Modeling and Design of a Suboptimal Controller for a Hydraulic System[1]

Prof. Dr. Rami A. Maher[2] **Dr. Eman S. Kareem[3]**

Abstract

Practical hydraulic control systems are nonlinear, high-order and parameters sensitive systems. On the other hand, usually the customer demands are difficult to achieve without some type of tradeoffs among these demands. Therefore, the burden of designing an optimal controller will be so complicated, and a suboptimal controller seems to be preferable. However, the validation of such design requires a detailed mathematical model of the hydraulic system and actual values of parameters. In this paper, a mathematical model of a hypothetical hydraulic system is derived first. Then, for the linearized model, a suboptimal controller is designed based on the LQR techniques. A Simulink model of the overall controlled system is utilized to simulate the closed-loop performance. The stable very fast response indicates the validity of the proposed procedure of design.

Key word:LQR Linear quadratic regulator ,EHS Electro hydraulic system.

الخلاصة

عمليا انظمة السيطرة الهيدروليكية انظمة لاخطية من الدرجة العالية high order وبراميترات حساسة جراء اي تغير يحصل بمدخلات النظام.ولتحقيق المطالب الصناعية يكون من الصعب اجراء تصميم مسيطر امثل لذا من الافضل اللجوء الى المسيطر شبه امثل suboptimal.وللتحقق من صحة التصميم المطلوب تم اعداد موديل رياضي لنظام هيدروليكي بقيم حقيقية لبراميترات المنظومة .في هذا البحث تم اشتقاق موديل رياضي افتراضي لنظام هيدروليكي اولا ثم استخدمت تقنية الـ simulink للنظام المسيطر عليه كليا لمحاكاة اداء الحلقة المغلقة للنظام.
نتائج البحث تشير الى ان الاستجابة مستقرة وسريعة جدا مما يدل على صحة الاجراء المقترح للتصميم.

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[2] Isra University/Amman-jordan

[3] Electromechanical department/ University of Technology, Baghdad, Iraq.

1. Introduction

In solving problems of optimal control systems, one may have the goal of finding a rule for determining the present control decision, subjected to certain constraints, which will minimize some measures of deviation from ideal behavior. Such a measures is usually provided by a criterion of optimization, or performance index. A performance index is a number which indicates the "goodness" of system performance. The performance index is important because it, to a large degree, determines the nature of the resulting optimal control. In other words, the resulting control may be linear, nonlinear, stationary, or time – varying, depending on the form of the performance index. [1]

Electrohydraulic systems (EHS) have been used in industry in a large number of applications due to their size-to-power ratio, and the ability to apply very large force and torque. However, the dynamics of hydraulic systems are highly nonlinear. The system may be subjected to non-smooth and discontinuous, nonlinearities due to control input saturation. Moreover, directional change of valve opening, friction, and valve overlap are affecting the operation [2]. Therefore, it is necessary to simulate the hydraulic actual-like system using its representative mathematical model. This should describe the dynamic of the hydraulic directional proportional valve and the cylinder unit of the hydraulic driver. A conventional controller has been designed using suboptimal control theory; as the results were within the required accuracy.

2. Mathematical Modeling of Electrohydraulic System:

The position electrohydraulic servomechanism consists of two parts [3]; the electro hydraulic proportional valve (proportional solenoid with stroke–to–current relationship), and a hydraulic driven unit cylinder. The output signal of this system is the position of the hydraulic cylinder piston, while the control signal is the output current of an electrical amplifier unit. Figure (1) shows the schematic of the proposed hydraulic system. The directional proportional valve converts the electrical signal to a translation motion, which in turn directs the position of the controlled sliding lever of the valve to control the fluid flow in a hydraulic cylinder. As a result, the hydraulic piston moves to translate the cylinders to the required position. With such description, if this control chain is closed through any type of position sensors, then a servo hydraulic system is obtained. However, the degree of complexity of the system is deliberately chosen to be near enough to practical systems.

