

Model Predictive Control of Blood Pressure by Drug Infusion

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Received on:2/2/2011

Accepted on :25/5/2011

Abstract

In this research the design of a Model Predictive Controller (MPC) is investigated for regulating Mean Arterial Pressure (MAP) by infusing the Sodium Nitro-Prusside (SNP) drug to the mathematical model of human patients during surgical operations where the blood pressure suffers from sudden rises due to multiple causes. The MPC is designed and compared with digital Proportional-Integral (PI) controller. MPC is chosen due to its known ability of estimating an optimal control action and dealing with input, state constraints and with disturbances. Simulation results for three patients' MAP models controlled by MPC and PI controllers that are designed for a sensitive patient model are analyzed and compared. Both controllers give a satisfactory response but MPC is preferred due to its optimally estimated actions.

Keywords: Model Predictive Control, Blood Pressure, MAP, SNP, PI.

سيطرة التنبؤ النموذجي على ضغط الدم بالحقن الدوائي

الخلاصة

في هذا البحث تم بناء جهاز سيطرة تنبؤي نموذجي (MPC) لتنظيم متوسط الضغط الشرياني (MAP) بواسطة حقن عقار الصوديوم نيتروبروسايد (SNP) لنموذج رياضي لمرضى البشر أثناء العمليات الجراحية حيث يعاني ضغط الدم من ارتفاعات مفاجئة لأسباب عديدة. سيقارن جهاز السيطرة التنبؤي النموذجي MPC مع جهاز السيطرة الرقمي التناسبي-التكاملي PI Controller. ان جهاز السيطرة التنبؤي النموذجي MPC تم اختياره لقدرته المعروفة على تخمين فعل السيطرة المثالية وللتعامل مع المدخل و قيود الحالة و الضوضاء. ان نتائج المحاكاة لثلاثة نماذج رياضية لضغط الدم لمرضى تمت السيطرة عليها بواسطة مسيطرا MPC و PI والذان صمما لنموذج رياضي لمرضى حساس تم تحليلها ومقارنتها. كلا المسيطرين اعطى استجابة مرضية لكن MPC مفضل لأجرائاته المخمنة بشكل مثالي.

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1.Introduction

Blood pressure plays an important role in many medical applications. For example, its change resembles the depth of anesthesia (i. e. unconsciousness) during surgical procedure. Precisely the Mean Arterial Pressure (MAP) is regarded the most reliable parameter to be controlled to maintain adequate anesthesia to ensure patient's safety, this fact greatly appears in the literature [1, 2, 3]. Also after completion of open-heart surgery, patients are sent to Cardiac surgical Intensive Care Units (CICU) for recovery. While being in the CICU, some patients develop high MAP. Such hypertension should be treated timely to prevent sever complications [4]. In either case the drug infusion to the hospitalized patient must be monitored carefully by the anesthetist who is in charge. Sodium NitroPrusside (SNP) can be automatically administered by a feedback control system. Actually researchers enriched this issue by different control designs.

In the early 70's PI controller was designed with parameters changing according to a decision table. A number of other automatic SNP delivery systems have also been developed and tested in computer simulation, in animal experiments, or with patient trials. The control schemes employed included PI control, optimal control, and adaptive control [4]. J. M. Arnsperger et al [5] presented two stochastic adaptive algorithms for the control of blood pressure. In the intelligent control field R. Meier et al [3] implemented a PI fuzzy controller which controls the MAP during anesthesia with isoflurane. A brief survey of other simple control applications to anesthesia is given by D. A. Linkenes [6] followed by a description of the use of generalized predictive control and fuzzy logic control for muscle relaxation. H. Ying and L. C. Sheppard [4] presented fuzzy control SNP delivery system and its clinical performance in regulating MAP in postsurgical patients in the CICU. C. W. Frie et al [7] studied

models used for the design of automatic controllers of MAP during anesthesia and suggested orders based on the linearization of the physiological compartment models.

An automatic drug delivery system was designed by K. E. Kwok et al [8] to assist the medical personnel in case management during intra-operative surgeries; the controller operates in an adaptive mode by using a recursive control-relevant identification algorithm for long-range predictive control. E. Furutani et al [9] developed a continuous feedback control of the MAP by a state-predictive servo controller and risk control based on the inference by fuzzy-like logics and rules using measured data.

Controlling MAP automatically requires an efficient controller that can handle sudden rises in MAP due to disturbances such as skin incisions and measurement errors like calibration errors and random stochastic noise. For example a skin incision can increase the MAP rapidly by 10 mmHg [1]. Classical controllers like PID when tuned perform well in the specific operating conditions they were designed for but for various other conditions they may not provide the desired behavior. Also, for problems where constraints on systems' inputs and states are exist the need for more powerful and efficient controllers to take the responsibility rises. MPC provides the optimal control action that is predicted for a certain prediction horizon to be applied on the manipulated variable. It also takes in account input disturbances and measurement noise in addition to the input constraints specified by the designer.

MPC proved a very efficient performance in industry where it is originally developed to meet the specialized control needs of power plants and petroleum refineries, MPC technology can now be found in a wide variety of application areas including chemicals, food processing, automotive, and aerospace applications [10].

In this work MAP regulation problem is analyzed by tuning a PI controller and then comparing the results with MPC.

2.System Model

Several models are presented in the literature to model the relationship between the drug infusion rate to the rate of MAP change [3, 5, 6]. In this work a well-established model that is widely used for designing various MAP controllers is utilized [4]:

$$\frac{\Delta \text{MAP} (s)}{\Delta \text{SNP} (s)} = \frac{k e^{-T_1 s} (1 + \alpha e^{-T_2 s})}{(1 + \tau s)} \quad (1)$$

Where,

ΔMAP : Change of MAP in mmHg.

ΔSNP : SNP infusion rate in ml/hr.

K: is the sensitivity to SNP, -0.72 for the typical patient, -0.18 for the insensitive patients, and -2.88 for the sensitive patients [4].

$T_1 = 30$ Sec. the transport time lag between a SNP injection site and the SNP receptors.

