Magnetohydrodynamics Opposing Mixed Convection in Two-Sided Lid-Driven Differentially Heated Parallelogrammic Cavity

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Abstract

Mixed convection in a two-sided anti-parallel lid-driven differentially heated parallelogrammic cavity with the presence of magnetic field has been investigated and solved numerically using the finite volume method. The top and bottom walls of the cavity are horizontal and thermally insulated, whereas the left and right moving walls are maintained at different hot and cold constant temperatures, respectively. Both the lid-driven left and the right sidewalls of the cavity are allowed to move in an anti-parallel motion with the same sliding velocity. Calculations have been made for a wide range of Richardson numbers from 0.01-100, Hartman numbers from 0-75 and various inclination angles with gravitational direction ($-60^{o}\leq\Phi\leq60^{o}$). Effort is focused on the interaction of force convection with natural convection in presence of magnetic field is subjected in the horizontal direction, while the gravitational force acts in the vertical direction. The working fluid is air, so that the Prandtl number equals to 0.71. Flow and heat transfer characteristics inside the cavity have been presented and discussed in terms of streamtraces, isotherms and average Nusselt number along the heated wall for various combinations of different governing parameters. The accuracy of the numerical method is checked by comparisons with previously published works and the results showed an excellent agreement. The obtained results showed that the positive values of Φ cause a greater increase in local Nusselt number than the same negative values of Φ, and both of them have great effects on the heat transfer and fluid flow phenomenon in the parallelogrammic cavity.
Keywords: Magneto-hydrodynamic, Mixed convection, two-sided, lid-driven, parallelogrammic cavity, finite volume, MHD.

الخلاصة

تم تقصي وحل مسألة انتقال الحرارة بالحمل المختلط في فجوة او حيز ذا شكل متوازي اضلاع متفاضلة التسخين ولها جدارين منزلقين ومتحركين في اتجاهين متعاكسين تحت تأثير وجود المجال المغناطيسي بطريقة الحجوم المحددة . الجداران العلوي والسفلي افقيان ومعزولان حرارياً بينما الجداران المتحركان الايسر والايمن عند درجات حرارة ساخنة وباردة على التوالي. كلا الجداران الايسر والايمن يتحركان بنفس السرعة و بصورة متوازية بحيث اتجاه حركة احدهما تعاكس الاخر . تم اجراء الحسابات العددية لمدى واسع من ارقام عدد ريكاردسون يتراوح ما بين (0.01−100) وارقام من عدد هارتمان (0−75) لمختلف زوايا الميلان للجارين الجانبيين بالنسبة لاتجاه التعجيل ($-60^{o}\leq\Phi\leq60^{o}$) . تم التركيز على التداخل ما بين انتقال الحرارة بالحمل الطبيعي والقسري بوجود المجال المغناطيسي الافقي بينما يكون اتجاه قوة التعجيل عمودياً . تم استخدام الهواء كمائع تشغيل وبعدد برانتدل مساوي الى 0.7. تم عرض ومناقشة معالم او متغيرات جريان المائع وانتقال الحرارة داخل الفجوة بدلالة خطوط الدوامية او الانسياب و درجات الحرارة واعداد نسلت المتوسطة على امتداد الجدار الساخن لمختلف القيم للمعاملات او المتغيرات الحاكمة للمسألة. تم تحقيق دقة النتائج والطريقة العددية المستخدمة في هذا البحث عن طريق المقارنة مع نتائج اخرى منشورة وتم الحصول على تطابق ممتاز في النتائج. النتائج المستحصلة في البحث بينت بأن قيم الارقام الموجبة لزاويا الميلان للجدارين الجانبيين ذات تأثير اكبر على توزيع اعداد نسلت الموضعي مما هو عليه بالنسبة لأرقام زوايا الميلان السالبة القيمة وكلتاهما ذات تأثير كبير على ظاهرة جريان المائع وانتقال الحرارة داخل الفجوة ذات الشكل المتوازي الاضلاع .

Nomenclature

Symbol	Description	Unit
B_{ox}	Magnetic induction in x-direction	Tesla
g	Gravitational acceleration	m/s^2
Gr	Grashof number	
H	Height of the parallelogrammic cavity	m
Ha	Hartmann number	
k	Thermal conductivity of fluid	$W/m.K$
n	Outward flux normal to boundary	
$\overline{Nu}$	Average Nusselt number	
P	Dimensionless pressure	
p	Pressure	N/m^2
Pr	Prandtl number	
Re	Reynolds number	
Ri	Richardson number	
T	Temperature	K
T_c	Temperature of the cold surface	K
T_h	Temperature of the hot surface	K
U	Dimensionless velocity component in x-direction	
u	Velocity component in x-direction	m/s
V	Dimensionless velocity component in y-direction	
v	Velocity component in y-direction	m/s
V_P	Lid-driven velocity	m/s
W	Width of the Parallelogrammic cavity	m
X	Dimensionless Coordinate in horizontal direction	
x	Cartesian coordinate in horizontal direction	m
Y	Dimensionless Coordinate in vertical direction	
y	Cartesian coordinate in vertical direction	m
Greek Symbols		
α	Thermal diffusivity	m^2/s
β	Volumetric coefficient of thermal expansion	K^{-1}
θ	Dimensionless temperature	
Φ	Sidewall inclination angle from vertical or skew angle	*degree*
ϑ	Kinematic viscosity of the fluid	m^2/s
μ	Dynamic viscosity of the fluid	$kg.s/m$
ρ	Density of the fluid	kg/m^3
σ	Fluid electrical conductivity	$W/(m.K)$
Subscripts		
h	Hot	
c	Cold	
p	Plate	
Abbreviations		
MHD	Magneto-hydrodynamics	
SIP	Strongly Implicit Procedure	

1. Introduction

Mixed convection flow and heat transfer in lid-driven cavities occurred due to forces mechanisms. The first force produces due to shear flow caused by movement of one or more of the walls in the cavity, while the second force is due to buoyancy force produced by thermal non-homogeneity of the cavity boundaries. There are many applications in engineering for mixed convection lid-driven with MHD effect such as cooling electronics, applications of material processing, solar ponds, food processing, dynamic of lakes, reservoir and cooling ponds, crystal growth, underground heat pumps, metal casting, galvanizing and metal coating.

Various numerical, analytical and experimental mixed convection studies related to the analysis of fluid flow phenomena and heat transfer in a square and rectangular with two or four sided, parallel or anti-parallel, facing or non-facing lid-driven differentially heated or thermally insulated wall(s) enclosures are found in literature [Aydin and Yang, 2000; Kuhlman et al., 2001; Oztop and Dagtekin, 2004; Mahapatra et al., 2006; Luo and Yang, 2007; Shah et al., 2007a & 2007b; Wahba, 2008 & 2009; Noor *et al*., 2009; Ouertatani *et al*., 2009; Perurnal and Dass, 2010a-b & 2011]. Moreover, very few studies have addressed the question of natural convection in nonrectangular or parallelogram type enclosures [Nakamura and Asako, 1980; Asako and Nakamura, 1982 & 1984; Naylor and Oosthuizen, 1994; Costa , 2004; Costa *et al*., 2005; Bairi *et al*., 2010; De Maria *et al*., 2010; Chamkha *et al*., 2012].

