Flood Frequency Analysis for Greater – ZAB River

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Abstract

The analysis of flood discharge for greater - Zab river have been conducted using different statistical models such models are , " log – normal type III , Pearson type III , log – Pearson type III and Gumbel distributions " . the models were applied to annual flood series for Greater - Zab River at Eski - Kelek . The magnitudes of flood computed for different return periods . The models were compared using statistical measures such as root mean square error , bias , and standard error. The goodness of fits of all these models was evaluated using the test of Chi-Square. According to this test the log – normal type III could be regarded as the best for flood series for this river . Its useful to determine the relation ships between the flood magnitude versus return period .

الخلاصة

تم تحليل التصاريف الفيضانية لنهر الزاب الأعلى عند منطقة أسكي كلك , باستخدام نماذج إحصائية مختلفة " التوزيع الطبيعي أللوغاريتمي النوع الثالث , توزيع بيرسن النوع الثالث , توزيع بيرسن أللوغاريتمي النوع الثالث و توزيع كامبل " لإيجاد قيم الفيضان ولفترات عودة مختلفة , قورنت هذه التوزيعات فيما بينها باستخدام مقاييس إحصائية مثل جذر معدل مربع الخطأ , ومقياس الانحدار , والخطأ القياسي . بعد ذلك تم تقييم دقة هذه التوزيعات باستخدام اختبار مربع كاي . وطبقا لهذا الاختبار يتبين أن التوزيع الطبيعي أللوغاريتمي النوع الثالث يمكن أن يعد أفضل توزيع بالنسبة للتصاريف الفيضانية لهذا النهر . هذا التوزيع ضروري لإيجاد العلاقة بين التصاريف الفيضانية وفترات العودة .

1-Introduction

Hydrologic systems were sometimes impacted by extreme events, such as severe storms, floods, and droughts. The magnitude of an extreme event is inversely related to its frequency of occurrence. The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions. The results of frequency analysis can be used for many engineering purposes: for the design of dams, bridge, culverts, and control structures.

2-Literature Review

[Todorovic,1971] Present's that the Gumbel (I) distribution is suitable for estimate maximum type events. [Burges,1978] discussed two methods for estimation of the third parameter (a) of log-normal type III. The estimator of (a) using sample mean, median, and standard deviation is found to be more variant and have larger bias for distributions of interest in operational hydrology than the estimator using sample mean , standard deviation , and skew. Analytical expression for the standard common probability distribution (PDS) were given by [Kite,1977] who discussed the use of moments to estimate event magnitude and standard error for several return periods.[Kuczera,1982] considered the performance of the following estimators on four-wake by parent distribution:

1- Log Pearson type III distribution , 2 -Gumbel distribution , 3 - Log –Gumbel distribution.

The general conclusion of all these models was that the Log Pearson type III distribution was better than the other models. [Haktanir,1993] discuss an evaluation of various stream flow frequency distributions using annual peak data , The three parameters log-normal, Gumbel, Pearson type III, log- Pearson type III were applied to annual peaks series of stream flow data. The parameters of most of these distributions were estimated by method of moments. It was found that the Gumbel and Log-normal III distributions better than the other distributions.

3-Theoretical background

The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions. the relationship between return period, T, probability of non-exceedance, F(x=X), and the magnitude of T-year peak value can be summarized for three-parameter distribution as follows .

Prob $(x \le X) = F(x = X) = \int f(x, \alpha, \beta, \gamma) dx = 1 - \frac{1}{T}$ ....(3 – 1)

where α , β and γ denote the shape, scale, and location parameters, respectively. Values of the population parameter are estimated using method of moments.

The analytical expressions of the classical first, second, third, and more central moments are equated to their estimates Solution of these equations yield method of moments parameters.

$$\mu = \int_1^u X\, f(x)\, dx \cong \bar{X} = \frac{\sum X_i}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. \quad (3-2)$$

$$\sigma^2 = \int_1^u (x-\mu)^2\, f(x) dx \cong sd^2 = \frac{\sum (x_i - \bar{X})^2}{n} \quad \ldots\ldots\ldots\ldots\ldots \quad (3-3)$$

$$G = \int_1^u (X-\mu)^3 f(x)\, dx / \sigma^3 \cong C_s = \frac{[\sum (X_i - \bar{X})]n}{(n-1)(n-2)sd^3} \quad \ldots\ldots.. \quad (3-4)$$

Where (μ) is the mean , (σ) is the standard deviation, (G) is the skews coefficient of the particular model, where ($\bar{X}$) , sd , and (Cs) are unbiased estimates computed from the observed sample series, (u) and (1) represent the integration limits.