2.1 Proportional Directional Valve Modeling

A proportional directional valve with electric feedback consists of the housing, two proportional solenoids, and inductive positional transducer. An electrical amplifier of a linear gain K_a characteristic is used. It is defined by the ratio of the electrical output current (solenoid current) i(mA) to the input voltage signal u_r (volt) for given load. Moreover, it is supposed that the current has no ripples, and the amplifier output signal u is the system control input. It is constrained as $u \in \langle -U_{\max}; +U_{\max} \rangle$, which actuates the hydraulic system. The value $U_{\max}$ is the maximum permissible value of the signal before saturation occurs. This means that the amplifier model will be as a nonlinear saturation element of gain K_a and $\pm U_{\max}$ (volt) cutoff values.

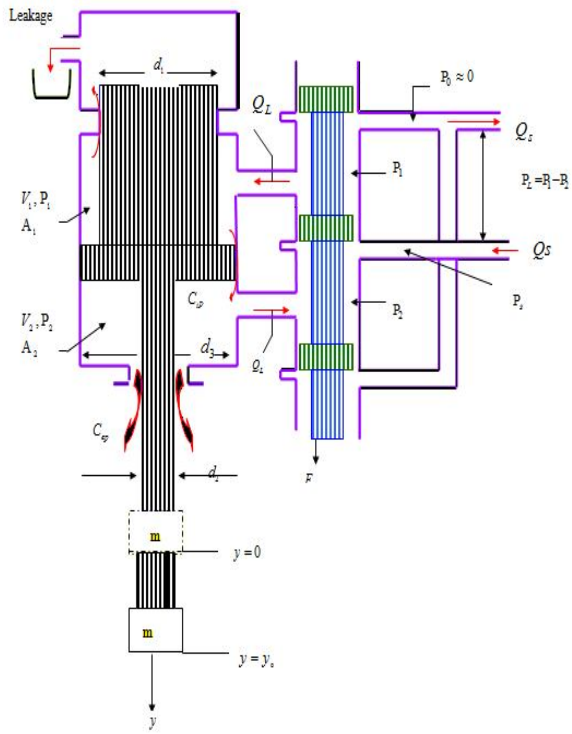


Figure (1) schematic diagram of hydraulic system

In the solenoid unit, the fixed solenoid performs a magnetic field of constant boosting, while the movable solenoid represents an electrical load of the electronic amplifier. The position of the moveable solenoid $d_s(mm)$ is a function of the current passing through it. A first order lag transfer function for the solenoid circuit can be used to model this electrical circuit, i.e.

$$\frac{i(s)}{u(s)} = \frac{1}{(T_s s + 1)} \qquad (1)$$

where T_s is the solenoid time constant.

Such overlap can be simply modeled by a dead–zone with saturation nonlinearity of 2δ sensitivity bandwidth and of a gain K_n, [2]. Therefore, the output of this hydraulic proportional valve will be defined as

The translation motion of the solenoid together with the mechanical part in the proportional valve can be represented by the following transfer function.

$$\frac{d_s(s)}{i(s)} = \frac{K_m}{(T_1 s + 1)(T_2 s + 1)} \qquad (2)$$

where K_m is the proportional valve gain T_1 and T_2 are two time constants.

If the position of the sliding lever $d(mm)$ is proportional to the movement of the movable solenoid then the total transfer function for the proportional valve with mechanical constraints on the sliding lever movement for $\pm\delta d(mm)$ becomes

$$G_P(S) = \frac{K_m}{(T_s s+1)(T_1 s+1)(T_2 s+1)(T_3 s+1)} \qquad (3)$$

However, due to the shelter of the sliding lever and the passive resistivity of sealing an overlap is expected as shown in figure 2.

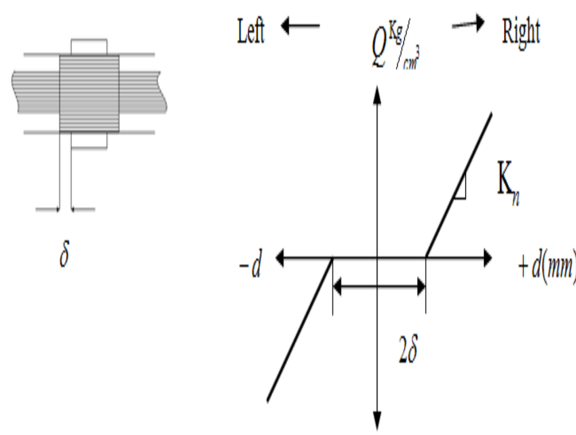


Figure 2 Overlap model [2]

$$f(d) = \begin{cases} 0 & ; |d| < \delta \\ dK_n & ; |d| > \delta \end{cases} \qquad (4)$$

2.2 Hydraulic Cylinder Modeling

It is known that the hydraulic cylinder can be represented in general by a third order differential equation [4, 5, and 6]. However, the nature of this equation (linear or nonlinear) and its coefficients depend on the piston shape and dimensions, fluid properties, the used pressure and sealing performance.

