δ- Continuous in bi topological Space By Gem-set

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Abstract

The study of a new type of continuity in bi topological space by Gem-set is introduced as .
1. Basic definition . 2. δ-☆-continuous on (δ-☆-open , δ-☆-closed , δ-☆-interior , δ-☆-closure) in bi topological space) , 3. δ-☆-continuous on separation Axioms in bi topological space . with proof of some theorem .
keywords: Gem-set, Bi topological, Theorem

الخلاصة

من خلال دراستي لمفهوم ((مجموعة الجوهرة)) توصلت الى نوع جديد من الاستمرارية في الفضاءات الثنائية التبولوجيا بواسطة المجموعة الجوهرة وقد قمت بأثبات بعض النظريات والخصائص الخاصة بالاستمرارية وكذلك بديهيات الفصل بالاعتماد على المجموعة الجوهرة

الكلمات المفتاحية: المجموعة ,الجوهرة الفضاءات, الثنائية التبولوجيا نظرية.

Introduction

This research establishes a relation between bi topological spaces , initiated by Kelly (1963) .

Defined as : A set equipped with two topologies is called a bi topological space , denoted by (X, τ, Ω) where (X, τ) , (X, Ω) are two topological space defined on X and $\propto$-Gem set in topological space , define A^{*^x} with respect to space (X, τ) as follows $A^{*^x} = \{y \in X; G \cap A \notin \mathrm{I}_x for\ every\ G \in \tau(y)\}$ is called " Gem-set " in topological space .

A new definition for δ- Gem-set in bi topological space define $A^{☆^x} = \{y \in X: G \cap A \notin \mathrm{I}_x, for\ every\ G \in \tau(y)\ or\ \Omega(\mathrm{y})\}$ from the relation above , the following generalization is formulated between $\propto$- Gem-set in topological space and $\delta^{☆}$-Gem-set in bi topological space .

And the research consists of basic definition and $\delta^{☆}$- continuous in bi topological space by Gem-sets

(δ-☆-open , δ-☆-closed , δ-☆-interior , δ-☆-closed)

List of symbols

Symbols	Description
A^{*^x}	$\propto$-Gem set in (X, τ)
$p_r^{*^x}(A)$	$A^{*^x} \cup A$
$A^{☆^x}$	$\propto$-Gem set in (X, τ, Ω)
$p_r^{☆^x}(A)$	$A^{☆^x} \cup A$
$\tau(y)$	The collection of all open subsets containing the point y

I_x	I deal at point x
$N(x)$	The neighborhood system at a point x in (X,τ)
$M(x)$	The neighborhood system at a point x in (X,Ω)
δ-☆-cl(x)	The collection of all δ -close subset in (X,τ,Ω)
δ-☆-o(x)	The collection of all δ -open subset in (X,τ,Ω)
τ-int(A)	The set of all interior point of A in (X,τ)
Ω-cl(A)	The set of all closure subsets A of (X,Ω)
$f: x \to y$	Single-valued function

1. Basic Definitions

1.1 Definition by Kelly (1963):

A set equipped with two topologies is called a bi topological space , denoted by (X,τ,Ω) where (X,τ) , (X,Ω) are two topological spaces defined on X.

1.2 Definition by Noiril (1974) :

Let (X,τ) be a topological space , and $A \subset X$, A is said to be $\propto$-open set iff $A \subset A^{o-o}$.

1.3 Definition

Let (X,τ,Ω) be a bi topological space . and A be a subset of X A is said to be δ-open set iff $A \subset \tau$-int (Ω-cl(τ-int(A))) .

1) **1.4 Definition (**Manoharan and Thangarelu , 2013) **:**

Let X be a non-empty set , A family I of subset of X is an ideal on X if :

i- $A \in I$ and $B \subseteq A$, then $B \in I$ (heredity)

ii- $A, B \in I$, then $A \cup B \in I$ (finite additivity)

1.5 Definition :

Let (X,τ) be a topological space with an ideal I on X , Then for any subset A of X , $A^*(I,\tau) = \{x \in X : U \cap A \in I \; for \; every \; U \in N(x)\}$ is called the local function of A with respect to I and τ , simply write $A^*(I,\tau)$ is case then is no chance for confusion . Also , cl$^*(A) = A \cup A^*$ defines kuratowski closere operator for topology τ^* which is a finer then τ for a topological space (X,τ) and $x \in X$, the ideal I_x [25][4] define by $I_x = \{G \in X : x \in G^c\}$.