Having selected a distribution and estimated its parameters , [Chow,1988] proposed a general equation to use this distribution in frequency analysis.

$$X_T = \mu + K\sigma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3-5)$$

where (X_T) is the event magnitude at a given return period, T. (μ) and (σ) are the population mean and standard deviation.

A measure of variability of the resulting event magnitudes is the standard error of estimate. Standard error (S_T), may be written by using method of moments [Kite,1977] as:

$$S_T = (\frac{\partial X}{\partial \alpha})^2 \text{ Var } \alpha + (\frac{\partial X}{\partial \beta})^2 \text{ Var } \beta + (\frac{\partial X}{\partial \gamma})^2 \text{ Var}\gamma \quad + 2\, (\frac{\partial X}{\partial \alpha})(\frac{\partial X}{\partial \beta}) \text{ Cov } (\alpha,\beta)$$

$$+ 2\; (\frac{\partial X}{\partial \alpha})(\frac{\partial X}{\partial \gamma}) \text{ Cov } (\alpha, \beta) + 2\; (\frac{\partial X}{\partial \beta})\left(\frac{\partial X}{\partial \gamma}\right) \text{Cov } (B,\gamma) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3-6)$$

where (α, β, and γ) are the estimated parameters.

3 – 1 Log - normal type III distribution

The three-parameter represents the normal distribution of the logarithms of the reduced variable (x-a), where a is lower boundary. The probability density distribution is given by:

$$P(X) = \frac{1}{(X-a)\,\sigma_y\,\sqrt{2\pi}} \exp\left[-\frac{Ln(x-a)-\mu_y)^2}{2\sigma_y{}^2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-7)$$

where (μ_y) and (σ^2_y) are the form and scale parameters, shown later to be the mean and variance of the logarithms of (x-a) . If the lower boundary, a, is known then the reduced variable (x-a) can be used together with the procedures described for the three-parameter log normal distribution[Pilon,1993].

By using method of moment (MOM):

$$(C_V)_{X-a} = \frac{\sigma}{\bar{x}-a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-8)$$

$(C_v)_{X-a}$ Can be found from equation as follows:

$$Cs = (C_V)^3_{x-a} + 3(C_V)_{x-a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-9)$$

The parameters σ_y and μ_y can be found by MOM method from : -

$$\sigma_y = \left[\ln((c_V)^2_{x-a}+1)\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-10)$$

$$\mu y = \ln\left[\frac{\sigma}{(C_V)_{x-a}}\right] - \frac{1}{2}\left[\ln(C_V)^2_{x-a}+1\right] \ldots\ldots\ldots\ldots\ldots(3-11)$$

Using the standard normal deviate as frequency factor the first expression obtained is:

$$y_T = \mu_y + Z\sigma_y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-12)$$

where μ_y and σ_y the mean and standard deviation of the series in (x-a) so that T-year event, X_T, is : -

$$X_T = a + e^{\mu y + Z\sigma y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-13)$$

By using MOM method, as in the three - parameter log normal distribution, with equation (3-12) gives **:**

$$ST^2 = \frac{\sigma_y{}^2}{N}\left[1+\frac{Z^2}{2}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-14)$$

3 - 2 Pearson type III distribution

The pdf of Pearson type III distribution is of the form**:**

$$P(X) = \frac{1}{\alpha\,\Gamma(\beta)}\left\{\frac{X-\gamma}{\alpha}\right\}^{\beta-1} \exp\left\{-\frac{x-\gamma}{\alpha}\right\} \ldots\ldots\ldots\ldots(3-15)$$

where (α), (β) and (γ) are parameters to be estimated and $\Gamma(\beta)$ is the gamma function .By using MoM method, the parameter is given as:

$$\beta = \left(\frac{2}{C_S}\right)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-16)$$

$$\alpha = \sigma/\sqrt{\beta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-17)$$

$$\gamma = \mu - \sigma\sqrt{\beta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3-18)$$

where β, α, and γ are parameters to be estimated. The frequency factor is given [Kite,1977] and [Chow,1988] as:

$$K \cong Z + (Z^2 - 1)\,\frac{C_S}{6} + \frac{1}{3}(Z^3 - 6Z)\,(\frac{C_S}{6})^2$$
$$-(Z^2 - 1)\,(\frac{C_S}{6})^3 + Z(\frac{C_S}{6})^4 + \frac{1}{3}(\frac{C_S}{6})^5 \quad ....(3-19)$$

