The Backbending Phenomenon in Deformed Even-Even Nuclei for $^{180,182}W$ Isotopes

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Abstract

$^{180,182}W$ isotopes near mass region A~180 which exhibit features of the SU(3)-O(6) symmetry at low energy and the backbending phenomenon at high spin , are studied in the framework of the interacting boson model (IBM-1) . Reasonable agreement was obtained between our theoretical calculations and the most recent experimental data. The backbending phenomenon was noticed both experimentally and theoretically and they were in good agreement.

الخلاصة

تمت دراسة نظيري التنكستن $^{180,182}W$ الواقعين قرب منطقة A~180 والتي تظهر سمات المنطقة الانتقالية SU(3)–O(6) التناظرية ضمن الطاقات الواطئة وظاهرة الأنحناء الخلفي عند الزخوم العالية ضمن أطار انموذج البوزونات المتفاعلة الأول (IBM–1). ومن خلال مقارنة النتائج المحسوبة نظرياً مع البيانات العملية المتوفرة حديثاً لظاهرة الانحناء الخلفي لوحظ توافق جيد.

1. Introduction

Investigations of the ground state bands of nuclei at A ~ 180 mass region have recently become a particularly interesting research in nuclear structure studies [Arima and Iachello, 1976] [Nomura *et al.*, 2011]. These nuclei exhibit a range of interesting features, including oblate and prolate deformations as well as rapid variations in shape as a function of both spin and particle number . The sudden disappearance of E2 strength at certain spins indicates a shape change and requires the inclusion of upper secondary shells (*pf)* configuration.

The effect of backbending [Ploszajczak *et al.* , 1982] has been observed experimentally [Grosse *et al.*, 1973] in the ground state rotational band of some deformed even-even nuclei. The effect occurs due to the rapid increase of the moment of inertia with rotational frequency towards the rigid value [Burcham , 1989]. When the rotational energy exceeds the energy needed to break a pair of nucleon, the unpaired nucleon goes into different orbits, which result in change of the moment of inertia [Krane , 1988]. An explanation of this effect is due to a disappearance of the pairing correlation [Mottelson and Valatin, 1960], [Bohr and Mottelson, 1969] by the action of Coriols forces, where the nucleus then undergoes a phase transition from a superfluid state to a state of independent particle motion. Other proposed explanations such as rotational alignment [Bohr and Mottelson,1975] [Stephens and Simon,1972] [Wyss and Pilloe,1991] [Bengtsson and Frauendorf , 1979] [Thieburger,1973] and centrifugal stretching[Feassler,], along with the former, could be described in terms of band crossing[Bertsch and Kurath,1980]; the case where the breakup of one pair of nucleons providing a large angular momentum, which may couple with the collective rotation to produce a new band.This effect makes up the backbending phenomena, which is experimentally observed in total angular momentum (J) , ω^2 rotational frequency plot as a radical change in the behavior of the angular momentum from linearity to a stretching and then back to linear again. When we plot rotational energy E_J with J (J+1) for low lying band (g-band) we can not recognize the change of energy, while when we make plot of the moment of inertia $2\vartheta/\hbar^2$ versus the square of the rotational frequency then the change in ϑ can be clearly seen as inverted Z letter as shown in figures (3),(4) , from this the name of the backbending is derived.

2. (IBM-1) Model

2.1. Energy Levels

The IBM-1 model describes the low-lying energy state of the even –even tungsten nuclei as a system of interacting s-bosons, and d-bosons. The π and υ bosons are treated as one boson. Introducing creation $(s^{\dagger}d^{\dagger})$ and annihilation $(sd^{\sim})$ operators for s and d bosons, the most general Hamiltonian [Feshband and Iachello, 1974] which includes one-boson term in boson-boson interaction has been used in calculating the levels energy is:

$$H = EPS.n_d + PAIR.(P.P) + \frac{1}{2}ELL.(J.J) + \frac{1}{2}QQ.(Q.Q) + 5OCT.(T_3.T_3) + 5HEX.(T_4.T_4) \quad (1)$$