$T_2 = 50$ Sec is the recirculation time delay of SNP in Sec.

$\alpha = 0.4$ is the recirculation fraction of SNP.

$\tau = 40$ Sec. is the time constant representing the uptake and distribution of SNP.

This model describes some aspects of MAP and SNP relation very well. The three different patients' sensitivities are investigated within the design of MPC and PI controllers to regulate MAP level to the desired baseline 80 mmHg.

3.Digital PI control design

The parameters of discrete PI controller are tuned for sensitive patient ($k = -2.88$) due to the fact that it is rapidly affected by small input changes. PI should automatically controls the infusion of SNP to a hospitalized patient model for a desired base line blood pressure of 80 mmHg. It is

assumed that the patient blood pressure is normal within 80 mmHg before a sudden rise of 160 mmHg due to skin incision during the surgery about 200 sec. of normal situation. The block diagram of the system and controller is given in figure (1).

The MAP samples are taken periodically every 5 sec. to release the control action by PI to the pump of SNP. The controller parameters are tuned to $K_p = -0.134$, $K_i = -0.0025$ work in a satisfying behavior for the sensitive patient, the MAP settles at 80mmHg within approximately seven minutes (from $t=200$ to 620 sec.) as illustrated in figure (2).

Figure (3) gives the control action for the sensitive case which settles at 20 ml/h and it is sufficient for it. To experience the effect of the PI controller previously tuned for other cases; like for the second patient case ($k = -0.72$) with the average sensitivity of MAP to SNP infusion rate, the response of figure (4) is yielded. The result shows a decay of MAP that settles at 80 mmHg in less than half of an hour ($t=200$ to 1620 sec.); and the SNP controlled rate that bring MAP to the desired level is 80 ml/h.

Finally, in figure (6) is the performance of the designed PI for the third case of insensitive patient's ($k = -0.18$).

This result has the slowest decay to the PI controlled SNP infusion $t = 6300$ sec. i.e. in one hour and 41 minutes from the sudden rise of MAP at $t=200$ sec and the control action of SNP is way more than before at 320 ml/h.

The results of the three cases present a good robustness by the PI controller which is tuned for one patient model and respond well for the rest two cases despite the slow ending transients. As a matter of fact the sudden rise of MAP for a hospitalized patient is expected at any moment for many reasons like pain or temperature rise, also the different cases leads to different systems.

In the following section the design of model predictive controller is investigated too then both results are analyzed.

4.Model Predictive Control

Model predictive control (MPC) is a process control technology that is being increasingly employed across several industrial sectors, at the heart of MPC is the process model and the concept of open-loop optimal feedback. The process model is used to generate a prediction of future subsystem behavior. At each time step, past measurements and inputs are used to estimate the current state of the system. An optimization problem is solved to determine an optimal open-loop policy from the present (estimated) state. Only the first input move is injected into the plant. At the subsequent time step, the system state is re-estimated using new measurements. The optimization problem is resolved and the optimal open-loop policy is recomputed. Figure (8) presents a conceptual picture of MPC [11]. In figure (9) a block diagram describing the main function of MPC.

To calculate its next move, u_k (k represents the current instant), the controller operates in two phases [12]:

1. Estimation: In order to make an intelligent move, the controller needs to know the current state. This includes the true value of the controlled variable $\bar{y}_k$, and any internal variables that influence the future trend $\bar{y}_{k+Ts}, \dots, \bar{y}_{k+P}$, where P (a finite integer ≥ 1) is the prediction horizon and T_s is the sampling period. In general the model of the plant is a linear time invariant system described by the equations:

$$\begin{aligned} x(k+1) &= Ax(k) + B_u u(k) + B_v v(k) + B_d d(k) \\ y_m(k) &= C_m x(k) + D_{vm} v(k) + D_{dm} d(k) \\ y_u(k) &= C_u x(k) + D_{vu} v(k) + D_{du} d(k) \end{aligned} \quad (2)$$

where $x(k)$ is the n_x -dimensional state vector of the plant, $u(k)$ is the n_u -dimensional vector of manipulated variables

(MV), i.e., the command inputs, $v(k)$ is the n_v -dimensional vector of measured disturbances (MD), $d(k)$ is the n_d -dimensional vector of unmeasured disturbances (UD) entering the plant, $y_m(k)$ is the vector of measured outputs (MO), and $y_u(k)$ is the vector of unmeasured outputs (UO). The overall n_y -dimensional output vector $y(k)$ collects $y_m(k)$ and $y_u(k)$

2. Optimization: Values of set points, measured disturbances, and constraints are specified over a finite horizon of future sampling instants, $k + T_s, k + 2T_s, \dots, k + P$. The moves are the solution of a constrained optimization problem. The controller computes M moves $u_k, u_{k+T_s}, \dots, u_{k+M-1}$, where M ($1 \leq M \leq P$) is the control horizon. The moves are the solution of a constrained quadratic optimization problem.

5.MPC for Blood Pressure

The first step in the design is the measurement of MAP taking place every five seconds (i.e. $T_s = 5$) which is less than one fifth of the dominant time constant in the system model. Then, the predictive moves are designed to be over $k+5, k+8$ (i.e. $P = 8$) while the controller computes one move (i.e. $M=1$). Increasing the predictive horizon above 8 leads to a sluggish MAP results. Also, MPC computes one action from the estimated actions, then the controller uses the new measurement values to make a new estimated output to predict the next scenario. Now the manipulated variable (MV) is SNP and it's constrained at ($0 \leq \text{SNP} \leq 350$), also the effect of unmeasured disturbance is experienced by the patient in addition to an integrated white noise is taken into account too to enhance the performance of the control action in each step. A block diagram of the controlled system is illustrated in figure (10).

MPC is designed basically for the sensitive patient model ($k = -2.88$) and the response is illustrated in figure (11). MAP settles within $t = 464\text{sec.}$ after the system is exposed to a disturbance that raised the

blood pressure to 160 mmHg at $t=200$ sec. when SNP records high injection level (see figure (12)), then 20ml/h is sufficient to keeps the MAP at its base line.

