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# Systematic study of even-even $^{20-32}$Mg isotopes

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## Abstract:

A systematic study for $2_1^+$ and $4_1^+$ energies for even-even $^{20-32}$Mg by means of large-scale shell model calculations using the effective interaction USDB and USDBPN with SD and SDPN model space respectively. The reduced transition probability $B(E2;\uparrow)$ were also calculated for the chain of Mg isotopes. Very good agreement were obtained by comparing the first $2_1^+$ and $4_1^+$ levels for all isotopes with the recently available experimental data and with the previous theoretical work using 3DAMP+GCM model, but studying the transition strengths $B(E2;0_{g.s.}^+ \rightarrow 2_1^+)$ for Mg isotopes using constant proton-neutron effective charges prove the limitations of the present large-scale calculations to reproduce the experiment in detail.

**Keywords:** Gamma transitions and levels energies, large scale shell model

## الخلاصة:

أجريت دراسة منهجية للطاقات $2_1^+$ و $4_1^+$ لنظائر المغنيسيوم $^{20-32}$Mg الزوجية – الزوجية بواسطة حسابات نموذج القشرة بنطاق واسع وغير مقيد عن طريق توظيف التفاعلات المؤثرة USDB و USDBPN مع فضائي النموذج SD و SDPN على التوالي. كما تم أيضاً حساب أحتمالية الأنتقال المختزله $B(E2;\uparrow)$ لسلسلة نظائر المغنيسيوم. تم الحصول على تطابق جيد عند مقارنة المستويات $2_1^+$ و $4_1^+$ المحسوبة نظريا ولكل النظائر قيد الدراسة مع البيانات العملية المتوفرة حديثاً ومع الحسابات النظرية السابقة بأستخدام أنموذج 3DAMP+GCM ولكن دراسة معدلات الأنتقال $B(E2;0_{g.s.}^+ \rightarrow 2_1^+)$ لنظائر المغنيسيوم قيمة ثابتة للبروتون–نيوترون أثبتت أن هناك قصور في حسابات نموذج القشرة حتى ضمن نطاق واسع وبدون قيود في مطابقة البيانت العملية بشكل تفصيلي.

## 1. Introduction

The low energy structure of magnesium nuclei has attracted considerable interests in the last decade, both experimental and theoretical. In particular, the sequence of isotopes $^{20-40}$Mg encompasses three spherical magic shell numbers : N=8, 20 and 28 and, therefore presents an excellent case for studies of the evolution of shell structure with neutron number, weakening of spherical shell closures, disappearance of magic numbers, and the occurrence of *islands of inversion* [Yao *et al*., 2011].

Extensive experimental studies of the low-energy structure of Mg isotopes have been carried out at the Institute of Physical and Chemical Research, Japan (RIKEN) [Iwasaki *et al*., 2001; Takeuchi *et al*., 2009], Michigan State University (MSU) [Pritychenko *et al*., 1999; Cook *et al*., 2007; Gade *et al*., 2007] , the Grand Accélérateur National d'Ions Lourds, France (GANIL) [Chisé *et al*., 2001] and CERN [Niedermaier *et al.*, 2005; Schwerdtfege, *et al.,* 2009]. In addition to numerous theoretical studies based on large-scale shell-model calculations [Caurier et al., 1998; Utsuno *et al*., 1999; Otsuka and

Fujimot *et al*.,2001; Otsuka and Utsuno, *et al*., 2001; Caurier *et al*., 2005, Maréchal, *et al*., 2005 ], the self-consistent mean-field framework, including the nonrelativistic Hartree-Fock-Bogolibov (HFB) model with Skyrme [Terasaki, *et al*., 1997] and Gogny forces [Rodríguez-Guzmán, *et al*., 2002 ] and the relativistic mean-field (RMF) model [Patra and Praharaj, 1991; Ren, *et al*., 1996] as well as the macroscopic-microscopic model based on a modified Nilsson potential [Zhi and Ren, 2006], have been used to analyze the ground-state properties (binding energies, charge radii, and deformations) and low-lying excitation spectra of magnesium isotopes.

The purpose of present work is to study the ground state $2_1^+$ and $4_1^+$ excitation energies and the reduced transition probabilities $B(E2;0_{g.s.}^+ \to 2_1^+)$ $(e^2\,fm^4)$ of the even-even $^{20\text{-}32}$Mg isotopes using the new version of Nushell@MSU for windows [Brown and Rae, 2007] and compare these calculations with the most recent experimental and theoretical work.