Where

$$P.P = \frac{1}{2}\left[\begin{array}{c}\left\{(s^{\dagger}s^{\dagger})_0^{(0)} - \sqrt{5}\,(d^{\dagger}d^{\dagger})_0^{(0)}\right\}x \\ \left\{(s\,s)_0^{(0)} - \sqrt{5}\,(\tilde{d}\,\tilde{d})_0^{(0)}\right\}\end{array}\right]_0^{(0)} \quad (2)$$

$$J.J = -10\sqrt{3}\left[(d^{\dagger}\tilde{d})^{(1)}x(d^{\dagger}\tilde{d})^{(1)}\right]_0^{(0)} \quad (3)$$

$$Q.Q = \sqrt{5}\left[\begin{array}{c}\left\{(s^{\dagger}\tilde{d} + d^{\dagger}s)^{(2)} - \frac{\sqrt{7}}{2}(d^{\dagger}\tilde{d})^{(2)}\right\}x \\ \left\{(s^{\dagger}\tilde{d} + \tilde{d}s)^{(2)} - \frac{\sqrt{7}}{2}(d^{\dagger}\tilde{d})^{(2)}\right\}\end{array}\right]_0^{(0)} \quad (4)$$

$$T_3.T_3 = -\sqrt{7}\left[(d^{\dagger}\tilde{d})^{(2)}x(d^{\dagger}\tilde{d})^{(2)}\right]_0^{(0)} \quad (5)$$

$$T_4.T_4 = 3\left[(d^{\dagger}\tilde{d})^{(4)}x(d^{\dagger}\tilde{d})^{(4)}\right]_0^{(0)} \quad (6)$$

Here two *d* bosons couple to produce angular momentum, *J*, equal to 0, 2, and 4, giving the six parameters: boson energy (ε'_d), coupling coefficients of the *d* bosons ($C_{0,2,4}$), and coefficients that describe the interaction between the *s* and *d* bosons (v_0 and v_2). The second group of terms describes the interaction between *d* boson pairs and does not alter the relative number of *s* and *d* bosons. The third and fourth groups of terms change the number of d bosons by ±1 or ±2, and therefore mix basis states. The $\tilde{d}$ operator is defined as $\tilde{d}_m = (-1)^m d_{-m}$ and introduces a phase factor. The term, *H.c.*, represents the Hermitian conjugates of the corresponding operators [Casten, 1990].

Table (1) show the magnitudes of IBM-1 parameters , the $\hat{n}_d$ operator gives the number of *d* bosons, $\hat{P}$ is the pairing operator for the *s* and *d* bosons, $\hat{J}$ is the angular momentum operator, $\hat{Q}$ is the quadrupole operator, $\hat{T}_3$ and $\hat{T}_4$ are the octupole and hexadecapole operators, respectively. The $\hat{n}_d$, $\hat{J}$, $\hat{T}_3$, and $\hat{T}_4$ operators have $\Delta n_d = 0$, while $\hat{P}^\dagger \cdot \hat{P}$ has $\Delta n_d = 0, \pm 2$ and $\hat{Q} \cdot \hat{Q}$ has $\Delta n_d = 0, \pm 1, \pm 2$.

Table (1) Parameters used in IBM-1 Hamiltonian (all in MeV)

nucleus	**N**	**EPS**	$\hat{P} . \hat{P}$	$\hat{J} . \hat{J}$	$\hat{Q} . \hat{Q}$	$\hat{T}_3 . \hat{T}_3$	$\hat{T}_4 . \hat{T}_4$	**CHI**	**SO6**	**E2DD (e b)**	**E2SD (e b)**
^{180}**W**	14	0.0000	0.05520	0.01149	-0.01380	0.0000	0.0000	-1.3228	1.0000	-0.292717	0.098957
^{182}**W**	13	0.0000	0.05980	0.01107	-0.01495	0.0000	0.0000	-1.3228	1.0000	-0.312213	0.105548

2.2. Transition Rates and Quadruple Moment

Another important property that can be deduced and calculated using the IBM-1 called the *reduced transition probability* B(E2).

