Totally sb* - Continuous Functions and sb*- Totally Continuous Functions in Topological Spaces

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Abstract
The aim in this work is to give some a new types of totally - continuous functions called (totally sb* - continuous functions and sb*- totally continuous functions) in topological spaces. As well as, will be investigated and discussion the relationships among these functions with other totally – continuous functions. Furthermore, will be introducing some of their properties.
Key words: totally continuous function, sb*-continuous ,sb*-open, sb*-closed

الخلاصة
الهدف الرئيسي في هذا ألعمل هو تقدم نوع جديد من الدوال التامة المستمرة تدعى (الدول المستمرة –sb* التامة و الدوال التامة المستمرة– sb*) في الفضاءات التبولوجية . ايضا سوف نبحث و نناقش العلاقات في ما بين تلك الدوال مع دوال مستمرة– تامة اخرى . علاوة على ذلك، سوف نقدم بعض من خواصها.
الكلمات المفتاحية: سالب sb* المستمرة، سالب sb* المفتوحة، سالب sb* المغلقة

1-Introduction :
Many different forms of continuous functions have been introduced over the years. Some of them are totally–continuous functions [Jain 1980], in 1995, Nour. T.M., introduced and studied the concepts of totally semi- continuous functions. Recently, Rajesh (2007), and Caldasm and Rajesh 2009), introduce the notions of totally ω-continuous functions and totally b- continuous functions respectively. In (2011), Raviet *et al.*, They give and investigated the concept of totally sg- continuous functions. While, the concepts (sb*- open sets , sb*- closed sets and sb*- continuous functions) were discussed and introduced by (Poongothai and Parimelazhagan , 2012) .

The purpose of this paper, we introduce and study a new types of totally– continuous functions which are (totally sb*-continuous functions and sb*-totally continuous functions) . Also, we give several properties of these functions are proved.

Throughout this paper (X, τ), (Y, σ) and (Z ,μ) (or simply X,Y and Z) represent non – empty topological spaces .For a subsets A of a spaces X. cl(A), int (A) and A^c denote the closure of A ,the interior of A and the complement of A respectively .

2-Preliminaries:

Some definitions and basic concepts related to this paper.

Definition (2-1):

A subset A of a space (X,τ) is said to be a :

1-**semi open set** [Levine, 1963] if $A\subseteq cL(int(A))$ and **semi closed set** if $int(cl(A)) \subseteq A$.

2- **b-open set** [Andrijevic, 1996] if $A \subseteq int(cl(A)) \cup cl(int(A))$ and **b-closed set** if $int(cl(A)) \cap cl(int(A)) \subseteq A$.

Definition (2-2):

The intersection of all semi-closed subsets of topological space (X,τ) containing a set *A* is called ***semi-closure*** [Levine, 1963] of *A* and is denoted by ***scl(A)* .**

Definition (2-3):

A subset A of a space (X,τ) is said to be a *:*

1-semi- generalized closed set (briefly , sg- closed) [Bhattacharyya1987] if scl(A) ⊆ U whenever A⊆ U and U is semi- open set in X .The complement of sg- closed set is called **sg-open** .

2-**ω- closed set** [Sundaram1995] if cl(A) ⊆ U whenever A⊆ U and U is semi- open set in X . The complement of sg- closed set is called **ω -open** .

3-**strongly b*- closed set** (briefly , sb*- closed) [Poongothai2012] if cl(int(A)) ⊆ U whenever A⊆ U and U is b- open set in X .The complement of sb*- closed set is called **sb*-open** .

The collection of all semi-open subsets (resp . b-open , sg-open , ω -open and sb*-open) subsets in (X,τ) is denoted by SO(X,τ) (resp . BO(X,τ) , SGO(X,τ) ,ωO(X,τ) and SB*O(X,τ)) and the collection all semi-closed subsets (resp . b-closed , sg- closed , ω - closed and sb*- closed) subsets in (X,τ) is denoted by SC(X,τ) (resp . BC(X,τ) , SGC(X,τ) ,ωC(X,τ) and SB*C(X,τ)) .

Remark(2-4),[Poongothai, 2012] :

Every open (resp . closed) subset in (X,τ) is a sb*- open (resp . sb*- closed) subset . But the converse need not be true in general.

Example (2-1):

Let X= {a,b} with the topology τ={X,∅}. Then the set A={a} is a sb*-open subsets but not open subsets in (X,τ) . And Also, A^c= {b} is sb*-closed subset in (X, τ). But is not closed.

Definitions (2-5):

A function f: (X, τ) → (Y, σ) is said to be

1- **Continuous function** [Levine, 1993] if for each open set A of (Y,σ), then f^{-1}(A) open in (X,τ) .

2- **b-continuous function** [El.Atik, 1997] if for each open set A of (Y,σ) , then f^{-1}(A) b-open in(X,τ) .

3- ***sb*-continuous function*** [Poongothai] if for each open set A of (Y,σ) ,then f^{-1}(A) sb*-open in (X,τ) .