Using MOM method [Kite,1977] standard error is given as:

$$S_T^2 = \frac{\sigma}{n}\begin{Bmatrix} 1 + KC_S + \frac{K^2}{2}\left[\left(3C_S^2/4\right)+1\right] + 3K\frac{\partial K}{\partial C_S}\left[C_S + {C_S}^3\!\!\big/4\right] \\ +3(\frac{\partial K}{\partial C_S})^2\left[2 + 3{C_S}^2 + 5C_S + 5{C_S}^4\!\!\big/8\right] \end{Bmatrix}...............(\ 3-20)$$

$$\frac{\partial K}{\partial C_S} \cong \frac{Z^2 - 1}{6} + \frac{4(Z^3 - 6Z)}{6^3}C_S - \frac{3(Z^2 - 1)}{6^3}{C_S}^2$$
$$+\frac{4Z}{6^4}{C_S}^3 - \frac{10}{6^6}{C_S}^4 \ ....(3-21)$$

where C_S is the skewness and K is the frequency factor

3 – 3 Log-person type III distribution

If the logarithms, Inx, of a variables x are distributed a Pearson type III variant then the variable x will be distribution as log-Pearson type III with pdf [Kite,1977] .

$$P(X) = \frac{1}{\alpha\ X\ \Gamma(\beta)}\left\{\frac{Ln\ X - \gamma}{\alpha}\right\}^{\beta-1}\exp\left[-\frac{LnX - \gamma}{\alpha}\right].........(\ 3-22)$$

where α, β, and γ the scale, shape and location parameters respectively.

[Bobee,1975] studied the theoretical properties of Log Pearson (LPIII) distribution and suggested an estimate method based on the moments of the real data to give direct application. The indirect method of moment has advocated an estimation based on the moment of log-transformed data using relationships given in equations [Kite,1977] :-

$$\bar{y} = \gamma + \alpha\ \beta \ ....(3-23)$$
$$\sigma_y = \alpha\sqrt{\beta} \ ....(3-24)$$
$$\gamma_y = 2\big/\sqrt{\beta} \ ....(3-25)$$

where $\bar{y}$: mean for y = lnx : σ_y: The standard deviation for y = Lnx : γ ,β, α Coefficients of skew of the event (location parameter), shape, and scale parameter respectively : γ_y :The coefficient of skew of the logarithms.

Using Pearson type III distribution to the logarithms of the sample events the T-year event can be computed from:

$$y_T = LnX_T = \mu_y + K\sigma_y \ ....(3-26)$$

where μ_y : mean for y = lx equal to $\bar{y}$

σ_y : The standard deviation for y = lnx : k = frequency factor .

The standard error by using method of moments may be computed using the same equation in Pearson type III distribution to obtain S_{ty} in log units from the normal deviate and coefficient of skew of the logarithms of observed events**.**

3 – 4 Gumbel distribution

If x is an unbounded variant of the maxima, the probability of occurrence of a variant value equal to or less than x is often given by the largest value [Kite,1977] as :

$$P(X \leq X) = e^{-e^{-\alpha(x-\beta)}} \ldots\ldots\ldots\ldots\ldots\ldots(3-27)$$

provided that $\alpha > 0$ .The parameters α and β are known as the scale and location parameters, respectively.The probability density funtion corresponds to equation (3-27)

$$P(X) = \alpha\, e^{\left\{-\alpha(x-\beta)-e^{-\alpha(x-\beta)}\right\}} \ldots\ldots\ldots\ldots(3-28)$$

By taking logarithms to the base (e) of equation (3-28) two times, it can be found that:

$$X = \frac{1}{\alpha}\left[\alpha\beta - \ln\{-\ln(p(x))\}\right] \ldots\ldots\ldots\ldots\ldots(3-29)$$

Since

$$Tr(X) = 1/(1-p(X)) \ldots\ldots\ldots\ldots\ldots\ldots(3-30)$$

where $T_r(X)$ is the return period. p(X) is the probability.

$$So \quad X = \frac{1}{\alpha}\left[\alpha\beta - Ln\left\{-\ln\frac{Tr(x)-1}{Tr(x)}\right\}\right] \ldots\ldots\ldots\ldots(3-31)$$