In this paper, a hydraulic cylinder of non-equal piston chambers will be considered; *see* figure 1. As it can be seen the piston area of the controlled pressure chamber, is greater than the area of the uncontrolled pressure chamber. For the considered piston chambers (the subscript 1 is used for uncontrolled pressure chamber, and the subscript 2 is used for controlled pressure chamber) the continuity relations are

$$Q_1 - C_{ip}(P_1 - P_2) - C_{ep}P_1 = \frac{dV_1}{dt} + \frac{V_1}{\beta_e}\frac{dP_1}{dt} \qquad (5)$$

$$-Q_2 - C_{ip}(P_1 - P_2) - C_{ep}P_2 = \frac{dV_2}{dt} + \frac{V_2}{\beta_e}\frac{dP_2}{dt} \qquad (6)$$

where:

- Q , Q_2 are the volumetric flow rates. ($m^3/\sec$)
- P_1, P_2 are the pressure. (N/m^3)
- V_1, V_2 are the volumes. (m^3)
- C_{ip} is the internal cross part leakage coefficient of piston. ($m^3.\sec^{-1}.pa^{-1}$)
- C_{ep} is the external leakage coefficient of piston. ($m^3.\sec^{-1}.pa^{-1}$)
- β_e is the effective bulk modulus. (N/m^3)

If the initial volumes for the two chambers are denoted by V_{o1} and V_{o2}, then the instantaneous volumes as a function of the piston movement y are given by

$$V_1(y) = V_{O1} + A_2 y$$
$$V_2(y) = V_{o2} + A_1 y \qquad (7)$$

where chambers effective area are given by

$$A_1 = \frac{\pi}{4}[d_3^2 - d_1^2]$$
$$A_2 = \frac{\pi}{4}[d_3^2 - d_2^2] \qquad (8)$$

where d_1, d_2, and d_3 are piston diameters as shown in Figure 1. The rates of change for the chambers volume are

$$\frac{dV_1}{dt} = A_2\frac{dy}{dt}$$
$$\frac{dV_2}{dt} = -A_1\frac{dy}{dt} \qquad (9)$$

By subtracting equation (6) from equation (5) and substituting for the volumes of the chambers, the following equation is obtained

$$Q_1 + Q_2 - 2C_{ip}(P_1 - P_2) - C_{ep}(P_1 - P_2)$$
$$= (A_1 + A_2)\frac{dy}{dt} + \frac{V_{o1}}{\beta_e}\frac{dP_1}{dt}\,\frac{A_2}{\beta_e}y\frac{dP_1}{dt}$$
$$-\frac{V_{o2}}{\beta_e}\frac{dP_2}{dt} + \frac{A_1}{\beta_e}y\frac{dP_2}{dt} \qquad (10)$$

Since we have,

$P_0 \approx 0, \quad P_L = P_1 - P_2, \quad P_s = P_1 + P_2$

Then,

$$Q_L = \frac{1}{2}(Q_1 + Q_2) = \frac{k_\mu}{\sqrt{\rho}}[\sqrt{P_s - P_L} - \sqrt{P_s + P_L}]f(d) \qquad (11)$$

where K_μ is the coefficient including orifice geometry and discharge coefficient, ρ is the oil density and $f(d)$ is the hydraulic valve output given by equation (4). Also,

$$\frac{dP_1}{dt} = \frac{1}{2}\frac{dP_L}{dt} \quad and \quad \frac{dP_2}{dt} = -\frac{1}{2}\frac{dP_L}{dt} \qquad (12)$$