A function f: (X, τ) → (Y, σ) is said to be

1- ***totally- continuous function*** [Jain 1980] if the inverse image of every open subset of (Y,σ) is a clopen (i.e., open and closed) subset of (X,τ) .
2- ***totally semi -continuous function*** [Nour1995] if the inverse image of every open subset of (Y,σ) is a semi- clopen (i.e., semi- open and semi-closed) subset of (X,τ) .
3- ***totally b-continuous function*** [El.Atik, 1997] if the inverse image of every open subset of (Y,σ) is a b- clopen (i.e.,b- open and b-closed) subset of (X,τ) .
4- ***Totally ω-continuous function*** [Rajesh, 2007] if the inverse image of every open subset of (Y,σ) is a ω- clopen (i.e., ω- open and ω- closed) subset of (X,τ) .
5- ***Totally sg-continuous function*** [Ravi, 2011] if the inverse image of every open subset of (Y,σ) is a sg-clopen (i.e.,sg- open and sg-closed) subset of (X,τ) .

Definition (2-7) :

A topological space (X,τ) is said to be ***b– space***[Nasef 1999] if every b-open subsets in (X,τ) is open.

3- Totally sb* - Continuous Functions and sb*- Totally Continuous Functions

In this section, we introduce a new type of totally-continuous functions namely (totally sb* -continuous functions and sb*- totally continuous functions) and studying the relations between them . Also, we give some of their propositions .

Definition(3-1):

A function $f : (X ,\tau) \to (Y ,\sigma)$ is said to be ***totally sb*- continuous*** if the inverse image of every open subset in (Y ,σ) is a sb*-clopen subset in (X ,τ).

Example (3-1):

Let X=Y={a,b,c} with the topologies τ ={X,∅, {c},{a,b}} and σ ={Y,∅,{c}} then SB*O(X,τ) = {X, ∅,{c},{a,b},{a,c},{b,c} } . Let $f : (X ,\tau) \to (Y ,\sigma)$ be an identity function . It observe that f is a totally sb*- continuous function .

Proposition (3-2):

A function $f : (X ,\tau) \to (Y ,\sigma)$ is totally sb*- continuous if and only if the inverse of every closed subset in (Y ,σ) is a sb*-clopen subset in (X ,τ).

Proof:

Assume that f is totally sb*- continuous . Let A be any closed subset in Y. Then A^c is a open subset in Y. , since f is totally sb*- continuous . Thus, $f^{-1}(A^c)$ is sb*- clopen subset in X(i.e., sb*- open and sb*-closed subset in X) . But $f^{-1}(A^c) = X - f^{-1}(A)$, and so $f^{-1}(A)$ is both sb*-closed and sb*-open subset . Hence, $f^{-1}(A)$ is sb*-clopen subset in X. ***Conversely,*** let G be an open subset in Y. Then $G^c$is closed subset in X .By assumption $f^{-1}(G^c)$ is sb*-clopen subset in X (i.e., sb*- open and sb*-closed subset in X) . But $f^{-1}(G^c) = X - f^{-1}(G)$,.and so $f^{-1}(G)$ is both sb*-closed and sb*-open subset . Hence, $f^{-1}(G)$ is sb*-clopen subset in X. Therefore, f is totally sb*- continuous function .

Proposition (3-3):

(i) Every totally– continuous function is totally sb* – continuous .

(ii) Every totally ω – continuous function is totally sb* – continuous .

Proof:

(i)Let $f : (X ,\tau) \to (Y ,\sigma)$ be a totally - continuous and A be an open subset in (Y ,σ) . Since f is a totally– continuous function . Thus, $f^{-1}(A)$ is clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both open and closed subset in (X,τ)) by using Remark(2-4) we get $f^{-1}(A)$ is both sb*-open and sb*-closed subset in (X,τ) . This implies that $f^{-1}(A)$ is sb*-clopen subset in (X,τ) .therefore , f is a totally sb* – continuous .

(ii) Let $f : (X ,\tau) \to (Y ,\sigma)$ be a totally ω-continuous and A be an open subset in (Y ,σ) . Since f is a totally ω – continuous function . Thus, $f^{-1}(A)$ is ω –clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both ω – open and ω – closed subset in (X,τ)) , since (Every ω – open set is a sb*-open , [Poongothai 2012] ,and every ω –closed set is a sb*-closed , [Poongothai2012]) we get $f^{-1}(A)$ is both sb*-open and sb*-closed subset in (X,τ) . This implies that $f^{-1}(A)$ is sb*-clopen subset in (X,τ) .therefore , f is a totally sb* – continuous .

The following example shows that the converse of Proposition (3-3) is not true.

Example (3-2): Let X=Y={a,b} with the topologies τ ={X,∅}and σ ={Y,∅,{a}},then SB*O(X,τ) =SB*C(X,τ) = {X, ∅,{a},{b}} . Let $f : (X ,\tau) \to (Y ,\sigma)$ be function defined by f(a)=b and f(b)=a . It observe that f is a totally sb*- continuous function, but f is not totally-continuous function and totally ω- continuous function , since for open set A={a} . f^{-1}(A)= f^{-1}({a})= {b}is not clopen (resp. ω- clopen) subset in (X, τ) .