The convection flow of fluid in lid-driven cavities in presence of magnetic field has been receiving a considerable attention in the literature. This attention comes from its importance in engineering and natural applications. Sposito and Michele (2006) studied the parallel, fully developed flow of an electrically conducting fluid between plane parallel walls under the simultaneous influence of a driving pressure head, buoyancy, and magnetohydrodynamic forces. It was found that the net electrical current is proportional to the mean velocity. The problems of combined free and forced convective magnetohydrodynamic flow in a vertical channels are analyzed by Umavathi and Malashetty (2005), Baletta and Celli (2008) and Prathap *et al*. (2011) by taking into account the effect of viscous and ohmic or Joule dissipations. The channels walls are maintained at equal or at different constant temperatures, or at an adiabatic wall and isothermal wall or filled with one region with conducting fluid and another region with non-conducting fluid are analyzed, respectively. Rahman et al. (2011a) has been studied numerically the development of magnetic field effect on mixed convection flow in a horizontal channel with a bottom heated open enclosure. The results indicate that the drag force and the average fluid temperature change from highest to lowest as Hartman number and Reynolds number increase while they grow up with large values of Rayleigh number. Later, Rahman *et al*. (2011b) performed the effect of Joule heating and magneto-hydrodynamic on the same previous horizontal channel with an open cavity by finite element analysis. The result show that the Joule heating parameter has a little effect on buoyancy-induced vortex in the streamlines and concentration contours. On the other hand, it has a striking effect on isotherms and density contours. The effect of an external magnetic field on three dimension unsteady natural convection in a cubical enclosure is carried out numerically by Kolsi *et al*. (2007). It was concluded that the three dimensional corner flow peculiar to the magnetic effect exists for a range of intermediate values of Hartman number. Oztop et al. (2011) considered laminar mixed convection flow in the presence of magnetic field in a top sided lid-driven cavity heated by a corner heater. In view of the obtained results, the temperature distribution inside the cavity mostly sterns from the right side of corner due to impinging air. Chamkha (2002) formulated the problem of unsteady, laminar, combined forced-free convection flow in a square cavity in the presence of internal heat generation or absorption and magnetic field. It was found that a significant reductions in the average Nusselt number were produced for both aiding and opposing flow situations as the strength of the applied magnetic field was increased. Later a numerical study of Sathiyamoorthy and Chamkha (2010) is presented for natural convection flow of electrically conducting liquid gallium in a square cavity. The results show that the average Nusselt number decreases non-linearly by increasing Hartmann number for any inclined angle. Nasrin and Parvin (2011) conducted to analyze the numerical simulation of mixed convection flow and heat transfer in a lid-driven cavity with sinusoidal wavy bottom surface in presence of transverse magnetic field. The results illustrated that the average Nusselt number at the heated surface increases with an increase of the number of waves as well as the Reynolds number, while decreases with increasing Hartmann number. Double diffusive convection flow in a rectangular enclosure with the upper and lower surfaces being insulated is studied numerically by Teamah (2008). The predicted results show that the heat transfer and fluid circulation were found to reduce due to magnetic field effect. Kaya (2011) studied numerically the problem of steady laminar Magnetohydrodynamic mixed convection heat transfer about a vertical slender hollow cylinder under the effect of wall conduction. It is determined that the local skin friction and the local heat transfer coefficients increase with increase the magnetic and buoyancy parameters and decrease with conjugate heat transfer parameter. A similarity analysis was performed to investigate the laminar boundary-layer flow in the presence of transverse magnetic field over a down-pointing and spinning core with mixed thermal boundary conditions by Ece and Öztürk (2009). The results show that the magnetic field retards the velocity profiles and expand the temperature profiles increasing both shear stress and heat flux. Kandaswamy *et al*. (2008) investigated numerically the magnetoconvection of an electrically conducting fluid in a square cavity with partially thermally active side walls. The active part of the left side wall is at a higher temperature than active part of the right side walls. The other walls are thermally inactive. It was concluded that the average Nusselt number decreases with an increase of Hartmann number and increases with increase of Grashof number. Later, they changed temperature of one of the thermally active regions of the side walls to periodic in time [Nithyadevi *et al*., 2009], while the opposite wall is isothermal. The other walls are remaining thermally inactive. It is observed that the average Nusselt number decreases with an increase of Hartmann number and increase with increase of Prandtl number and Grashof number. Sivasankaran and Ho (2008) studied numerically the natural convection of water near its density maximum in the presence of magnetic field in a cavity with temperature dependent properties. It is observed that the average Nusselt number decreases with increase of Hartman number. Later Sivasakaran *et al*. (2011) subjected the same square cavity with sinusoidal boundary temperatures at the side walls in the presence of magnetic field too and investigated numerically. The results show that the heat transfer rate increases with phase deviation up to $\pi/2$ and then it decreases for further increase in the phase deviation. Jung and Tanahashi (2008) perform numerically the natural convection between concentric spheres in electromagnetic fields for two different working fluids. This study examines the relationship between the velocity field and the electromagnetic field in the presence of Coriolis forces. The hydrodynamic natural convection flow and heat transfer characteristics in square cavity with a solid circular heated obstacles located at the center have been investigated numerically by Nasrin (2011). The outcome of this analysis, the heat transfer rate grows up with rising of

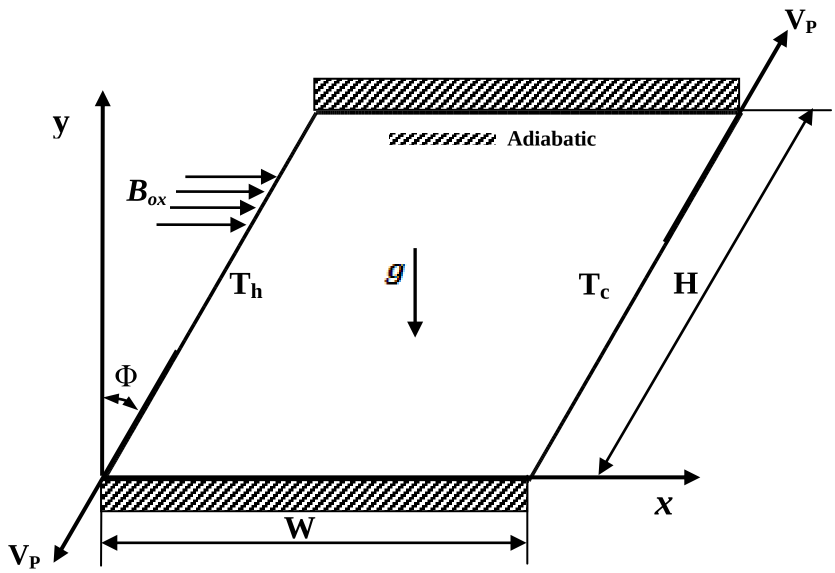

Fig. 1. Schematic diagram and coordinate system of the physical domain with boundary conditions.

prandtl number while it devalues for mounting Hartmann number and Joule heating parameter. Finally Rahman *et al*. (2010) investigated numerically the conjugate effect of Joule heating and magnetic forces, acting normal to the left vertical wall of a circular heat-conducting solid obstacle lid-driven square cavity, saturated with an electrically conducted fluid. Later, Rahman *et al*. (2011) centered heat-conducting horizontal square block instead of circular obstacle to understand the effect of magnetic field and Joule heating on the flow and thermal fields in a lid-driven cavity and solved numerically. It was drawn from the results of Rahman *et al*. (2010 & 2011) that the values of Nusselt number are always lower for the highest value of Hartman number. Also, the temperature of the fluid in the cavities increased due to the increase of Joule heating parameters and thus that negate the heat transfer from the heated surfaces.

In the light of the above literature, it has been noted that there is no enough information about MHD effect on mixed convection problems in a parallelogram cavity. Depending on the applications, various interactions between the magnetic field and force, mixed and natural convection should be known. The present work addresses with solving numerically two-dimensional mixed convection flow equations in a two sided lid-driven parallelogram-shaped cavity filled with air and subjected to a horizontal magnetic field. Also, the left wall is considered to be hot and sliding downwards and the right to be cold and sliding upwards. Numerical solution are carried out over a wide range of Richardson numbers of (0.01 to 100), sidewall inclination angles 60° $\leq\Phi\leq$ -60° and Hartmann numbers of 0, 25, 50 and 75. Results are presented graphically in terms of streamtraces and isotherms. Finally, the average Nusselt numbers at the left heated wall are calculated.

