Cantor Fractal Linear Antenna Array with Koch Fractal Elements

Assist. Prof. Refat Talib Hussein (Ph.D.)* Eng. Mohannad Ahmad*

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Abstract

Cantor fractal linear array, with '101' generator from second stage of growth, is analyzed here with two types of current amplitude excitation coefficients (Dolph and Fractal). The Koch fractal dipole element with 2^{nd} iteration and $\theta=60^o$ will be used here as the array elements. Kaiser-Bessel window will be used as the generating function, to calculate the fractal amplitude excitation coefficients. The benefit from using fractal antenna element in the design of fractal antenna array will be clearly deduced from the results. The radiation pattern and impedance were calculated by using software package MATLAB version 7.6 (R2008a) and software package 4NEC2 respectively.

Keywords: Fractal antenna array, Fractal antenna element, fractal amplitude excitation coefficients, 2^{nd} iteration Koch dipole fractal element.

الخلاصة

صَفّ صورةِ نمطي هندسي متكرر Cantor الخطيّ، بمولّد 101 مِنْ المرحلةِ الثانيةِ مِنْ النمو، مُحَلَّلُ هنا بإثنان مِنْ أنواعِ معاملاتِ إثارةِ الغزارةِ الحاليةِ (دولف وصورة النمطي هندسي متكرر).صورة نمطي هندسي متكرر عنصرِ Koch dipole التكرارِ الثانيِ و $\theta = 60^\circ$ سَيَكُونُ مستعمل هنا كعناصر الصَفّ. نافذة قيصر Bessel سَتَكُونُ مستعملة كوظيفة التَوليد، لحِساب معاملاتِ إثارةِ غزارةِ صورةَ النمطي هندسي متكرر.المنفعة مِنْ إستعمال عنصرِ هوائي صورةِ النمطي هندسي متكرر في تصميمِ صَفّ هوائي صورةِ النمطي هندسي متكرر سَتَستنتجُ بشكل واضح من النَتائِجِ. ظ الإشعاعَ والمعاوقة الكهربائيةَ حُسِبا بإستعمال مجموعةِ برامج MATLAB نسخة 7.6 (R2008a) ومجموعة برامج 4NEC2 على التوالي.

*Electrical and Electronic Engineering Department, University of Technology, Baghdad-IRAQ

1. Introduction

The objective of fractal antenna synthesis is to obtain a multiband behavior in which the radiation characteristics are held constant at several frequency bands [1]. A comprehensive overview of recent developments in the field of fractal antenna engineering with particular emphasis placed on the theory and design of fractal arrays are found in [2]. An efficient recursive procedure for evaluating the impedance matrix of linear and planar fractal arrays is exploited in [3]. In this paper, Cantor linear fractal antenna array with 101 generator and second stage of growth will be simulated with second iteration Koch fractal elements. The system has a total of 4 active elements. Uniform, Dolph and fractal current amplitude excitation coefficients will be used to feed the antenna array elements. The design frequency is chosen to be 2250MHz and minimum spacing between elements is assumed to be $d = \lambda o/4$. Finally, the driving point impedance for the array elements for all current amplitude excitations stated above and different resonant frequencies will be introduced.

2. Cantor fractal antenna array theory

Cantor fractal array can be formed through the repetitive application of a generating sub-array.

A generating sub-array is a small array at scale one (P=1) used to construct larger arrays at higher scales (P>1) [1]. The array factor can be expressed in the general form [2].

$$AF_p(\Psi) = \prod_{p=1}^{p} GA(\delta^{p-1}\Psi) \quad (1)$$

Where

$AFp(\Psi)$ = Array factor associated with the generating sub-array.

δ = Scaling (Expansion) factor.

$\Psi = k\, d\, cos(\theta)$.