Proposition (3-4):

Every totally sb*– continuous function $f : (X ,\tau) \to (Y ,\sigma)$ is a

(i) totally b– continuous function .

(ii) b- continuous function .

(iii) sb*- continuous function .

Proof:

(i)Let A be an open subset in (Y ,σ) . Since f is a totally sb*– continuous function . Thus, $f^{-1}(A)$ is sb*-clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both sb*-open and sb*closed subset in (X,τ)) since (Every $sb*-$ open set is a b-open , [Poongothai 2012] ,and every$sb*$–closed set is a b-closed , [Poongothai 2012]) we get $f^{-1}(A)$ is both b-open and b-closed subset in (X,τ) . This implies that $f^{-1}(A)$ is b-clopen subset in (X,τ) .therefore , f is a totally b – continuous .

(ii) Let A be an open subset in (Y ,σ) . Since f is a totally sb* – continuous function . Thus, $f^{-1}(A)$ is sb* –clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both sb* – open and sb* – closed subset in (X,τ)) , since Every $sb*-$ open set is a b-open , [Poongothai 2012] ,and every$sb*$–closed set is a b-closed , [Poongothai 2012]) we get $f^{-1}(A)$ is both b-open and b-closed subset in (X,τ) . and so, $f^{-1}(A)$ is b-open subset in (X,τ) .therefore , f is a b – continuous .

(iii) Let A be an open subset in (Y ,σ) . Since f is a totally sb* – continuous function . Thus, $f^{-1}(A)$ is sb* –clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both sb* – open and sb* – closed subset in (X,τ)) , thus $f^{-1}(A)$ is sb*-open subset in (X,τ) .therefore , f is a sb* – continuous .

The following example shows that the converse of Proposition (3-4) is not true.

Example (3-3):

Let X=Y={a,b,c} with the topologies τ ={X,∅, {a},{a,c}} and σ ={Y,∅,{b,c}},then SB*O(X,τ)={X, ∅ ,{a},{a,c},{a,b}}, SB*C(X,τ) ={X, ∅ ,{c},{a,c},{b,c}}and BO(X,τ) ={X,∅, {a},{b},{a,b},{a,c},{b,c}}. Let $f : (X ,\tau) \to (Y ,\sigma)$ f(a)=a, f(b)=b and f(c)=c . It observe that f is a totally b- continuous function ,but f is not totally sb*- continuous. Since for open subset A={a,c}in (Y,σ) . f^{-1}(A)= f^{-1}({a,c})= {a,c}is sb*-closed but is not sb*- open subset in (X, τ) . Hence, f^{-1}(A)= f^{-1}({a,c})= {a,c}is not sb*-clopen subset in X .

Example (3-4):

Let X=Y={a,b,c}with the topologies τ={X ∅ ,{a},{b},{a,b}} and σ={Y, ∅ ,{a},{b},{a,b},{b,c}},then SB*O(X,τ) ={X, ∅ ,{a},{b},{a,b}}, SB*C(X,τ) ={X,∅,{c},{a,c},{b,c}}, BO(X,τ)={X ,∅, {a},{b},{a,b},{a,c},{b,c}}}and BC(X,τ)={ X, ,∅,{a},{b},{c},{a,c},{b,c}}, Let $f : (X ,\tau) \to (Y ,\sigma)$ f(a)=a, f(b)=c and f(c)=c . Then , f is a b- continuous function ,but f is not totally sb*- continuous. Since for open subset A={a,b}in (Y,σ) . f^{-1}(A)= f^{-1}({a,b})= {a,c}is sb*-closed but is not sb*- open subset in (X, τ) . Hence, f^{-1}(A)= f^{-1}({a,b})= {a,c}is not sb*-clopen subset in X .

Example (3-5):

Let X=Y={a,b,c} with the topologies τ ={X,∅, {a},{b},{a,b}} and σ ={Y,∅,{a,c}},then SB*O(X,τ) ={X,∅,{a},{b},{a,b}}, SB*C(X,τ)={X,∅,{c},{a,c},{b,c}}and Let $f : (X ,\tau) \to (Y ,\sigma)$ f(a)=c, f(b)=b and f(c)=a . Then , f is a sb*- continuous function ,but f is not totally sb*- continuous. Since for open subset A={b,c}in (Y,σ) . f^{-1}(A)= f^{-1}({b,c})=

{a,b}is sb*-open but is not sb*- closed subset in (X, τ) . Hence, f^{-1}(A)= f^{-1}({b,c})= {a,b}is not sb*-clopen subset in X .