This type is often used for maximum type events. The parameters α, and β can be found using method of moments [Kite,1977] as the following equations **:**

$$\alpha = \frac{1.2825}{\sigma} \ldots\ldots\ldots\ldots\ldots\ldots(3-32)$$

$$\beta = \mu - 0.45\,\sigma \ldots\ldots\ldots\ldots\ldots\ldots(3-33)$$

where α and β are the scale and location parameters. α and β are the μ and σ are the mean and standard deviation. The frequency factor for the type (I) extreme distribution can be found by substituting α and β in equation (3-41) and compare with standard frequency equation [Chow,1988] yield :-

$$K = -\left\{0.45 + 0.7797\ln(-\ln(1-\frac{1}{T}))\right\} \ldots\ldots\ldots\ldots(3-34)$$

By using method of moments to estimate the standard error [Kite,1977] . The standard error is given in the equation :-

$$S_T = \left[(1 + 1.1396\,K + 1.1\,K^2)\frac{\sigma^2}{n}\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots(3-35)$$

3 – 5 Chi-Square test

This test has been applied to check the differences between the observed and computed event magnitudes. [Levin,1994] define the general expression for Chi-Square as: -

$$\chi^2 = \sum_{i=1}^{K}\frac{(Q_0 - Q_C)^2}{Q_C} \ldots\ldots\ldots\ldots\ldots\ldots(3-36)$$

where k is the number of class intervals, Q_o is the observed and Q_c is the estimated (according to the distribution being tested) number of observation in the class interval. The distribution of χ_C^2 is a chi-square distribution with k-r-l degrees of freedom where r is the number of parameters estimated from the data.

Many statisticians such as[Haan,1977] and [Yevjevich,1999] recommended that classes be combined if the expected number in a class is less than 3, therefore this modification was included in this test. Standard error(SE),root mean square error(RMSE)and standard bias (BIAS) can be used for the comparison between fitted distributions, these measures were computed as :

$$SE = \left[\frac{\sum (Q_o - Q_c)^2}{N-M} \right]^{0.5} \qquad (3-37)$$

$$RMSE = \sum \left[\frac{Q_o - Q_c}{Q_o} \right]^2 \qquad (3-38)$$

$$BIAS = \sum \left| \frac{Q_o - Q_c}{Q_o} \right| \qquad (3-39)$$

Where : N = The sample size : Q_o = Observed discharge : Q_c = Computed discharge: M = The number of parameter distribution .

4-Observation data

A series of flood discharges of Greater – Zab River at Eski - Kelek were set up for a period of (32) years (1975 – 2006) as shown in Table (1) [11] :-

Table (1) Maximum water discharge (cumecs) for Greater – Zab river , at Eski - Kelek , for (32) years .

Year	Discharge (m^3 / sec)	Year	Discharge (m^3 / sec	Year	Discharge (m^3 / sec
1975	458	1986	567	1997	426
1976	397	1987	741	1998	175
1977	476	1988	121	1999	183
1978	385	1989	247	2000	220
1979	439	1990	127	2001	450
1980	481	1991	145	2002	489
1981	440	1992	650	2003	423
1982	398	1993	375	2004	314
1983	329	1994	630	2005	434
1984	501	1995	345	2006	366
1985	316	1996	480		

5-Results and Discussion

5 – 1 Predicted flood magnitudes

Four statistical models , Log – Normal type III, Pearson type III, log- Pearson type III and Gumbel distribution were used to estimate the flood magnitude for various return periods. A computer program is used to compute the parameters of the distributions. These parameters were estimated by method of moment; and this value as are given in table (2). This program gives also the magnitude of floods for various return periods, upper limit, lower limit and the magnitude of χ^2 (chi-square) and gives the decision according to (χ^2 test) whether the model results are accepted or not .The results are shown in table (3) . In table (2) the parameters give indication about which distribution should be accepted . The skewness is greater than zero and the Kurtosis is computed from the equation $C_K = 3+1.5(C_S)^2 = 8.02$ [Kite,1977] is close to the Kurtosis that computed from data table (1) , so Log-Normal III could be accepted as the best. The skewness of logarithm of data should be greater than zero for log-Pearson III distribution [Kite,1977] so it could not be accepted as the best.

As for Gumbel distribution (or EVI) the skewness and Kurtosis should be 1.14 and 5.4 respectively [Kite,1977] so this distribution could not be regarded as the best.