1.6 Definition(AL-Swidi and AL-Nafee ,2013) **:**

Let (X,τ) be a topological space , $A \subseteq X$ and $x \in X$ Define A^{*^x} with respect to space (X,τ) as follows : $A^{*^x} = \{y \in X : G \cap A \notin I_x \; for \; every \; G \in \tau(y)\}$. A set A^{*^x} is called "Gem-set" we write the $pr^{*^x}(A) = A^{*^x} \cup A$. Thus $pr^{*^x}(F) \subseteq cl(F)$

A new definition for δ-☆-open set in bi topological space.

1.7 Definition :

Let (X,τ,Ω) be a bi topological space with an ideal I on X. for any subset A of X, $A^{☆}(I,\tau,\Omega)=\{x\in X; U\cap A\notin I\ for\ every\ U\in N(x)\ in\ (X,\tau)\ or\ U\in M(x)\ in\ (X,\Omega)\}$ is called the local function of A with respect to I and τ or with respect to I and Ω

Also Ω-cl$^{☆}(A)=A\cup A^{☆}$ defines kuratowski chosere operator for τ^{*} is finer than τ .

For a bi topological space (X,τ,Ω) and $x\in X$, the ideal I_x define by $I_x=\{G\subseteq X, x\in G^{c}\}$.

1.8 Definition :

Let (X,τ,Ω) be a bi topological space , $A\subseteq X$ and $x\in X$, Define $A^{☆^{x}}$ with respet to space (X,τ,Ω) as follows :

$$A^{☆^{x}}=\{y\in X: G\cap A\notin I_x\ , for\ every\ G\in\tau(y)\ or\ G\in\Omega(y)\}$$

A set $A^{☆^{x}}$ is called "Gem-set" . we write the $pr^{☆^{x}}(A)=A^{☆^{x}}\cup A$ Thus $pr^{☆^{x}}(F)\subseteq$ Ω-cl(F) .

1.9 Definition

A subset A of an bi topological space (X,τ,Ω) is called

i. δ-open if $A\subset\tau$-int(Ω-cl(τ-int(A)))

ii. δ-open if $A\subset\tau$-int(Ω-cl$^{☆}$(τ-int(A)))

The collection of all δ-open sets in (ii) is denoted by δ-☆-o(x) and the collection of all δ-open sets is denoted by δ-o(x)

1.10 Definition

A bi topological space (X,τ,Ω) is said to be δ-☆-closed space if and only if each non-empty subset A of X is δ-☆-closed subset .

1.11 Definition

A bi topological space (X,τ,Ω) is said to be δ-☆-perfected space if and only if each non-empty subset A of X is δ-☆- perfected subset .

1.12 Lemma

Let (X,τ,Ω) be a bi topological space with I and J being ideal son X, and let A and B be two subset X then

i- $A\subseteq B$ then $A^{☆}\subseteq B^{☆}$

ii- $I\subseteq J$ then $A^{☆}(J)\subseteq B^{☆}(I)$

iii- $A^{☆}=$ Ω-cl$\left(A^{☆}\right)\subseteq$ Ω- cl(A)

iv- $\left(A^{☆}\right)^{☆}\subseteq A^{☆}$

v- $(A\cup B)^{☆}=A^{☆}\cup B^{☆}$

vi- $A^{☆}-B^{☆}=(A-B)^{☆}-B^{☆}\subseteq(A-B)^{☆}$

vii- For every $I_1\in I$, $(A-I_1)^{☆}=(A-I)^{☆}$

If this is casy to prove by the properties in Lemma (1.12) with respect to Gem-sets .