The general linear E2 operator of the IBM-1 is the *J*=2 tensor operator give by [Casten and Warner, 1988]:

$$T^{(E2)} = E2SD.\left(s^\dagger\tilde{d} + d^\dagger s\right)^{(2)} + \frac{1}{\sqrt{5}} E2DD.\left(d^\dagger\tilde{d}\right)^{(2)} \tag{7}$$

The important physical quantities calculated with the E2 operator are the reduced transition probability $B(E2; J_i \rightarrow J_f)$, which is defined by the expression [Arima and Iachello, 1987]:

$$B(E2\,;\,J_i \rightarrow J_f) = \frac{\left[<J_f \| T^{(E2)} \| J_i>\right]^2}{2J_i + 1} \tag{8}$$

where $[<J_f \| T^{(E2)} \| J_i>]$ is the matrix element of E2 transition,

$$Q_J = \beta_2 \sqrt{\frac{16\ \pi}{5}} \sqrt{\frac{J}{14}} \tag{9}$$

Therefore, $Q_{2_1^+}$ becomes [Arima *et al.*, 1977];

$$Q_{2_1^+} = \beta_2 \sqrt{\frac{16\pi}{35}} \tag{10}$$

where β_2 can be defined as [Arima, and Iachello, 1976];

$$\beta_2 = -\frac{0.7}{\sqrt{5}} \alpha_2 \tag{11}$$

3. Results and Discussion

3.1. Energy Levels

IBM-1 model has been used in calculating the energy of the positive low -lying levels of tungsten isotopes. A comparison between the experimental data [Singh, 2002][

Singh , 2003] and our calculations, using the values of the model parameters given in table 1 for the ground bands, is illustrated in figures (1) and (2) . The agreements between the theoretical and their correspondence experimental values for all the nuclei are slightly higher but reasonable.

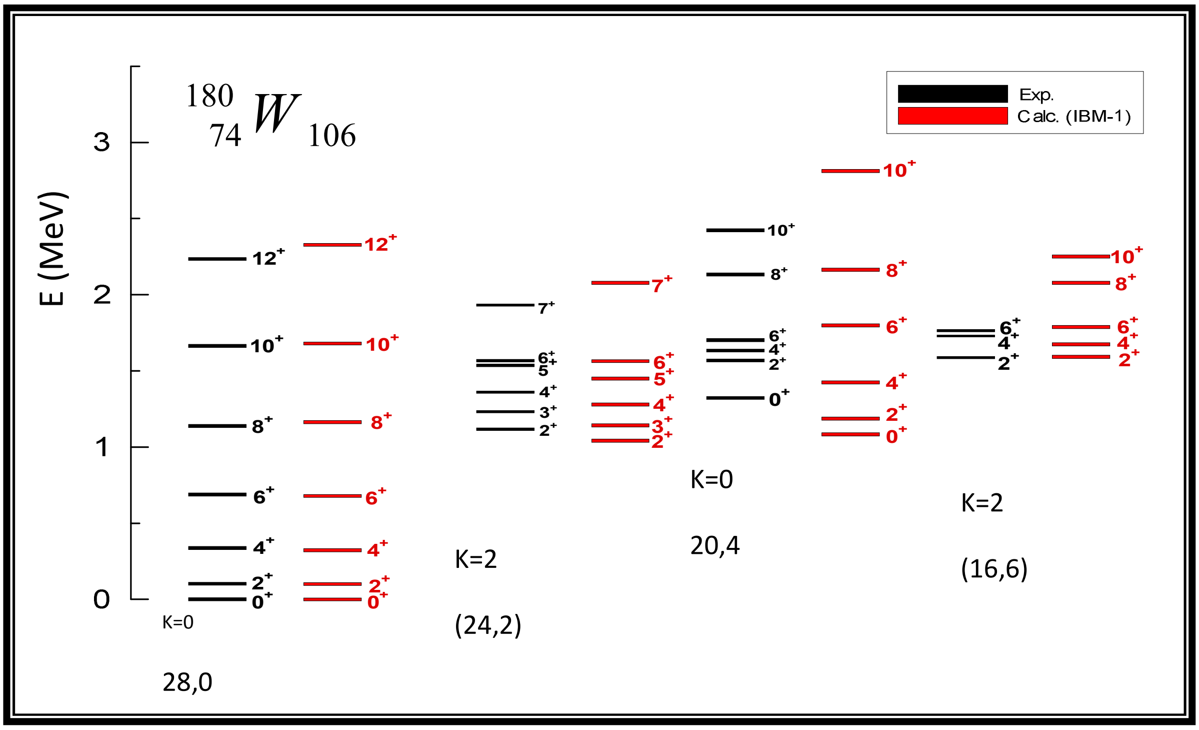

Figure 1: Comparison between Exp. [Tuli,2011] **and Calculated (IBM-1) Energy Levels**