The discrete state space of MPC when constraints are not active can be obtained as follows:

$$\begin{aligned} x_{MPC}(k+1) &= ax_{MPC}(k) + by_m(k) \\ u(k) &= cx_{MPC}(k) + dy_m(k) \end{aligned} \quad (3)$$

a =

	xMPC1	xMPC2	xMPC3	xMPC4	xMPC5	prev.MV
xMPC1	2.874	-1.489	0.6538	-0.2034	-0.06668	0.003858
xMPC2	1.947	0.07744	-0.07102	0.04128	-0.3292	0
xMPC3	-0.07272	1.106	-0.09715	0.05646	-0.4502	0
xMPC4	-0.04481	0.06527	0.4401	0.03479	-0.2774	0
xMPC5	-0.1554	0.2263	-0.2076	0.1206	0.03807	0
prev.MV	-0.4099	0.5248	-0.4399	0.2407	0.03477	0.003858

b =

	meas.Y
xMPC1	1.011
xMPC2	0.3292
xMPC3	0.4502
xMPC4	0.2774
xMPC5	0.9619
prev.MV	0.91

c =

	xMPC1	xMPC2	xMPC3	xMPC4	xMPC5	prev.MV
MV	-0.4099	0.5248	-0.4399	0.2407	0.03477	0.003858

d =

	meas.Y
MV	0.91

For a moderate patient model $k=-0.72$ controlled by MPC for the specifications desired earlier gives the response as illustrated in figure (13). Obviously the transient response is slower than before, $t=1190$ sec. since this patient model has less sensitivity and this is translated by the SNP level infused is 80 ml/h (see figure (14)).

Finally for the insensitive patient model the MPC controlled MAP is obtained in figure (15), due to very low sensitivity MAP slowly decays to steady state at $t=4500$ sec. and the control action of MPC needed to inject 320 ml/h SNP to guide MAP into the base line desired 80mmHg as shown in figure (16). Table (1) summarizes the

results of PI and MPC action for the different cases.

Both controllers reside SNP infusion rate at the same level for the three model cases but with different settling times as seen by the table. PI controlled sensitive case is faster as compared to MPC response, while MPC leads MAP to its base line considerably faster than PI for the remaining cases. However, estimating the constrained optimal control action for the measured MAP each step time by MPC and taking measurement noise and unmeasured disturbance into account throughout the design operation puts this strategy ahead. It is good to mention that PI controller made a very satisfactorily behavior for the three cases despite the fact that it is tuned originally once, although it would give a better response when incorporating adaptive design like investigated in the literature.

6.Conclusion

A model predictive controller is designed to automatically control blood pressure for hospitalized patients' models for three different sensitivities and compared with the results of digital PI controller that is tuned for a sensitive patient model. Although PI controller gave a satisfactory response but MPC is preferred for its optimally estimated actions for any circumstances and different cases and handles disturbances all the way in the design. This is a very favorable property associated with actuator constraints. PI controller could be enhanced if designed to be adaptively tuned for variable models. Simulation results are compared especially from the time point of view since it is a key factor in such case. Automatic control of blood pressure takes the burden off the human supervisor that handles the operation manually and gives him space to look after other aspects during surgery.

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Table (1): Settling times for PI and MPC actions.

Controller Type	Settling Time (Min) for 2% error criteria			Control action (SNP(ml/h))		
	Sensitive case $k=-2.88$	Moderate case $k=-0.72$	Insensitive case $k=-0.18$	Sensitive case $k=-2.88$	Moderate case $k=-0.72$	Insensitive case $k=-0.18$
PI	10.333	23.667	105	19.84	79.36	316.5
MPC	7.333	19.833	75.0	19.84	79.33	316.88

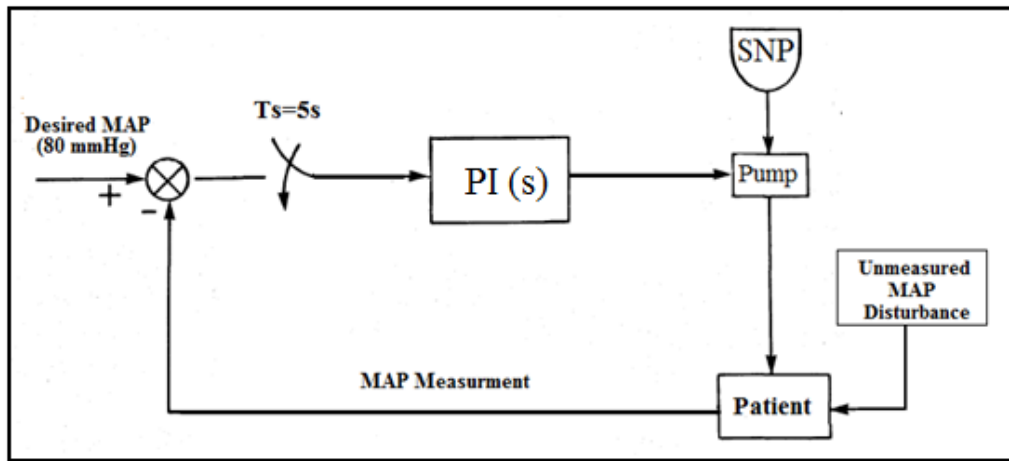
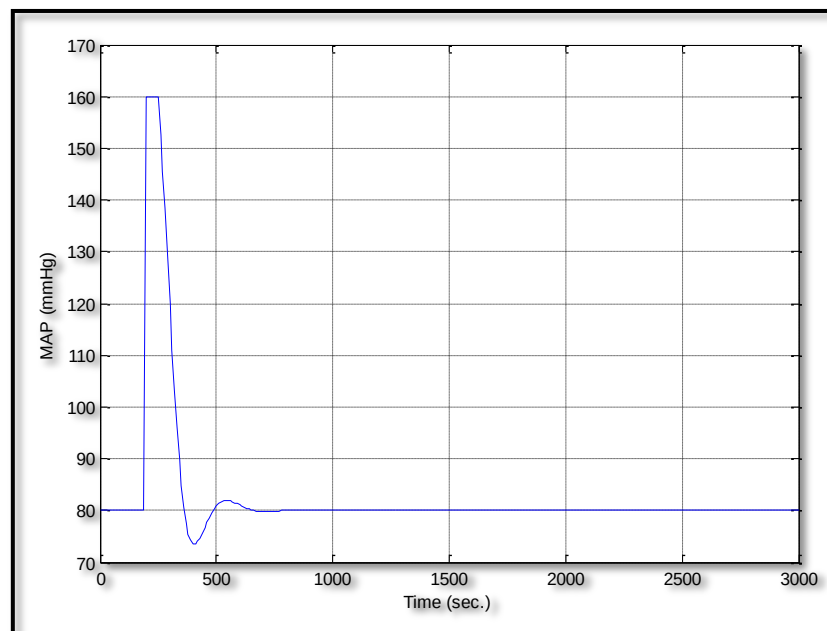