If a leakage constant C is defined by

$$C = C_{ip} + \frac{1}{2}C_{ep} \qquad (13)$$

then a final equation can be written

$$Q_L = C\mathrm{P}_1 + \mathrm{A}\frac{dy}{dt} + \frac{V_o + ay}{4\beta_e}\frac{dP_l}{dt} \qquad (14)$$

where

$$\mathrm{A} = \frac{\mathrm{A}_1 + \mathrm{A}_2}{2} \quad , \quad a = \mathrm{A}_2 - \mathrm{A}_1 \quad , \quad V_o = V_{o1} + V_{o2}$$

For certain constant load force F_L , the load pressure can be calculated from the principle equation

$$\mathrm{P}_L = \frac{F_L}{\mathrm{A}} \qquad (15)$$

Applying Newton's law to the forces acting on the piston yields

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} = \mathrm{AP}_L \qquad (16)$$

where

- y is the piston displacement (mm).
- m is the total mass of piston & the load referred to piston (kg).
- b is the viscous damping coefficient of piston to the load ($N.\sec.m^{-1}$).

Arranging both of equation (14) and equation (16) in one relation by eliminating the load pressure P_L , yields

$$\frac{V_\circ + ay}{4\beta\mathrm{A}}m\frac{d^3y}{dt^3} + \left[\frac{V_\circ + ay}{4\beta\mathrm{A}}b + \frac{Cm}{\mathrm{A}}\right]\frac{d^2y}{dt^2} + \left[\mathrm{A} + \frac{Cb}{\mathrm{A}}\right]\frac{dy}{dt} = Q_L \qquad (17)$$

Next, we define the following eight coefficients; only two of them $c_2\,, c_3$ are function of the piston displacement y .

$$c_1 = \frac{1}{2}\frac{\mathrm{K}_\mu}{\sqrt{\rho}}[\sqrt{\mathrm{P}_s - \mathrm{P}_L} + \sqrt{\mathrm{P}_s + \mathrm{P}_L}]$$

$$c_2 = \frac{4\beta\mathrm{A}}{m(V_\circ + ay)}, \quad c_3 = \frac{1}{mc_2} = \frac{V_\circ + ay}{4\beta\mathrm{A}}$$

$$c_4 = \frac{m}{\mathrm{A}}, \quad c_5 = \frac{b}{\mathrm{A}}, \quad c_6 = \mathrm{A}, \quad c_7 = C, \quad c_8 = b \qquad (18)$$

The specified 3rd-order nonlinear differential equation governs the piston translation movement (displacement y) will have the form

$$\frac{d^3y}{dt^3} + c_2[c_3c_8 + c_7c_4]\frac{d^2y}{dt^2} + c_2[c_6 + c_7c_5]\frac{dy}{dt} = c_1c_2 f(d) \qquad (19)$$

The 4th order transfer function of equation (3) and the 3rd-order differential equation of equation 19 represent the complete model of the electrohydraulic system.

3. Linearized Model

In order to design the controller via the optimal LQR theory, we have to determine first a linear state space model. A state vector $X(t)$ representing the electrohydraulic system will have seven states, and are selected as follows:

- The current of the movable solenoid, $x_1(t) = i(m\mathrm{A})$.
- The velocity of the movable solenoid, $x_2(t) = \dot{d}_s(mm.\sec^{-1})$.
- The position of the movable solenoid, $x_3(t) = d_s(mm)$.
- The position of the sliding lever, $x_4(t) = d(mm)$.
- The dynamic load pressure, $x_5(t) = P_L(Pa)$
- The velocity of piston $x_6(t) = y'(mm.\sec^{-1})$
- The position of piston, $x_7(t) = y(mm)$

As it is mentioned in section (2.1) the proportional value has been linearized by a transfer function of four lags constants $T_S, T_1, T_2, \& \mathrm{T}_3$ and total gain K_m, while the hydraulic cylinder has not yet been linearized. For constant F_L and assumed initial position of the piston $y_\circ$ (the piston is moved before any control but only due to pressure balance), the two nonlinear coefficient functions c_2 *and* c_3 can be calculated at $y = y_{\circ}$ to obtain constant values $\bar{c}_2 \, and \, \bar{c}_3$. In this way, it is possible to represent the differential equation (21) by a transfer function of the form