The generating sub-array in our case has three uniformly spaced elements with the center element removed, i.e. 101. The array factor with this representation is

$$GA(\Psi) = 2cos(\Psi) \quad (2)$$

Substituting (2) in (1) and choosing δ=3 results in the following expression for Cantor fractal linear array

$$AF_p(\Psi) = \prod_{p-1}^{p} GA(3^{p-1}\Psi) \quad (3)$$

If the array elements were point sources, the array will operate at n resonant frequencies according to the following equation

$$f_n = f_o / \delta^n \quad (4)$$

Where n is the band or iteration number (n=0, 1, 2…)

3. Koch curve fractal antenna

The Koch curve fractal geometries were originally introduced by the Swedish mathematician Helg Von Koch in 1904 [4]. It is among the first antennas based on a fractal geometry designed as small sized antennas with multiband characteristics [5]. The Koch curve is generated by starting with a straight line ‘initiator’ with an indentation angle of θ=0°.Then, the straight line is divided into three equal parts and the middle part is replaced by two linear segments with the same length at angle

$\theta=\theta_o$ 'generator'. The process is repeated for these four line segments until the required curve is obtained, as shown in figure (1). This will produces the other iterations of standard Koch curve.
The length of the nth iteration of the Koch dipole $L_{\theta,n}$ with indentation angle θ is given by

$$L_{\theta,n} = \left(\frac{2}{1+\cos\theta}\right)^n L_o \quad (5)$$

With L_0 is the length of the linear dipole (initiator) which remains constant through the iterations with the same end-to-end length. $L_{\theta,n}$s is increased with each iteration , and called the total curve length.

Fractal dimension, another important fractal property, is a measure that can only be applied to fractal objects. It can be calculated by applying the following equation [6].

$$FD = \frac{\log(N)}{\log(1/r)} \quad (6)$$

r is related to the indentation angle by the following relation [7].

$$r = 2(1+\cos\theta) \quad (7)$$

Since there are four identical copies (N =4) of the original geometry that are scaled down by a factor of 3 (r =3), then

$$FD = \frac{\log(4)}{\log(3)} = 1.26186 \quad (8)$$

4. Fractal amplitude distribution

Fractal amplitude distribution was originally used to obtain a self-similar fractal radiation with Koch and Weierstrass fractal arrays [8]. A unified approach to the design of multiband arrays via the synthesis of fractal radiation patterns was introduced [9].
Here, it is assumed that the Cantor fractal array is composed of two uniformly spaced sub-arrays with I_{pn} feeding currents at each sub-array. Then, the contribution of these currents will be added to obtain the final fractal amplitude excitation coefficients. I_{pn} can be calculated as follows

$$I_{pn} = \left(\frac{1}{\gamma\delta}\right)^{P-1} I_n \quad (9)$$

Where

$$I_n = \left(\frac{d}{\lambda}\right)\int_{-\frac{\lambda}{2d}}^{\frac{\lambda}{2d}} f(\omega)e^{-j2\pi n(\frac{d}{\lambda})\omega}\, d\omega \quad (10)$$

f (ω) is the desired generating function (Kaiser-Bessel window in this case), P is the number of stages used in the construction of the Cantor fractal antenna array, δ is the scale factor, and γ is the current amplitude parameter. Kaiser-Bessel window function can be expressed as follows [5]

$$f(\omega) = \begin{cases} \dfrac{I_o\left(\pi\alpha\sqrt{1-(\frac{2\omega}{\Delta})^2}\right)}{I_o(\pi\alpha)} & -\dfrac{\Delta}{2} \le \omega \le \dfrac{\Delta}{2} \\ 0 & \text{Otherwise} \end{cases} \quad (11)$$

Where

Δ = first-null beam width.

$I_o(\chi)$ = modified Bessel function of the first kind of order zero and argument χ.

$\propto$ = Kaiser independent factor.

An optimization is made to choose these design parameters as follows (δ=3, γ=0.9, Δ=1.4, α=50).