Figure (1): Automatic control of MAP using digital PI controller.

Figure (2): MAP response to SNP infusion utilizing PI controller for $k = -2.88$.

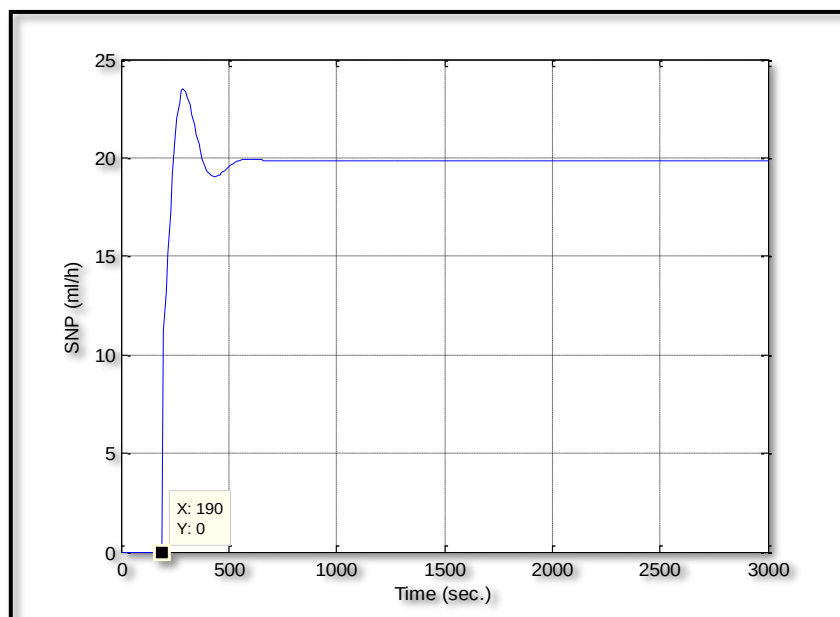


Figure (3): PI control action of SNP infusion rate for $k = -2.88$.

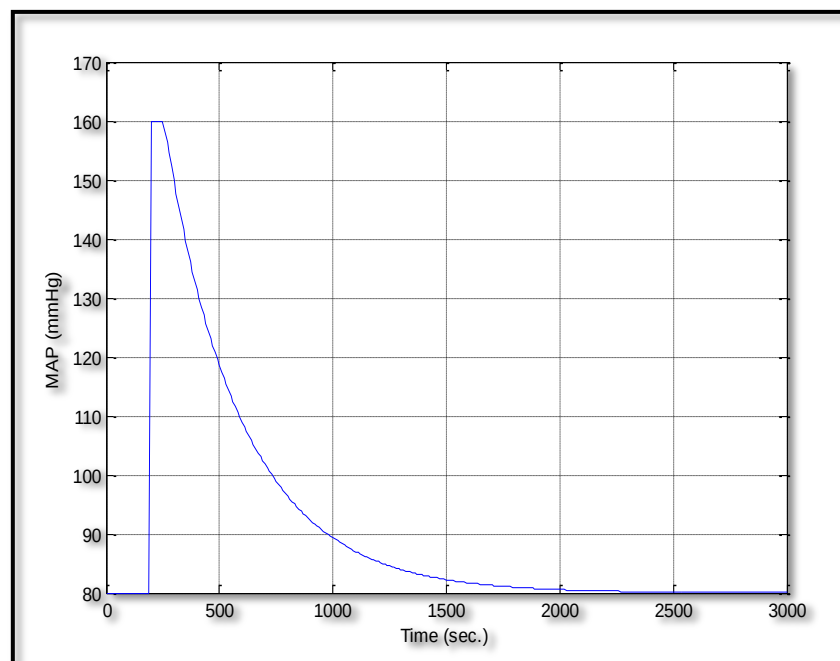


Figure (4): MAP response to SNP infusion utilizing PI controller for $k = -0.72$.

