

Deriving And Calculation Of Point Spread Function For Array Of Obscured Circular Synthetic Apertures

Ban Hussein Ali AlrueshdyM , Adnan Falih Hassan Aldehadhawe
University of Kufa/ College of Sciences / Department of Physics

Abstract

This research includes the equation of point spread function (PSF) which has been derived for array of obscured circular synthetic apertures; it studies the effect of changing the obscuration ratios ($\varepsilon=0, 0.25, 0.5, 0.75$), MathCAD programs were used to simulate the equation of the point spread function, we obtain from the result increasing the intensity of secondary peaks and the best resolving power comparison with resolving power for obscured individual circular aperture.

الخلاصة

تضمن البحث اشتقاق معادلة دالة الانتشار النقطية لصف من الفتحات المركبة الدائرية المعاقة ودراسة تأثير تغيير نسب الإعاقة ($\varepsilon=0,0.25,0.5,0.75$) وذلك باستخدام برنامج الماتكاد ، ووجد من خلال النتائج حصول زيادة في شدة القمم الثانوية وقدرة التحليل أفضل مقارنة مع قدرة تحليل الفتحة الدائرية المنفردة المعاقة.

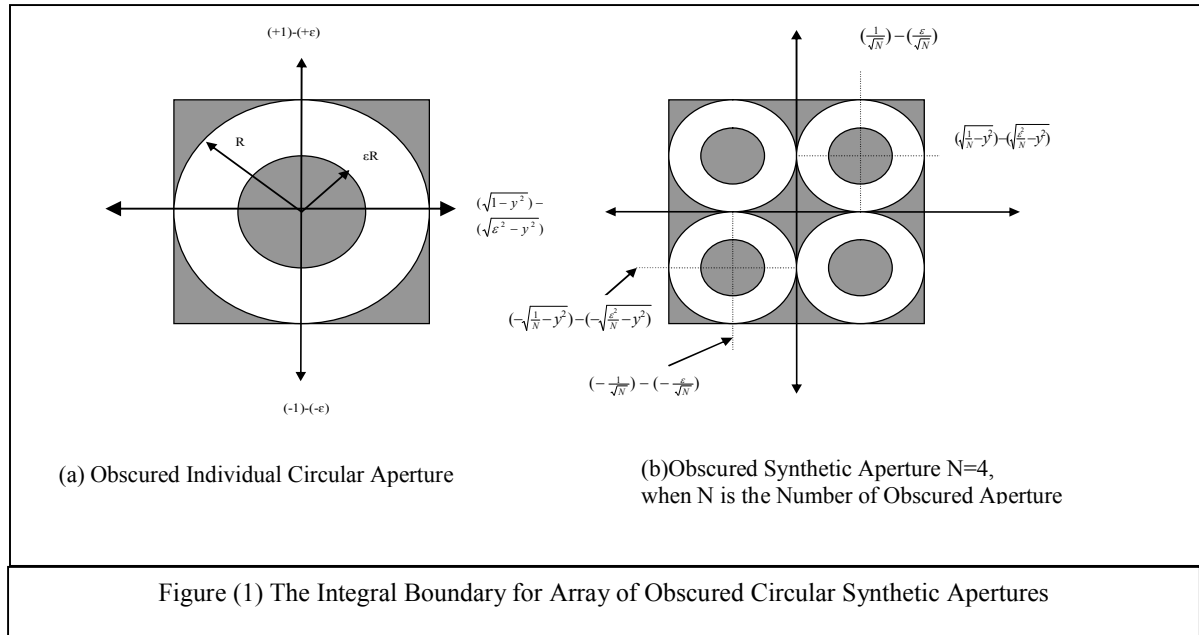
Introduction

Most optical systems are used to create images: eyes, cameras, microscopes, telescopes ,etc. These image forming instruments use lenses or mirrors whose properties, in terms of geometrical optics(A. Lipson *et al.*,2011). The image of a point source object formed by a lens system is known as the point spread function (PSF) of the lens, it is one of the most complete function for describing the performance of an optical system and can be extended to include the effects of an obstructed aperture, apodization and any factor external to the optical system(A.F. Aljebory, 2004). The properties of the optical system can be described by a point spread function, point spread function is related to the phase at these exit pupil apertures through a Fourier transform (K.K. Sharma, 2004). A PSF is an irradiance distribution representing the image of an ideal point source , namely a PSF is a diffraction limited intensity distribution in response to a point source such as a distant star (Soon-Jo Chung,2002). Diffraction plays an important role in optical systems (R. Murugesan, 2008). An imperfection or distortion in an image is called an aberration, an aberration can be produced by a flaw in a lens or mirror, but even with a perfect optical surface some degree of aberration is unavoidable (B. Crowell,2007). The method adapted to seeing the close objects as separate objects is called resolution. The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power(N. Subrahmanyam *et al.