Remark(3-5):

The concepts of totally semi- continuous functions and totally sg- continuous functions are independent to totally sb*- continuous functions . In Example(3-2) it is clear that *f* is totally sb*- continuous functions , but *f* is not totally semi – continuous function . Also , in Example (3-3) *f* is totally semi- continuous functions and *f* is totally sg- continuous functions , but *f* is not totally sb* – continuous function .

Remark(3-6):

The concept of continuous functions and totally sb*- continuous functions are independent . As, shows in the following example .

Example (3-6):

(i) Let X={a,b,c} with the topology={X, ∅, {a},{a,c}}, then SB*O (X,τ) = {X, ∅ ,{a} {a,c},{a,b}}, SB*C(X,τ) ={X,∅,{c},{b},{b,c}}. Let *f : (X ,τ) →(X ,τ)* be the identity function .Then , *f* is a continuous function ,but *f* is not totally sb*- continuous. Since for open subset A={a}in (X ,τ) . f^{-1}(A)= f^{-1}({a})= {a}is sb*-openbut is not sb*- closed subset in (X, τ) . Hence, f^{-1}(A)= f^{-1}({a})= {a}is not sb*-clopen subset in (X,τ) .

(ii) Let X=Y={a,b} with the topologies τ ={X,∅}and σ ={Y,∅,{b}}, then SB*O (X,τ) = SB*C(X,τ) ={X, ∅,{a},{b}}.Let *f :(X ,τ)→(Y,σ)* be function defined by *f* (a)=b and *f*(b)=a . It observe that *f* is a totally sb*- continuous function, but *f* is not continuous function, since for open set A={b} . f^{-1}(A)= f^{-1}({b})= {a}is not open subset in (X, τ) .

The following proposition give the condition in order to every sb*-clopen is clopen .

Proposition(3-7):

If a space (X,τ) is b- space , then

(i) Every sb*-open subset of space (X,τ) is an open .

(ii) Every sb*-closed subset of space (X,τ) is an closed .

(iii) Every sb*-clopen subset of space (X,τ) is an clopen .

Proof:

(i)Let A is a sb*- open subset in (X,τ) , since (Every sb*- open set is b-open). This implies that A is a b-open subset in(X,τ) . By hypotheses (X,τ) is a b-space and by using definition (2-7) we get A is an open subset in (X,τ) .

(ii) Let A is a sb*- closed subset in (X,τ) , then $A^c$is an sb*- open subset in (X,τ), since (X,τ) is a b-space and by step-i- we get A^c is an open subset in (X,τ) .Thus , A is a closed subset in (X,τ) .

(iii) Let A is a sb*- clopen subset in (X,τ) ,(i .e., A is both sb*-open and sb*closed subset in (X,τ)) . Since (X,τ) is a b-space and by step-i- and –ii- we get A is an open and closed subset in (X,τ) .Hence, A is clopen subset in (X,τ) .

Next, in the following proposition we give the conditions to make the converse of a proposition (3-3) ,(3-4), Remark(3-5)and Remark(3-6) are true:

Proposition(3-8):

If *f: (X ,τ) →(Y, σ)* is a totally sb*- continuous function and *X* is a b- space , then *f* is a

(i) totally - continuous function .

(ii) totally ω- continuous function .

Proof:

(i)Let A is an open subset of (Y,σ) . Since *f* is a totally sb*- continuous function . Thus, $f^{-1}(A)$ is sb* –clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both sb* – open and sb* – closed subset in (X,τ)) . By hypotheses (X,τ) is a b-space and by using Proposition (3-7) step-iii- we get $f^{-1}(A)$ is clopen subset in (X,τ) . Therefore, *f* is totally- continuous function.

(ii) Let A is an open subset of (Y,σ) . Since *f* is a totally sb*- continuous function . Thus, $f^{-1}(A)$ is sb* –clopen subset in (X,τ) By hypotheses (X,τ) is a b-space and by using Proposition (3-7) step-iii- we get $f^{-1}(A)$ is clopen subset in (X,τ)) (i.e., $f^{-1}(A)$ is both open and closed subset in (X,τ)), since (Every open set is an *ω*-open ,[Sundaram 1995] and every closed set is a *ω*- closed [Sundaram1995] .This implies $f^{-1}(A)$ is *ω*- clopen subset in (X,τ). Therefore, *f* is totally *ω*- continuous function.

Proposition(3-9):

Let *f: (X ,τ) →(Y, σ)* be any function from discrete space (X,τ) and topological space (Y,σ) , then *f* is a totally sb*- continuous function if and only if

(i) *f* is totally b-continuous function .
(ii) *f* is totally semi-continuous function .
(iii) *f* is totally sg-continuous function.
(iv) *f* is continuous function .
(v) *f* is b-continuous function.
(vi) *f* is sb*-continuous function .

