Numerical Solution of Heat Transfer and Flow of Nanofluids in Annulus With Fins Attached on the Inner Cylinder

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Abstract

Natural convection heat transfer enhancement by utilizing various nanofluids (Ag, Cu and TiO_2)flow in a three – dimensional horizontal and an inclined annulus with two heated fins attached on the inner cylinder have been studied numerically .The inner cylinder and two fins are maintained at constant wall temperature (CWT), while the outer cylinder is diathermal (adiabatic). The problem was solved numerically using Alternating Direction Implicit (ADI) method.The numerical results represent streamlines, isotherms, Local Nusselt number around inner cylinder and fins surfaces, axial profile of the peripheral average Nusselt number and average skin friction coefficient. The two types of nanoparticles used in this study metallic silver(Ag), copper (Cu) and nonmetallic titanium oxide (TiO_2). This study indicating to the effect the Rayleigh number, volume fraction, fin length, fin inclination angles and annulus inclination angles. The numerical results show that as the solid volume fraction increases, the heat transfer is enhanced for all values of Rayleigh number. This enhancement is more significant at high Rayleigh number. The lowest heat transfer was obtained for TiO_2 (50 nm) due to domination of conduction and large nanoparticles .whereas Ag (20 nm), Cu (30 nm) – distilled water nanofluids has the highest heat transfer, respectively. As well as the enhancement in heat transfer of nanofluids for annulus with fins attached to the inner cylinder at (5 %) volume concentration of (Ag , Cu and TiO_2) nanoparticles increases (41.5 %, 35%,19 %) respectively compared with the base fluid (distilled water). The average Nusslet number increases with increasing both Rayleigh number and the volume fraction of nanoparticles. The average Nusselt number decreases by increasing fins' length. Moreover the increase of the average Nusslet number is attributed to the increase of the thermal conductivity of nanofluid with increasing the volume fraction of the nanoparticles.

Keywords: laminar natural convection, Nanofluid,Fin, Heat transfer enhancement

الخلاصة

تم دراسة تحسين انتقال الحراة بالحمل الحر عدديا بواسطة استعمال موائع نانوية مثل الفضة والنحاس وأوكسيد التيتانيوم من خلال جريان ثلاثي الابعاد في انبوب حلقي افقي ومائل مع زعنفتين مثبته على الاسطوانة الداخلية ولحالة ثبوت درجة حرارة السطح على الأسطوانة الداخلية والزعنفتين بينما كانت الاسطوانة الخارجية معزولة. تم حل المسالة باستخدام طريقة الاتجاه الضمني المُتَنَاوِب (ADI). النتائج العددية تم تمثيلها بمخططات دالة الجريان ودرجة الحرارة وعدد نسلت الموضعي حول الأسطوانة الداخلية وسطوح الزعانف ومعدل عدد نسلت المحوري ومعامل الاحتكاك السطحي. تم استخدام نوعين من الجزئيات النانوية المعدنية مثل الفضة والنحاس وغير المعدنية مثل أوكسيد التيتانيوم. هذه الدراسة اشارت الى تأثير عدد رالي، الكسر الحجمي، طول الزعنفة، زوايا ميلان الزعنفة، وزوايا ميلان الانبوب الحلقية. نتائج الحل العددي اظهرت أنه عند زيادة تركيز الجزئيات النانوية فان انتقال الحرارة يزداد الى كل قيم عدد رالي. وهذا التحسين يكون اكثر اهمية عند عدد رالي الكبير. واقل انتقال للحرارة يحصل للأوكسيد التيتانيوم بسب هيمنة التوصيل وكبر الجزئيات النانوية له. ان نسب التحسين في انتقال الحرارة للموائع النانوية للأنابيب الحلقية مع زعانف مثبتة على الاسطوانة الداخلية وبتركيز (5%) من الفضة والنحاس وأوكسيد التيتانيوم كانت بمقدار (41.5%, 35%, 19%) بتتابع مقارنة مع الماء. ومعدل عدد نسلت يزداد مع زيادة عدد رالي والكسر الحجمي للجزئيات النانوية بينما يتناقص مع زيادة طول الزعنفة علاوة على ان زيادة معدل عدد نسلت تساهم في زيادة الموصلية للمائع النانوية مع زيادة نسب الحجوم لجزئيات النانوية.

الكلمات المفتاحية: الحمل الحراري الطبيعي الطباقي،Nanofluid ، فنلندا، وتعزيز نقل الحرارة

1. Introduction

Heat transfer within horizontal annuli has many engineering applications especially in heat exchangers, solar collectors, thermal storage systems, and electronic components.Several applications use natural convection as the main heat transfer mechanism. Therefore, it is worth to understand the thermal behavior of such systems when natural convection is important.The first extensive research on this flow geometry was done by Kuhen and Goldstein [1976,1978].In their work, an experimental and numerical investigation was done when the ratio of gap width to inner cylinder diameter was 0.8 for water and air in the annulus. In their experimental study, it was found that transition from laminar to turbulent flow occurs at Rayleigh number of 10^6. Kumar [1988]studied natural convection in horizontal annuli, where the inner cylinder is heated by the application of a constant heat flux and the outer cylinder is isothermally cooled. Ho *et al*. [1989] investigated the natural convection in concentric and eccentric horizontal cylindrical annuli with mixed boundary conditions using numerical methods.They realized that the heat transfer and fluid flow are dependent on the Rayleigh number and eccentricity of the annulus.A numerical investigation for three dimensional natural convection inside horizontal concentric annulus with specified wall temperature and heat flux was done by Chun lang Yeh[2002]. An essential restriction in natural convection in an annulus is that the heat transfer rate is limited. Using radial fins on the outer surface of inner cylinder has significant effect on heat transfer rate in the annulus. Having such fins, improves the flow and heat transfer characteristics. Chai and Patankar [1993] investigated natural convection heat transfer in an annulus having six radial fins attached to the inner cylinder arranged in two different configurations. It was observed that orientation of the internal fins has no important effect on the average Nusselt number, even though the blockage due to the fins has significant effect on the flow and temperature fields.

Turbulent natural convection between two horizontal concentric cylinders in the presence of radial fins was studied numerically by Rahnama and Farhadi [2004]. They observed that higher fins have a blocking effect on flow and cause a lower heat transfer rate, so that there is a reduction in heat transfer rate compared to the case of no fin, at the same Rayleigh number. Nowadays, enhancement of heat transfer attracts the researchers' attention. The low thermal conductivity of conventional heat transfer fluids, such as water, is considered a primary limitation in enhancing the heat transfer performance. Maxwell's study [1873] showed the possibility of increasing the thermal conductivity of a fluid–solid mixture by increasing the volume fraction of solid particles.Thus,the particles with micrometer or even millimeter dimensions were used. Those particles caused several problems such as abrasion,clogging, and pressure losses.

During the past decade the technology of producing particles in nanometer dimensions was improved and a new kind of solid–liquid mixture that is called nanofluid, was established [1995].The dispersion of a small amount of solid nanoparticles in conventional fluids such as water or Ethylene glycol changes their thermal conductivity remarkably.In general,in most recent research areas, heat transfer enhancement in forced convection is desirable[2007–2009], but there is still a debate on the effect of nanoparticles on heat transfer enhancement in natural convection applications Natural convection of Al_2O_3–water and CuO–water nanofluids inside a cylindrical enclosure heated from one side and cooled from the other side was studied by Putra *et al*. [2003]. They found that the natural convection heat transfer coefficient was lower than that of pure water. Wen and Ding [2005] investigated the natural convection of TiO_2 –water in a vessel composed of two discs.Their results showed that the natural convection heat transfer coefficient decreases by increasing the volume fraction of nanoparticles. Jou and Tzeng [2006] conducted a numerical study of the natural convection heat transfer in rectangular enclosures filled with a nanofluid using the finite difference method with the stream function vorticity formulation. They investigated the effects of the Rayleigh number, the aspect ratio of the enclosure, and the volume fraction of the nanoparticles on the heat transfer inside the enclosures. Their results showed that the average heat transfer coefficient increased with increasing the volume fraction of the nanoparticles.