Substituting (11) in (10) yields the following equation for the fractal amplitude distribution with Kaiser-Bessel generating function.

$$I_n \approx \frac{kd}{2\pi}\frac{\Delta}{I_o(\pi\alpha)}\frac{\sinh(\sqrt{(\pi\alpha)^2-(\frac{2nkd}{\Delta})^2})}{\sqrt{(\pi\alpha)^2-(\frac{2nkd}{\Delta})^2}} \quad (12)$$

So,

$$I_{pn} = [I_1,\ I_2+I_1/(\delta * \gamma),\ I_2/(\delta * \gamma)+I_1/(\delta * \gamma)^2,\ I_2/(\delta * \gamma)^2]$$

5. Design of Koch curve dipole fractal wire antenna

Second iteration dipole Koch fractal antenna with 60° indentation (rotation) angle will be designed around a design frequency of 750MHz with feed point located at the center of the geometry shown in figure (2).

From the method of moment, the 4NEC2 software package will be able to compute real and imaginary parts of the input impedance. The resulting input impedances are listed in table (1).

Since the designed Koch fractal antenna is of second order, it has only two resonant frequencies (545 MHz, 1580 MHz), where at these frequencies the input impedance is approximately (50+j0) Ω.

6. Proposed model design

Second iteration (P=2) Cantor fractal linear antenna array with 4 active second order Koch fractal elements will be designed $at\ 3 \times f_{element}$= 2250MHz design frequency. The array is symmetrically fed with uniform and non-uniform current amplitude feeding coefficients (Dolph and Fractal). The total array length (L) is $2\lambda_o$ (26.6667cm) as shown in figure (3).

The input impedance is calculated for each array element for the purpose of impedance matching using MoM based antenna software package (4NEC2 software) with uniform and non-uniform current amplitude excitation coefficients at each resonant frequency and the results are shown in table (2).

The normalized array factor plots for uniform, Dolph and fractal current amplitude excitation coefficients are calculated from equation (3) with different resonant frequencies resulting from 4NEC2 simulation as shown in figure (4).

The results of the radiation properties (D, SLL and HPBW) for the proposed model are listed in table (3).

7. Conclusions

From the above analysis the following points can be concluded:

1. The number of resulting resonant frequencies at all models is greater than the number of resonant frequencies of the fractal element and fractal array together, where some of these resonant frequencies are close to each other.

2. Multiple groups of resonant frequencies appear with each current amplitude feeding coefficients type, where the radiation pattern and input impedance are approximately the same at each group of frequencies.
3. The best field pattern is obtained with all current amplitude feeding coefficient types when the resonant frequencies are around the element resonant frequencies and equal approximately (4.8 to 6) times the first resonant frequency of the element.
4. The proposed model with Uniform and Dolph current amplitude feeding coefficients is the best design model, since it has a large number of frequency groups that are matched between radiation pattern and input impedance.
5. SLL is reduced when the resonant frequencies are less than the array design frequency since the distance between array elements will be less than one wavelength. The SLL is increased when the resonant frequency becomes higher than the array design frequency since the distance between array elements will be higher than one wavelength.

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Table (1): Resonant frequencies and their impedances

Resonant frequency (MHz)	Impedance (Ω)
545	**43-j0.94**
954	1555-j3.58
1580	**61+j1.82**

Table (2a): Input impedance for array elements with **Uniform** amplitude feeding coefficients

Array Resonant Frequency (MHz)	Impedance (Ω)	
	$Z1_{Driving}$	$Z2_{Driving}$
574	94.94-j0.4	135.56-j8.87
590	100.6+j12.61	145.6-j0.03
1540	142.43+j0.22	111.4-j8.46
1556	157.7+j6.17	121.87+j0.41
1618	228.66+j0.56	171.84+j27.9
1720	240.95-j97.76	339.29-j0.89
2452	114.06-j10.34	125.3+j0
2476	124.07+j0.05	139.48+j8.64
2592	232.52+j21.1	241.31-j0.51
2622	259.13+j0.95	267.36-j26.45
3186	58.48-j0	59.09-j15.19
3216	66.69+j17.46	64.44-j0.07
3630	279.18-j0.07	271.68+j74.49
3790	289.91-j26.28	375.29+j0.79