1.13 Definition

Let (X,τ,Ω) be a bi topological space and $A\subseteq X$, defined $pr^{☆^{x}}(A)=A^{☆^{x}}\cup A$, for each $x\in X$.

1.14 Definition

A subset A of a bi topological space (X,τ,Ω) is called perfected set if $A^{☆^x} \subseteq A$, for each $x \in X$.

1.15 definition

A bi topological space (X,τ,Ω) is said to be perfected space if and only if , each non-empty subset A if y is perfected subset .

2. δ-☆- continuous map in bi topological space

2.1 δ-☆- continuous on (δ-☆- open , δ-☆- closed , δ-☆-interior , δ-☆- closure) in bi topological space

2.1.1 Definition

Let (X,τ,Ω) and $(Y,\acute{\tau},\acute{\Omega})$ be a bi topological space , A mapping $f\colon X \longrightarrow Y$ is said to be δ-☆-continuous at $x_\circ \in X$ iff for every δ-☆-open set V in Y containing $f(x_\circ)$ there exist δ-☆-open set U in Y containing $x_\circ$ such that $f(U) \subset V$.

2.1.2 Definition :

Let $f\colon X \longrightarrow Y$ be a mapping , then

(a) f is said to be δ-☆-open mapping iff $f(G)$ is δ-☆-open in Y for every δ-☆-open set G in X .

(b) F is δ-☆-closed iff $f(F)$ is δ-☆-closed in Y for every δ-☆-closed set F in X .

(c) f is δ-☆-continuous iff f is δ-☆-open and δ-☆-closed .

(d) f is δ-☆-homeomorphism iff

i- f is bijective (1-1 , onto)

ii- f and f^{-1} are δ-☆-continuous where (X,τ,Ω) , $(Y,\acute{\tau},\acute{\Omega})$ are two bi topological space .

2.1.3 Example (1)

Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\}\}$, $\Omega=\{\emptyset,X\}$

(X,τ) , (X,Ω) are two topologies on X

Then (X,τ,Ω) is a bi topological space , such that

δ-☆-o$(x)=\{\Phi,X,\{a\},\{a,b\},\{a,c\}\}$

And let $Y=\{1,2,3\}$, $\acute{\tau}=\{\emptyset,Y,\{1\}\}$, $\acute{\Omega}=\{\emptyset,Y\}$

$(Y,\acute{\tau})$, $(Y,\acute{\Omega})$ are two topologies on Y

Then $(Y,\acute{\tau},\acute{\Omega})$ is a bi topological space , such that

δ-☆-o$(y)=\{\emptyset,Y,\{1\},\{1,2\},\{1,3\}\}$

Define $f\colon(X,\tau,\Omega)\to(Y,\acute{\tau},\acute{\Omega})$ by $f(a)=1, f(b)=2, f(c)=3$

Then f is δ-☆-continuous and δ-☆-open set because

$f^{-1}(c)=\{a,b,c\}=X$ is δ-☆-open in X , and $f^{-1}(\emptyset)=\emptyset$ is δ-☆-open in X , similarly the other cases $f^{-1}(\{1\}), f^{-1}(\{1,2\}), f^{-1}(\{1,3\})$ are δ-☆-open in Y , there fore f is δ-☆-continuous . and since $f(\{a\})=\{1\}$ is δ-☆-open in Y and $f(\{\emptyset\})=\emptyset$ is δ-☆-open in Y , similarly the other cases $f(\{x\})$, $f(\{a,b\})$, $f(\{a,c\})$ are δ-☆-open in Y . therefore f is δ-☆-open mapping .