Recently, an experimental study of natural convection heat transfer of nanofluid in vertical square enclosures was studied by Ho *et al.*[2010]. They concluded that the average heat transfer rate depend on the thermophysical properties of the nanofluid and the heat transfer enhancement for nanofluid is more than that of for the pure water at high Rayleigh number. Aminossadati and Ghasem[2011] studied natural convection in an isosceles triangular enclosure filled with a nanofluid. It is found that the heat transfer rate enhanced by increasing Rayleigh number and solid volume fraction and the variation of heat transfer rate with respect to the enclosure apex angle and heat source position is different at high and low Rayleigh number. Cho *et al.* [2012] studied Natural convection heat transfer in complex-wavy wall enclosed cavity filled with nanofluid. They understood that the Nusselt number increases by increasing the volume fraction of nanoparticles and Heat transfer performance can be optimized by tuning the geometry parameters. Arefmanesh *et al.*[2012] researched Buoyancy – driven heat transfer analysis in two-square duct annuli filled with a nanofluid. The calculated result of them showed that the average Nusselt number increases by increasing the volume fraction of the nanoparticles.Alsomultiple eddies are developed in the gap between the top walls of the square ducts by enhancement of the width of the gap between the ducts. Mokhtari Moghari *et al.*[2011] quested two phase mixed convection Al_2O_3-water nanofluid flow in an annulus. They found that the Nusselt number increases by increasing volume fraction of nanoparicles at inner and outer cylinder, but it did not have any significant effect on the friction factor. Also, the value of the Nusselt number on the inner cylinder is more than that of on the outer cylinder.Thermal conductivity variation on natural convection flow of water – alumina nanofluid in an annulus is investigated by Parvin *et al.*[2012] Two thermal conductivity models namely, the Chon et al. model and the Maxwell-Garnett model, are used to evaluate the heat transfer enhancement in the annulus. It is found that heat transfer rate enhancement is carried out by increasing volume fraction of nanoparticles and the Prandtl number at moderate and large Grashof number using both models but for chon model, it had greatest value. Soleimani *et al.*[2012] studied natural convection heat transfer in a nanofluid filled semi – annulus enclosure. It is found that the angle of turn had significant effect on the streamlines, isotherms and maximum or minimum values of local Nusselt number.

Abu – Nada *et al.* [2008] studied natural convection heat transfer enhancement in horizontal concentric annuli using nanofluids. They showed that the enhancement of heat transfer in natural convection depends mainly on Rayleigh number and for certain Rayleigh numbers, $Ra=10^4$, the heat transfer was not sensitive to nanoparticles concentration whereas at higher Rayleigh numbers an enhancement in heat transfer was taking place. Abu – Nada [2009] investigated the effect of variable viscosity and thermal conductivity of Al_2O_3–water nanofluid on heat transfer enhancement in natural convection. It was observed that heat transfer enhancement is dependent of Rayleigh number and volume fraction of nanoparticles.Different models for estimating the effective viscosity have different effects on heat transfer rate.

In the present study, the natural convection in annuli with two fins filled with Ag, Cu and TiO_2– water nanofluids are studied numerically using the finite difference method. A parametric study is performed and the effects of pertinent parameters, such as, the location of the fins on the inner cylinder, fin length, Rayleigh number and the volume fraction of the nanoparticles on the nanofluid flow and heat transfer inside the annuli are investigated.

2. Problem Description and Governing Differentional Equations

In view of the annular geometry of the problem a cylindrical coordinate system and due to geometrical symmetry, Fig.1, only one half of the annulus is simulated with case CWT. Two fins of length l_f and thickness d are attached on the inner cylinder. The fin thickness d is kept constant in current analysis d=2mm. the angle between the two fins is 180^0. The annulus between the two cylinders is filled with water based nanofluid. Three types of nanoparticles(Ag,Cu and TiO_3)are investigated. It is assumed that the base fluid (water) and nanoparticles are in thermal equilibrium and no slip occurse between them. The thermo – physical properties of the base fluid (water) and the three types of nanoparticles forming the nanofluids are given in Table 1.The governing equations (continuity, momentum and the energy equations in the polar three–dimensional coordinate) for the case of single phase fluid (Homogenous model), laminar and steady flow in cylindrical coordinates are as follows [1966].

Continuity Equation

$$\frac{\rho_{nf}}{r}(ru)_r + \frac{\rho_{nf}}{r} V_\theta + \rho_{nf} W_Z = 0 \quad (1)$$

Momentum Equation

r – Component

$$\rho_{nf}\left(VV_r + \frac{W}{r} V\theta - \frac{W^2}{r} \right) = -P_r + \mu_{nf}\left(V_{rr} + \frac{1}{r} V_r + \frac{1}{r^2} V_{\theta\theta} - \frac{V}{r^2} - \frac{2}{r^2} W_\theta \right) - \rho_{nf} g(\cos\alpha\cos\theta) \quad (2)$$

θ – Component

$$\rho_{nf}\left(VW_r + \frac{W}{r} W\theta - \frac{VW}{r} \right) = -\frac{1}{r} P_\theta + \mu_{nf}\left(W_{rr} + \frac{1}{r} W_r + \frac{1}{r^2} W_{\theta\theta} - \frac{W}{r^2} + \frac{2}{r^2} W_\theta \right) - \rho_{nf} g(\sin\alpha\sin\theta) \quad (3)$$

Z – Component

$$\rho_{nf}\left(VU_r + \frac{W}{r} U\theta \right) = -P_Z + \mu_{nf}\left(U_{rr} + \frac{1}{r} U_r + \frac{1}{r^2} U_{\theta\theta} \right) - \rho_{nf} g(\sin\alpha) \quad (4)$$

Energy equation

$$\rho_{nf} Cp_{nf}\left(Vt_r + \frac{W}{r} t_\theta + Wt_z \right) = k_{nf}\left[-\frac{1}{r}\frac{\partial}{\partial r}(rt_r) + \frac{1}{r^2} t_{\theta\theta} + t_{zz} \right] \quad (5)$$

The properties of nanofluid (fluid containing suspended nanoparticles) are defined as follows:

Effective thermal conductivity [2007]

$$\frac{k_{nf}}{k_f} = \left[\frac{k_s + (n-1)k_f - (n-1)(k_f - k_s)\Phi}{k_s + (n-1)k_f + (k_f - k_s)\Phi} \right] \quad (6)$$

Where n is a shape factor and equal to 3 for spherical nanoparticles.

Thermal diffusivity [1985].

$$\alpha_{nf} = \frac{k_{nf}}{(1-\Phi)(\rho Cp)_f + \Phi(\rho Cp)_s} \quad (7)$$

Thermal expansion coefficient [2003].

$$\beta_{nf} = \left[\frac{1}{1 + \frac{(1-\Phi)\rho_f}{\Phi\rho_s}} \frac{\beta_s}{\beta_f} + \frac{1}{1 + \frac{\Phi}{(1-\Phi)} \frac{\rho_s}{\rho_f}} \right] \quad (8)$$

Specific heat [2003].

$$Cp_{nf} = \frac{(1-\Phi)(\rho Cp)_f + \Phi(\rho Cp)_s}{(1-\Phi)\rho_f + \Phi\rho_s} \tag{9}$$

Effective viscosity [1985].

$$\mu_{nf} = \left[123\Phi^2 + 7.3\Phi + 1\right] \tag{10}$$

Table .1.Thermo – physical properties

Base fluid	Pr	ρ (Kg/m³)	Cp (J/kg K)	K (W/m K)	β X10^{-5} (K^{-1})	α X10^{-5} (m²/s)
Distilled water	6.2	997.1	4179	0.613	21	
Nanoparticles						
Copper (Cu)		8933	385	401	1.67	11.7
Silver (Ag)		10500	235	429	1.89	17.4
Titanium Oxide (TiO_2)		4250	686.2	8.9538	0.9	0.31

3. Boundary Conditions

Inlet (z = 0) :

$U = V = 0$, $W_Z = W_{Zi}$; and $T = T_i$

$U= V = W = 0$ R=0 , 1

T=1 R=0

T=0 R=1

$U = V = W = 0$ on fin's surfaces

$U = V = W_\Theta = 0$ and $T_\Theta = 0$ on the left and right side of the domain

$$k_r = \left.\frac{\partial T}{\partial R}\right|_{fin} = -\frac{k_{nf}}{k_f}\left.\frac{\partial T}{\partial R}\right|_{nf} \qquad R= l_{fin}$$

Where, k_r is the ratio of fin conductivity to the conductivity of base fluid.