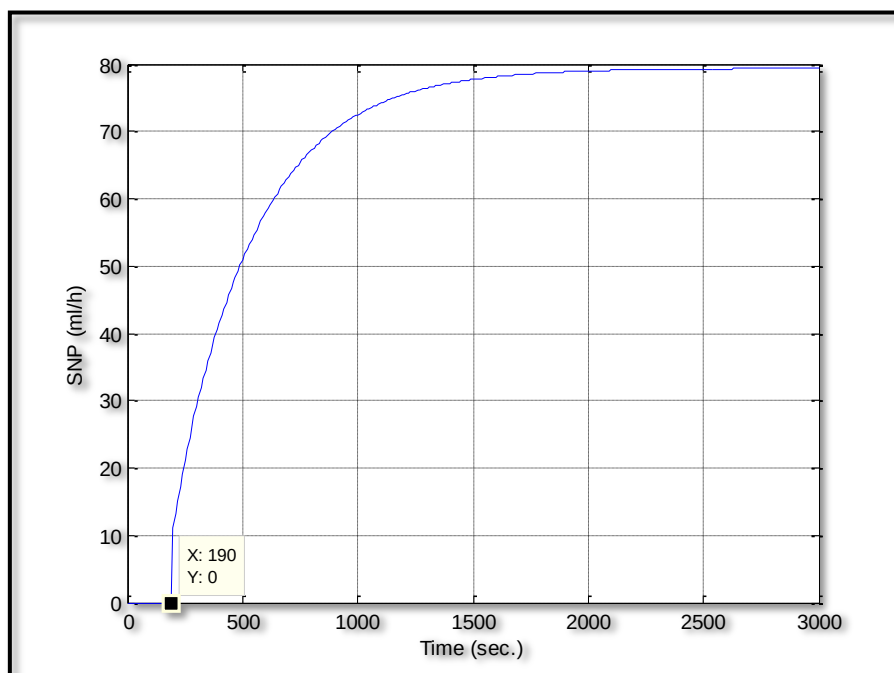


Figure (5): PI controlled SNP infusion rate for $k = -0.72$.

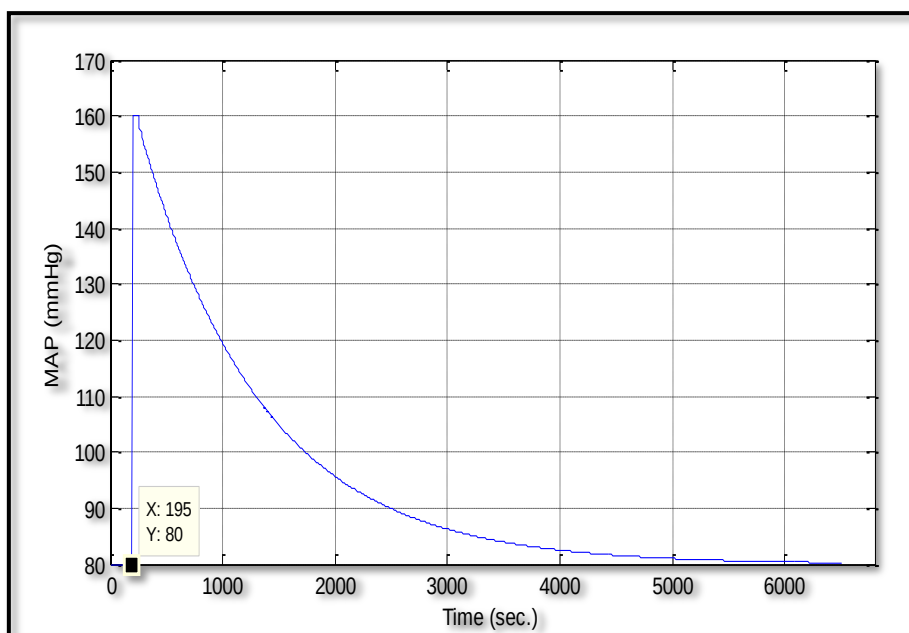


Figure (6): MAP response to SNP infusion utilizing PI controller for $k = -0.18$

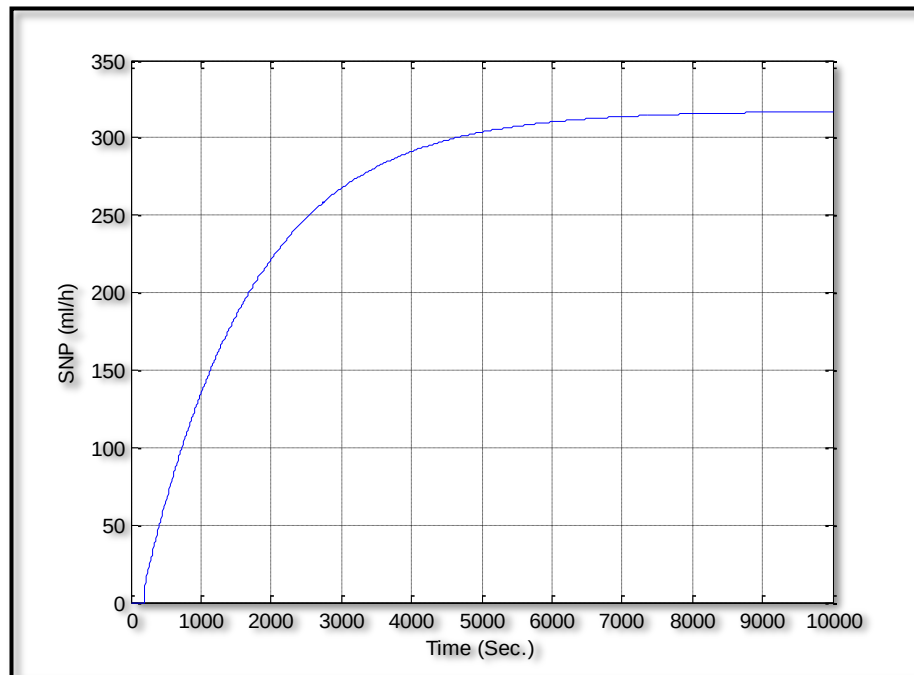


Figure (7): PI controlled SNP infusion rate for $k=-0.18$.