Table (2b): Input impedance for array elements with **Dolph** amplitude feeding coefficients

Array Resonant Frequency (MHz)	Impedance (Ω)	
	$Z1_{Driving}$	$Z2_{Driving}$
572	92.71-j0.78	135.93-j11.3
592	147.29-j1.47	148.59-j0.43
1536	138.18-j0.13	109.67-j1.25
1558	159.08+j8.58	123.91-j0.42
1624	236.09+j0.15	177.01+j26.43
1720	248.43-j98.76	331.59+j0.16
2454	116.73-j8.74	124.45-j0.21
2474	125.34-j0.29	135.95+j7.44
2594	227.31+j17.95	241.91+j0.55
2620	257.49+j0.59	265.38-j21.53
3188	58.84+j0.46	59.45-j13.4
3214	63.8-j1.28	64.23-j0.27
3632	278.3+j0.28	273.89+j73.12
3790	290.14-j25.59	375.03+j0.08

Table (2c): Input impedance for array elements with **Fractal** amplitude feeding coefficients

Array Resonant Frequency (MHz)	Impedance (Ω)	
	$Z1_{Driving}$	$Z2_{Driving}$
571	119.91-j1.81	121.02+j0.5
593	119.72-j0.54	133.91+j13.37
1534	145-17.91	101.78-j0.22
1718	160-j90.39	397.91-j2.07
2484	102.78-j0.14	163.97+j13.84
2644	282.58+j1.33	279.59-j72
2562	166.17+j45.68	229.66-j0.08
3229	73.18+j33.98	65.38-j0.3
3604	289.98-j0.05	245.93+j85.33
3798	289.06-j33.51	381.42-j0.34

Table (3a): Radiation properties for the proposed model with **Uniform** amplitude feeding coefficients

Array Resonant Frequency (MHz)	Radiation Property		
	D (dB)	SLL (dB)	HPBW (deg.)
574	3.1700	-∞	56.5627
590	3.2507	-∞	55.3118
1540	3.7179	-0.5661	24.8470
1556	6.0727	-5.0938	19.0778
1618	3.4223	0	22.8328
1720	5.0725	0	13.5624
2452	8.6091	-11.8828	13.772
2476	8.7273	-13.0690	13.7390
2592	9.0645	-19.2355	13.4838
2622	9.1176	-20.2110	13.3834
3186	9.4884	-11.7106	10.8653
3216	9.8032	-13.9707	11.4974
3630	7.7823	0	--------
3790	7.3362	0	13.6680

Table (3b): Radiation properties for the proposed model with **Dolph** amplitude feeding coefficients

Array Resonant Frequency (MHz)	Radiation Property		
	D (dB)	SLL (dB)	HPBW (deg.)
572	3.1628	-∞	56.6704
592	3.2372	-∞	55.5138
1536	6.3583	-6.4333	19.7250
1558	6.0618	-5.0471	19.0596
1624	4.4217	-0.9383	16.9476
1720	5.065	0	13.5844
2454	8.6081	-12.0134	13.8118
2474	8.7059	-12.9989	13.7848
2594	9.0546	-19.3718	13.5124
2620	9.1005	-20.4364	13.4342
3188	9.4692	-11.6539	10.8886
3214	9.3677	-10.3941	10.7324
3632	7.7396	0	--------
3790	7.2915	0	13.8826

Table (3c): Radiation properties for the proposed model with **Fractal** amplitude feeding coefficients

Array Resonant Frequency (MHz)	Radiation Property		
	D (dB)	SLL (dB)	HPBW (deg.)
571	3.2579	-∞	55.2378
593	3.3882	-∞	53.7608
1534	4.3253	-2.3313	22.2140
1718	5.2345	0	13.4746
2484	8.8356	-11.6240	13.4924
2644	9.061	-14.8512	13.3100
2562	9.2078	-17.5557	13.0750
3229	9.3702	-9.2185	10.3084
3604	7.9958	0	-------
3798	7.5678	0	12.3038