2. Mathematical Formulation of The Problem

Consider steady, laminar, mixed convection flow in a two-dimensional parallelogrammic cavity of height (H) and width (W) filled with air. The physical model and coordinate system considered in this investigation are shown in Fig 1. Both the top and bottom walls of the cavity are insulated, while the left and the right.

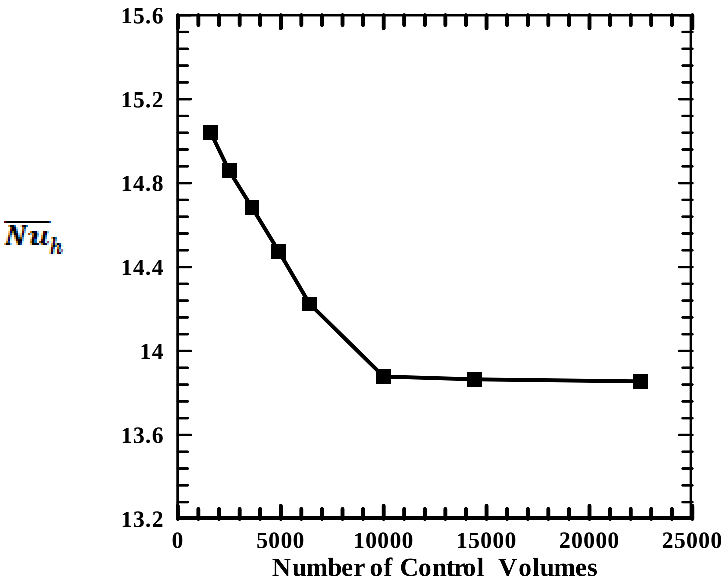

Fig. 2. Convergence of average Nusselt number along the heated left side wall with grid refinement at Re=1000, Pr = 0.71,Ri= 0.01, Ha=75 and Φ = - 60°.

magnetic field of strength (B_{ox}) is subjected in the horizontal direction, while the gravitational force acts in the vertical direction. Both the lid-driven left and the right sidewalls of the cavity are allowed to move in an anti-parallel motion with the same sliding velocity (V_p). The lid-driven left wall is moving downwards while the lid-driven right wall is allowed to move in upwards direction from bottom to top. The flow inside the parallelogrammic cavity is assumed to be laminar, two-dimensional, incompressible, Newtonian and steady. Hartmann numbers are varied as $0 \leq Ha \leq 75$, the Richardson numbers ($Ri = Gr/Re^2$) are varied as $0.01 \leq Ri \leq 100$.This range of Richardson number is produced by considering the Grashof number (Gr) constant at $Gr = 10^4$, while the Reynolds number is varied as ($10 \leq Re \leq 1000$) by changing the sliding velocity (V_p). Sidewall inclination angle from vertical or skew angle is varied as $-60° \leq \Phi \leq 60°$. The fluid properties are considered constant except for the density variation, which is modeled according to Boussinesq approximation while viscous dissipation effects are assumed to be negligible. In this work, the effect of magnetic field is considered whereas Joule heating is neglected. The governing non-dimensional mass, momentum and energy equations are as follows [Oztop *et al.*, 2011]:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (1)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad (2)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ha^2\,\mathrm{Pr}\,V + Ri\theta \qquad (3)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (4)$$

The previous dimensionless governing equations are converted into a non-dimensional form by using the following dimensionless parameters [Oztop et al., 2011]:

$$\theta = \frac{T - T_c}{T_h - T_c}, X = \frac{x}{H}, Y = \frac{y}{H}, U = \frac{u}{V_p}, V = \frac{v}{V_p}, V_p = \frac{\alpha}{H} \text{ and } P = \frac{p}{\rho V_p^2} \qquad (5)$$

The previous dimensionless numbers are defined as:-

$$Pr = \frac{\vartheta}{\alpha}, Gr = \frac{g\beta(T_h - T_c)H^3}{\vartheta^2}, Re = \frac{V_p H}{\vartheta}, Ri = \frac{Gr}{Re^2} \quad \text{and } Ha = B_{ox} H \sqrt{\frac{\sigma}{\mu}} \qquad (6)$$

The Hartmann number (Ha) represents the effect of the electromagnetic field. While, the Richardson number (Ri) represents the relative strength of the natural convection and forced convection for the considered problem. The rate of heat transfer is represented in terms of average Nusselt number at the hot left sidewall ($\overline{Nu}_h$) as follows [Nasrin and Parvin, 2012]:

$$\overline{Nu}_h = -\frac{1}{S}\int_0^S \left[\frac{\partial \theta}{\partial n}\right]_{X=0} dn \qquad (7)$$

where S, n are the non-dimensional length and coordinate along the inclined heated surface respectively.

2.1 Boundary Conditions

On the solid walls no-slip boundary conditions are applied and the relevant non-dimensional boundary conditions of the present problem are expressed as follows:

1. The left sidewall of the parallelogrammic cavity is maintained at uniform hot temperature (T_h), and sliding in downwards direction at a uniform sliding velocity (V_p) so that:

$$\theta = 1, V = -\cos(\Phi) \text{ and } U = -\sin(\Phi)$$

2. The right side wall of the parallelogrammic cavity is maintained at uniform cold temperature (T_c), and sliding in upwards direction at a uniform sliding velocity (V_p) so that:

$$\theta = 0, V = \cos(\Phi) \quad \text{and} \quad U = \sin(\Phi)$$

3. The upper and lower horizontal walls of the parallelogrammic cavity are considered adiabatic, so that:

$$\frac{\partial \theta}{\partial Y} = 0, \quad U = V = 0$$

Table (1): Comparison of the average Nusselt number at the heated left moving upwards sidewall for aiding flow conditions (Gr=100, Pr=0.71, Φ= **0**° and Re=1000) with those of previous study.

Ha	Average Nusselt number		Error %
	Chamkha (2002)	Present work	
0	2.2692	2.2588	-0.45
10	2.1050	2.1132	+0.38
20	1.6472	1.6587	+0.69
50	0.9164	0.9211	+0.51

3. Method of Solution and Verification

The dimensionless governing equations associated with their boundary conditions are solved numerically using the finite volume method. The details of this method are
well presented in Ferzinger and Peric (1999).The hybrid-scheme, which is a combination of the central difference scheme and the upwind scheme, is used to discretize the convection terms whereas central differencing is used to discretize the diffusion terms. The momentum and energy balance equations are the combinations of mixed elliptic-parabolic system of partial differential equations and the resulted set of algebraic equations are solved sequentially using **S**trongly **I**mplicit **P**rocedure (**SIP**). In order to couple the velocity field and pressure in the momentum equations, the well-known SIMPLE-algorithm is utilized. To satisfy the required convergence an under-relaxation factor of **0.2-0.85** is used in the computational scheme. The present code utilizes the collocated variable arrangement. The computation process is terminated when the residuals for the continuity and momentum equations get below 10^{-6} and the residual for the energy equation gets below 10^{-9}. In order to capture the flow and thermal fields inside the parallelogrammic cavity accurately especially adjacent to the sharp corners, a grid clustering procedure close to solid boundaries is adopted. This is due to the strong expected temperature gradient adjacent the two sided lid-driven sidewalls. To check the sensitivity of the solution to the grid used, numerical experiments are performed with different sizes of non-uniform grids. Finally, a grid size of (**10000** node) is used in generating all solutions presented in this work as shown in Fig.2. In order to verify the accuracy of the numerical code, comparisons with the previously published results are necessary. But due to the lack of availability of experimental data on the particular problems along with its associated boundary conditions investigated in this study, validation of the predictions could not be done against experiment. The present numerical code is verified against a documented numerical study. Namely, the numerical solutions reported by Chamkha (2002), which is based on finite volume scheme. However, we recall here some results obtained by our code in comparison with those reported in Chamkha (2002). The physical problem studied by Chamkha (2002) was a two-dimensional lid-driven square cavity filled with an electrically conducting fluid. The left moving upwards wall was maintained at the hot temperature (T_h) while the nonmoving vertical right wall was kept at the cold temperature T_c (aiding flow) and the two horizontal walls were under adiabatic condition. A uniform magnetic field (B_{ox}) was applied in the horizontal direction normal to the moving sidewall. Using our code the present numerical predictions have been obtained Hartmann numbers between **0** to **50** and Grashof number at 100. The comparison of the results obtained by the present

numerical code with those of Chamkha (2002) with respect to average Nusselt number (at the hot wall) are shown in Table1. The computed results are in very good agreement with the Chamkha (2002) solution which verifies the present computations indirectly.