*, 2009; R. Murugesan, 2008). Synthetic aperture techniques are commonly used to obtain high resolution from data acquired using low resolution sensors, these techniques are commonly used in modern sonar and radar systems, being designated by Synthetic Aperture Sonar (SAS) and by Synthetic Aperture Radar (SAR) systems, respectively; this kind of systems is presently used in civil and military applications (P. Marques *et al.*,2004). " Synthetic Aperture " Can be defined as a structure to separate optical systems of large individual aperture function sometimes called " Mosaic" or "Segmented

Mirror", Synthetic aperture is an image system for independent optical system which are together sharing the image domain (A.F. Aljebory, 2004).

Deriving the Equation of Point Spread Function for Array of Obscured Circular Synthetic Apertures

The representation of the array of circular synthetic apertures with radius (R) inside it array of obscured circular synthetic apertures with radius (ϵR) as shown in Figure (1)



The ratio between the radius of obscured circular synthetic apertures and the radius of circular synthetic apertures is called obscuration ratio and it is symbolized (ϵ) (G.C. Sloon et al., 2003; G.S. Alshabaan, 2001; I.M. Alkhafaji, 2011)

The equation for array of circular synthetic apertures is given by the relation (see Figure(1)):

$$x^2 + y^2 = \left(\frac{R}{\sqrt{N}}\right)^2 \dots\dots\dots(1)$$

and when $R = 1$, then:

$$x = \pm \sqrt{\frac{1}{N} - y^2} \quad \text{and} \quad y = \pm \frac{1}{\sqrt{N}} \dots\dots\dots(2)$$

The equation for array of obscured circular synthetic apertures is given by the relation

$$x'^2 + y'^2 = \left(\frac{\epsilon R}{\sqrt{N}}\right)^2 \dots\dots\dots(3)$$

$$x' = \pm \sqrt{\frac{\epsilon^2}{N} - y'^2} \quad \text{and} \quad y' = \pm \frac{\epsilon}{\sqrt{N}} \dots\dots\dots(4)$$

The intensity in point spread function for array of obscured circular synthetic apertures, can be calculated by the relation (G.C. Sloon et al., 2003).

$$PSF = |F|^2 = |F_1 - F_2|^2 \dots\dots\dots(5)$$

where

F_1 : is the complex amplitude function for array of circular apertures.

F_2 : is the complex amplitude function for array of obscured circular apertures.

We can express the complex amplitude in the point (u', v') in the image plane by using Fourier transform to pupil function(*B.J. Davis et al.,2004; G.A. Cirino et al.,2006; J.W. Goodman,1998; K.J. Gasvik,2002; S. Wang et al.,2008*).

$$f(x', y') = \tau(x', y')e^{ikw(x', y')} \dots\dots\dots(6)$$

where

$\tau(x', y')$: represents the real amplitude distribution in exit pupil and which is called " pupil transparency " or " transmission function " and often chooses equal one unit .

$e^{ikw(x', y')}$: wave front of aberration function.

$w(x', y')$: aberration factor

(x', y') : exit pupil coordinates (*G. Chartier, 2005; M. Bass et al.,2010*).