Proof:

(i) If *f* is totally sb*- continuous function, then by using Proposition (3-4) step-i- we get *f* is a totally b- continuous function . Suppose that *f* is a totally b- continuous function and A is an open subset of (Y,σ) . Thus, $f^{-1}(A)$ is b–clopen subset in (X,τ) i.e., $f^{-1}(A)$ is both b–open and b –closed subset in (X,τ) . By hypotheses (X,τ) is a discrete space we get every subset of (X,τ) is both open and closed subset in (X,τ) . B y Remark (2-4) we get $f^{-1}(A)$ is both sb*-open and sb*-closed subset in (X,τ) . This implies $f^{-1}(A)$ is sb*-clopen subset in (X,τ) .therefore , *f* totally sb* –continuous

(ii)Let *f* is totally sb*- continuous function and A be an open subset in (Y,σ) .Thus , $f^{-1}(A)$ is sb*–clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both sb*– open and sb* –closed subset in (X,τ)) . By hypotheses (X,τ) is a discrete space we get every subset of (X,τ) is both open and closed subset in (X,τ) ,and since (Every open set is semi-open [Levine 1963] , and every closed set is semi- closed , [Levine,1963]), this implies $f^{-1}(A)$ is both semi- open and semi– closed subset in (X,τ). Hence, $f^{-1}(A)$ is semi-clopen subset in (X,τ) . and so *f* totally semi – continuous function . **Conversely** , assume that *f* is a totally semi- continuous function , and A be an open subset in (Y,σ) .Thus , $f^{-1}(A)$ is semi-clopen subset in (X,τ) (i.e., $f^{-1}(A)$ is both semi- open and is semi – closed subset in (X,τ)) . By hypotheses (X,τ) is a discrete space we get every subset of (X,τ) is both open and closed subset in (X,τ) , . By Remark(2-4) we get $f^{-1}(A)$ is both sb*-open and sb*-closed subset in (X,τ) This implies $f^{-1}(A)$ is sb*-clopen subset in (X,τ) .therefore , *f* totally sb* – continuous.

The proof of steps-iii- , -iv-,-v-and –vi- is similar to step –ii- .

Next, we give other type of totally sb*- continuous function is called sb*- totally continuous function.

Definition(3-10):

A function *f : (X ,τ) →(Y ,σ)* is said to be ***sb*- totally continuous*** if the inverse image of every sb*-open subset in *(Y ,σ)* is a clopen subset in *(X ,τ)*.

Example (3-7):

Let X={a,b,c},Y={a,b} with the topologies τ={X,∅,{a},{b,c}}and σ={Y,∅,{a}} , then the clopen subset in (X,τ) are {X,∅,{a},{b,c}} and SB*O(Y,σ) ={X,∅,{a}}. Let *f : (X ,τ) →(Y ,σ)* be a function defined by *f* (a)=b and *f* (b)=*f* (c)=b . It observe that *f* is a sb*-totally continuous function .

Proposition (3-11):

A function *f: (X, τ) →(Y, σ)* is sb*- totally continuous if and only if the inverse of every sb*-closed subset in (*Y, σ)* is a clopen subset in (*X, τ)*.

Proof:_ This proof is similar to that of proposition (3-2)

Proposition(3-12):

If f*: (X ,τ) →(Y ,σ)* is sb*- totally continuous function , then *f* is a

(i) Totally – continuous function.
(ii) Totally semi – continuous function.
(iii) Totally *ω* – continuous function.
(iv) Totally sg – continuous function.
(v) Totally b – continuous function.
(vi) Totally sb* – continuous function.

Proof:

(i)Let A is an open subset in (Y,σ) . By Remark (2-4) we get A is a sb*- open subset in (Y,σ) . Since *f* is a sb*- totally continuous function, this implies that $f^{-1}(A)$ is clopen in (X,τ) . Therefore, *f* totally – continuous function.

(ii) Let A is an open subset in (Y,σ) . By Remark (2-4) we get A is a sb*- open subset in (Y,σ) . Since *f* is a sb*- totally continuous function. Thus , $f^{-1}(A)$ is clopen in (X,τ) , since (Every open set is a semi- open , [Levine 1963] and every closed set is a semi-closed , [Levin 1963]) . this implies that $f^{-1}(A)$ is both semi- open and semi–closed subset in (X,τ). Hence, $f^{-1}(A)$ is semi-clopen subset in (X,τ) . Therefore, *f* totally semi – continuous function.

(iii)) Let A is an open subset in (Y,σ) . By Remark(2-4) we get A is a sb*- open subset in (Y,σ) . Since *f* is a sb*- totally continuous function . Thus , $f^{-1}(A)$ is clopen in (X,τ) , since (Every open set is a*ω*- open , [Sundaram 1995] and every closed set is a*ω*- closed , [Sundaram 1995]) . this implies that $f^{-1}(A)$ is both *ω*- open and semi–closed subset in (X,τ). Hence, $f^{-1}(A)$ is *ω* -clopen subset in (X,τ). therefore, *f* totally*ω*– continuous function .

The proof of steps, -iv-,-v-and –vi- are similar to step –iii- .

The converse of Proposition(3-12) need not be true in general , the following example shows that .