4. Results and Discussion

The mixed convection in two sided hot and cold walls lid-driven in parallelogram cavity with effect of magnatohydrodynamic (MHD) is investigated. Also stramtraces and isotherms inside the parallelogram cavity and the average Nusselt number distrubution along the heated surface have been examined and disscused for the Richardson number, Ri, varied from 0.01 to 100, and Hartman number, Ha, varied from **0** to **75**. The working fluid is chosen as air with Prandtle number, Pr=0.71. The sidewall inclination angles (Φ) are ranging from **-60^o** to **60^o**.

4.1 Flow Structure and Thermal Field

Figure 3 shows the streamtraces for **-60^o ≤Φ≤ 60^o** at Ri= **0.01, 0.1, 1, 10** and **100** for case of without effect of magnetic field, Ha=0. For Ri=**0.01** forced convection plays a dominant role and the circulatin flow is mostly created only by moving lids. The flow field consist of main single cell with counterclockwise direction and two secondary cells are seen in the upper left and lower right corners due to clash of fluid to the two inclined sidewalls. It is noted that when the angles of inclination are positive the secondary cells are greater than that angles are nagative due to the lids moving toward acute angles, this render of fluid impingement violence in top and bottom walls and take opposite flow in direction. For Ri=**0.1**, the cavity is occupied by single main cell rotates with counter-clockwise direction and some disturbance are also observed in the upper left and lower right corners for all inclination angles except when Φ=**60^o,** a new bahaviour is appeared, a strong recirculation result in two cells due to collision of fluid in two acute angles. For Ri=**1** a new behaviour is appered too, three circulation cells are occupied the cavity, middle, right and left cells. The right and left cells induced due to shear effect while the middle cell induced due to bouyancy effect. With increasing skew angle, this new behavior is vanished and shear effect still the dominant mode. As Richardson number is increased (**10** and **100**), natural convection becomes the dominant mode, which appeared by increasing of the size middle cell and squeeze the left and right cells toward the inclined sidewalls.

The isotherms pattern is shown in Fig.4 for different values of Ri (**0.01,0.1,1,10** and **100**) and sidewall inclination angles Φ (**0^o,30^o,-30^o,60^o** and **-60^o**) for Hartman number Ha=0. At Ri=**0.01** a sigle large recirculation cell is observed, the fluid is pulled up and down due to drag force created by the motion of the lids. The low Richardson number refers to high Reynold number due to definition of Richardson number as $Ri=Gr/Re^2$. It means that the forced convection heat transfer is dominated compared to the other modes of heat transfer. With changing inclination angle the recirculation behaviour are observed with some notices at Φ=**30^o** and Φ=**60^o** where the isotherms seperated from inclined walls and clustered at the core of the cavity. With increasing of Richardson number (**0.1** to **100**) the bouyancy effect clearly observed on the isotherms patterns and the horizontal isothermal lines are observed in the core of the cavity and recirculation behaviour is vanished. This means that the conduction becomes the dominant mode.

Figure 5 depicts the streamtraces for different Richardson numbers and angles of inclination with effect of magnetic field (Ha=**25**). The presence of magnetic field has significant effect on the flow structure in the forced and mixed convection but has slight effect on the flow structure in the natural convection. For Ri=**0.01** and Φ=**0^o** the flow structure characterized by a three recirculating cells, one large primery cell rotates with counter-clockwise direction and two small secondary cells rotate with clockwise direction. The effect of magnetic field is obviously in this case when compared with the case of no magnetic field effect. When changing inclination angle to Φ=**30^o** and **-30^o** the small secondary cells merges with the primary cell and become comparatively large due to movement of the right and left walls. With varying sidewall angle of inclination to Φ=60^o, a major cell occupies the cavity generated by buoyancy force and minor cells at the top right and the bottom left corners while at Φ=-60^o the minor cells distributed horizontally up and down the main cell, this is due to impengment of fluid into acut angles. As wall inclination angles increase to 30^o, -30^o and -60^o there is no change in flow structure are noticed, only the streamtraces move to the left or to the right according to the movement of inclination angles, but for Φ=60^o the center of cell straiten along the horizontal center line of the cavity. As Richardson number increase (Ri=0.1) the single large cell noticed in the cavity rotates with counter-clockwise direction. For mixed convection (Ri=1) the cavity occupied by three cells, two small cells near the inclined lid walls and third cell bounded them. For the different angles of inclination (Φ=30^o, -30^o and -60^o), it is evident that the small side cells become high in strength and the center of it shifted to the mid horizontal line, but for Φ= -60^o the main circullar cell occupied the cavity. It can be noticed the effect of magnetic field obviously by vanishing the middle cell (represent the bouyancy effect) as compared with previouse case in Fig.3. When the Richardson increase (10 to 100), the natural convection begine to dominate gradually, which is reflected as enlarge the midle cell then squeese the side cells. The main effect of magnetic field which can observed to reduce the effect of natural convection mode. The size of natural recirculation cell (middle cell) is smallest as compared with the same Richardson number without effect of magnetic field appears espatially for Ri=10 as shown in Fig.3. With increasing angle of inclination to 30^o and -30^o there is no dramatic change observed, only the streamtraces distorted to the left or to the right with changing sidewalls angle, while for Φ=60^o and -60^o the cell at the core of the cavity is diminshed and flow structure is characterized by a large counter-clockwise direction vortex generated by movement of the left and right walls and two minor clockwise vortices generated by the bouyancy force. Furthrmore, the size of the counter-clockwise cell increases and clockwise cells decrease. This is because the magnetic field slow down the movement of bouyancy force.

The influnce of Hartmman number on the temperature field for five selected values of Ri (Richardson number) and Φ (inclination angle) is shown in Fig.6 from this figure, it is observed that the isotherms rotations are vanishe and take a wavy trend for forced and mixed convection regimes while the isothermal lines take straight trend as the natural convection appear (Ri=10 and 100). This indicating that most of heat transfer process is carried out by conduction mode. At low Richardson number (Ri=0.01 and 0.1), the effect of inclination angles that when angles are nagative values the isotherm lines clustered along the inclined walls while when angles are positive values the isothermal lines meets toward the core of the cavity. For Ri=1 the effect of inclination angles that the wavy trend becomes less in curvature and empty space is observed at the acute angles for Φ=30^o and 60^o because the isothermal lines transferse along shortest dioganal line. At high Richardson number (Ri=10 and 100) the wavy behaviour of isothrmal lines is disapeared and takes straight behaviour since the magnetic field quench the heat transfer process.