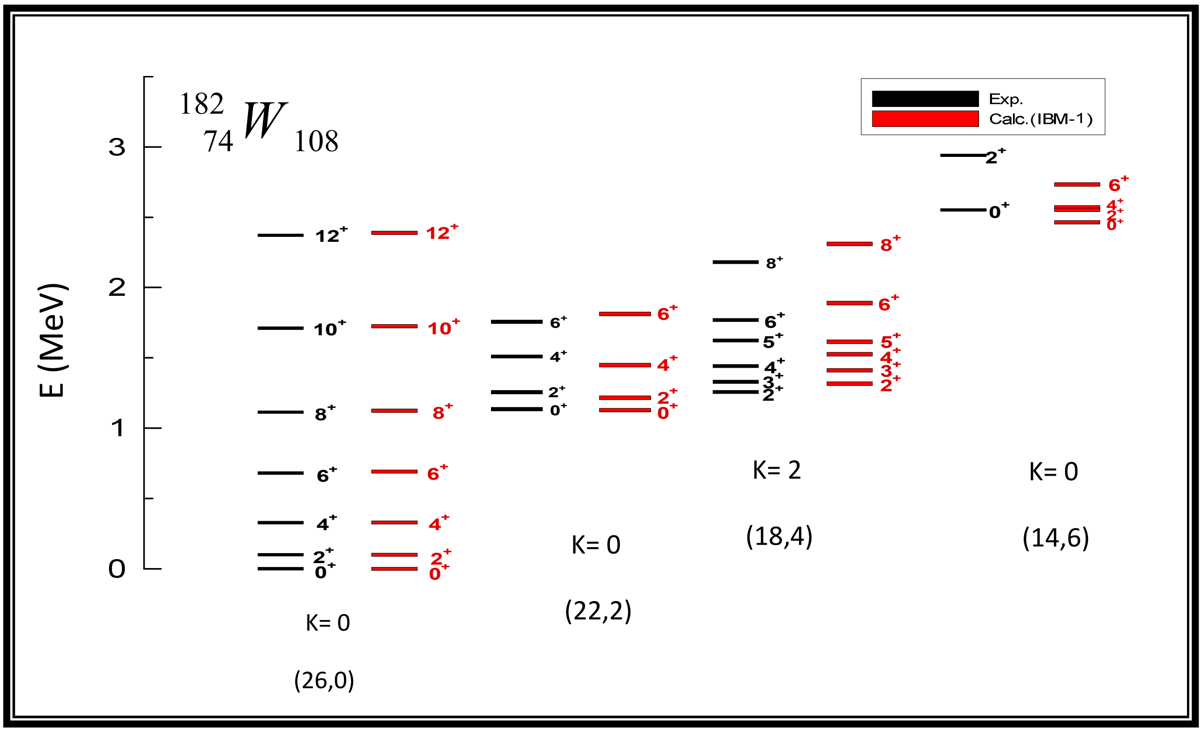

Figure 2: Comparison between Exp. [Tuli, 2011] **and Calculated (IBM-1) Energy Levels**

3.2. Electromagnetic Transition Rates and Quadrupole

There is no enough measurements of B(E2) rates for these series of nuclei. The only measured $B(E2;\ 0_1^+ \rightarrow 2_1^+)$ are presented, in Table 2, for comparison with the calculated values. The parameters E2SD and E2DD used in the present calculations are determined by normalizing the calculated values to the experimentally known ones and displayed in table 1.