We can express point spread function for circular aperture in its integral form, by putting the exit pupil coordinates of form (x, y) instead of (x', y') , for simplicity .

$$F_1(u', v') = n \cdot f \int \int_{y \ x} f(x, y) e^{i2\pi(u'x + v'y)} dx dy \dots\dots\dots(7)$$

and for N from obscured circular apertures (Figure(2)), we get:

$$\because x = x' + x_j \quad y = y' + y_j \dots\dots\dots(8)$$

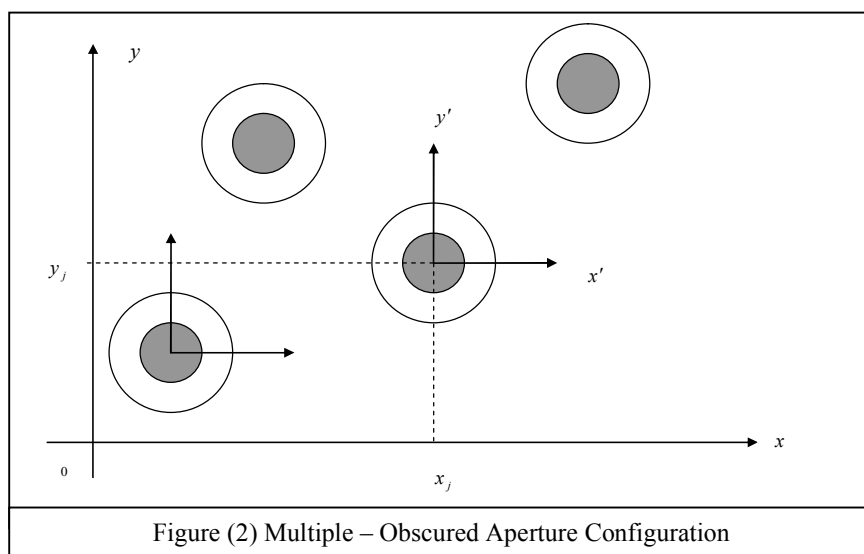


Figure (2) Multiple – Obscured Aperture Configuration

$$F_1(u', v') = n.f \sum_{j=1}^N \int_y \int_x f(x, y) e^{i2\pi[u'(x'+x_j)+v'(y'+y_j)]} dx dy \dots\dots\dots(9)$$

$$F_1(u', v') = n.f \sum_{j=1}^N \int_y \int_x f(x, y) e^{i2\pi(u'x'+v'y')} .e^{i2\pi(u'x_j+v'y_j)} dx dy \dots\dots\dots(10)$$

$$F_1(u', v') = n.f \int_y \int_x f(x, y) e^{i2\pi(u'x'+v'y')} dx dy . \sum_{j=1}^N e^{i2\pi(u'x_j+v'y_j)} \dots\dots\dots(11)$$

By substituting the equation (6) in equation (11), we obtain:

$$F_1(u', v') = n.f \int_y \int_x \tau(x, y) e^{ikw(x, y)} .e^{i2\pi(u'x'+v'y')} dx dy . \sum_{j=1}^N e^{i2\pi(u'x_j+v'y_j)} \dots\dots\dots(12)$$

Let $\tau(x, y) = 1$

$$F_1(u', v') = n.f \int_y \int_x e^{ikw(x, y)} .e^{i2\pi(u'x'+v'y')} dx dy . \sum_{j=1}^N e^{i2\pi(u'x_j+v'y_j)} \dots\dots\dots(13)$$

where $k = \frac{2\pi}{\lambda}$

$$F_1(u', v') = n.f \int_y \int_x e^{i2\pi[w(x, y)+(u'x'+v'y')]} dx dy . \sum_{j=1}^N e^{i2\pi(u'x_j+v'y_j)} \dots\dots\dots(14)$$