Figure 7 presents the effect of incresing Hartmman number (Ha=50) on the streamtraces, the figure show the streamtraces for different values of Ri (0.01, 0.1, 1, 10 and 100) and angles of inclination Φ (0^o, 30^o, -30^o, 60^o, -60^o). As seen from the figure for lower Ri number Ri (0.01, 0.1) and $\Phi=0^o$ the cavity contains more than one recirculation cell, the entire cells stretch horizontally due to influnce of magnetic field (direction effect of magnetic field). As changing inclination angle, the cavity occupies by one large cell with counter-clock wise direction due to movement of lids except when $\Phi= -30^o$ where the cavity containing two entire cells and one outer cell icluded them. For Ri=1 the flow structure changing dramatically at $\Phi=0^o$ the flow consist of four cells, two side cells due to shear force effect, one small cell appears at the center of the cavity due to bouyancy force and one large cell included other three cells. As th inclination anlgles varying to 30^o and -30^o the small cell at the center of the cavity disappeared, but for $\Phi= 60^o$ and -60^o one large cell occupied the cavity while othe cells are vanished. For Ri=10 and $\Phi=0^o$, four different size cells are observed in the cavity with small cell at the top center of the cavity, also it is observed that the center cell of bouyancy effect stretch longtudinally. When varying angle of inclination to $\Phi= 30^o$ and -30^o the two small cells at the center and top center disappeared while when $\Phi= 60^o$ and -60^o the entire cells merge gradually. At Ri=100 the bouyancy force appears as a large cell occupied the mid space of the cavity with two side cells due to lid force. As inclination angle changing to 30^o and -30^o the three cell inclined with inclination of side walls to the right or to the left. For $\Phi=60^o$ and -60^o, the middle cell squeezed by side cells.

Figure 8 shows the effect of Richardson number and inclination angles on the isotherrms for Ha=50. From this figure, it is seen obviously that the wavy behaviour noticed befor for Ha=25 is damped here due to the influence of magnetic field at low Richardson number except when $\Phi=60^o$, -60^o where the recirculation on the previous case becomes wavy here, this means that magnetic field reduces the convection heat transfer therefore; the isotherms become either uniform and parallel to the horizontal walls at low Richarson number Ri=0.01 and 0.1 (forced convection) or uniform and parallel to the inclined walls at middle and high Richardson number Ri=1, 10 and 100 (mixed and natural convection) for all values of inclination angles. This is because the magnetic field tend convection to retard the flow and it makes the conduction is dominant mode of heat transfer process.

Figure 9 shows the streamtraces for different values of Richardson number Ri=0.01, 0.1, 1, 10 and 100 and different angles of inclination $\Phi=0^o$, 30^o, -30^o, 60^o and -60^o at Ha=75. As seen from the left column of this figure Ri= 0.01 and 0.1 the cavity occupied by a primary circulating cell generated by the movement of the inclined walls and three secondary recirculation cells have an elliptical shape appears due to influence of magnetic field. With varying angles of inclination the three secondary cells reduces to two or one cell. As the Richardson number increase Ri=1, the effect of bouyancy force appears as a small cell located at the center of cavity. The small cell deformed with changing inclination angles and vanish when $\Phi=-60^o$. At higher Richardson number Ri= 10 and 100, the effect of bouyancy force appears as a cell at the midplane of the cavity. It is noted that the recirculating cell caused by bouyancy force and becomes smaller in size as the strength of the magnetic field increases further. On the other hand, corresponding isothermal lines is shown in Fig.10 are similar behaviour has been seen with previous case with more cluster of isothermal lines here. Also the thermal boundary layer near the hot and cold walls becomes less compressed while the isothermal lines at the center of cavity compressed due to imposed of magnetic fields. As the Richardson number increase, isothermal lines inside the cavity approaches more and more towards the conduction like distribution pattern of isothermal line. For large Richardson number, convection is almost supressed and the isotherms are almost parallel to the inclined wall, indicating that quasi conduction regime is reached.

The variation of the average Nusselt number at the hot wall in the cavity against Richardson number for different angles of inclination are indicated in Figs.11(a, b, c and d) for Hartmann number (0, 25, 50 and 75) respectively. It is observed that average Nusselt number decreases slightly with increasing the Richardson number up to Ri=1 (pure mixed convection), but beyond this value the average Nusselt number decreases slowly with increasing the Richardson number for various values of Hartmann number, except for the cases when Ha=0, $\Phi=30^o$ and Ha=25, $\Phi=60^o$ takes different trend where the average Nusselt number increases slightly with increasing Richardson number up to Ri=0.1 then decreasing slightly up to Ri=1, beyond this value same trend apears with the other cases. Also in the case of Ha=0 and $\Phi=30^o$ has minimum value of Nusselt number, since the shear force equall to the bouyancy force and have opposite effect. As the Hartmann number increase Ha=50, 75 this irregular bahaviour is damped. Also it is observed that for high Richardson number 10<Ri<100, the lines of average Nusselt number match especialy when effect of magnetic field increased because magnetic field put out the heat transfer at the natural convection mode. Fnally, from these figures it is obviously that the average Nusselt number have highest values when forced convection dominated (Ri=0.01 and 0.1) and the average Nusselt number have lowest values when natural convection domiated (Ri=100). It is noticed that the irregular trend of Nusselet number appears at Ri=0.01 for $\Phi=30^o$ at Ha=0 and $\Phi=60^o$ at Ha=25 due to the seperation of thermal boundary layer from the inclined moving heating sidewalls but with increasing of Richardson number Ri=0.1 the thermal boundary layer created along the inclined sidewalls.

5. Conclusions

Mixed convection in two-sided lid driven parallelogram cavity with a magnetic field effect is very charming to enhance or reduce the heat transfer process. The problem of laminar combined convection flow and heat transfer of conducting fluid (air with Pr= 0.71) in two-sided lid-driven parallelogram cavity in the presence of horizontal magnetic field was formulated numerically. The governing parameters that effect are Richardson number, Hartmann number and sidewalls inclination angle. The following conclusions may be drawn from the present study:

► In the forced convection regime at (Ri = 0.01) for (Ha = 0), the maximum heat transfer occurs at the square cavity ($\Phi = 0^o$), while the minimum heat transfer occurs at the parallelogrammic cavity at the positive angle ($\Phi = +30^o$). And at (Ri = 0. 1) for (Ha =0), the maximum heat transfer occurs at the angles ($\Phi = 0^o$ and $+30^o$), while the minimum heat transfer occurs at the parallelogrammic cavity at the positive angle ($\Phi = +60^o$).

► At mixed convection regime at (Ri =1) and (Ha = 0), the negative angles ($\Phi = -30^o$ and -60^o) minimizes the natural convection effect and precipitates in merging the three cells into one.

► In the natural convection regime at (Ri =10) for (Ha = 0), the maximum heat transfer occurs in the square cavity ($\Phi = 0^o$) and in the parallelogrammic cavity at ($\Phi = - 60^o$). And at (Ri =100) for (Ha =0), the maximum heat transfer occurs in the square cavity ($\Phi = 0^o$).

► In the absence of magnetic field, the isothermal line have recirculation behavior at low Richardson number (Ri=0.01) for all inclination angles, with increasing Richardson number the wavy trend appears instead of recirculation behavior, except when Ri=0.01, Φ=30^o and 60^o where the isothermal lines separated from the inclined sidewalls and clustered at the core of the enclosure.

►Magnetic field has significant effect on the flow structure for three regimes (forced, mixed and natural convection), for forced convection Ri=0.01 and 0.1 the number of vortices increased and stretch horizontally with increasing Hartmann number. For mixed convection Ri=1, the vortices changing in shape and size while for natural convection Ri=10 and 100 the middle vortex becomes small in size with increasing strength of magnetic field.

►The thermal boundary layer is decreased with increasing Richardson number with and without magnetic field, therefore the average Nusselt number decreases with increasing Richardson number for all values of inclination angles except when Ri=0.1 and Φ=30^o and Φ=60^o.

►The average Nusselt Number has maximum and different values at lower Richardson number and minimum and same values at higher Richardson number especially at higher Hartmann number.