Table 2 : The Experimental and Calculated B(E2)↓ Using IBMT-Code and the Quadrupole Moment $Q2_1^+$ for $^{180,182}W$ Isotopes

$J_i - J_f$	B(E2)↓ e^2b^2 ^{180}W		B(E2)↓ e^2b^2 ^{182}W		
	Exp. [Tuli, 2011]	**Calc.**	**Exp.** [Tuli, 2011]	**Calc.**	**Previous Work**[Navrátil,et al, 1996]
$2_1^+ \rightarrow 0_1^+$	**0.850(5)**	**0.847**	**0.839 (18)**	**0.829**	**0.850**
$2_1^+ \rightarrow 0_2^+$	------	**0.001**	------	**0.001**	------
$2_2^+ \rightarrow 0_1^+$	------	**0.000**	**0.021(1)**	**0.006**	**0.006**
$2_2^+ \rightarrow 0_2^+$	------	**0.686**	------	**0.007**	------
$2_2^+ \rightarrow 2_1^+$	------	**0.001**	**0.041(1)**	**0.010**	**0.010**
$2_3^+ \rightarrow 0_1^+$	------	**0.000**	**0.006(1)**	**0.001**	**0.001**
$2_3^+ \rightarrow 0_2^+$	------	**0.004**	**1.225(368)**	**0.682**	**0.486**
$2_3^+ \rightarrow 2_1^+$	------	**0.000**	**0.0039(5)**	**0.0021**	**0.0023**
$2_4^+ \rightarrow 0_3^+$	------	**0.550**	------	**0.507**	------
$2_1^+ \rightarrow 2_2^+$	------	**0.001**	------	**0.001**	------
$4_1^+ \rightarrow 2_1^+$	------	**1.199**	**1.201(61)**	**1.197**	**1.199**
$4_1^+ \rightarrow 2_3^+$	------	**0.000**	------	**0.002**	------
$4_2^+ \rightarrow 2_1^+$	------	**0.000**	------	**0.001**	------
$4_2^+ \rightarrow 2_2^+$	------	**0.968**	------	**0.385**	------
$4_1^+ \rightarrow 2_3^+$	------	**0.000**	------	**0.002**	------
$6_1^+ \rightarrow 4_1^+$	------	**1.290**	**1.225(135)**	**1.256**	**1.290**
$Q(2_1^+)$	$-1.966^{+0.04}_{-0.04}$	-1.92	$-2.00^{+0.04}_{-0.08}$	-1.986	-1.988

3.3. Backbending Effect

In the present work an attempt is made to study the behavior of the backbending phenomena in the ground state bands for $^{180-182}$W even–even nuclei using the first version of the interacting boson model (IBM-1).

Figures (3) and (4) show the comparison of the calculated and experimental study of the backbending phenomena for $^{180-182}$W even–even nuclei. From these figures it is shown that the backbending happen for these isotopes due to level crossing between the ground and negative parity states. The moment of inertia ϑ and squared rotational frequency ω2 are related to the spin derivative of the energy [Lin and Chern, 1979].

$$\frac{2\vartheta}{\hbar^2} = \frac{4J-2}{\Delta E_\gamma} \qquad (12)$$

$$(\hbar\omega)^2 = (J^2 - J + 1)\left[\frac{\Delta E_\gamma}{2J-1}\right] \qquad (13)$$

where $\Delta E_\gamma = E(J) - E(J-2)$ (14)