4. Grid Testing and Code Validation

Fig.(1) shows the geometry of the considered problem. Basically, the flow region associated with the polar coordinates (R,Θ) is divided into a grid network which contains the following dimensions (ΔR×ΔΘ) for one division as shown in Fig.(2). The number of divisions and nodal points in this case will be (mt×nt) and [(mt+1) × (nt+1)] , respectively, where mt refers to the number of divisions in R direction which changes from (m=1) to (m=mt) and equal to (1/ΔR) , while (nt) refers to the number of divisions in – direction which changes from (n=1) to (n=nt) and is equal to(π/ΔΘ) for one half of the annulus gap because of flow symmetry about the vertical line of the annulus. Fig (3) demonstrates the influence of number of grid points for a test case of fluid confined within the present configuration at $Ra=10^4$ and Φ =0, it is clear that, the grid system of (65*75) is enough to obtain accurate results and guarantees. This grid mesh is shown in fig (4).Fig.(5) shows the variation of normalized local Nusselt number on the inner and outer cylinder surfaces. The results of Kuhen and Goldstien [1976] and Sheikhzadeh *et al.*, [2013] for the same problem are also shown in this figure. In this comparison, the normalized local Nusselt number is defined as [1995]:

$$k_{eq} = \frac{Nu_{i,o}}{\mathrm{lin}\left(\frac{Do}{Di}\right)} \qquad (11)$$

The excellent agreement is observed between the present results and the results of Kuhen and Goldstien[1976] and Sheikhzadeh *et al.*,[2013]. In Fig (6) the variation of temperature versus radial direction for various angles are compared with the experimental results of Kuhen and Goldstien [1976]and numerical result of Sheikhzadeh *et al.*,[2013]. Good agreement between the present results and the previous results of Kuhen and Goldstien[1976] and Sheikhzadeh *et al.*,[2013].

5. Numerical implementation

The governing equations in the cylindrical coordinates (equations 1, 2, 3,4and 5) as well as boundary conditions were discretized by finite difference method. In this study the finite difference equations were derived by using central difference approximation for the partial derivatives except the convective terms for which upwind difference formula was employed. Derivative at the boundary were approximated by three point forward difference. The alternating direction implicit (ADI) method was employed for the solution of energy, while the momentum and continuity equations were combined as the pressure correction formula and solved by the simple algorithm. A time increment $\Delta t = 10^{-5}$ has been used for $Ra=10^3, 10^5$ and 10^6. In order to evaluate how the presence of the nanofluids affect . the heat transfer rate around the perimeter of inner cylinder for various values of Rayleigh number, nanoparticles volume fraction, and inclination angles of the annulus it is necessary to observe the variation of the Local Nusselt number on the perimeter of inner cylinder . In generalized coordinate the local and average Nusslet number is defined:

The local Nusselt number at the heated wall (1)is calculated from the temperature gradient at the wall for each time step and takes the following form:

$$Nu_n^k = \frac{T_R\big|_{1,n}^{k}}{T_1^k} \qquad (12)$$

The derivative at the wall $T_R\big|_{1,n}^{k}$ is approximated by using three – points forward difference with error order $O(\Delta R)^2$ and takes the following form

$$T_R\Big|_{1,n}^{k} = \frac{Nu^k}{2\Delta R}\left[3T_{1,n}^{k} - 4T_{2,n}^{k} + T_{3,n}^{k}\right] \qquad (13)$$

But, the value of T_1 at the location (k) can be calculated from the value of mean Nusselt number around the perimeter of inner cylinder after finishing the step of location (k). As a result, it can be written as follows:

$$T_1^{k+1} = \frac{2\left(\frac{r_2}{r_1} - 1\right)}{Nu^k} \qquad (14)$$

and, by substituting Eq.(14) & Eq.(13) into Eq.(12), the Nusselt number around the perimeter of inner cylinder will be as follows:

$$Nu_{nt}^{k+1} = \frac{Nu^k}{4\left(\frac{r_2}{r_1} - 1\right)\Delta R}\left[3T_{1,n}^{k} - 4T_{2,n}^{k} + T_{3,n}^{k}\right] \qquad (15)$$

The mean Nusselt number around the perimeter of inner cylinder at location (k) is deduced by integrating local Nusselt number as follows:

$$Nu^{k+1} = SNu^K + (1-S)\frac{2}{\pi}\int_0^{\pi} Nu_{nt}^{k+1} d\theta \qquad (16)$$

The Nusselt number is used to calculate the surface temperature at the location (k+1), but it is found that the boundary conditions cause unstable state in the solution at the value of relaxation factor (S = 0). Therefore, the relaxation factor (S = 0.8) is used for stability considerations. The above integral was calculated using Simpson's 1/3 rule method. To show the effect of the nanofluids on heat transfer rate, a variable called Nusselt number ratio (NuR) is introduced with its definition given as:

$$NuR = \frac{Nuave|_{\text{with nanofluid}}}{Nuave|_{\text{pure fluid}}} \qquad (17)$$

If the value of NuR greater than 1 indicated that the heat transfer rate is enhanced on that fluid, whereas reduction of heat transfer is indicated when NuR is less than 1. The local and average nusslet numbers on the surface of the fins is defined as:

$$Nu_{fin} = -\frac{2r_i k_{eff}}{1\, k_f}\frac{\partial T}{R\,\partial\theta} \qquad (18)$$

$$Nu_{fin\ avg} = \frac{1}{L_{fin}}\int_0^{l_{fin}} Nu_{fin}\, dR \qquad (19)$$

Where , k_{eff} is defined as:

$$k_{eff} = \frac{2k_{fin}k_{nf}}{k_{fin} + k_{nf}} \qquad (20)$$

6. Results and discussion

In the present study laminar natural convection of nanofluids between two horizontal and an inclined concentric cylinders with fins attached to the inner cylinder is studied numerically. The inner cylinder is hot (maintained at constant temperature) and the outer cylinder is daithermal (adiabatic). The annulus is filled with three types of nanofluids (Ag (20nm),Cu (30nm),TiO_2 (50nm) – distilled water). The results are obtained for various Rayleigh numbers($Ra=10^3,10^4,10^5$),volume fractions of nanoparticles (0.25,0.5,1,3,5%vol), fins' length(0.3,0.4,0.6,0.8m) and fin inclination angle (30^0, 60^0, 90^0).

Fig. (7) shows the secondary flow and isotherms plots for different Rayleigh numbers, Φ = 0.3%vol, L_{fin} = 0.3. In general, by increasing Rayleigh number the center of vortexes moves to the upper zone of the annulus. It is observed that the streamline concentration in the upper part of the annuli is more than the lower part for three types of nanofluids Ag (20nm),Cu(30nm),TiO2(50nm)–distilled water, convection strength is weak in the lower part of annuli. It is to be noted that the buoyancy– induced motion in the lower part of the annulus is very weak due to the fact that high temperature surface (inner cylinder surface) is over the low temperature one (outer cylinder surface).In fact, there is a very weak fluid motion in the lower part of the annulus. The secondary flow for the three types of the nanofluids are greater than distilled water at all Rayleigh number due to nanoparticle and convection strength is a very high. The vortex strength increases with Rayleigh number, and finally the vortex breaks up into two vortices at $Ra=10^5$. Fig.(7) on the right side, depict isotherms. In general, by increasing Rayleigh number, convection heat transfer increases and the plume region becomes more obvious. The isotherm concentration in lower part of annuli around the inner cylinder is more than other places between two cylinders. The gradient temperature becomes smaller and the boundary layer becomes thicker at Ra = 10^4 and 10^5. Minimum temperature of the nanofluid is smaller than distilled water due to nanoparticle.