►The magnetic field has different influence on heat transfer for three regimes, generally increase Hartmann number decreases heat transfer except for the case of Φ=60^o , Ha=50 and Ri=0.01, where it has maximum average Nusselt number because the magnetic field effect on the temperature distribution and thermal boundary layer near the inclined heated moving sidewalls.

► Inclination angle has a significant opposite influence on heat transfer processes inside the cavity. Whereas, when Φ=0^o the cavity has maximum Nu number at lower Ha number and lower Nu number at maximum Ha number.

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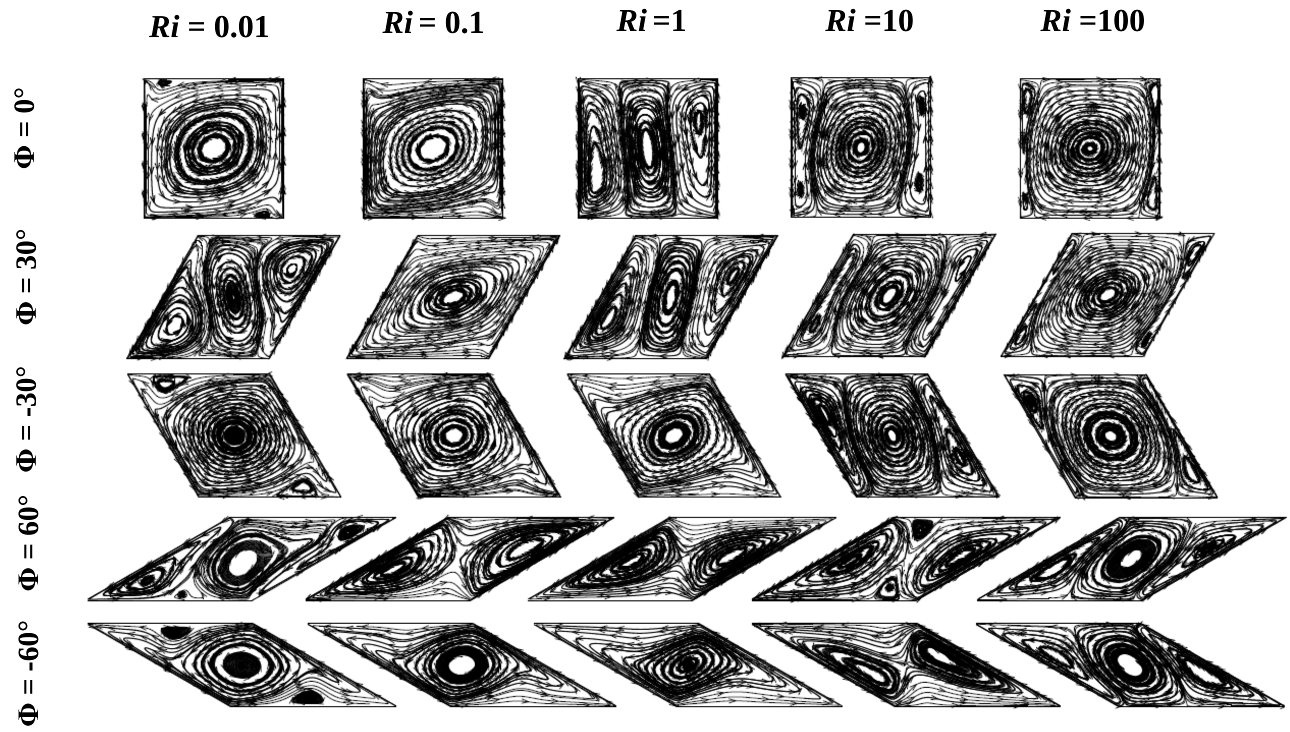