## 2. Theory

In a non-relativistic approximation, nuclear properties are described by the Schrödinger equation for (A) nucleons [Brussaard and Glaudemans, 1977];

$$\hat{H}\,\Psi(1,2,3,\cdots,A) = E\,\Psi(1,2,3,\cdots,A) \tag{1}$$

where $\hat{H}$ contains nucleon kinetic energy operators and interactions between nucleons of a two-body and, eventually, of a three-body character, i.e.

$$\hat{H} = \sum_{i=1}^{A}\left(-\frac{\hbar^2}{2m}\Delta_i\right) + \sum_{i<j=1}^{A} W(i,j) + \sum_{i<j<k=1}^{A} W(i,j) \tag{2}$$

$\Psi(1,2,3,\cdots,A)$ is an A-body wave function, while i denotes all relevant coordinates $\vec{r}_i,\vec{s}_j,\vec{t}_i$ of a given particle ($i$=1,2, . . . ,$A$). Although the three-body forces are proved to be important [Brussaard and Glaudemans, 1977], in the present work we will consider only the two-body interaction.

We can re-write the Hamiltonian (2.1), adding and subtracting a one-body potential of the form $\sum_{i=1}^{A} U(i)$ as [Brussaard and Glaudemans, 1977]

$$\hat{H} = \sum_{i=1}^{A}\left[-\frac{\hbar^2}{2m}\Delta_i + U(i)\right] + \sum_{i<j=1}^{A} W(i,j) - \sum_{i=1}^{A} U(i) = \hat{H}^{(0)} + \hat{V} \tag{3}$$

where we denoted a sum of single-particle Hamiltonians as $\hat{H}^{(0)}$

$$\hat{H}^{(0)} = \sum_{i=1}^{A}\left[-\frac{\hbar^2}{2m}\Delta_i + U(i)\right] = \sum_{i=1}^{A}\hat{h}(i) \tag{4}$$

and $\hat{V}$ is called a residual interaction. Existence of a nuclear average potential allows to assume that we can find such a potential $\sum_{i=1}^{A} U(i)$, that the residual interaction V is small.

In terms of the single particle energies the Hamiltonian of equation (2) can be written as [Brown and Richter 2006];

$$\hat{H} = \sum_i \varepsilon_i a_i^\dagger + \sum_{ijkl} V_{ijkl}\, a_i^\dagger a_j^\dagger a_i a_j \tag{5}$$

where

$\varepsilon_i$: is the single particle energies (SPE) which can be found from A=closed core+1 nuceli.

$V_{ijkl}$: is the two-body matrix element (TBME) couple to good JT values.

$a_i^\dagger a_j^\dagger$: is the creation operators to create pair of fermions.

$a_i a_j$: is the annihilation operators to annihilate pair of fermions.

The two-body matrix element (TBME) coupled to good JT can be written as [Brussaard and Glaudemans, 1977]

$$V_{j,j'}^{T} = \frac{\sum_J (2J+1)\left\langle jj' \middle| V \middle| jj' \right\rangle_{JT}}{\sum_J (2J+1)} \tag{6}$$

The two-body matrix element represents the effective interaction which is codenamed USDB and USDBPN is diagonalized in the sd-model space to get the eigenvalues (energy levels) and the eigenvectors (the wavefunctions) which the later can be used to calculate the reduced transition probability which is defined as [Bohr and Mottelson, 1998];

$$B(E\lambda; J_i \to J_f) = \frac{1}{(2J_i+1)} \left\langle \Psi_f \middle\| M(E\lambda) \middle\| \Psi_i \right\rangle \tag{7}$$

where $M(E\lambda)$ is the electric multipole operator and $E\lambda$ refers to the case of an electric transition and its multipolarity, λ. $J_i$ and $J_f$ are the spin of the initial and final states, respectively.

## 3. Shell model calculations

The calculations were carried out in the SD and SDPN model spaces with the USDB and USDBPN effective interactions [Brown and Richter, 2006] using the shell model code Nushell@MSU for windows [Brown and Rae, 2007].

The core is taken as $^{16}$O with 4 valence protons and 4, 6, 8, 10, 12, 14, 16 valence nucleons for $^{20}$Mg, $^{22}$Mg, $^{24}$Mg, $^{26}$Mg, $^{28}$Mg, $^{30}$Mg and $^{32}$Mg respectively distributed over $1d_{5/2}$ , $2s_{1/2}$, $1d_{3/2}$.

The effective interaction USDB with model space SD where used in the calculation of the $^{20\text{-}30}$Mg isotopes, while USDBPN in pn formalism where employed with SDPN model space for $^{32}$Mg nucleus.

## 4. Results and discussion

The test of success of large-scale shell model calculations is the predication of the low-lying $2_1^+$ and $4_1^+$ and the transition rates $B(E2;0_{g.s.}^+ \rightarrow 2_1^+)$ using the optimized effective interactions for the description of *sd*-shell nuclei.

Figure 1 presents the comparison of the calculated $E_x(2_1^+)$ energies from the present work (P.W.) with the experiment [ENSDF, 2012], the work of [Yao et al., 2011] using 3DAMP+GCM model with the relativistic density functional PC-F1.