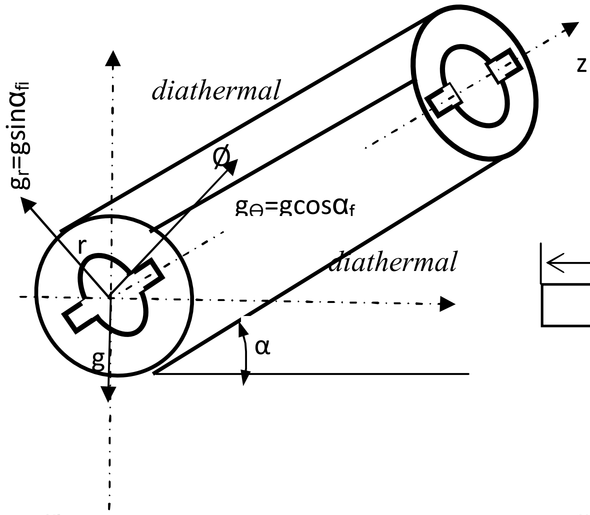
Fig.(8) When adding Ag (20nm),Cu (30nm) and TiO_2 (50 nm) nanoparticles in distilled water increases the effective thermal conductivity of the nanofluids and therefore the molecular heat diffusion is augmented. The vortex strength is similar approximately between nanofluids and distilled water.It can be noticed small second eddy formed below fin which pushed the main eddy to upward and making unsymmetrical. The secondary flow does not significantly change despite of higher heat flux needs to keep the Rayleigh number constant for higher particles concentration.

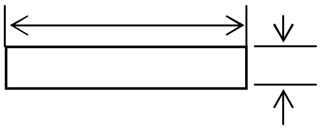
Fig.(9) reveals the effect of fin length on stream lines and isotherms at different fin length (L_{fin}= 0.3,0.4,0.6,0.8),Φ=0.3%vol,$Ra=10^5$. In this figure, there are two circulation loops. The fin length of 0.4 and 0.6 the vortex loop become thin in the level of fins until they are divided into four loops at a fin length 0.8. The results also indicate that the upper two loops are stronger than the lower loops. The isotherms in the annulus show that the heat transfer concentrated around the bottom of the inner cylinder. The existence of fin increases the thermal boundary layer along the bottom part of the inner cylinder and lower surface of attached fin. These results indicate that the heat transfer will increase as fin length increases.

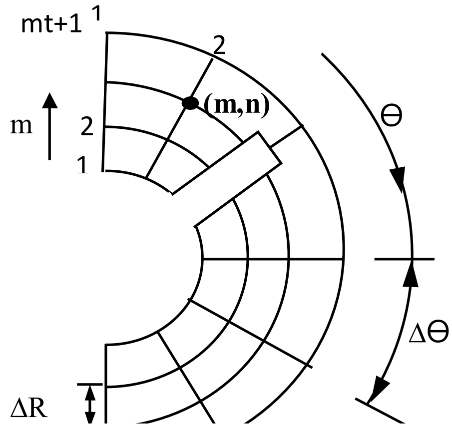
Fig.(10) The effects of fin inclination angle on stream lines and isotherms for nanofluids (Ag(20nm),Cu(30nm),TiO2 (50nm)–distilled water) at fin inclination angles ($\theta=30^0$,60^0,90^0), Φ = 0.3 % vol, L_{fin}= 0.3,and Ra =10^5. For fin inclination angle of $\theta = 90^0$,there are two symmetric circulation loops.As the fin angles decreases, the fins shift one loop down and the other up. This shift increases the resistance to the flow.As a result, circulation velocity decreases and thermal boundary layer gets thick along the upper faces of fins and upper surface of the inner cylinder. The vortex strength increases with increases fin inclination angles and minimum temperature of the nanofluid increases too, while the minimum temperature of the distilled water is greater the minimum temperature of the nanofluid.

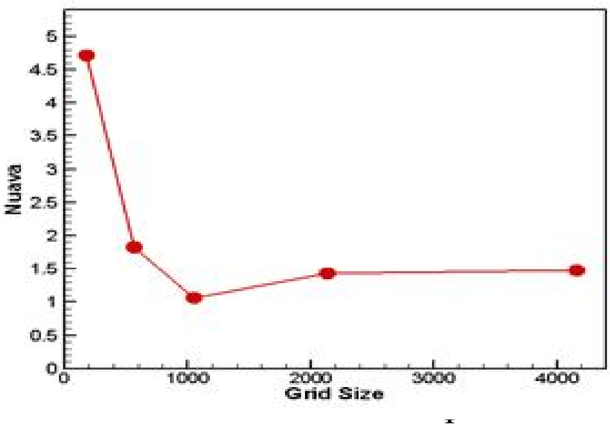
Fig.(11) shows the variations of local Nusselt number on the fins' surfaces and inner cylinder surface for nanofluids (Ag(20nm),TiO2 (50nm)–distilled water) at different Rayleigh numbers (Ra=10^3, 10^4, 10^5),volume fractions of nanoparticles(0.25, 0.5,1,3, 5%vol). from this figure can be seen that , by increasing the Rayleigh number, the Nusselt number on the inner cylinder and fins' surfaces increases. From this figure on the left side at Ra= 10^5 by increasing angular position,the Nusselt number increases to a maximum value at about $\Theta = 53^0$ and then decreases. This behavior is due to variations of temperature gradient on the inner cylinder. By increasing the angle, the temperature gradient increases first and causes the Nusselt number to increase. The temperature gradient is then decreased due to the lower fin, and causes the Nusselt number to decrease. The temperature gradient becomes a maximum at Θ= 53^0. The Nusselt number increases along the fin surface from the base to the tip and it decreases along the fin from the tip to the base. The Nusselt number increases by increasing Θ to 180^0 and it does not decrease because the temperature gradient only increases. Variation of local Nusselt number on the fins' surfaces and inner cylinder surface at Ra = 10^4 and 10^3 is similar to Ra = 10^5. It is to be mentioned that maximum Nusselt number is occurred at $\Theta = 49$ and $\Theta = 47$ for Ra = 10^4 and 10^3, respectively. The same behavior for variation of local Nusselt number to nanofluid TiO2 (50nm) – distilled water.

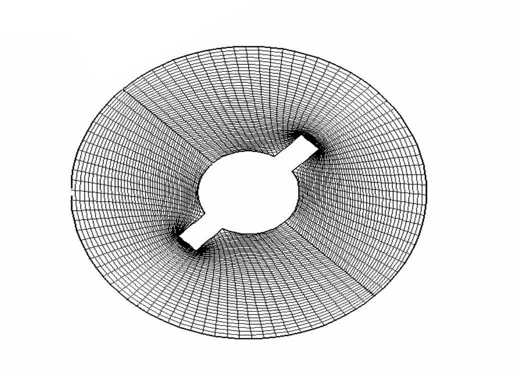
Fig.(12) Shows the axial profile of the peripheral average for Nusselt number with different inclination angles of the annulus ($\alpha=0^0$, 30^0,45^0,60^0, 90^0), Rayleigh number (Ra=10^5), fin length (L_{fin} =0.3,0.8) and volume fraction (Φ= 5 %vol). on the left side this figure show clearly that the increasing of the Nusselt number at horizontal position. the Nusselt number at fin length L_{fin}=0.3 is greater than at fin length L_{fin} = 0.8 to three types of nanofluids. As well as the Nusselt number int horizontal position is higher than the vertical position at the same values of the Rayleigh number (Ra = 10^5) and fin length due to the considerable influence of free convection currents at the horizontal position. As it is clearly seen increasing the Rayleigh number augments the buoyancy force and enhances Nusselt number. The enhancement heat transfer for the nanofluid (Ag (20nm) - distilled water) is higher than both nanofluids (Cu (30 nm)–distilled water) and (TiO_2(50 nm) – distilled water) due to small particles size for silver, ,the random motion is larger and the convection like effect become dominant. The three types of nanofluids showed higher heat transfer rate than the base fluid. The metallic nanoparticles give higher heat transfer enhancement than nano metallic particles (oxides) due to the high thermal conductivity of metallic nanoparticles. Fig. (13) represents the peripherally average skin friction coefficient for nanofluid (Ag (20nm)– distilled water) at constant Rayleigh number (Ra=10^5), fin length (L_{fin} = 0.3,0.8), volume fraction (Φ=0, 1, 5 % vol) and different inclination angles of the annulus ($\alpha=0^0$, 30^0,45^0,60^0, 90^0). In spite of augmenting the Nusselt number by increasing the nanoparticles concentration, the skin friction does not change. As it was seen in the previous figures, the secondary flow are not significantly affected by the nanoparticles concentration. Also at the high of the Rayleigh number by increasing the nanoparticles volume fractions does not have significant effect on Cf. In general, increasing the annulus inclinations augments the flow acceleration near wall and consequently higher skin friction occurs.