2.1.3 Example (2)

Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\}\}$, $\Omega=\{\emptyset,X,\{b\},\{a,b\}\}$

(X,τ) , (X,Ω) are two topologies on X
Then (X,τ,Ω) is a bi topological space , such that
δ-☆-o$(x) = \{\emptyset, X, \{a\}\}$
And let $Y = \{1,2,3\}$, $\acute{\tau} = \{\emptyset, X, \{2\}, \{1\}, \{1,2\}\}$,
$\acute{\Omega} = \{\emptyset, X, \{1\}, \{1,2\}\}$
$(Y,\acute{\tau})$, $(Y,\acute{\Omega})$ are two topologies on Y
Then $(Y,\acute{\tau},\acute{\Omega})$ is a bi topological space , such that
δ-☆-o$(y) = \{\emptyset, Y, \{1\}, \{2\}, \{1,2\}\}$
Define $f: (X,\tau,\Omega) \to (Y,\acute{\tau},\acute{\Omega})$ by $f(a) = 1, f(b) = f(c) = 2$

Then f is δ-☆-open but not δ-☆-continuous because

$f^{-1}(Y) = \{a,b,c\} = X$ is δ-☆-open in X and $f^{-1}(\emptyset) = \emptyset$ is δ-☆-open in X . but $f^{-1}(\{2\}) = \{b,c\}$ is not δ-☆-open in X , Hence f is δ-☆-continuous .

And since $f(x) = \{1,2\}$ is δ-☆-open in Y , $f(\{\emptyset\}) = \emptyset$ is δ-☆-open in Y . $f(a) = (\{1\})$ is δ-☆-open in Y therefore f is δ-☆-open .

2.1.5 Theorem

Let (X,τ,Ω) and $(Y,\acute{\tau},\acute{\Omega})$ be bi topological space , then a mapping $f: X \to Y$ is δ-☆-continuous iff for every $x \in X$ the inverse image under f of every δ-☆-open $V \circ f\ f(x)$ is δ-☆-open set of Y .

Proof

Let f is δ-☆'-continuous and V is δ-☆-open in Y to prove $f^{-1}(V)$ is δ-☆-open in X . If $f^{-1}(V) = \emptyset$ so it is δ-☆-open in X . If $f^{-1}(V) \neq \emptyset$, Let $x \in f^{-1}(V)$, then $f(x) \in V$, by definition of δ-☆-continuous there exists δ-☆-open G_x in X containing x such that $f(G_x) \subset V$.

$\therefore x \in G_x \subset f^{-1}(V)$, Hence $f^{-1}(V)$ is δ-☆-open set in X .

Conversely

Let $f^{-1}(V)$ is δ-☆-open set in X , for each V is δ-☆-open set in Y to prove f is δ-☆-continuous .

Let $x \in X$ and V is δ-☆-open set in Y containing $f(x)$ so $f^{-1}(V)$ is δ-open in X containing x and $f(f^{-1}(V)) \subset V$. Then f is δ-☆-continuous on X .

2.1.6 Theorem

Let (X,τ,Ω) and $(Y,\acute{\tau},\acute{\Omega})$ be bi topological space , a mapping $f: X \to Y$ is δ-☆-continuous iff the inverse image under $f \circ f$ every δ-☆-closed set in Y is δ-☆-closed set in X .

Proof : (Obvious)

2.1.7 Theorem

A mapping $f: X \to Y$ is δ-☆-continuous iff $f(\delta$-☆-cl$(A)) \subset \delta$-☆-cl$(f(A))$ for every $A \subset X$, where (X,τ,Ω) and $(Y,\acute{\tau},\acute{\Omega})$ are two bi topological space .

Proof

Let f be δ-☆-continuous . Since δ-☆-cl$(f(A))$ is δ-☆-closed set in Y $\therefore f^{-1}(\delta$-☆-cl$(f(A)))$ is δ-☆-closed set in X by [2.1.6] therefore δ-☆-cl$(f^{-1}(\delta$-☆-cl$(f(A)))$ $= f^{-1}(\delta$-☆-cl$(f(A)))$…(1)

Now
$f(A) \subset \delta$-cl$(f(A))$, $A \subset f^{-1}(f(A)) \subset f^{-1}(\delta$-☆-cl$(f(A)))$.
Then δ-☆-cl$(A) \subset f^{-1}(\delta$-☆-cl$(f(A)))= f^{-1}(\delta$-☆-cl$(f(A)))$by1