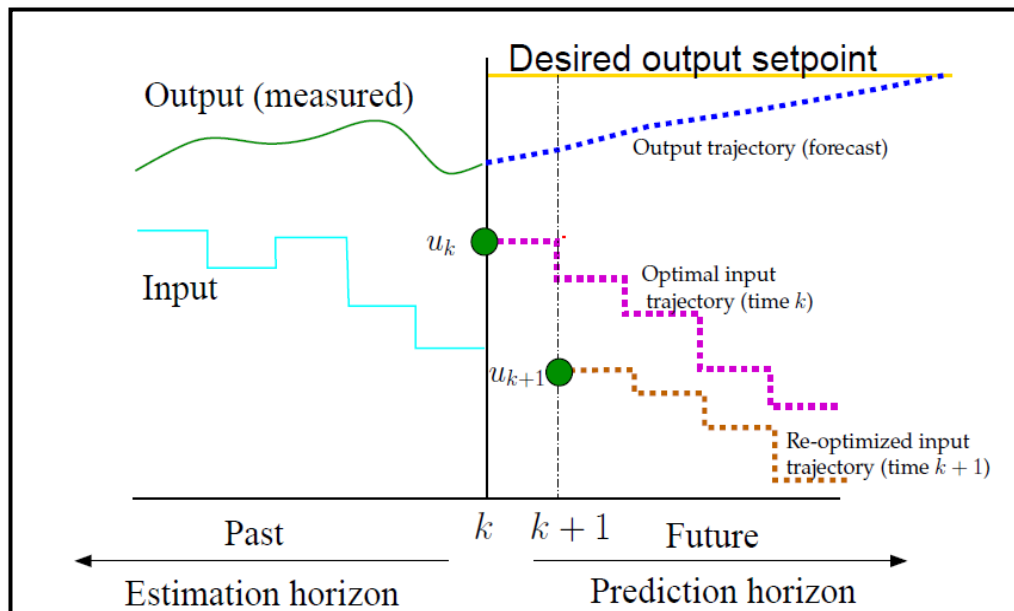


Figure (8): A conceptual picture of MPC Only u_k is injected into the plant at time k . At time $k + 1$, a new optimal trajectory is computed.

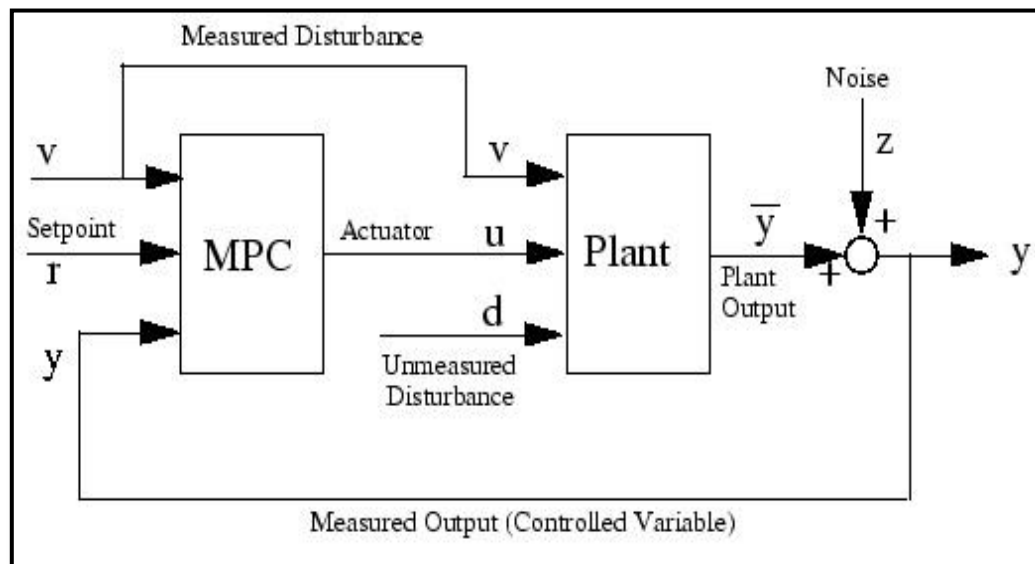


Figure (9): Block Diagram of a SISO Model Predictive Control [12].

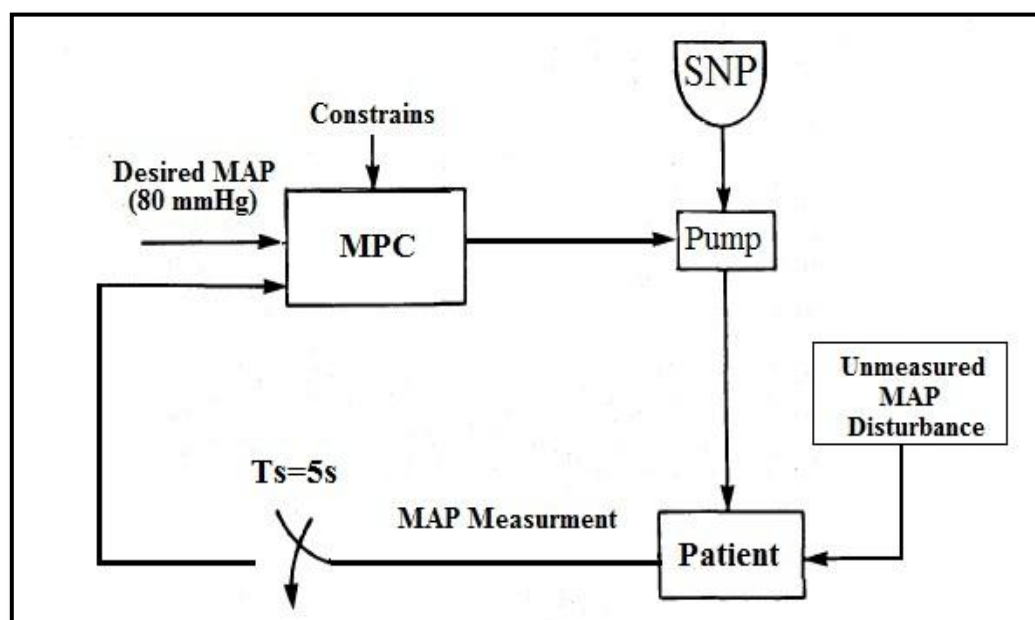


Figure (10): Automatic control of MAP using MPC.