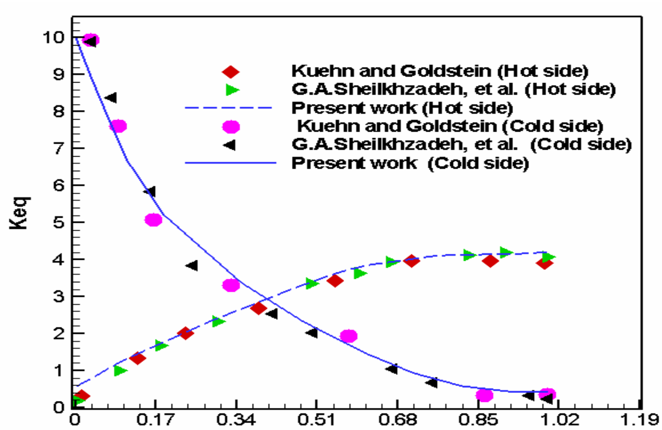












$\frac{\theta}{\pi}$

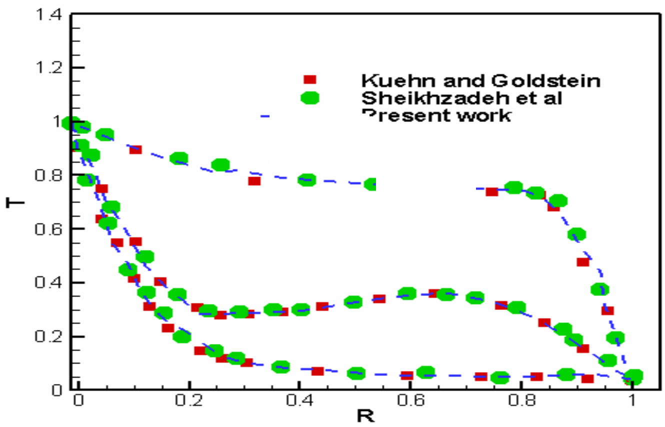


Fig.(6)Variation of temperature versus radial direction: comparison between the Present work and the results [1, 29]

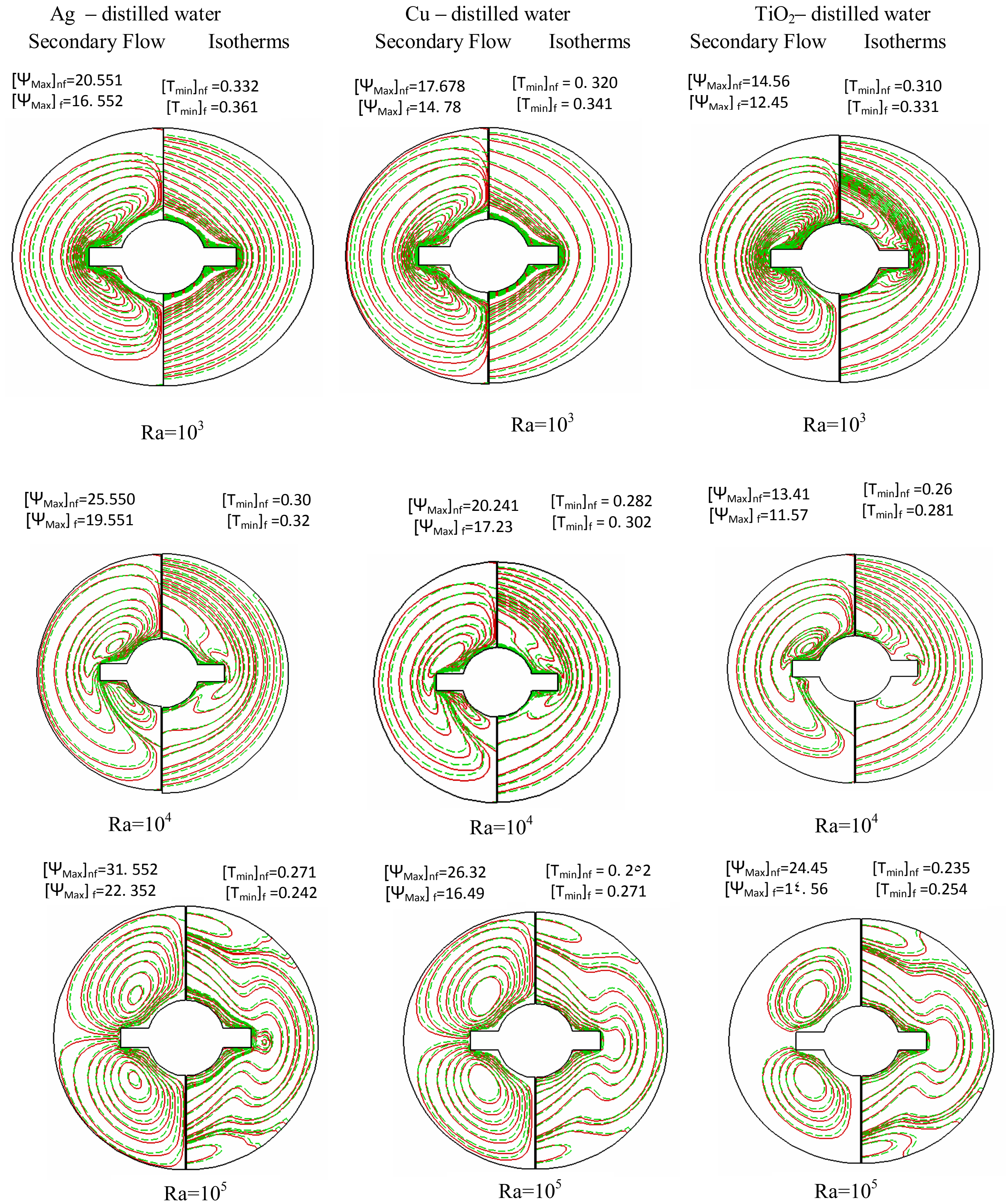


Fig.(7) : Secondary flow (on the left) and Isotherms (on the right) for Ag, Cu , TiO_2 – distilled water nanofluids (——) and water (- - - -) with different Ra, L_{fin} = 0.3, Φ =0.3, and α =0^0 (Horizontal)