Table (2) Parameter estimation for peak flood data

Mean ($\bar{x}$) (cumecs)	**standard deviation sd (cumecs)**	**Skewness Cs**	**Kurtosis Ck**	**mean for logarithm of data $\bar{y}$ (cumecs)**	**standard deviation for logarithm of data sdy**	**skewness for logarithm of data Csy**	**Kurtosis for logarithm of data Cky**
391.5	147.99	1.83	8.34	2.55	0.198	- 0.07	2.64

Tables (3) Return periods versus flood magnitudes, standard error, upper limit and lower limit

Type : Log - Normal Distribution III				
Return Periods (Years)	Floods Magnitudes (M^3 / Sec)	Standard Error (M^3 / Sec)	Upper Limit (M^3 / Sec)	Lower Limit (M^3 / Sec)
1	152.435	0.15	203.24	148.74
2	452.112	0.16	587.17	236.73
5	715.482	0.18	764.28	421.41
10	825.470	0.20	1283.83	699.25
20	1248.455	0.23	1529.56	781.923
50	1398.245	0.26	1745.22	1148.36
100	1622.146	0.29	2182.79	1366.47
Type : Pearson Distribution III				
1	215.758	147.58	385.97	0
2	426.244	194.36	572.35	247.36
5	712.348	214.77	855.13	388.76
10	934.568	299.71	1312.45	447.37
20	1256.894	382.46	1627.28	725.61
50	1588.345	587.38	2072.11	793.75
100	1984.656	743.21	2832.74	944.81
Type : Log - Pearson Distribution III				
1	212.576	43.68	314.73	186.49
2	458.348	92.76	546.21	293.86
5	688.534	178.37	807.95	352.67
10	821.734	286.16	1086.44	474.83
20	1058.681	366.72	1683.19	689.34
50	1369.258	611.89	2138.55	823.89
100	1749.648	948.49	3174.97	792.58
Type : Gumble Distribution				
1	218.827	72.13	314.98	174.57

2	433.510	90.38	524.83	284.64
5	684.287	125.47	798.62	437.29
10	824.764	174.28	1185.61	693.37
20	1149.834	219.82	1438.48	753.91
50	1285.429	286.37	1568.92	915.47
100	1472.834	368.79	1938.45	1387.09

When we use measures such as standard error (SE), root mean square error (RMSE) and (BIAS), the smallest values of these measures lead to the best fit. From Table (4) the Log-Normal type III .

Table (4) Standard error (SE), root mean square error (RMSE) and bias (BIAS) of four models for peak flood data.

Type	SE	RMSE	BIAS
Log-normal III	10.49	0.027	0.21
Pearson III	14.92	0.016	0.36
Log-Pearson III	17.81	0.031	0.30
Gumbel I	13.88	0.028	0.29

5 – 2 Goodness of Fits

Mean and standard deviation are used to describe a set of data or observations. These statistics are estimated from samples. Some times the samples may be unrepresentative and may, therefore, lead to estimates that are too high or too low.

This estimation will be of no use if they differ from expected values by more than certain prescribed limits. It is therefore necessary to test the statistics to see whether their difference is significant or not. Such tests are called the tests of significance.

The more important one is χ^2– test .The χ^2– test can be carried out by using the numerical integration to find the value of P (Probability of deviation) if this probability

is equal to or less than a given probability value, then that the deviation is significant at the given probability level.

The computed value of χ^2 is used to determine the probability that the deviation would be larger than or equal to computed value. This can be calculated by using the χ^2 distribution as follows**:**

$$P = P_r\left(\chi^2 \geq C\right) = \int_0^{\infty} f(\chi^2)\,dx = 1 - \int_0^{C} f(\chi^2)\,dx^2$$

$$\text{Where } f(\chi^2) = \frac{1}{2^{d/2}\,\Gamma\left(\frac{d}{2}\right)}\left[\left(\chi^2\right)^{\left(\frac{d-2}{2}\right)}\right]e^{-\chi^2/2}$$

$f\left(\chi^2\right)$: is the probability density function of χ^2: d = degree of freedom:

The result given in table (5) shows that the log-Normal III, Log – Pearson III and Gumbel (EVI) are better models, but the Pearson distribution III could not be regarded as the best .

Table (5) Chi-square goodness of fit (χ^2) for peak flood data

Type	(χ^2) computed	Decision
Log-normal III	6.72	**Accepted**
Pearson III	28.37	Rejected
Log-Pearson III	3.24	**Accepted**
Gumbel I	4.94	**Accepted**

6- Conclusions

As the analysis of frequency for flood and after analysis the results by comparison of fits , using measures such as (SE, RMSE and BIAS) and Goodness of fit , from these methods the Log- Normal III could be regarded as the best for flood data for the Greater Zab River .

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