Example(3-8):

Let X=Y={a,b} with the topologies τ ={X,∅,{a}}and σ ={Y,∅}, then SO(X,τ)= *ω*O(X,τ)=SGO(X,τ)=BO(X,τ) =SB*O(X,τ) = {X, ∅,{a}} . Let *f : (X ,τ) →(Y ,σ)* be function defined by *f* (a)=b and *f* (b)=a . It observe that *f* is a totally- continuous(resp . totally semi – continuous , totally *ω* - continuous , totally b- continuous and sb*-continuous) function, but *f* is not sb*-totally continuous function , since for sb*-open subset A={a} . f^{-1}(A)= f^{-1}({a})= {b}is closed subset in (X,τ) , but is not open . Hence, f^{-1}(A)= f^{-1}({a})= {b}is not clopen subset in (X, τ) .

Proposition(3-13):

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is sb*- totally continuous function , then f is a

(i) Continuous- function.
(ii) b- continuous- function.
(iii) sb*-continuous- function.

Proof: It is clear.

The converse of Proposition (3-13) need not be true in general, the following example shows that.

Example(3-9):

Let X={a,b,c} with the topology τ ={X,∅,{a}}, then BO(X,τ)=SB*O(X,τ)={ X, ∅, {a},{a,b},{a,c}} . Let $f : (X, \tau) \rightarrow (X, \tau)$ be an identity function . It observe that f is a continuous function (resp. b- continuous function and sb*- continuous function),but f is not sb*- totally continuous function, sinc for sb*-open subset A={a}. f^{-1}(A)= f^{-1}({a})= {a}is an open subset in (X,τ) , but is not closed . Hence, f^{-1}(A)= f^{-1}({a})= {a}is not clopen subset in (X, τ) .

Next, in the following proposition we give the conditions to make the converse of a proposition (3-12) and (3-13) are true:

Proposition(3-14):

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be any function from discrete space (X,τ) into b- space (Y,σ) , then f is a sb*- totally continuous function if f is a

(i) Totally- continuous function.
(ii) Totally -semi continuous function.
(iii) Totally ω- continuous function.
(iv) Totally sg- continuous function.
(v) Totally b- continuous function.
(vi) Totally sb*- continuous function.

Proof:

(i) Let A be a sb*- open subset of (Y,σ) . Since (Y,σ) is b-space and by using Proposition (3-7) step-i- we get A is an open subset in (Y,σ). By hypotheses f is a totally – continuous function, this implies that $f^{-1}(A)$ is a clopen subset in (X,τ) . Therefore, f sb*- totally continuous function.

(ii) Let A be a sb*- open subset of (Y,σ) . By hypotheses (Y,σ) is b-space and by using Proposition (3-7) step-i- we get A is an open subset in (Y,σ). Since f is a totally semi – continuous function, this implies that $f^{-1}(A)$ is a semi- clopen subset in (X,τ) . Also , since f is a discrete space , then every subset in (X,τ) is both open and closed . Hence, $f^{-1}(A)$ is a clopen subset in (X,τ), therefore , f sb*- totally continuous function .

(iii) Let A be a sb*- open subset of (Y,σ) . By hypotheses (Y,σ) is b-space and by using Proposition (3-7) step-i- we get A is an open subset in (Y,σ). Since f is a totally ω – continuous function, this implies that $f^{-1}(A)$ is a ω-clopen subset in (X,τ) . Also, since f is a discrete space, then every subset in (X,τ) is both open and closed . Hence, $f^{-1}(A)$ is a clopen subset in (X,τ), therefore , f sb*- totally continuous function.

The proof of steps, -iv-,-v-and –vi- are similar to step –iii- .

The proof of the following Proposition it is easy. Hence, it is omitted.

Proposition(3-15)

Let f: $(X,\tau) \to (Y,\sigma)$ be any function from discrete space (X,τ) and topological space (Y,σ), then f is a totally sb*- continuous function if and only if

(i) Continuous function.
(ii) b-continuous function.
(iii) sb*- continuous function.

Proposition (3-16)

Let $f: (X,\tau) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (Z,\mu)$ be any two function, then $g \circ f: (X,\tau) \to (Z,\mu)$ is totally sb*- continuous function if

(i) f is a totally sb*- continuous function and g is a continuous function .
(ii) f is a totally sb*- continuous function and g is a totally- continuous function .
(iii) f and g are two totally – continuous functions .

Proof:

(i) Let A be an open subset in Z , since g is a continuous function . Thus $g^{-1}(A)$ is an open subset in Y .Also ,since f is a totally sb*- continuous function ,then $f^{-1}(g^{-1}(A))$ is sb*-clopen subset in X . But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is sb*- clopen subset in X .Hence, $g \circ f: (X,\tau) \to (Z,\mu)$ is a totally sb*- continuous function .

(ii) Let A be an open subset in Z , since g is a totally- continuous function . Thus $g^{-1}(A)$ is clopen subset in Y , this implies that $g^{-1}(A)$ is an open subset in Y By hypotheses f is a totally sb*- continuous function ,then $f^{-1}(g^{-1}(A))$ is sb*- clopen subset in X . But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is sb*- clopen subset in X .Hence, $g \circ f: (X,\tau) \to (Z,\mu)$ is a totally sb*- continuous function .