Fig. 3. Variation of streamtraces for different Ri and Φ with Ha= 0

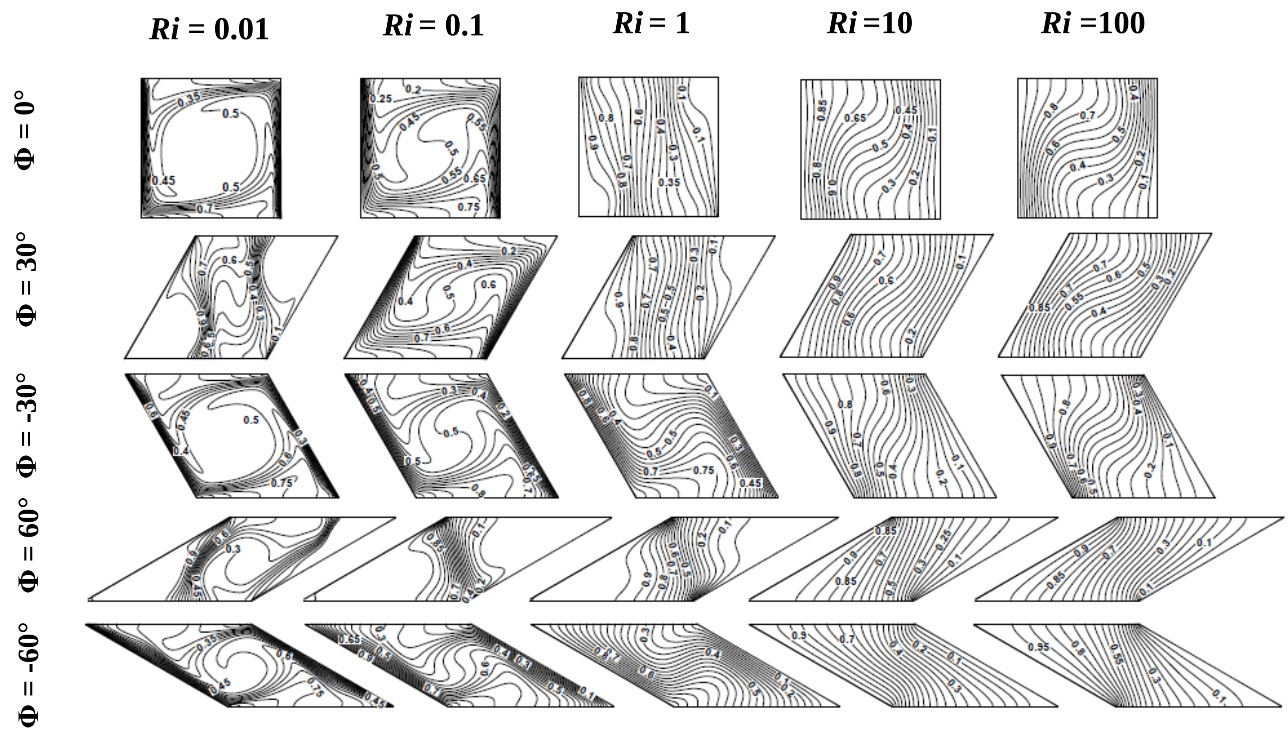

Fig. 4. Variation of isotherms for different Ri and Φ with Ha= 0

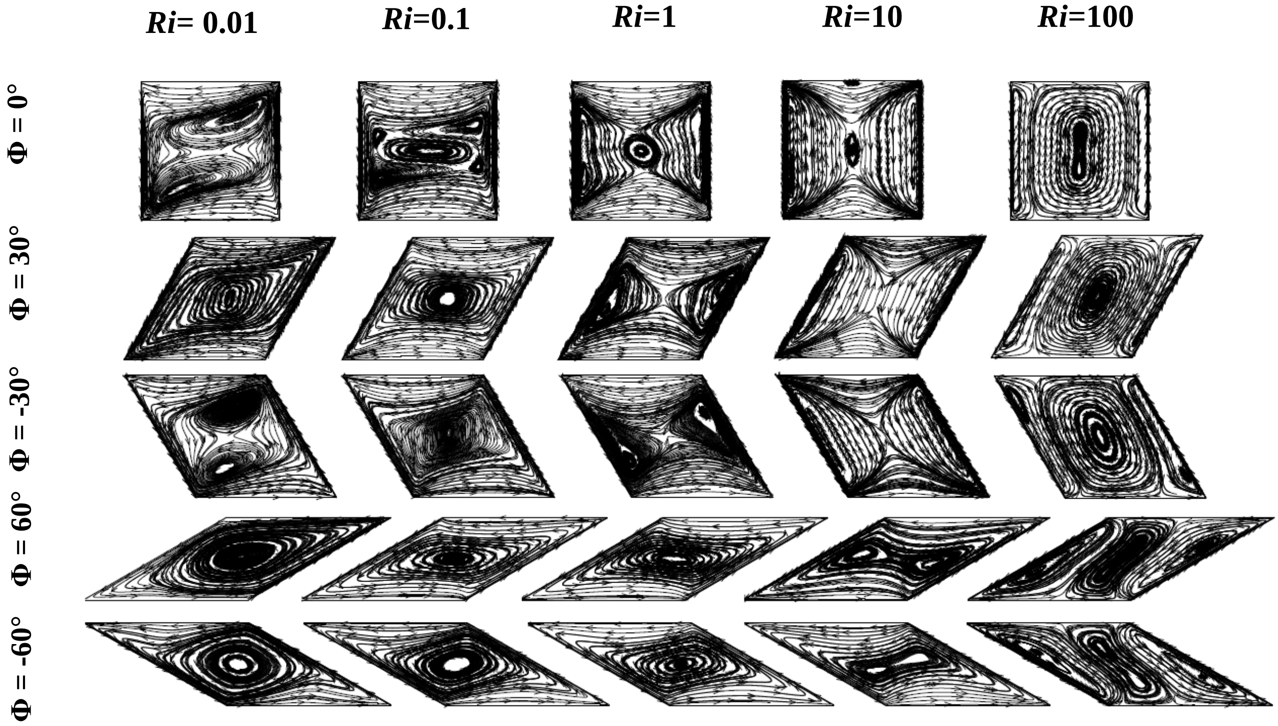

Fig. 7. Variation of streamtraces for different Ri and Φ with Ha=50

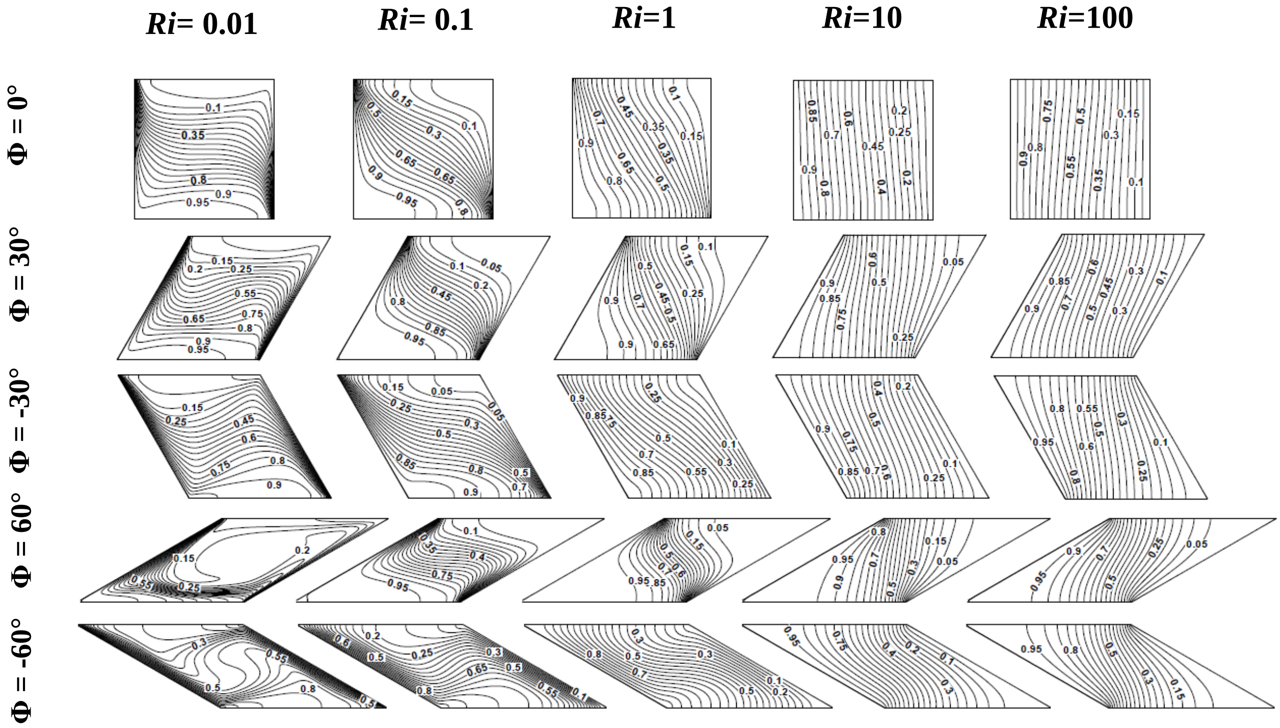

Fig. 8. Variation of isotherms for different Ri and Φ with Ha=50

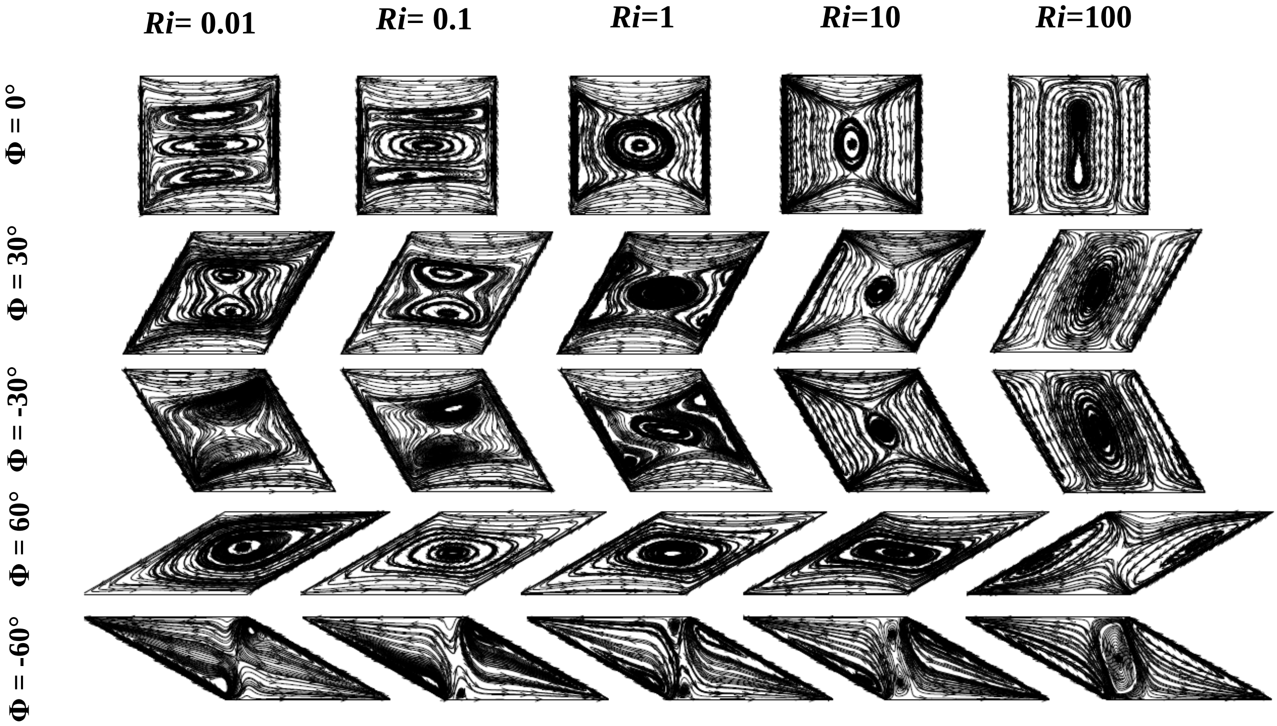