By substituting the integral boundary from equation (2) the area of synthetic aperture includes

$$F_1(u', v') = n.f \int_{-\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}}^{\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}} \int_{-\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}}^{\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}} e^{i2\pi[w(x, y)+(u'x'+v'y')]} dx dy . \sum_{j=1}^N e^{i2\pi(u'x_j+v'y_j)} \dots\dots\dots(15)$$

where $e^{i\theta} = \cos \theta + i \sin \theta$

$$F_1(u', v') = n.f \left[\int_{-\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}}^{\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}} \int_{-\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}}^{\frac{1}{\sqrt{N}} - \sqrt{\frac{1}{N}-y^2}} [\cos \{2\pi[w(x, y) + u'x' + v'y']\} + i \sin \{2\pi[w(x, y) + u'x' + v'y']\}] dx dy \right] \\ \cdot \left[\sum_{j=1}^N \cos \{2\pi(u'x_j + v'y_j)\} + i \sin \{2\pi(u'x_j + v'y_j)\} \right] \dots\dots\dots(16)$$

Let $z' = 2\pi u'$ and $m' = 2\pi v'$ become :

$$F_1(z', m') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\cos \{2\pi w(x, y) + z'x' + m'y'\} + i \sin \{2\pi w(x, y) + z'x' + m'y'\}] dx dy \right] \cdot \left[\sum_{j=1}^N \cos \{z'x_j + m'y_j\} + i \sin \{z'x_j + m'y_j\} \right] \dots \dots \dots (17)$$

The intensity distribution on the two axes (z', m') is symmetric, so; we can reduce it to one axis only; let ($m' = 0$), then equation (17) will take a new form .

$$F_1(z') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\cos \{2\pi w(x, y) + z'x'\} + i \sin \{2\pi w(x, y) + z'x'\}] dx dy \right] \cdot \left[\sum_{j=1}^N [\cos(z'x_j) + i \sin(z'x_j)] \right] \dots \dots \dots (18)$$

Equation (18) represents the complex amplitude for array of circular synthetic apertures .

The complex amplitude function for array of **obscured** circular apertures from equation (6) can be found as:

$$F_2(u', v') = n.f \int \int_{y \ x} f(x', y') e^{i2\pi(u'x' + v'y')} dx' dy' \dots \dots \dots (19)$$

By using the same steps and substituting the integral boundary from the equation (4) we obtain .

$$F_2(z') = n.f \left[\int_{-\frac{\epsilon}{\sqrt{N}}}^{\frac{\epsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\epsilon^2}{N}-y^2}}^{\sqrt{\frac{\epsilon^2}{N}-y^2}} [\cos \{2\pi w(x', y') + z'x'\} + i \sin \{2\pi w(x', y') + z'x'\}] dx' dy' \right] \cdot \left[\sum_{j=1}^N [\cos(z'x_j) + i \sin(z'x_j)] \right] \dots \dots \dots (20)$$

By using the physical conception of the equation (5), we can obtain the following:

$$PSF = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\cos \{2\pi w(x, y) + z'x'\} + i \sin \{2\pi w(x, y) + z'x'\}] dx dy \right] \cdot \left[\sum_{j=1}^N [\cos(z'x_j) + i \sin(z'x_j)] \right] \left[\int_{-\frac{\epsilon}{\sqrt{N}}}^{\frac{\epsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\epsilon^2}{N}-y^2}}^{\sqrt{\frac{\epsilon^2}{N}-y^2}} [\cos \{2\pi w(x', y') + z'x'\} + i \sin \{2\pi w(x', y') + z'x'\}] dx' dy' \right] \cdot \left[\sum_{j=1}^N [\cos(z'x_j) + i \sin(z'x_j)] \right] \dots \dots \dots (21)$$