(iii) Let A be an open subset in Z , since g is a totally- continuous function . Thus $g^{-1}(A)$ is clopen subset in Y , this implies that $g^{-1}(A)$ is an open subset in Y By hypotheses f is a totally -continuous function ,then $f^{-1}(g^{-1}(A))$ is clopen subset in X and by using Remark(2-4) we get $f^{-1}(g^{-1}(A))$ is sb*- clopen subset in X . But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is sb*- clopen subset in X .Hence, $g \circ f: (X,\tau) \to (Z,\mu)$ is a totally sb*- continuous function .

Next, we give some propositions about the composition of these types of functions

Remark(3-17):

If $f: (X,\tau) \to (Y,\sigma)$ and $g: (Y,\sigma) \to (Z,\mu)$ are two totally sb*- continuous function , then $g \circ f: (X,\tau) \to (Z,\mu)$ is not necessarily totally sb*- continuous function , as shows in the following example .

Example(3-10):

Let X=Y=Z={a,b} with topologies $\tau=\{X,\emptyset,\{a\}\}$, $\sigma=\{Y,\emptyset\}$ and $\mu=\{Z,\emptyset,\{b\}\}$.Then SB*O(X,τ) $=\{X,\emptyset,\{a\}\}$, SB*O(Y,σ) $=\{Y,\emptyset\{a\},\{b\}\}$ and $\mu=\{Z,\emptyset,\{b\}\}$. Define $f: (X,\tau) \to (Y,\sigma)$ by f (a)=a, f (b)=b and $g: (Y,\sigma) \to (Z,\mu)$ defined by g(a)=a,g(b)=b. Then clearly f and g are two totally sb*- continuous function , but $g \circ f: (X,\tau) \to (Z,\mu)$ is not totally sb*- continuous function .Since for open subset A ={b} in (Z,μ) . $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A)) = f^{-1}(g^{-1}(\{b\})) = f^{-1}(\{b\})=\{b\}$ .is a sb*- closed subset in (X,τ) but is not open . this implies $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A)) = f^{-1}(g^{-1}(\{b\})) = f^{-1}(\{b\})=\{b\}$ is not sb*- clopen subset in (X,τ).

Proposition (3-18):

Let $f: (X,\tau) \to (Y,\sigma)$ be a sb*-totally continuous function and $g: (Y,\sigma) \to (Z,\mu)$ be any function , then $g \circ f: (X,\tau) \to (Z,\mu)$ is totally sb*- continuous function if

(i) g is a totally sb*- continuous function .
(ii) g is a totally - continuous function .
(iii) g is a totally ω-continuous function .

Proof:

(i)Let A be an open subset in Z , since g is a totally sb*- continuous function . Thus $g^{-1}(A)$ is sb*-clopen subset in Y , this implies that $g^{-1}(A)$ is an sb*- open subset in Y By hypotheses f is a sb*- totally continuous function ,then $f^{-1}(g^{-1}(A))$ is clopen subset in X and by using Remark(2-4) we get $f^{-1}(g^{-1}(A))$ is sb*- clopen subset in X . But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is sb*- clopen subset in X .Hence, $g \circ f: (X, \tau) \to (Z, \mu)$ is a totally sb*- continuous function .

The proof of step-ii- and –iii- are similar to step-i- .

Similarly, we prove the following corollary.

Corollary(3-19):

Let $f: (X, \tau) \to (Y, \sigma)$ be a sb*-totally continuous function and $g: (Y, \sigma) \to (Z, \mu)$ be any function , then $g \circ f: (X, \tau) \to (Z, \mu)$ is totally sb*- continuous function if

(i) g is a continuous function
(ii) (ii) g is a sb*- continuous function .

Proposition (3-20):

Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \mu)$ be any two function, if g is a sb*- totally continuous function and

(i) f is a totally – continuous function ,then $g \circ f: (X, \tau) \to (Z, \mu)$ is sb*- totally continuous function .
(ii) f is a totally semi – continuous function ,then $g \circ f: (X, \tau) \to (Z, \mu)$ is totally semi-continuous function.
(iii) f is a totally b – continuous function ,then $g \circ f: (X, \tau) \to (Z, \mu)$ is totally b- continuous function .
(iv) f is a totally ω - continuous function ,then $g \circ f: (X, \tau) \to (Z, \mu)$ is totally sb*- continuous function.
(v) f is a totally sg– continuous function ,then $g \circ f: (X, \tau) \to (Z, \mu)$ is totally sg-continuous function .