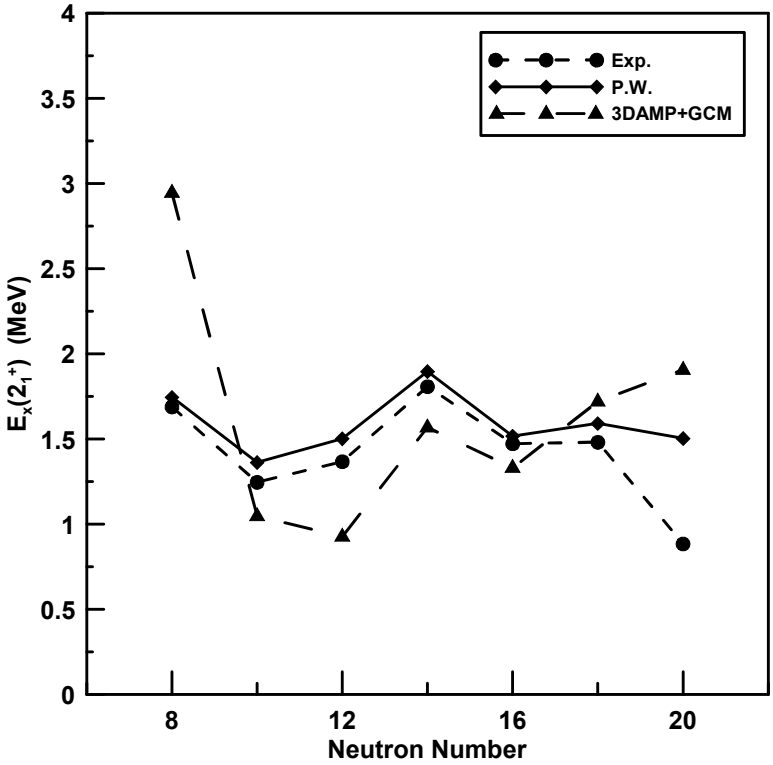


**Fig. 1:** Systematics of $E_x(2_1^+)$ for eve-even $^{20\text{-}32}$Mg isotopes. Experimental data (closed circles) are compared with present work (solid line), the previous work using 3DAMP+GCM model (long dashed line) [Yao *et al*., 2011]. Experimental data are taken from Ref. [ENSDF, 2012].

Figure 2 shows the comparison of the calculated low-lying $E_x(4_1^+)$ excitation energies from present work (P.W.) with the experiment [ENSDF, 2012], the work of [Yao et al., 2011] using 3DAMP+GCM model with the relativistic density functional PC-F1.

The comparison shows very clear that our prediction for the $E_x(4_1^+)$ are in better agreement with the experiment.

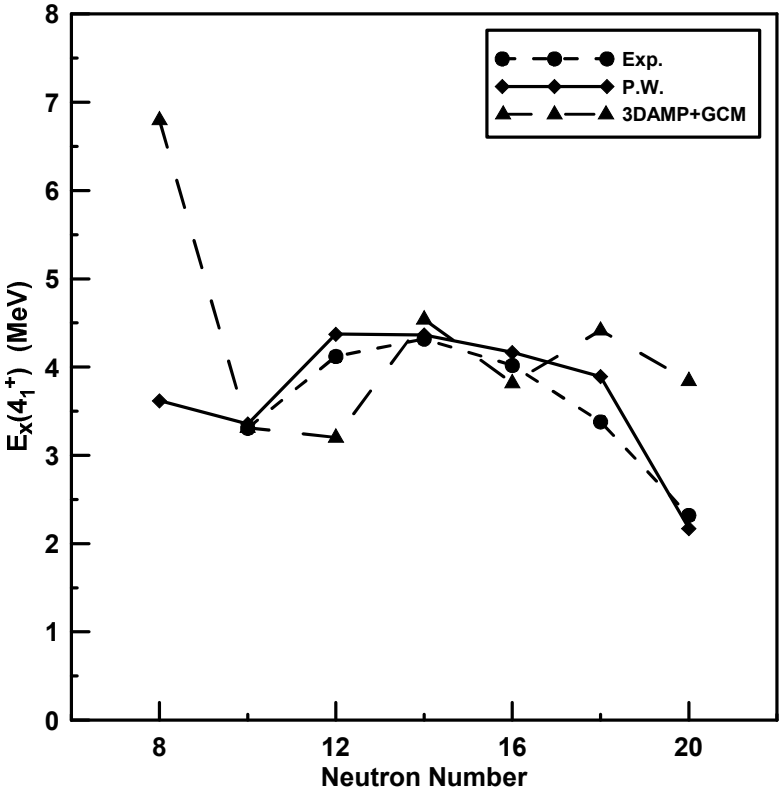


**Fig.2:** Systematics of $E_x(4_1^+)$ for eve-even Mg isotopes. Experimental data (closed circles) are compared with present work (solid line), the previous work using 3DAMP+GCM model (long dashed line) [Yao, *et al*., 2011]. Experimental data are taken from Ref. [ENSDF, 2012].