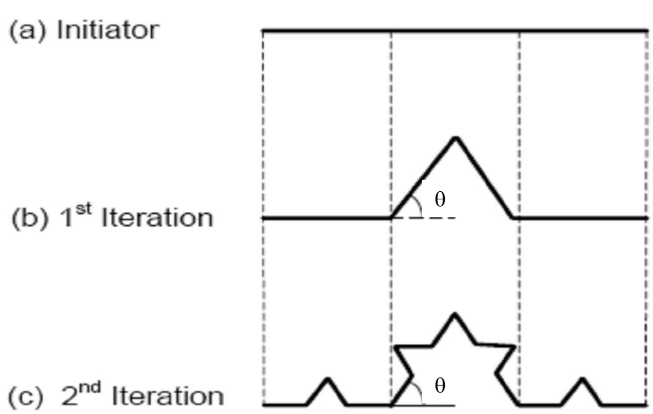
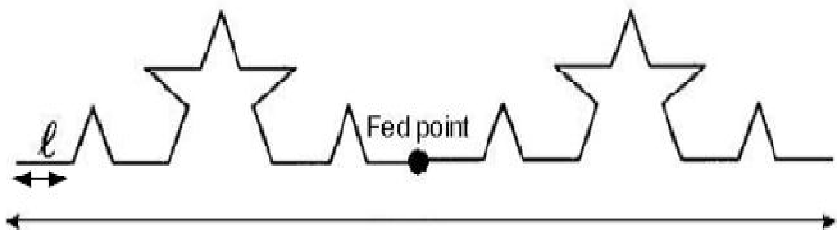