Proof:

(i) Let A be a sb*-open subset in Z , since g is a sb*- totally continuous function . Thus $g^{-1}(A)$ is clopen subset in Y, this implies that $g^{-1}(A)$ is an open subset in Y . By hypotheses f is a totally- continuous function ,then $f^{-1}(g^{-1}(A))$ is clopen subset in X and by using Remark(2-4) we get $f^{-1}(g^{-1}(A))$ is sb*- clopen subset in X . But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is sb*- clopen subset in X .Hence, $g \circ f: (X, \tau) \to (Z, \mu)$ is a sb* totally - continuous function .

(ii) Let A is an open subset in (Z,μ) . By Remark(2-4) we get A is a sb*- open subset in (Z,μ) . Since g is a sb*- totally continuous function , $g^{-1}(A)$ is clopen subset in Y , this implies that $g^{-1}(A)$ is an open subset in Y . By hypotheses f is a totally semi-continuous function ,then $f^{-1}(g^{-1}(A))$ is semi- clopen subset in X .But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is semi- clopen subset in X Hence, $g \circ f: (X, \tau) \to (Z, \mu)$ is a totally semi - continuous function .

(iii) Let A is an open subset in (Z,μ) . By Remark (2-4) we get A is a sb*- open subset in (Z,μ) . Since g is a sb*- totally continuous function, $g^{-1}(A)$ is clopen subset in Y, this implies that $g^{-1}(A)$ is an open subset in Y. By hypotheses f is a totally b-continuous function, then $f^{-1}(g^{-1}(A))$ is b- clopen subset in X . $f^{-1}(g^{-1}(A))$ =$(g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is b-clopen subset in X Hence, $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is a totally b - continuous function .

The proof of step-iv- and –v- are similar to step-iii- .

Proposition (3-21):

Let $f:(X,\tau) \rightarrow (Y,\sigma)$ and $g:(Y,\sigma) \rightarrow (Z,\mu)$ be any two function, if g is a sb*- totally continuous function and

(i) f is a continuous function ,then $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is continuous function .

(ii) f is a b – continuous function ,then $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is b- continuous function.

(iii) f is a sb*- continuous function ,then $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is sb*- continuous function.

Proof: It is observe.

Proposition (3-22):

Let $f:(X,\tau) \rightarrow (Y,\sigma)$ and $g:(Y,\sigma) \rightarrow (Z,\mu)$ be two sb*- totally continuous function, the $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is also sb*- totally continuous function.

Proof:

Let A be a sb*-open subset in Z , since g is a sb*- totally continuous function . Thus $g^{-1}(A)$ is clopen subset in Y , this implies that $g^{-1}(A)$ is an open subset in Y . By Remark (2-4) we get $g^{-1}(A)$ is sb*-open subset in Y , since f is a sb*- totally continuous function ,then $f^{-1}(g^{-1}(A))$ is clopen . But $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ and so $(g \circ f)^{-1}(A)$ is clopen subset in X .Hence, $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is a sb*-totally continuous function .

References

Andrijevic, D., "on b-Open Sets ", Matema Bech., Vesnisk, Vol. 48, pp. 59-64, (1996).

Bhattacharyya, P.and lahiri, B.K.;" Semi-Generalized Closed Sets in Topology ," Indian J. Math.,29(1987).

Caldas.M.,Jafari.S., and Rajesh.N.,"Properties of Totally b-Continuous Functions", Analele Stiintifice Ale University . Math , 2009, 120-130 .

EL.Atik.A.A.,"A Study of Some Types of Mapping on Topological Spaces", M.S.C .Thesis. Tanta University . Egypt, 1997 .

Jain.R.C.," The Role of Regularly Open Sets in General Topology ", Ph.D. Thesis . Meerut University Institute of a Duanced Studies . Meerut .Indian , 1980 .

Levine, N.;"Semi-open Sets and Semi-continuity In Topological Spaces" Amer.Math.Monthly.70(1963),36-41.

Nasef.A.A., and Farrag.A.S.," Completely b- Irresolute Functions ", Proc . Soc .Egypt , 74(1999),73-86 .

Nour.T.M.," Totally Semi-Continuous Functions ", Indian . J. Pure appl . Math. ,26(7), July 1995, 675-678 .

Poongothai .A., and Parimelazhagan .R.," sb*-Closed Sets in Topological Spaces ", Int .Journal of Math. Analysis , Vol.6,2012, No. 47., 2325-2333.

Poongothai .A., and Parimelazhagan .R.," Strongly b- Continuous Functions in Topological Spaces ", Int .Journal of Computer Applications , Vol.58, No.14, November 2012, 8-11 .

Rajesh.N.,"On Totally ω- Continuity , Strongly ω- Continuity and Contra ω- Continuity ", Soochow Journal of Math ., Vol.33, No.4, October 2007, 679-690 .

Ravi.O.,Ganesan.S., and Chandrasekar.S.,"On Totally sg- Continuity , Strongly sg-Continuity and Contra sg-Continuity ", Gen . Math ,Notes, Vol.7, 2011, 13-24.

Sundaram.P.,Sheik John.M.," Weakly Closed Sets and Weak Continuous Maps in Topological spaces " , Indian .Sci, Cang . Calcutta, 1995, p-49 .