Figure 3 presents the comparison of the calculated $B(E2;0^+_{g.s.} \rightarrow 2^+_1)$ $(e^2 fm^4)$ from present work (P.W.) with the experimental data taken from the Institute of Physical and Chemical Research, Japan (RIKEN) [Iwasaki *et al*., 2001; Takeuchi *et al*., 2009] the Grand Accélérateur National d'Ions Lourds, France (GANIL) [Chisé et al., 2001] and CERN [Niedermaier *et al*., 2005, Schwerdtfeger *et al*., 2009], the previous theoretical work of [Yao *et al*., 2011] using 3DAMP+GCM model and with the work of R. Rodríguez-Guzmán [Rodríguez-Guzmán *et al*., 2002] using HFB-Gogny force. The effective charges were taken to be $e_\pi = 1.25e$ for proton and $e_\nu = 0.8e$ for neutron. With these effective charges our prediction for the reduced transition probability $B(E2;0^+_{g.s.} \rightarrow 2^+_1)$ are closer to the experimental values than the previous work of Refs. [Yao *et al*., 2011; Rodríguez-Guzmán *et al*., 2002].

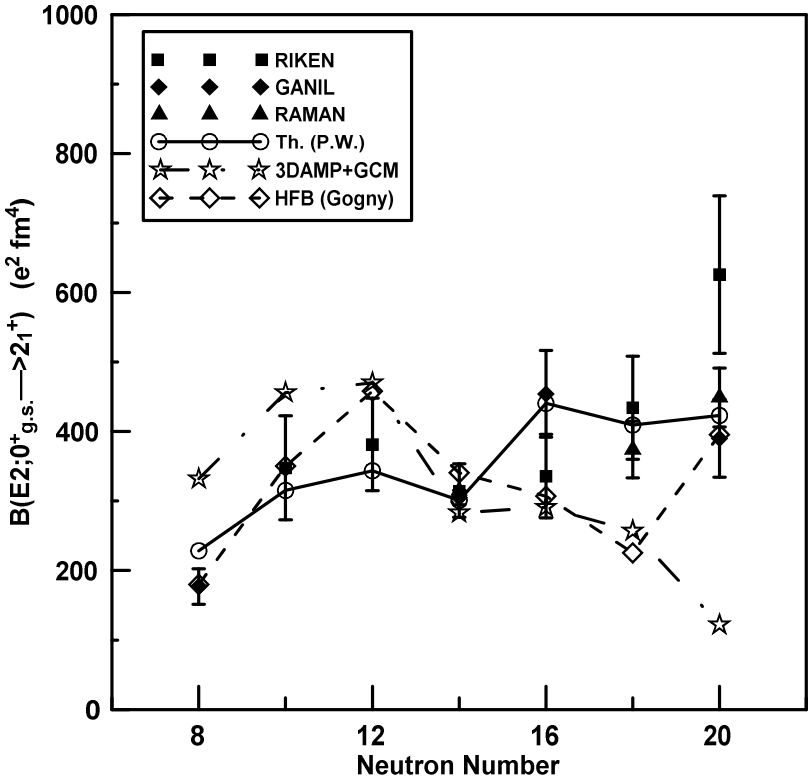


**Fig.3:** Comparisons between the calculated $B(E2;0^{+}_{g.s.} \rightarrow 2^{+}_{1})$ $(e^2\,fm^4)$ of the even-even $^{20\text{-}32}$Mg isotopes (solid line) (P.W.), 3DAMP+GCM model (dashed-dotted line [Yao *et al*., 2011] and HFB (Gogny) (dashed line) [Rodríguez-Guzmán *et al*., 2002]. Experimental data taken from Refs. [Iwasaki *et al*., 2001; Takeuchi *et al*., 2009; Chisé et al., 2001; Niedermaier *et al*., 2005, Schwerdtfeger *et al*., 2009].

## 5. Summary and conclusions

Unrestricted large scale-shell model calculations were performed using the effective interactions USDB and USDBPN in pn formalism with the model space SD and SDPN to study the low lying $2^{+}_{1}$ and $4^{+}_{1}$ energies for even-even $^{20\text{-}32}$Mg isotopes and the transition strengths $B(E2;0^{+}_{g.s.} \rightarrow 2^{+}_{1})$ $(e^2\,fm^4)$ for the mass region A=20-32. Good agreement were obtained in comparing our theoretical work with the recent available experimental data and with the most recent theoretical work of Ref. [Yao *et al*., 2011] using 3DAMP+GCM model with the relativistic density functional PC-F1 for both excitation energies and transition strengths.

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