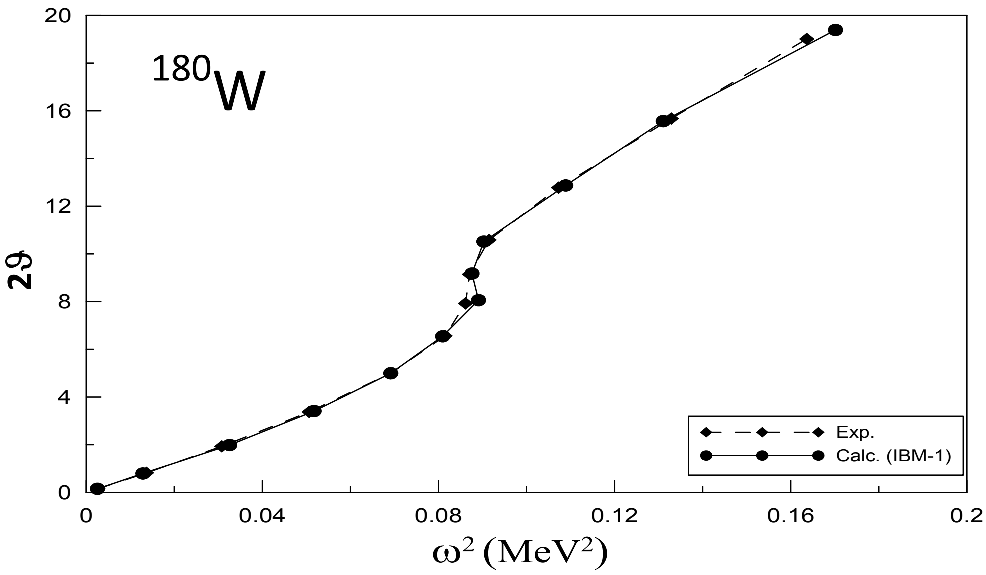

Figure 3: Backbending Plot for the Yrast Band in ^{180}W. The Experimental Data [Wu, and Niu,2003][Singh and Roediger,2010] are Compared with the Theoretical (IBM-1) Calculations. Moment of Inertia is Defined as ϑ=(2J-1)/2ω (MeV^{-1}), while Rotational Frequency as ω=[E(J)-E(I-2)]/2 (MeV)

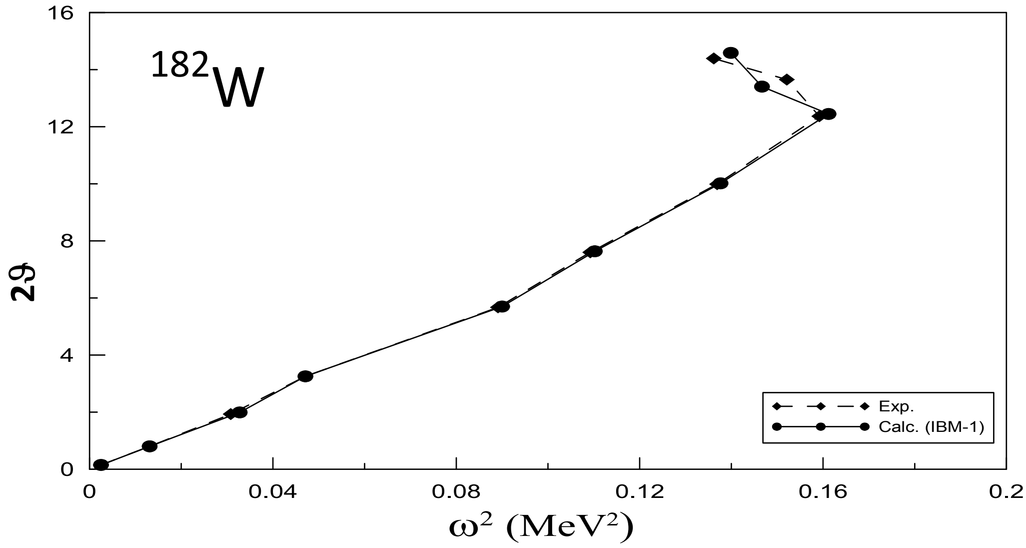

Figure 4: Backbending Plot for the Yrast Band in ^{182}W. The Experimental Data [Wu, and Niu,2003][Singh and Roediger,2010] are Compared with the Theoretical (IBM-1) Calculations. Moment of Inertia is Defined as $\vartheta=(2J-1)/2\omega$ (MeV^{-1}), while Rotational Frequency as $\omega=[E(J)-E(J-2)]/2$ (MeV)

Conclusions

The IBM-1 model has been applied successfully to ^{180,182}W isotopes and we have got:

1. The ground state bands are successfully reproduced.
2. Electromagnetic transition rates B(E2) are calculated .
3. Quadrupole $Q(2_1^+)$ moment also is calculated.
4. The effect of backbending has been studied.
5. The results show a good agreement with experimentally , but in upper levels a little difference has been found ,because the IBM-1 do not distinguish between protons and neutrons .
6. Also the results show that the energy levels calculated in this work have excellent agreement with experimentally data for the g-band (low) and in reasonable agreement with β-band (middle) and γ-band (high) , also some of energy levels calculated in this work are not calculated experimentally .

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