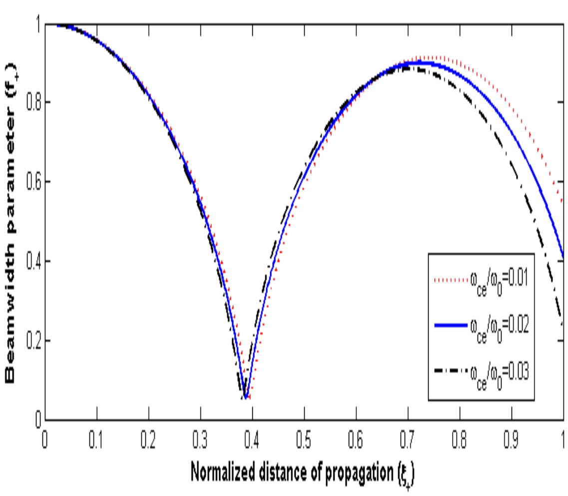

FIG.1. Variation of beam width parameter f_+ with normalized distance ξ_+ when $\vec{B} \parallel \vec{k}$. Where dotted red line, blue solid line and semi-dotted black line represent $\frac{\omega_{ce}}{\omega_0} = 0.01, \frac{\omega_{ce}}{\omega_0} = 0.02$ and $\frac{\omega_{ce}}{\omega_0} = 0.03$ respectively.

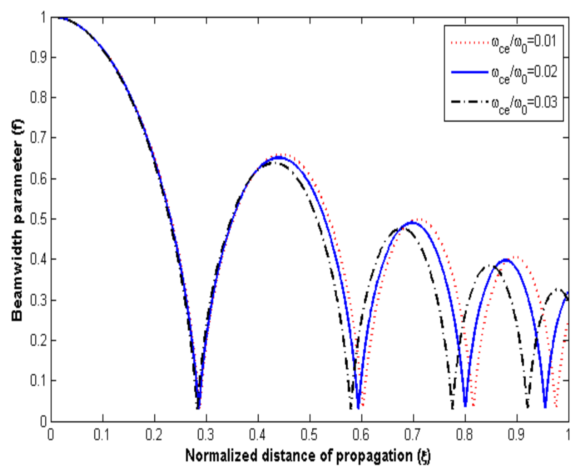

FIG.2. Variation of beam width parameter f with normalized distance ξ when $\vec{B} \perp \vec{k}$. Where dotted red line, blue solid line and semi-dotted black line represent $\frac{\omega_{ce}}{\omega_0} = 0.01, \frac{\omega_{ce}}{\omega_0} = 0.02$ and $\frac{\omega_{ce}}{\omega_0} = 0.03$ respectively.

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