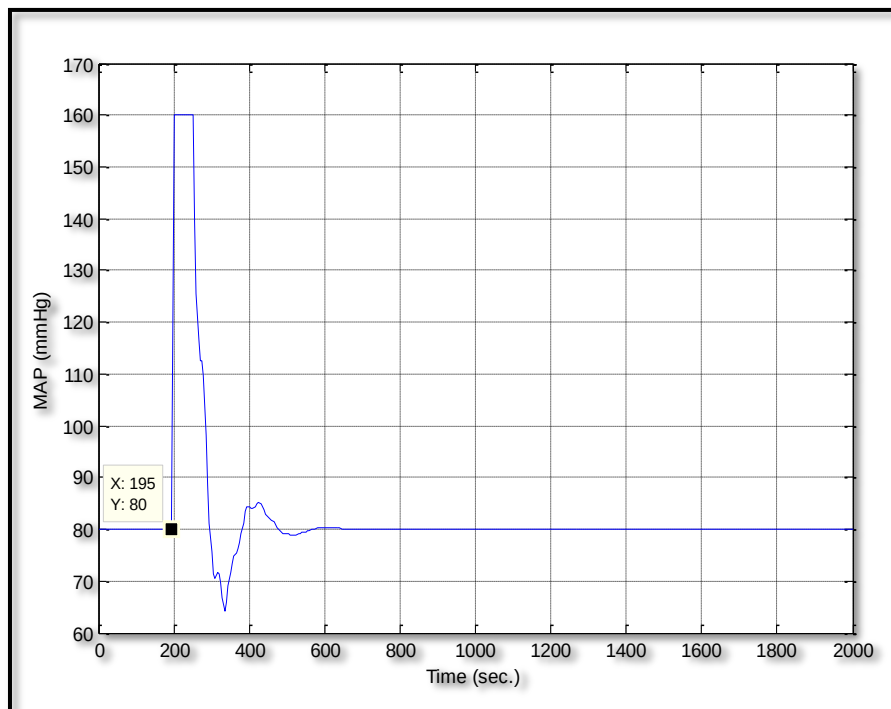


Figure (11): MAP response for MPC controlled SNP infusion for $k = -2.88$.

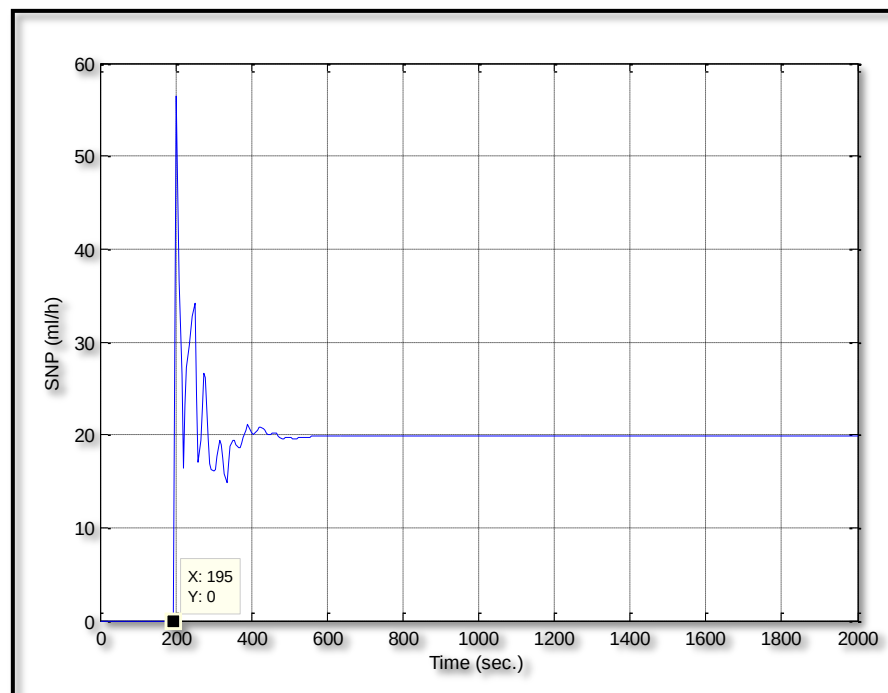


Figure (12): MPC controlled SNP infusion rate for $k = -2.88$.

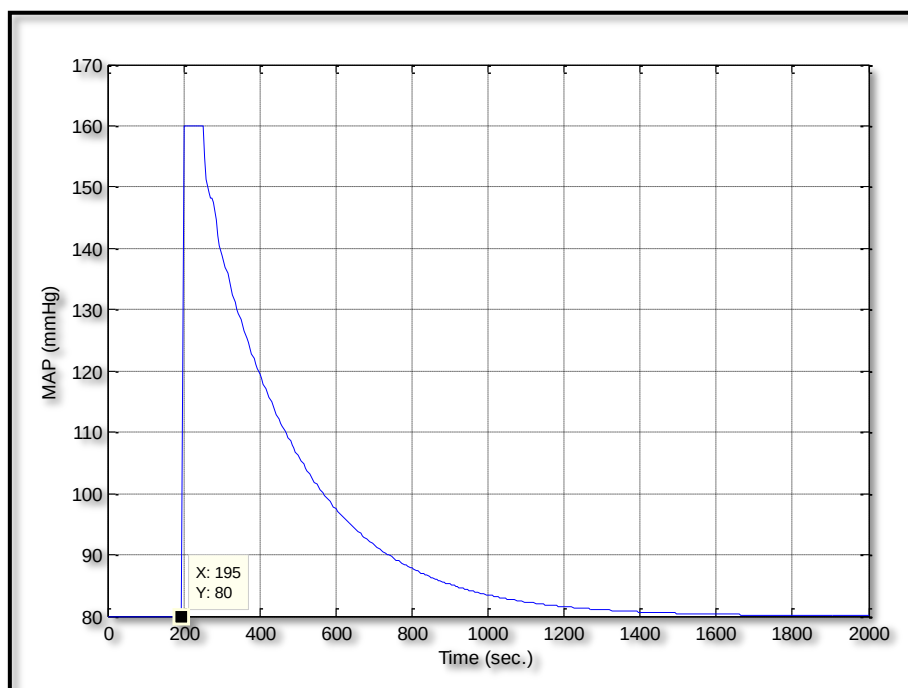


Figure (13): MAP response for MPC controlled SNP infusion for $k = -0.72$.