Fig. 9. Variation of streamtraces for different Ri and Φ with Ha=75

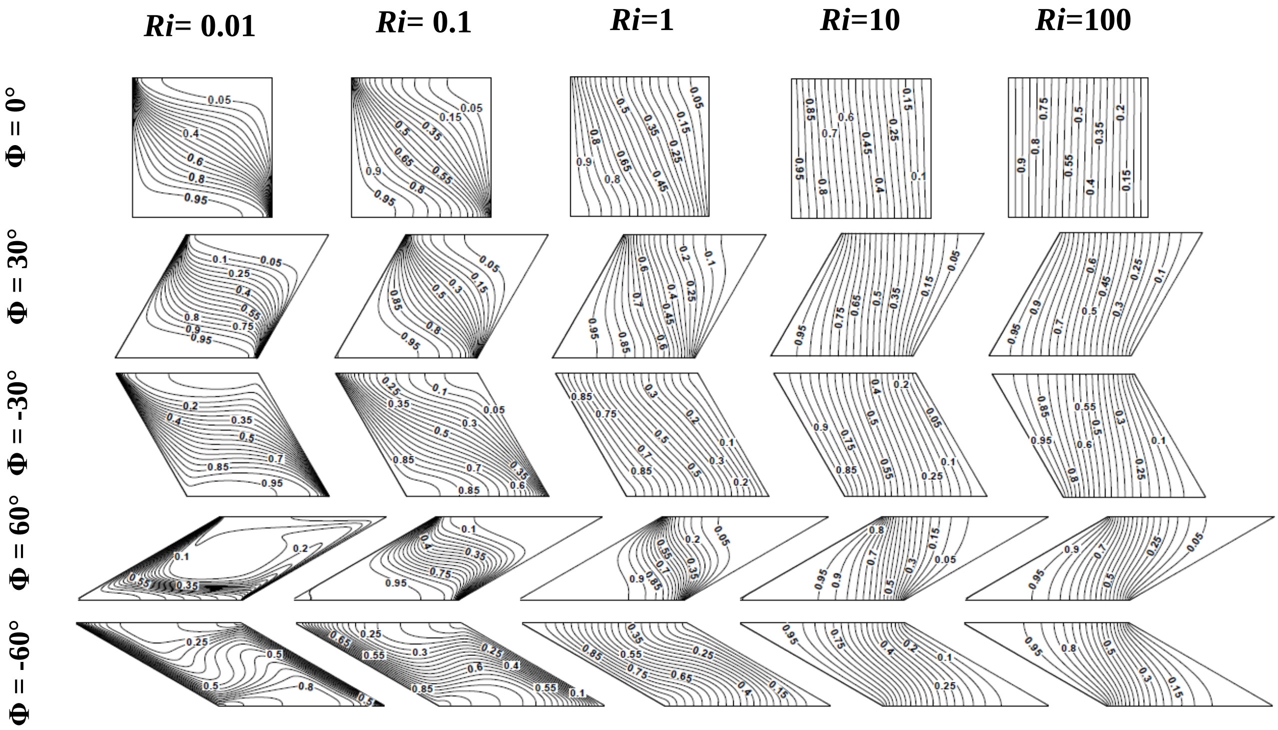

Fig. 10. Variation of isotherms for different Ri and Φ with Ha=75

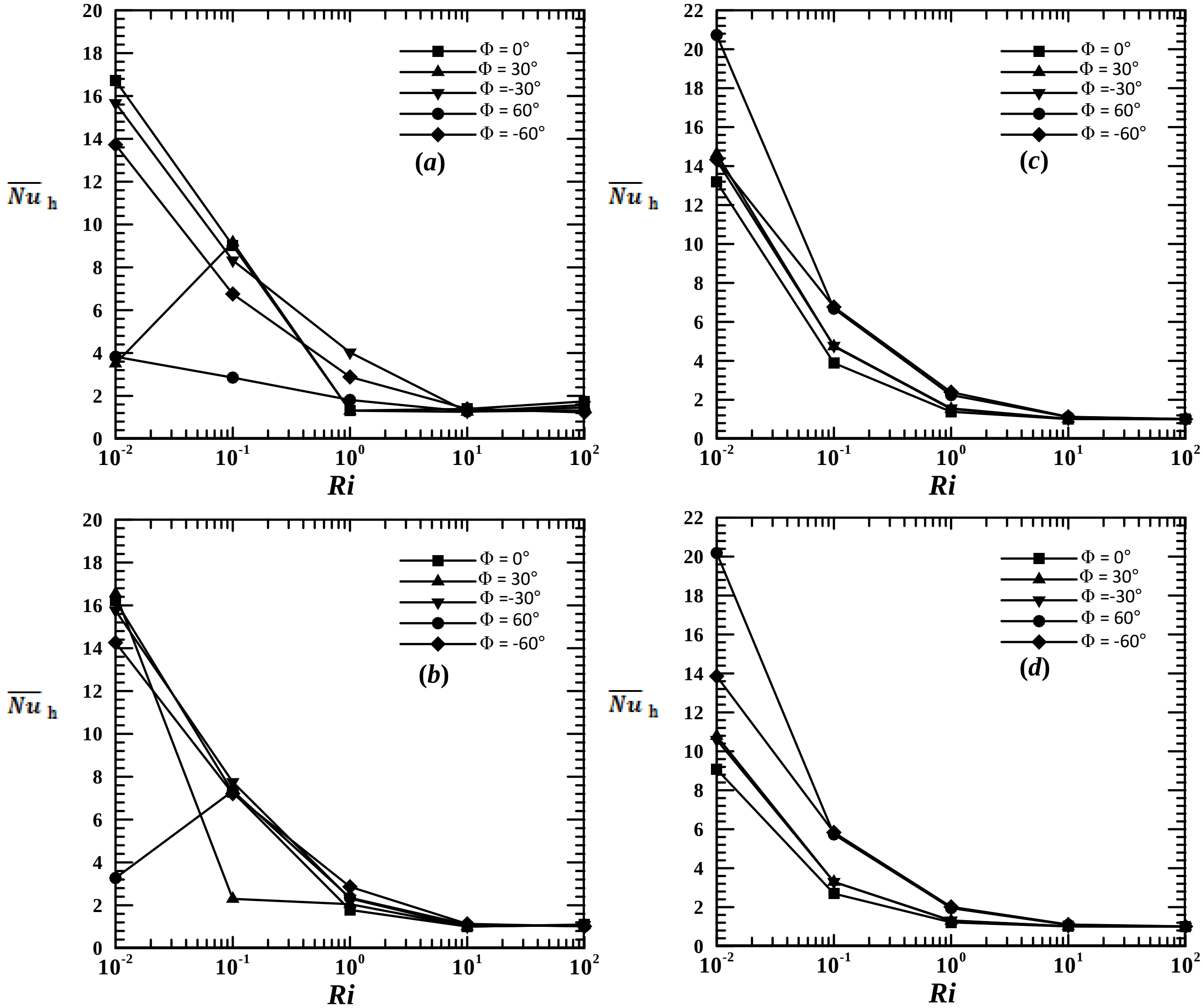

Fig.11 Average Nusselt number along the left heated wall for different skew angles as a function of Richardson number at (***a***) Ha= 0 , (***b***) Ha= 25, (***c***) Ha= 50 and (***d***) Ha= 75.