$$G_c(s) = \frac{\mathrm{K}_c}{s(s^2 + as + b)} \tag{20}$$

where

$$\mathrm{K}_c = c_1 \bar{c}_2 \mathrm{K}_n$$

$$a = \bar{c}_2(\bar{c}_3 c_8 + c_7 c_4)$$

$$b = \bar{c}_2(c_6 + c_7 c_5)$$

With the above definition for state variable and the two transfer functions given in equation (3) and equation (22), the linearized model can be represented by the state equation

$$X' = \begin{bmatrix} -\frac{1}{T_s} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\mathrm{K}_m}{T_1 T_2} & -\frac{(T_1 + T_2)}{T_1 T_2} & -\frac{1}{T_1 T_2} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T_3} & -\frac{1}{T_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4 \mathrm{K}_c & -\bar{c}_2 c_4 c_7 & -\bar{c}_2 c_4 c_6 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{c_4} & -\frac{c_5}{c_4} & 0 \\ 0 & 0 & 0 & 0 & 0 & k & 0 \end{bmatrix} X + \begin{bmatrix} 1/T_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \tag{21}$$

where k is equal to 10 (1cm = 10mm). Finally, the nonlinear elements are modeled in the linearized model only by their gain K_a *and* K_n ; however it is expected that the dead zone with saturation element may cause unstable limit cycle or at least long regulating time and unaccepted overshoot. Therefore, a sort of compensation is necessary to be included. A standard technique is to use a dither signal.

Due to the expected sensitivity problem of the considered hydraulic system, the system simulation has to be carefully carried. This in turn requires that the system parameters must be correct to physical point of view. To overcome such circumstances, the numerical model is readjusted several times based on the suitable literatures [5, 6, and 7] to reach a final physical meaningful model. The final numerical values taken to construct the hydraulic system model are listed in Table (1) of the appendix. The hydraulic cylinder parameters and coefficients c_i are calculated and listed in Table (2) of the appendix . For the linearized model, the coefficient matrix, A and the input matrix, B are calculated, see the appendix.

The eigenvalues set of the A matrix contain one zero. Theoretically, if there is zero eigenvalue in the linearized model, then the stability of the nonlinear system (according to second Lyapunov theory) cannot be deduced from the linearized model. However, as expected the simulation of the open loop nonlinear electrohydraulic system shows unstable response for arbitrary step input.

4. Suboptimal Controller Design

For the linearized model, the linear optimal control theory is invoked; specifically, the infinite horizon LQR. Therefore, for this SISO constrained system, the problem is stated as,

$$\min_{|u| \le U} \{ J(u) = \int_0^\infty (X^T Q X + r u^2) \, dt \}, \quad r = 1$$

$$subjected \quad to:$$

$$X'(t) = AX + Bu, \quad X(0) = 0_{7 \times 1} \tag{22}$$

Accordingly, a state feedback gain can be obtained to perform this task using the Matlab function ***lqr***. However, first the weight matrix Q has to be determined. Since in this system, only the piston position of interest, the Q matrix should have only one non zero element q, i.e. Q has the form $Q = \begin{bmatrix} 0_{6\times6} & 0_{6\times1} \\ 0_{1\times6} & q \end{bmatrix}$.

Using the known thumb of rules of selecting q, it could be possible to set a range of values [q_{min}, q_{max}] and based on the system performance (the regulating time, settling time and the maximum overshot) one specified value could be obtained. To reduce the effort of such ad hoc searching, a SIMULINK model of the system as shown in figure (3) is set (see the appendix for the complete electrohydraulic model), in which a direct measure of the regulating time and settling time is recorded for different values of input q in the stable operation range of values. Moreover, a Matlab m. program is running simultaneously as a function block in the Simulink set up to solve the LQR problem and supplies the model with the state feedback gain vector K. For $2\times10^{7} \le q < 2\times10^{8}$, the simulation reveals that one can choose the value $q = 2.5\times10^{7}$ as an optimal value, for which the step response has a compatibly good response. Figure (4) illustrates the idea of how to select specific q, where the vertical lines represent the crossing of 1 ± 0.01 value. The regulating settling time is read at the first and last vertical lines respectively. The gain matrix k for $q = 2.5\times10^{7}$ is

$$K = [0.1029 \quad 0.1642 \quad 141.8653 \quad 26.3514 \quad 0.0007 \quad 0.2598 \quad 192.3077]$$