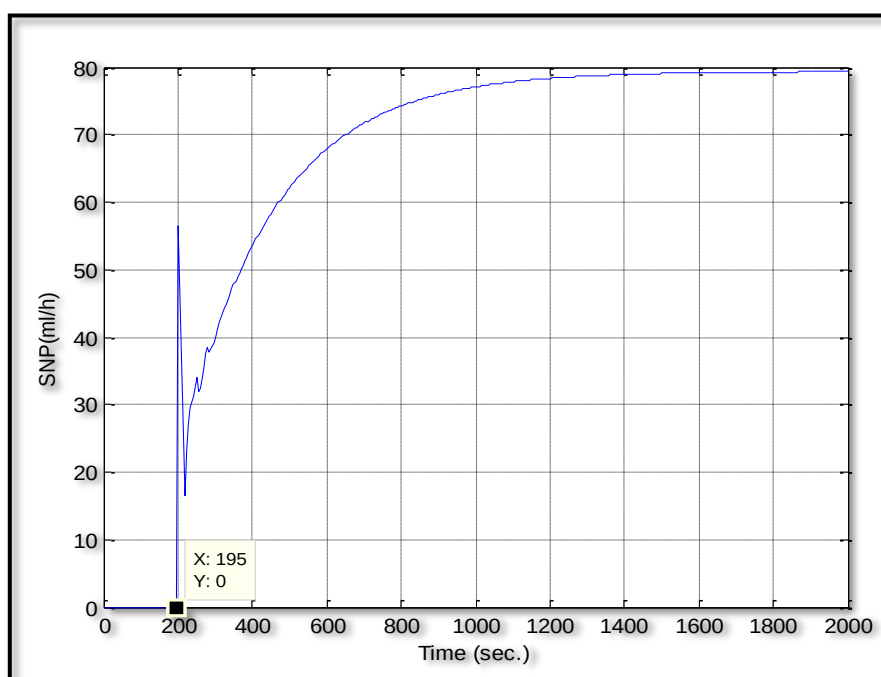


Figure (14): MPC controlled SNP infusion rate for $k = -0.72$.

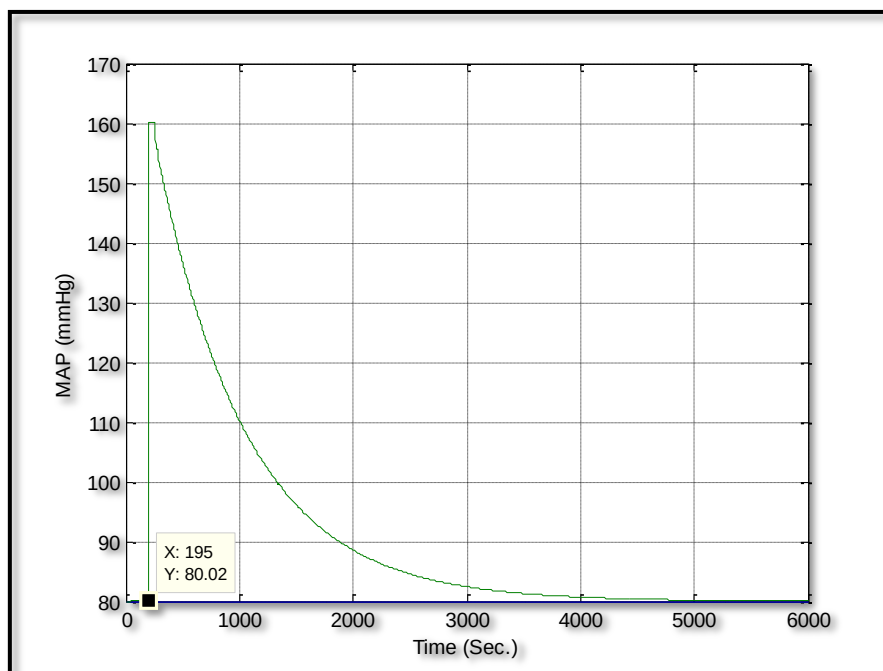


Figure (15): MAP response to MPC controlled SNP infusion for $k = -0.18$.

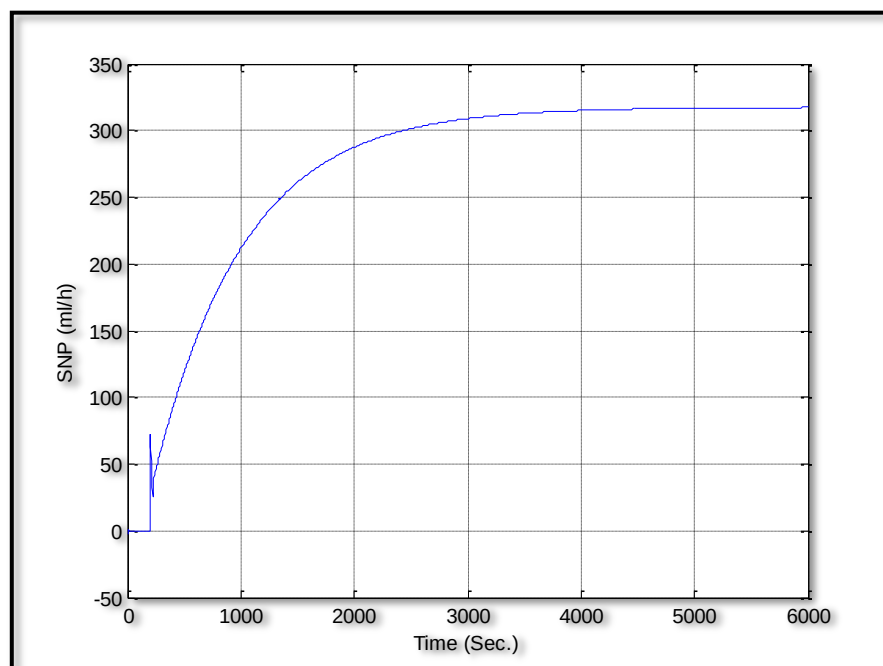


Figure (16): MPC controlled SNP infusion rate for $k = -0.18$.