Then $f(\delta$-☆-cl$(A)) \subset \delta$-☆-cl$\big(f(A)\big)$.
Conversely :
Let $f(\delta$-☆-cl$(A)) \subset \delta$-☆-cl$\big(f(A)\big)$ for every $A \subset X$
Let F be any δ-☆-closed set in Y , So that δ-cl$(F) = F$
Now , $f^{-1}(F) \subset X$, by hypothesis .
$f(\delta$-☆-cl$\big(f^{-1}(F)\big))\subset \delta$-☆-cl$\Big(f\big(f^{-1}(F)\big)\Big) \subset \delta$-☆-cl$(F) = F$
Therefore δ-☆-cl$\big(f^{-1}(F)\big) \subset f^{-1}(F)$
But $f^{-1}(F) \subset \delta$-☆-cl$\big(f^{-1}(F)\big)$ always
Hence δ-cl$\big(f^{-1}(F)\big) \subset f^{-1}(F)$ and $f^{-1}(F)$ are δ-☆-closed set in X Hence f is δ-☆-continuous by theorem [2.1.6]

2.1.8 Theorem

A mapping $f: X \to Y$ is δ-☆-continuous iff . δ-☆-cl$\big(f^{-1}(B)\big) \subset f^{-1}(\delta$-☆-cl$(B))$ for every $B \subset Y$, Where (X, τ, Ω) and $\big(Y, \acute{\tau}, \acute{\Omega}\big)$ are two bi topological space .
Proof : (Obvious)

2.1.9 Theorem

A mapping $f: X \to Y$ is δ-☆-continuous iff
$f^{-1}(\delta$-☆-int$(B)) \subset \delta$-☆-int$\big(f^{-1}(B)\big)$ for every $B \subset Y$, where (X, τ, Ω) and $\big(Y, \acute{\tau}, \acute{\Omega}\big)$ are two bi topological space .
Proof : (Obvious)

2.1.10 Theorem

Let X, Y and Z be a bi topological space and the mappings $f: X \to Y$ and $g: Y \to Z$ be δ-☆-continuous then the composition map $g \circ f: X \to Z$ is δ-☆-continuous .
Proof : (Obvious) (using definition 2.1.2 c+d)

2.2 δ-☆-continuous on separation Axioms in Bi topological space

2.2.1 Theorem

Let $\big(Y, \acute{\tau}, \acute{\Omega}\big)$ be δ-☆-To space , if $f: (X, \tau, \Omega) \to \big(Y, \acute{\tau}, \acute{\Omega}\big)$ is δ-☆-continuous 1-1 function . Then (X, τ, Ω) is δ-☆-To space .

Proof :

Let $x_1, x_2 \in X , x_1 \neq x_2$, since f is 1-1 function , then $f(x_1) \neq f(x_2)$, $f(x_2) \in Y$, and Y is δ-☆-To space , then there exists δ-☆-open set G in Y such that $f(x_1) \in G , f(x_1) \notin G$ So $x_1 \in f^{-1}(G) , x_2 \in f^{-1}(G)$.

$\therefore f^{-1}(G)$ is δ-☆-open set in Y , Then (X, τ, Ω) is δ-☆-To space .

2.2.2 Theorem

Let $f: (X, \tau, \Omega) \to \big(Y, \acute{\tau}, \acute{\Omega}\big)$ be an δ-☆-continuous δ-☆-open 1-1 and onto function , If (X, τ, Ω) is δ-To space then $\big(Y, \acute{\tau}, \acute{\Omega}\big)$ is δ-☆-To space .

Proof :

Suppose that $y_1, y_2 \in Y , y_1 \neq y_2$, since f is onto , there exists $x_1, x_2 \in X$, such that $y_1 = f(x_1) , y_2 = f(x_2)$ and since f is 1-1 , then $x_1 \neq x_2$, since X is δ-To space . There exists δ-☆-open set G , such that $x_1 \in G , x_2 \notin G$.

Hence $y_1 = f(x_1) \in f(G) , y_2 = f(x_2) \notin f(G)$, since f is δ-☆-open function , then $f(G)$ is δ-☆-open set Y . there fore $\big(Y, \acute{\tau}, \acute{\Omega}\big)$ is δ-☆-To space .