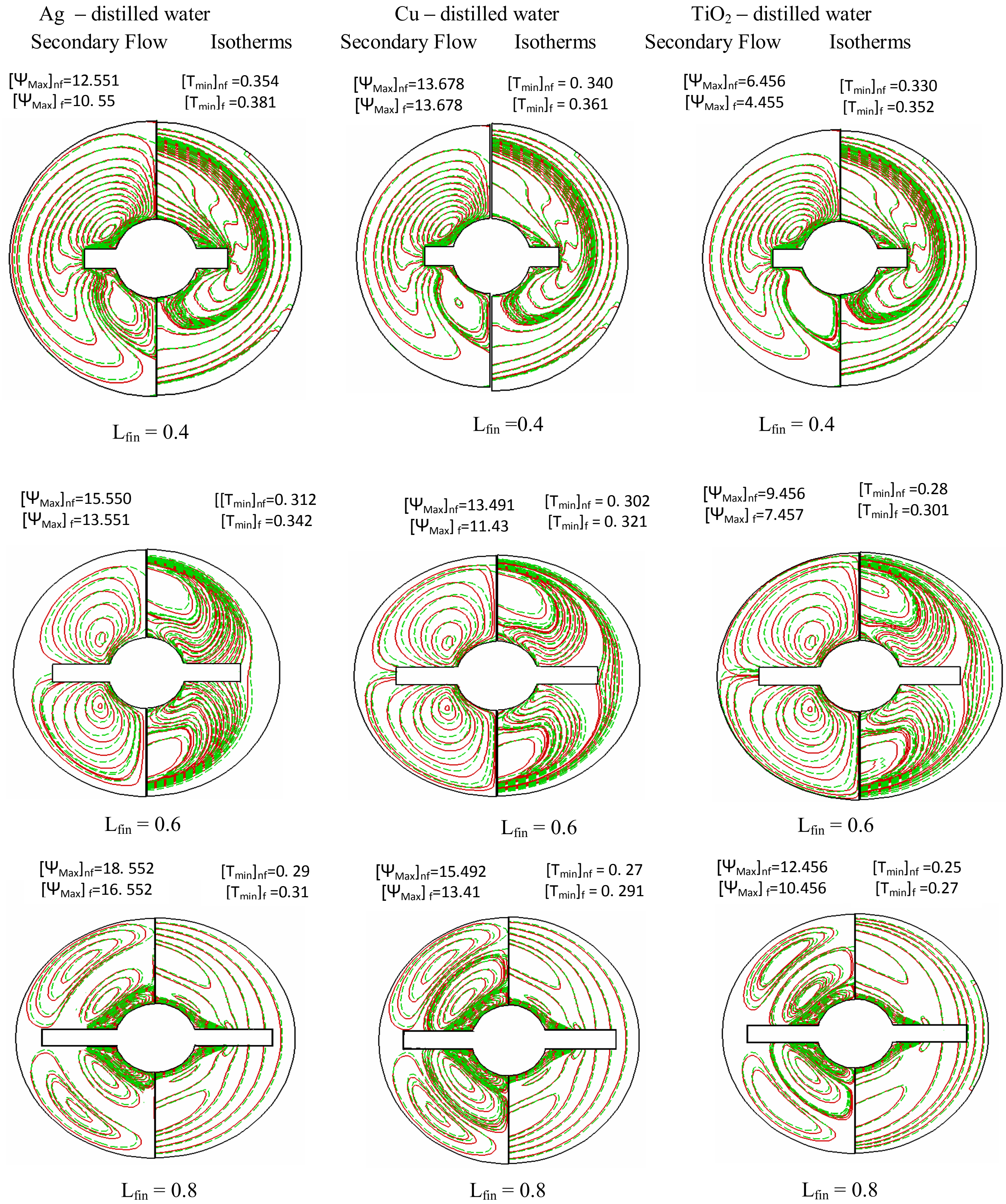


Fig.(9) : Secondary flow (on the left) and Isotherms (on the right) for Ag, Cu , TiO_2 – distilled water nanofluids (——) and water (- - - -) with different fin length, Φ =0.٣, Ra=10^5 and α =0^0 (Horizontal)

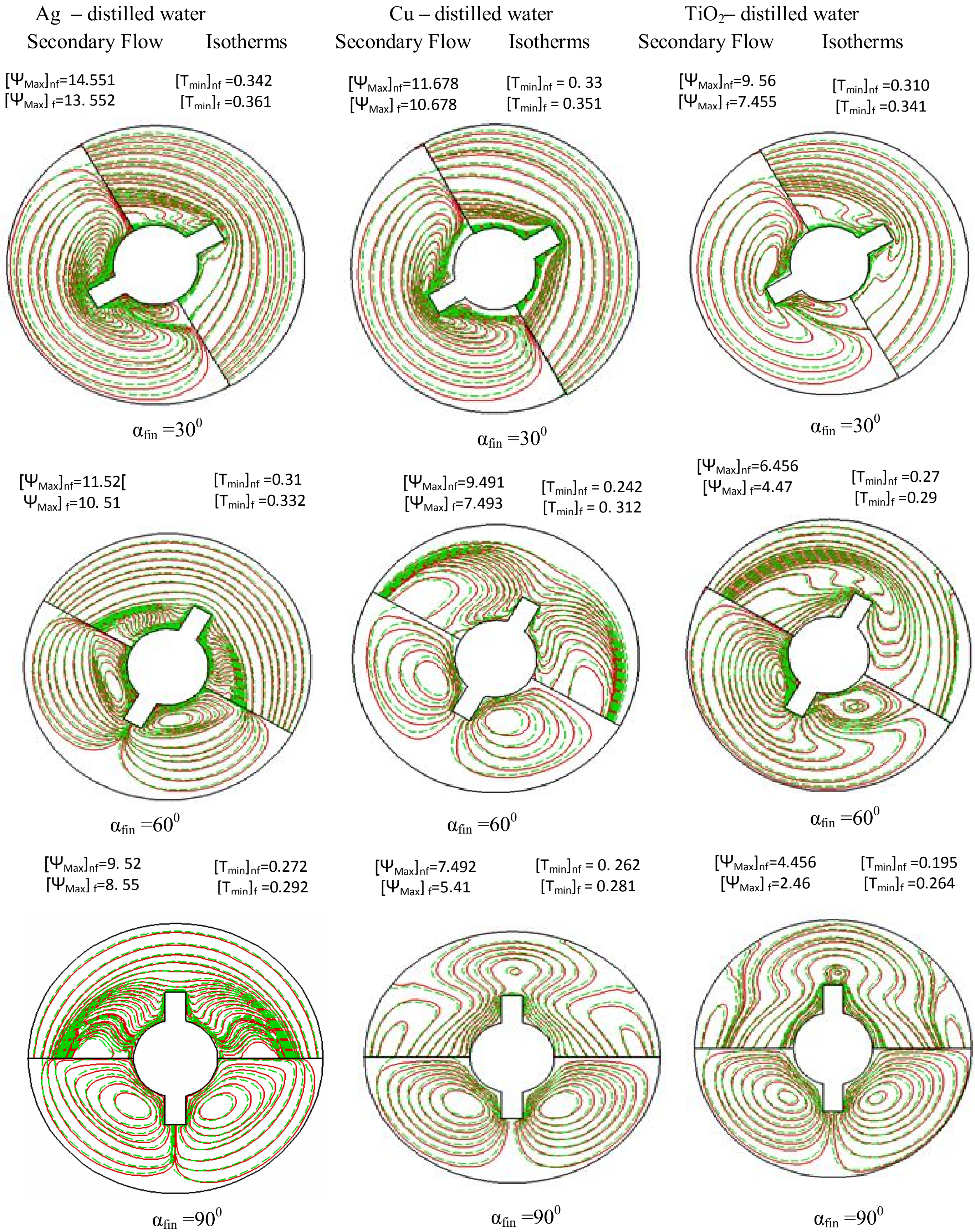


Fig.(10) : Secondary flow (on the left) and Isotherms (on the right) for Ag, Cu , TiO_2 – distilled water nanofluids (——) and water (- - -) with different fin inclination angles, Φ =0.3 %vol, L_{fin} =0.3 and $Ra=10^5$