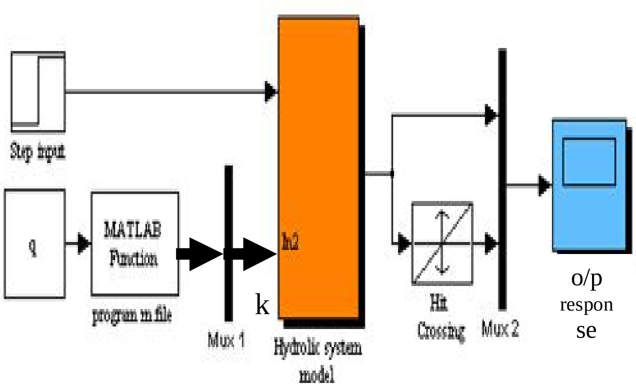
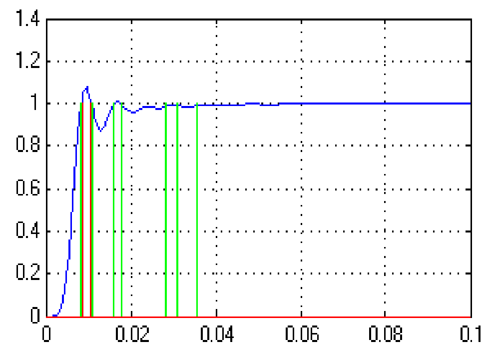


Figure (3) The Simulink Model illustrates how to reach the optimal value of q

Output response y(t)(mm)



Time (sec)

Figure (4) Step response for values of $q = 2.5\times10^{7}$

For implementing the optimal state feedback control law, the position and velocity of the movable solenoid ($x_2(t)$, $x_3(t)$) are not measurable while the other states are all measurable. Therefore, a certain type of reduced-order state estimator should be incorporated. The general theory of design reduced –order Luenberger observer will be utilized [8]. The estimator uses states $x_1(t)$, and $x_4(t)$ to estimate $x_2(t)$, and $x_3(t)$. The design of the estimator is performed so as to have a fast response with minimal possible estimation errors [9]. The result is shown by the dynamic system in figure (5), where,

$$A = \begin{bmatrix} -525 & 84761 \\ 1 & -214.8 \end{bmatrix},$$

$$B = \begin{bmatrix} 586.3339 & -40071 \\ 0 & 92.12 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad D = 0_{2\times2}$$

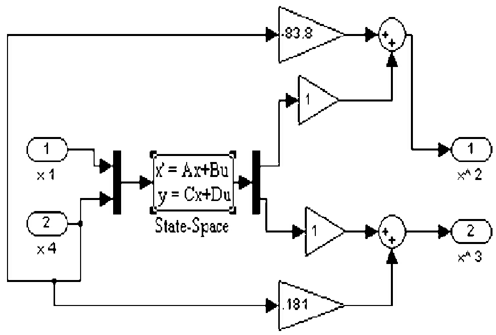


Figure (5) Block diagram of reduced order Lunberger observer

Running the simulation of the complete design electrohydraulic with state estimator and state feedback control, we can determine the system response. Figure (6) depicts the output response for different input set with the optimal value of q; the simulation time is 0.1 second. In all three cases, the output reaches the required position within a settling time less than approximately 0.02 second. However, the dither signal is experimentally decided to be with amplitude 17, and frequency (2*pi*100); these values give the smallest amplitude of the existing limit cycle.