2.2.3 Theorem

Let $(Y, \acute{\tau}, \acute{\Omega})$ be δ-T_1 space . if $f: (X, \tau, \Omega) \to (Y, \acute{\tau}, \acute{\Omega})$ is δ-✫-continuous 1-1 function , then X is δ-✫-T_1 space .

Proof

Let $x_1, x_2 \in X$, $x_1 \neq x_2$, since f is 1-1 , $f(x_1) \neq f(x_2)$, $f(x_1)$, $f(x_2) \in Y$, Y is δ-✫-T_1 space , then there exists U_1, U_2 δ-✫-open set in Y such that $f(x_1) \in U_1$, but $f(x_2) \in U_1$ and $f(x_2) \in U_2$ but $f(x_1) \notin U_2$.

Then $x_1 \in f^{-1}(U_1)$but $x_2 \notin f^{-1}(U_1)$; and $x_2 \in f^{-1}(U_2)$, but $x_1 \notin f^{-1}(U_2)$; and $f^{-1}(U_1)$, $f^{-1}(U_2)$ are δ-✫-open set in X Hence (X, τ, Ω) is δ-✫-T_1 space .

2.2.4 Theorem

Let $f: (X, \tau, \Omega) \to (Y, \acute{\tau}, \acute{\Omega})$ be an δ-✫-continuous 1-1 and onto , δ-✫-open function . If (X, τ, Ω) is δ-✫-T_1 space then $(Y, \acute{\tau}, \acute{\Omega})$ is δ-✫-T_1 space .

Proof

Suppose $y_1, y_2 \in Y$, $y_1 \neq y_2$, since f is onto , there exists $x_1, x_2 \in X$, Such that $y_1 = f(x_1)$, $y_2 = f(x_2)$, since f is 1-1 then $x_1 \neq x_2 \in X$, $f(x_1) \neq f(x_2)$, and X is δ-✫-T_1 space , there exists δ-✫-open sets G , H such that $x_1 \in G$ but $x_2 \notin G$ and $x_2 \in H$ but $x_1 \notin H$.

Hence $f(x_1) \in f(G)$, $f(x_2) \in f(H)$, since f is δ-✫-open function , Hence $f(G)$, $f(H)$ are δ-✫-open sets of Y .

$y_1 \in f(G)$, but $y_2 \notin f(G)$ and $y_2 \in f(H)$, but $y_1 \notin f(H)$

Then $(Y, \acute{\tau}, \acute{\Omega})$ is δ-✫-T_1 space .

2.2.5 theorem

Let $(Y, \acute{\tau}, \acute{\Omega})$ be δ-✫-T_2 space . if $f: (X, \tau, \Omega) \to (Y, \acute{\tau}, \acute{\Omega})$ is δ-✫-continuous 1-1 function , then (X, τ, Ω) is δ-✫-T_2 space .

Proof

Let $x_1 \neq x_2 \in X$, since f is 1-1 , $f(x_1) \neq f(x_2)$

Let $y_1 = f(x_1)$, $y_2 = f(x_2)$, $y_1 \neq y_2$. since Y is δ-✫-T_2 space , there exists two δ-✫-open sets G . H in Y , such that $y_1 \in G$, $y_2 \in H$, $G \cap H = \emptyset$.

Hence $x_1 \in f^{-1}(G)$, $x_2 \in f^{-1}(H)$ since f is δ-✫-continuous and $f^{-1}(G)$, $f^{-1}(H)$ δ-✫-open sets in X .

Also $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\emptyset) = \emptyset$

Thus (X, τ, Ω) is δ-✫-T_2 space .

2.2.6 Theorem

Let $f: (X, \tau, \Omega) \to (Y, \acute{\tau}, \acute{\Omega})$ be an δ-✫-continuous 1-1 and onto , δ-✫-open function . If (X, τ, Ω) is δ-✫-T_2 space then $(Y, \acute{\tau}, \acute{\Omega})$ is δ-✫-T_2 space .