Figure (1): Koch curve geometry construction



L_i=20cm
Figure (2): 2nd iteration Koch curve dipole antenna

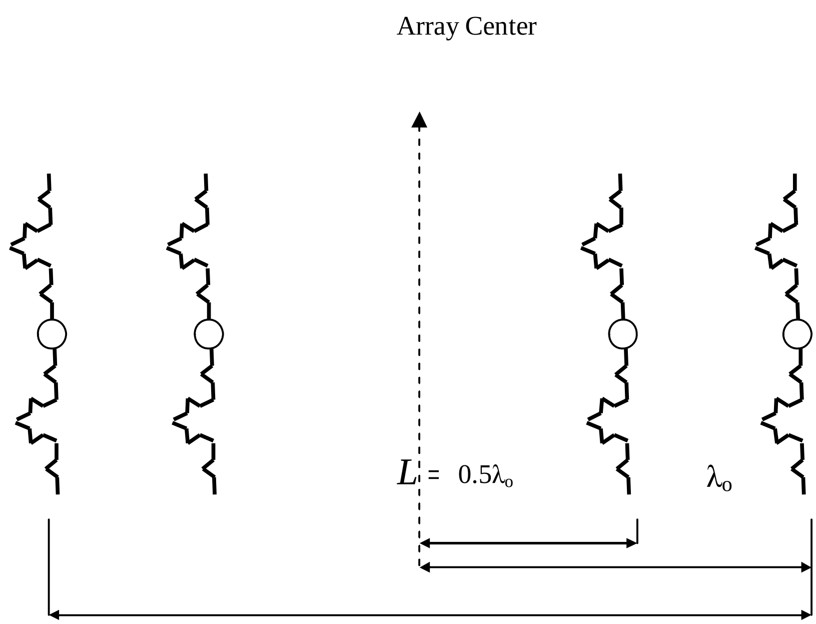
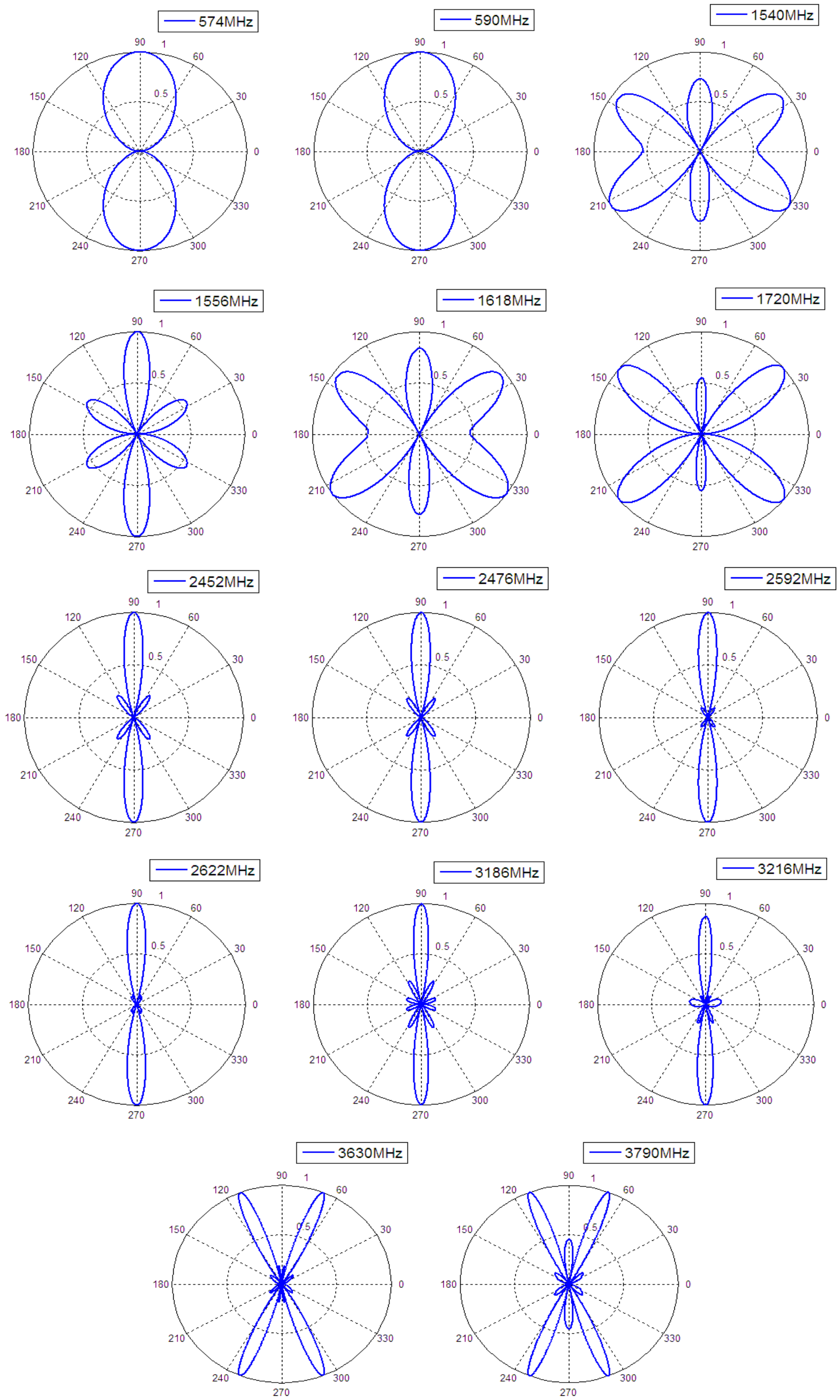
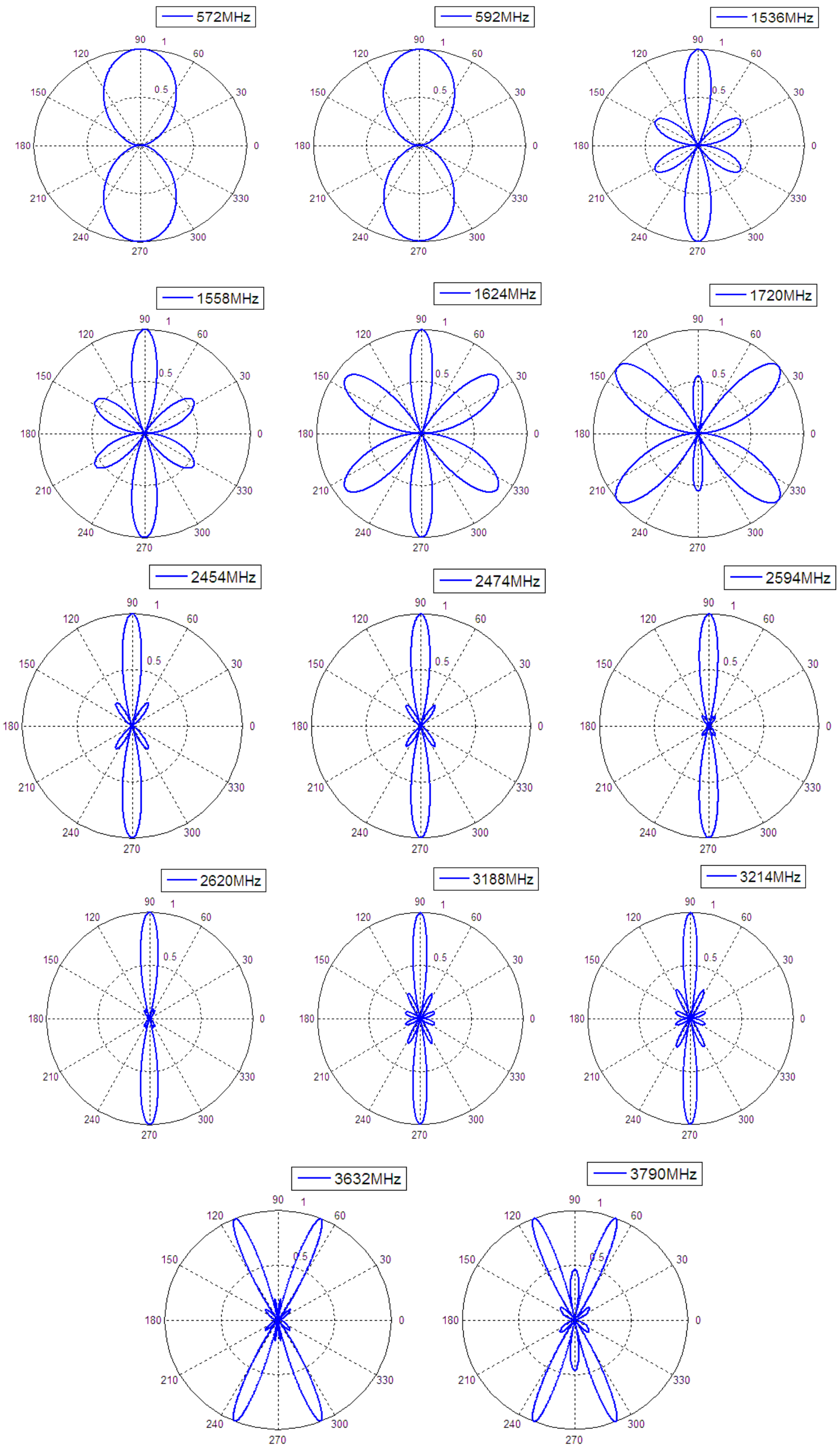