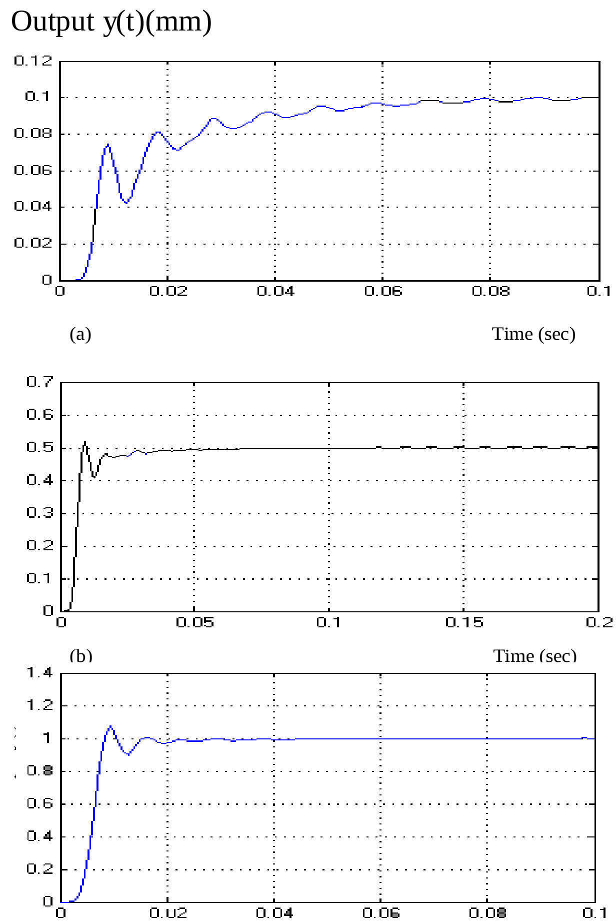


Figure (6) the output response for different input set with the optimal value of $q=2.5*10^7$
a)i/p=0.1 unit step b)i/p=0.5unit step c)i/p=1 unit step

5. Conclusions

The following points summarize the main conclusions drawn from this research:

1. The mathematical modeling of both the proportional valve and the hydraulic cylinder and hence transfer functions are presented. However, the degree of complexity of the system is deliberately chosen to be near enough to practical systems. A Hdraulic cylinder of non -equal piston chambers is considered.
2. A suboptimal controller can be designed for nonlinear system by applying linear quadratic regulator (LQR) technique to a linearized model and adjusting the controller parameters based on nonlinear system performance.
3. The dead zone element causes unstable limit cycle, long regulating time and unaccepted overshoot, therefore a conventional technique dither signal is used.

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Appendix:- Table (1&2) Numerical values of parameters and coefficients