From the relation $|x + iy|^2 = x^2 + y^2$ the equation above become:

$$PSF = n.f \left[\begin{aligned} & \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \cos \{2\pi w(x, y) + z'x'\} dx dy \cdot \sum_{j=1}^N e^{iz'x_j} \\ & - \int_{-\frac{\epsilon}{\sqrt{N}}}^{\frac{\epsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\epsilon^2}{N}-y^2}}^{\sqrt{\frac{\epsilon^2}{N}-y^2}} \cos \{2\pi w(x', y') + z'x'\} dx' dy' \cdot \sum_{j=1}^N e^{iz'x_j} \\ & + \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \sin \{2\pi w(x, y) + z'x'\} dx dy \cdot \sum_{j=1}^N e^{iz'x_j} \\ & - \int_{-\frac{\epsilon}{\sqrt{N}}}^{\frac{\epsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\epsilon^2}{N}-y^2}}^{\sqrt{\frac{\epsilon^2}{N}-y^2}} \sin \{2\pi w(x', y') + z'x'\} dx' dy' \cdot \sum_{j=1}^N e^{iz'x_j} \end{aligned} \right]^2 \dots\dots\dots (22)$$

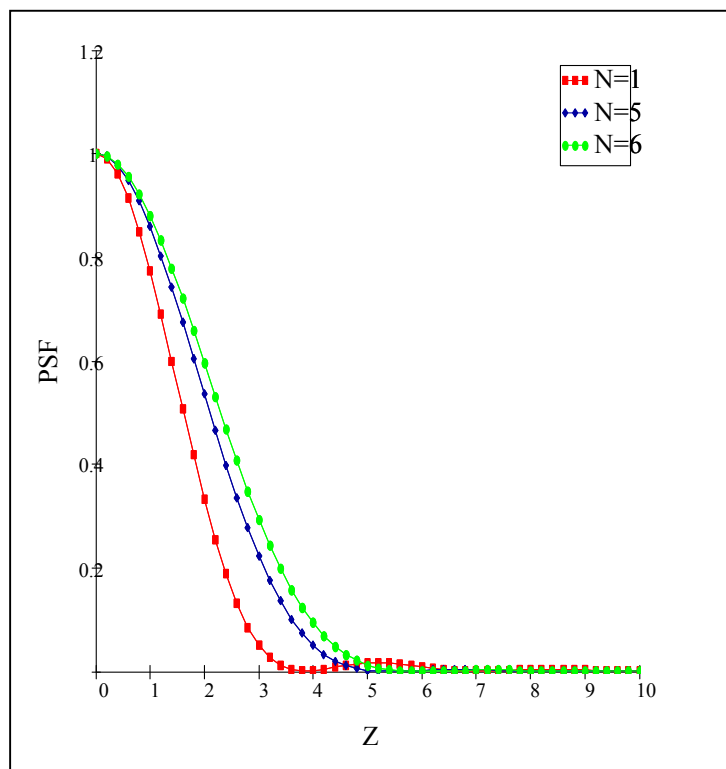
This equation represents the point spread function for array of obscured circular synthetic apertures.

By substituting the value of normalizing factor $(\frac{1}{(\pi - \epsilon^2 \pi)^2})$ (B.H. Alrueshd ,2011).in

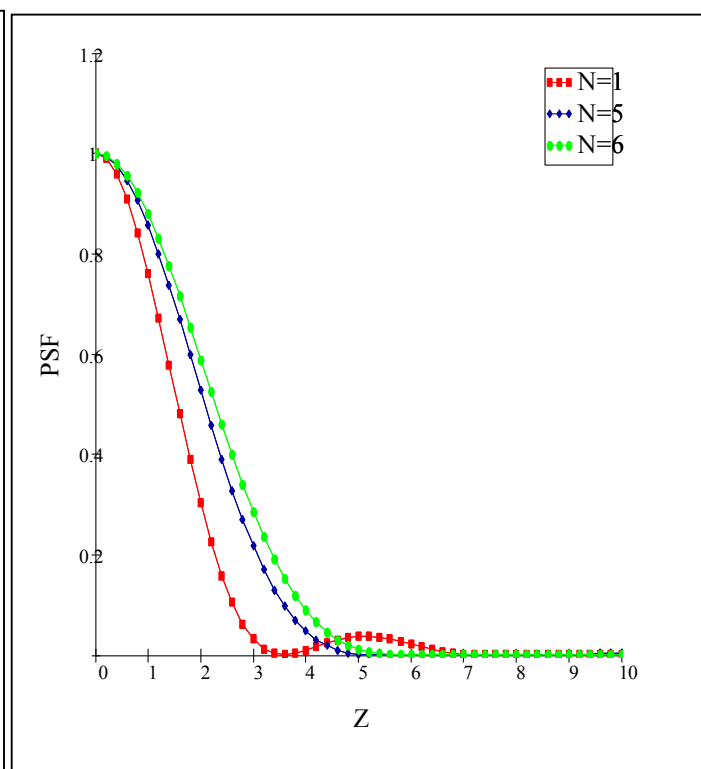
equation (21),
we obtain:

$$PSF = \frac{1}{(\pi - \epsilon^2 \pi)^2} \left[\begin{aligned} & \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \cos(z'x') dx dy \cdot \sum_{j=1}^N e^{iz'x_j} \\ & - \int_{-\frac{\epsilon}{\sqrt{N}}}^{\frac{\epsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\epsilon^2}{N}-y^2}}^{\sqrt{\frac{\epsilon^2}{N}-y^2}} \cos (z'x') dx' dy' \cdot \sum_{j=1}^N e^{iz'x_j} \\ & + \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \sin(z'x') dx dy \cdot \sum_{j=1}^N e^{iz'x_j} \\ & - \int_{-\frac{\epsilon}{\sqrt{N}}}^{\frac{\epsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\epsilon^2}{N}-y^2}}^{\sqrt{\frac{\epsilon^2}{N}-y^2}} \sin(z'x') dx' dy' \cdot \sum_{j=1}^N e^{iz'x_j} \end{aligned} \right] \dots\dots\dots (23)$$

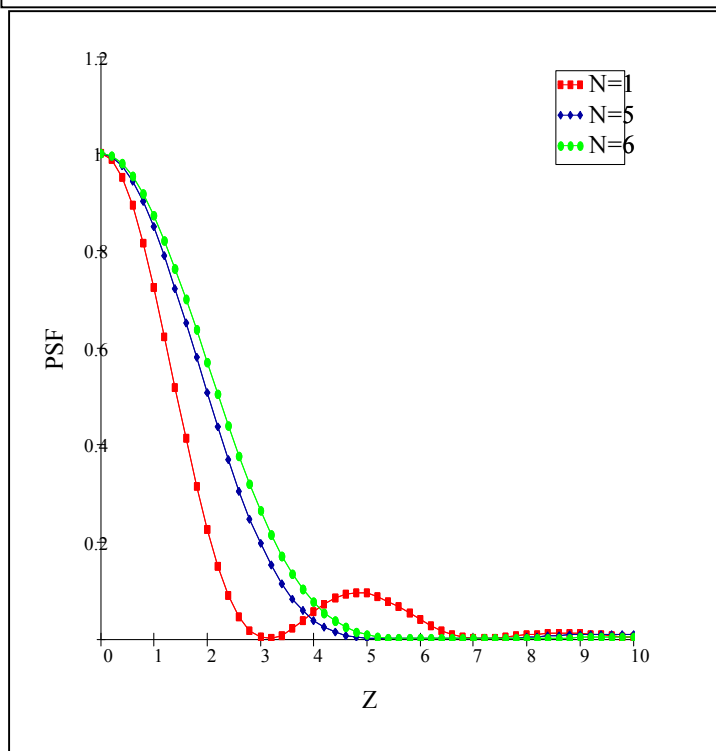
Equation (23) represents the point spread function for array of obscured circular synthetic apertures with diffraction – limited system .