Ag – distilled water

TiO_2 – distilled water

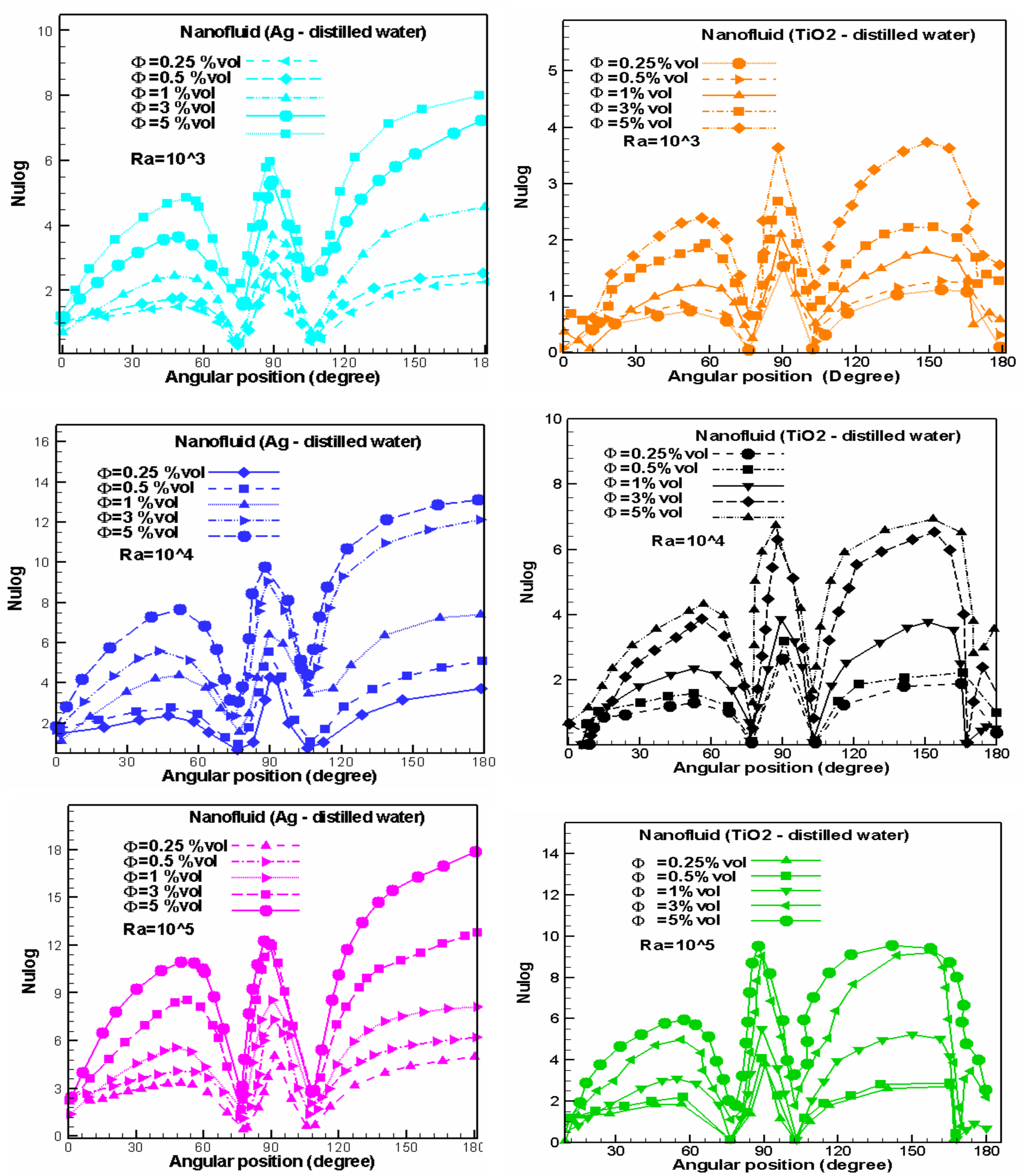


Fig. (11): variation of local Nusselt number around the inner cylinder and fins surfaces for Ag and TiO_2 – distilled water nanofluids with different Φ , Ra and L_{fin} =0.3

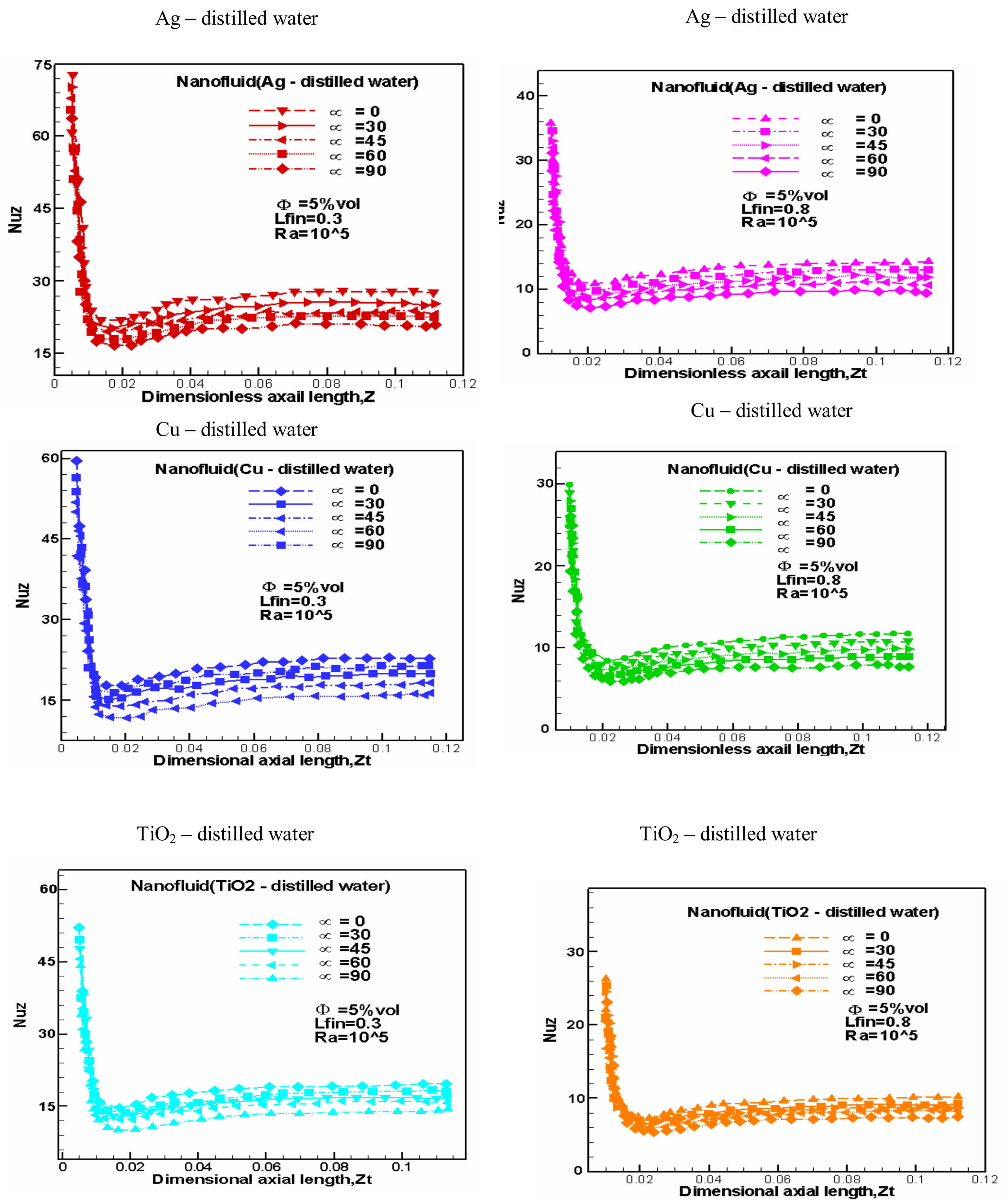
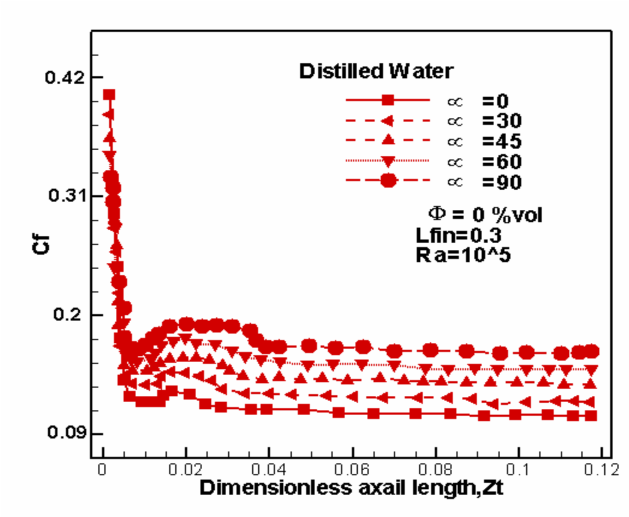
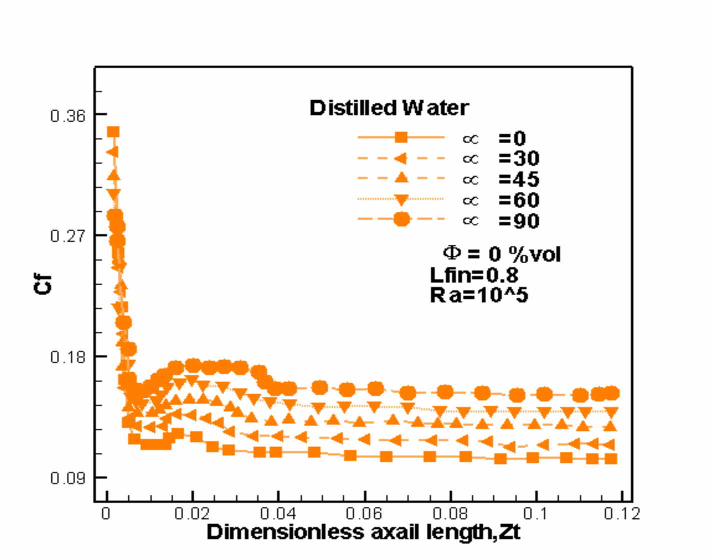