Table (1) Numerical values of parameters for an electrohydraulic system

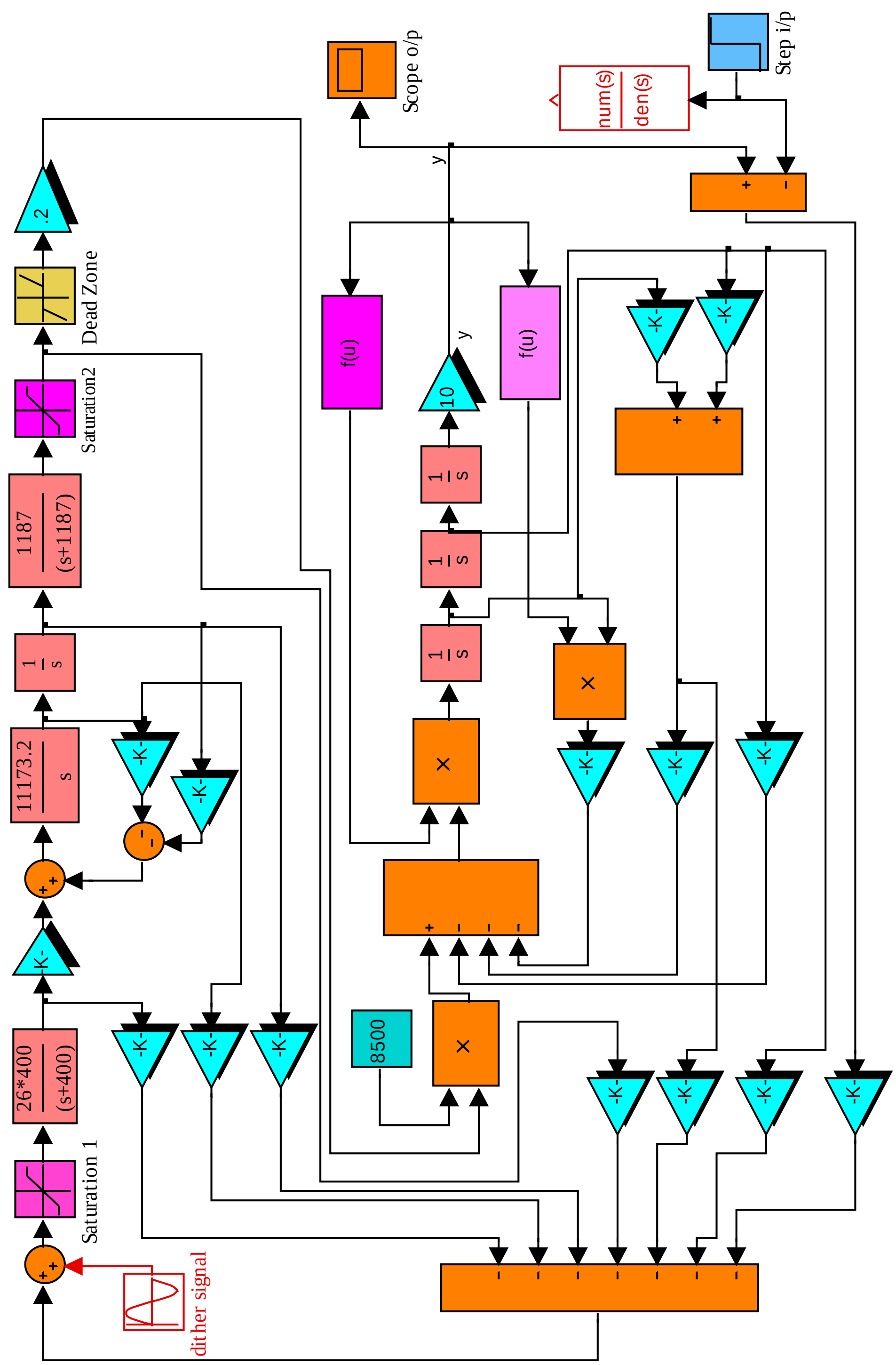
Unit	value	symbol
$mA.v^{-1}$	26	K_a
volt	300	U_{max}
sec.	2.5×10^{-3}	T_s
$mm.A^{-1}$	0.04	K_m
sec.	2.018×10^{-3}	T_1
sec.	3.3806×10^{-2}	T_2
sec.	8.424×10^{-4}	T_3
mm	0.1	δ
mm	4	δ_d
$mm.v^{-1}$	0.2	K_n
cm^3	500	$V_\circ$
$m^3.\sec^{-1}Pa^{-1}$	10^{-8}	C
mm	100	d_1
mm	25	d_2
mm	136	d_3
M Pa	12	P_s
M Pa	2.73×10^{5}	β
$m^2.\sec^{-1}.Pa^{-0.5}$	2.57×10^{-4}	$\frac{K_m}{\sqrt{\rho}}$
kg	297	m
$N.\sec.m^{-1}$	1.66×10^{6}	b
kN	70	F_L
mm	55	y_o

Table (2) the hydraulic cylinder parameters &coefficients

Values and units	Parameters & coefficients	Values and units	Parameters & coefficients
$4.48\times10^{-8}+6.64\times10^{-9}y$	c_3	$66\ cm^2$	A_1
$4.1\times10^{-7}\ cm^3.N^{-1}$	$\bar{c}_3$	$140\ cm^2$	A_2
$2.88\ Kg.cm^{-2}$	c_4	$103\ cm^2$	A
$161\ N.sec.cm^3$	c_5	$74\ cm^2$	a
$103\ cm^2$	c_6	$6.8\ MPa$	P_L
$0.01\ cm^3.sec^{-1}.Pa^{-1}$	c_7	$8500\ cm^2.sec^{-1}$	c_1
$1.66\times10^4\ N.sec.cm^{-1}$	c_8	$5.22\times10^6(69.5+10.3y)^{-1}$	c_2
		$8207\ cm^{-2}.sec^{-2}$	$\bar{c}_2$

- The matrices A and B

$$A=\begin{bmatrix} -400 & 0 & 0 & 0 & 0 & 0 & 0 \\ 586.3339 & -525.1 & -14658 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1187 & -1187 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\times10^7 & -236.4 & -2.43\times10^6 & 0 \\ 0 & 0 & 0 & 0 & 0.347 & 55.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 \end{bmatrix},\quad B=\begin{bmatrix} 400 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



The complete block diagram with suboptimal controller for HS