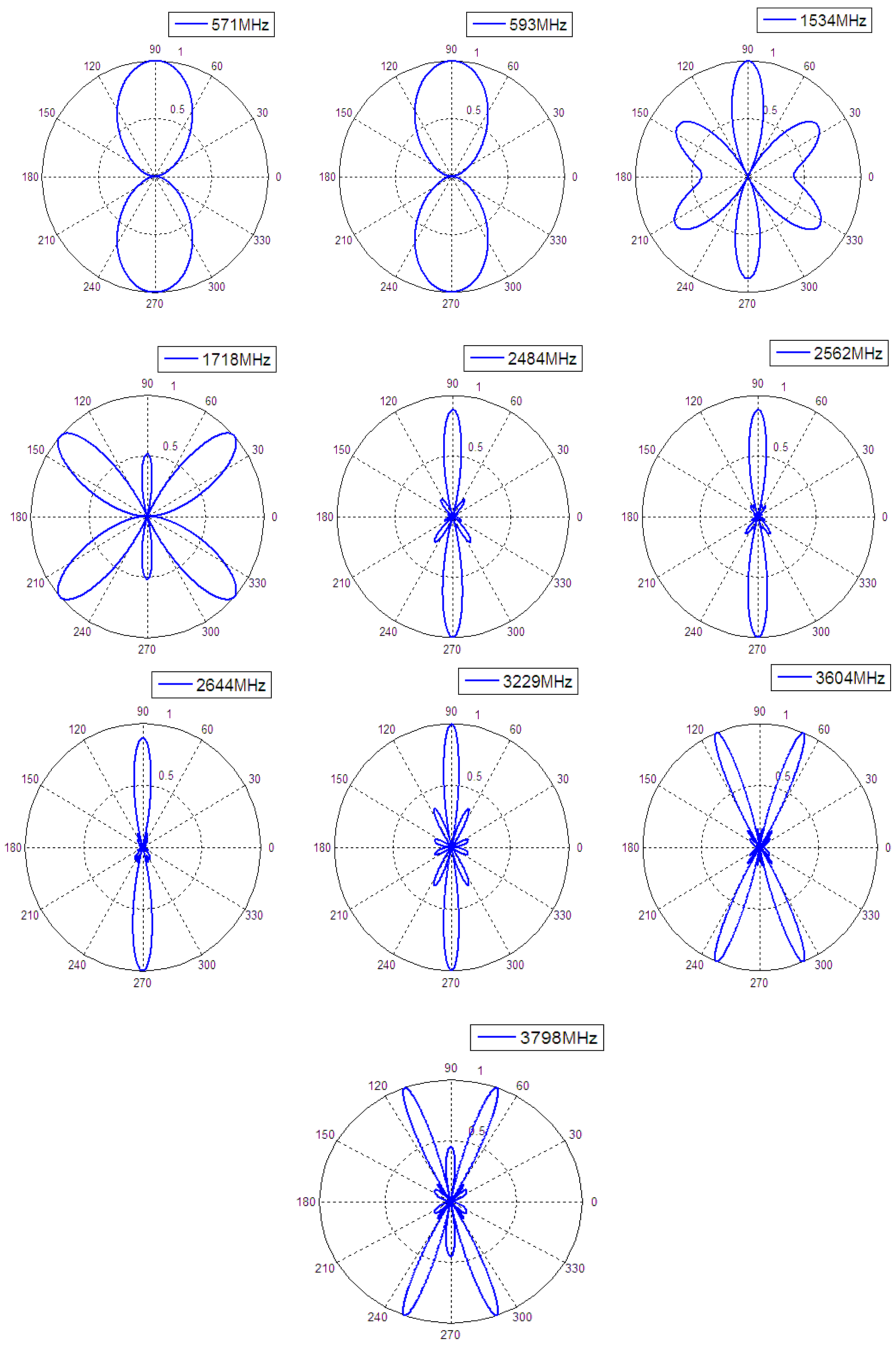
Figure (3): Cantor fractal linear antenna array with Koch fractal elements



(a) **Uniform** amplitude feeding coefficients



(b) **Dolph** amplitude feeding coefficients



(c) **Fractal** amplitude feeding coefficients
Figure (4): Array factor plots for proposed model