Fig. (12): Axial profile of the peripheral average Nusselt number to Ag, Cu and TiO_2 – distilled water nanofluids with different angles, fin length ,Ra=10^5, Φ = 5vol%

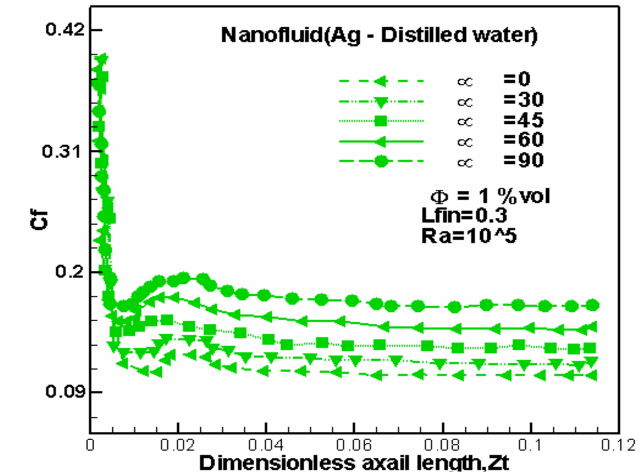
distilled water



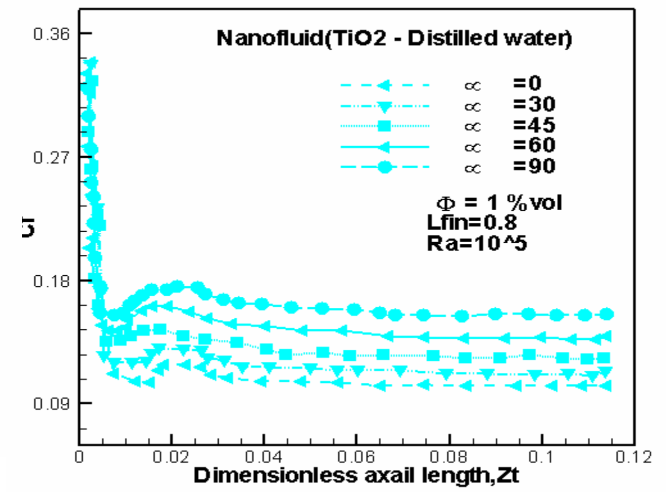
distilled water



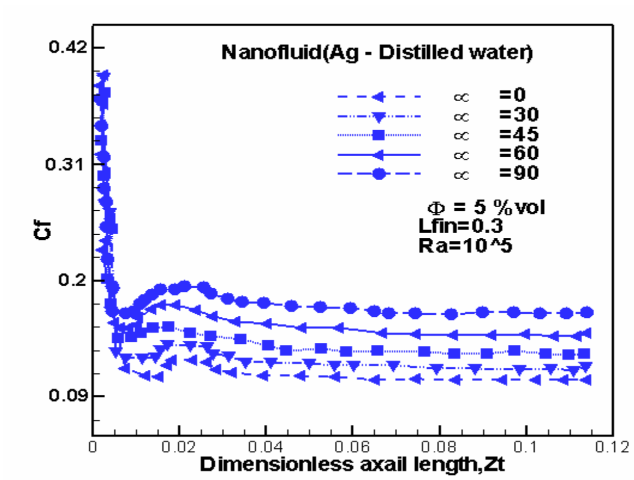
Ag – distilled water



TiO_2 – distilled water



Ag – distilled water



TiO_2 – distilled water

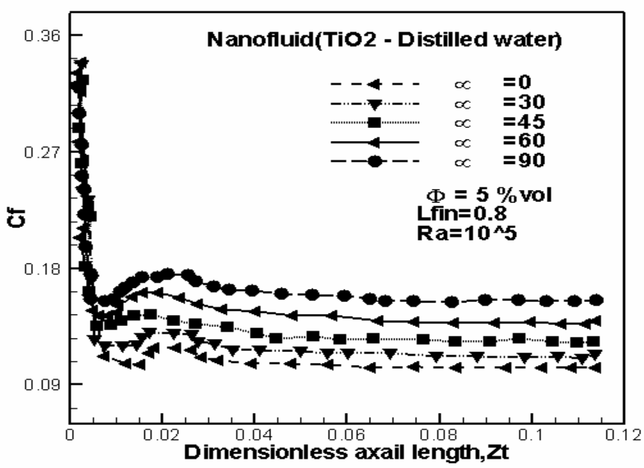


Fig. (13) : Axial profile of the peripheral average skin friction coefficient to for nanofluid (Ag, TiO_2 – distilled water) with different Φ ,Ra=10^5 and Lfin=0.3,0.8

7. Conclusion

The main conclusions of the present study are:

1. As the solid volume fraction increases, the heat transfer is enhanced for all values of the Rayleigh number. This enhancement is more significant at high Rayleigh number.
2. The lowest heat transfer was obtained for TiO_2 (50 nm) due to domination of conduction and large nanoparticles .whereas Ag (20 nm), Cu (30 nm) – distilled water nanofluids has the highest heat transfer, respectively.
3. The enhancement in heat transfer of nanofluids for annulus with fins attached to the inner cylinder at (5%vol) volume concentration of (Ag , Cu and TiO_2) nanoparticles increases (41.5 %, 35%,19) respectively compared with the base fluid (distilled water). There the choice of nanoparticles is very important in the convective heat transfer application.
4. Heat transfer rate increases by increasing the fins length while the average Nusselt number decreases by increasing fins' length.
5. The metallic nanoparticles give higher heat transfer enhancement than nonmetallic particles (oxides) due to the high thermal conductivity of metallic nanoparticles.
6. The average Nusslet number increases with increasing both Rayleigh number and the volume fraction of nanoparticles. More over the heat transfer rate increases with increasing the volume fraction of nanoparticles at all Rayleigh number.
7. The increase of the average Nusslet number is attributed to the increase of the thermal conductivity of nanofluid with increasing the volume fraction of the nanoparticles. However, the viscosity of nanofluid also increases with increasing the volume fraction of nanoparticles.
8. Skin friction coefficient is augmented by increasing inclinations of the annulus with fins attached to the inner cylinder.

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9. Nomenclature

m	The annulus inner radius	r_i
m	The annulus outer radius	r_0
kJ/kg.k	Specific heat of nanofluid at constant pressure	Cp_{nf}
m/s^2	Gravity acceleration	g
–	Dimensionless gravity acceleration	G
m	The fin thickness	d
m	Fin length	L_{fin}
–	Dimensionless cylindrical coordinates	R, Θ,Z
–	Nusselt number	Nu
–	Relaxation factor	S
°C	Temperature	T
m/s	Radial velocity component (r)	u
m/s	Tangential velocity component (Θ)	v
m/s	Axial velocity component (z)	w
W/m.k	Thermal conductivity of the nanofluid	k_{nf}
–	Dimensionless pressures	P
nm	Particle diameter	d_p
–	Constant wall temperature	CWT
–	Alternating direction implicit	ADI
		Creak Symbols
degree	Angle of inclination of tube	α
m^2/s	Thermal diffusivity of the nanofluid	α_{nf}
1/k	Thermal Expansion Coefficient of the nanofluid	β_{nf}
kg/m.s	Dynamic viscosity of the nanofluid	μ_{nf}
m^2/s	Kinematic viscosity of the nanofluid	ν_{nf}
kg/m^3	Density of the nanofluid	ρ_{nf}
m^2/s	Stream function	Ψ
Vol%	Volume fraction	Φ
		Subscripts
Number of radial points in the numerical mesh network		mt
Number of tangential points in the numerical mesh network		nt
	Wall	w

$\frac{\partial}{\partial r}$	r
$\frac{\partial^2}{\partial r^2}$	rr
$\frac{\partial^3}{\partial r^3}$	rrr
$\frac{\partial}{\partial \theta}$	θ
$\frac{\partial^2}{\partial \theta^2}$	θθ
$\frac{\partial^3}{\partial \theta^3}$	θθθ
$\frac{\partial}{\partial z}$	z
$\frac{\partial^2}{\partial z^2}$	zz
$\frac{\partial^3}{\partial z^3}$	zzz