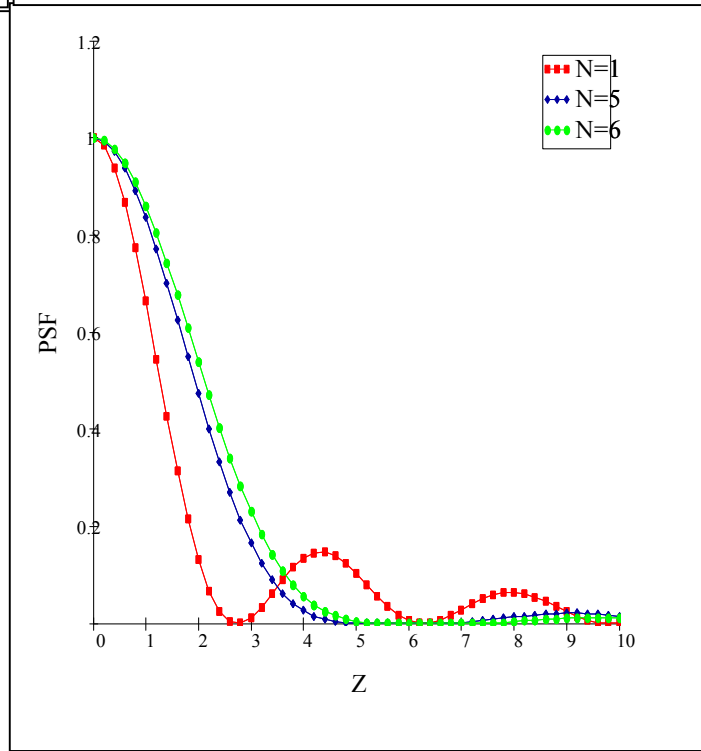
Figure(1) Point Spread Function When ($w_{20}=0$, $R=1$, $\varepsilon=0$)



Figure(2) Point Spread Function When ($w_{20}=0$, $R=1$, $\varepsilon=0.25$)



Figure(3) Point Spread Function When ($w_{20}=0$, $R=1$, $\varepsilon=0.5$)



Figure(4) Point Spread Function When ($w_{20}=0$, $R=1$, $\varepsilon=0.75$)

Discussion

Through out noticing the results that we have obtained from the MathCAD program for the point spread function(PSF) for array of obscured circular synthetic apertures, which has the circular synthetic apertures ($N=1,5$ and 6) with different obscuration ratios ($\epsilon=0,0.25,0.5$ and 0.75), for diffraction limited system and the separating distances ($R=1$) (where R represents distance from the center of synthetic aperture to the center of Cartesian coordinates), we can conclude that : By using the obscured circular apertures system the one part transfers from central spot energy (Airy disk) to secondary peaks in diffraction pattern. The number of secondary peaks increase with increasing the obscuration ratio (ϵ), and the resolving power for obscured aperture with obscuration ratio $\epsilon=0.75$, is better than resolving power with obscuration ratio $\epsilon=0.5$ and $\epsilon=0.25$. This is shown in Figures(1),(2),(3) and(4), which represent the values of PSF for different values of obscuration ratios ($\epsilon=0,0.25,0.5$ and 0.75) for diffraction limited system having individual circular aperture. We notice from the Figures above, with increasing the obscuration ratio increasing the secondary peaks with clear shape, because increase the diffraction ratio of the light rays, and increment of deviation of the light passing through it and appearance the another secondary peaks as result. Figures (1),(2),(3)and (4) represent the intensity distribution curves of PSF for diffraction limited system with obscuration ratios ($\epsilon=0,0.25,0.5$ and 0.75), when the number of apertures ($N=5$ and 6). We notice that by increasing the number of apertures we see the positions of secondary peaks were moved inside (near the PSF axis), This means increasing the intensity of secondary peaks. We notice from the Figures above an increase in the resolving power with increasing the obscuration ratio which is equivalent with increment for the number of apertures which was became best from the obscured individual aperture.

The cause of increase in resolving power when we use a design having the obscured circular synthetic apertures is the presence of obscuration in circular synthetic apertures which in turn causes the diffraction of the light passing through it and when increasing obscuration ratio. The diffraction will increase which means redistribution of the intensity in the image plane.

The intensity for PSF in this research (array of obscured circular synthetic apertures) represents the intensity distribution of an array of similarly oriented identical circular aperture equal intensity distribution of an individual aperture function multiplied by intensity sum result from a set of point sources arrayed in the same configuration.

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