Proof

Let $y_1 \neq y_2 \in Y$, since f is 1-1 and onto , there exists $x_1 \neq x_2 \in X$, Such that $y_1 = f(x_1)$, $y_2 = f(x_2)$, since X is δ-✫-T_2 space , then there exists δ-✫-open sets G, H such that $x_1 \in G$, $x_2 \in H$, $G \cap H = \emptyset$, since f is δ-✫-open mapping , then $f(G)$, $f(H)$ are two δ-✫-open set in Y and $f(G \cap H) = f(G) \cap f(H) = f(\emptyset) = \emptyset$.

Also $y_1 = f(x_1) \in f(G)$, $y_2 = f(x_2) \in f(H)$.

Hence $(Y, \acute{\tau}, \acute{\Omega})$ is δ-✫-T_2 space .

2.2.7 Theorem

Let (X, τ, Ω) be δ-✫-regular space and

$f: (X, \tau, \Omega) \to (Y, \acute{\tau}, \acute{\Omega})$ be δ-✫-homeomorphism , Then $(Y, \acute{\tau}, \acute{\Omega})$ δ-✫-regular.

Proof

Let F be δ-☆-closed set in Y , $q \notin F , q \in Y$. since f is 1-1 and onto map , then there exists $p \in X$ such that $f(p) = q$, $p = f^{-1}(q)$. since f is δ-☆-continuous so $f^{-1}(F)$ is δ-☆-closed in X , $q \notin F , p = f^{-1}(q) \notin f^{-1}(F)$. since (X, τ, Ω) is δ-☆-regular there exists δ-☆-open sets G, H such that $p \in G , f^{-1}(F) \subset H$ and $G \cap H = \emptyset$.

So $q = f(p) \in f(G) , F \subset f\left(f^{-1}(F)\right) \subset f(H)$, since f is δ-☆-open map , hence $f(G) , f(H)$ are δ-☆-open sets in Y and $f(G \cap H) = f(G) \cap f(H) = f(\emptyset) = \emptyset$.

Therefore $\left(Y, \acute{\tau}, \acute{\Omega}\right)$ is δ-☆-regular .

2.2.8 Theorem

δ-☆-normality is bi topological property .

Proof

Let (X, τ, Ω) be δ-☆-normal space and Let $\left(Y, \acute{\tau}, \acute{\Omega}\right)$ be δ-☆-homeomorphic image of (X, τ, Ω) under δ-☆-homeomorphic f to show that $\left(Y, \acute{\tau}, \acute{\Omega}\right)$ is also . δ-☆-normal space .

Let L, M be a pair of dis joint δ-☆-closed subsets of Y . since f is δ-☆-continuous map , then $f^{-1}(L)$ and $f^{-1}(M)$ are f is δ-☆-closed subsets of X . Also $f^{-1}(L) \cap f^{-1}(M) = f^{-1}(L \cap M) = f^{-1}(\emptyset) = \emptyset$.

Thus $f^{-1}(L), f^{-1}(M)$ are disjoint pair of δ-☆-closed subsets of X . since the space (X, τ, Ω) is δ-☆-normal , then there exist δ-☆-open set G and H such that $f^{-1}(L) \subset G , f^{-1}(M) \subset H$ and $G \cap H = \emptyset$ but $f^{-1}(L) \subset G$ then $f\left(f^{-1}(L)\right) \subset f(G) , L \subset f(G)$.

Similarly

$M \subset f(H)$, Also since f is an δ-☆-open mapping $f(G)$ and $f(H)$ are δ-☆-open subset of Y , such that

$f(G) \cap f(H) = f(G \cap H) = f(\emptyset) = \emptyset$

Thus there exists δ-open subset in Y , $G_1 = f(G)$ and $H_1 = f(H)$ such that $L \subset G_1$, $M \subset H_1$, and $G_1 \cap H_1 = \emptyset$.

It follows that $\left(Y, \acute{\tau}, \acute{\Omega}\right)$ is also δ-☆-normal space .

Accordingly , δ-☆-normality is a bi topological property .

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