

A Combined H_2 /Sliding Mode Controller Design for a TORA System

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Received: 14-May-2018

Revised: 10-Oct.-2018

Accepted: 14-Nov.-2018

<http://doi.org/10.29194/NJES.21040501>

Abstract

In this work, the control of Translational Oscillations with a Rotational Actuator (TORA) system is presented in this paper. The optimal sliding mode controller is proposed to control the two DOF underactuated mechanical system. The nonlinear coupling from the rotational to the translational motion is the main problem that faces the controller design. The H_2 sliding mode controller is designed to give a better performance if only sliding mode control is used. The results illustrate that the proposed H_2 sliding mode controller can achieve the stabilization of the system with the variation in system parameters and disturbance.

Keywords: TORA System, Robust Control, Sliding Mode Control, H_2 Statefeedback, Optimal Control.

1 Introduction

The Translational Oscillations with a Rotational Actuator (TORA) or Rotational/Translational Actuator (RTAC) system is one of the nonlinear underactuated benchmark systems. TORA system was proposed in the early 1990s, Bupp *et al* [1]. TORA consists of the motion control of a cart possessing one translational degree of freedom, which is actuated by an eccentric rotational mass actuator mounted on the cart. TORA system is one of the complex control problems of a dual-spin aircraft [2]. The interaction between spin and nutation represents the complicated part in the algorithm design task for the control system engineers [3]. However, it is a quite difficult task to prove the versatility of a control algorithm on an actual dual-spin aircraft, and it may even lead to a catastrophic system failure.

Recently, many nonlinear controller design methods applied to stabilize the TORA system equation. Hung *et al* [4] in 2007 designed self-tuning fuzzy sliding mode control for TORA system using the decoupled method. Morillo *et al* [5] in 2008 applied interconnection and damping assignment control to stabilize the TORA system using passivity based control approach. Li *et al* [3] in 2009 proposed a design of Single Input Rule

Modules (SIRMs) based Type-2 Fuzzy Logic controller for TORA system and GA was used for all parameter tuning to improve the control performance. Chen *et al* [6] in 2010 presents a smooth switching control for TORA system via LMI in a linear parameter varying. Lee and Chang [1] in 2012 implement a wavelet-based neural network to develop an adaptive backstepping controller design for TORA system. Chang *et al* [7] in 2016 presented a hybrid algorithm to control TORA system. Sliding mode controller is combined with a fuzzy controller to achieve robust performance. The result compared with a previous work to show the effectiveness of the proposed controller but the settling time still too long. Lin and Chang [8] in 2017 developed a Takagi-Sugeno fuzzy model-based for controlling TORA system. They used two rules and triangular memberships. The results show the presence of high oscillation in the states time responses.

In this work, the model of the TORA system is developed by Euler Lagrange equation of motion methods. The design of H_2 sliding mode controller is presented to achieve an optimal performance in presence of system parameter variation and disturbance.

2 System Mathematical Model

TORA system schematic representation is shown in Figure 1. This system consists of a rotating arm with mass (m) which added to a rotary disk of inertia (I). This arm is riding on a cart constrained to move horizontally. A spring of stiffness (k) is used to attach the moving cart to a wall. M represents the total mass of the disk and the cart and e denote the distance between the center of the disk and the unbalanced mass. In the horizontal plane the cart and pendulum will move, where x_c and \dot{x}_c denote the normalized translational position and velocity of the cart, respectively, θ and $\dot{\theta}$ denote the angular position and velocity of the rotational actuator where $\theta = 0$ corresponds to the 90-degree rotation from the spring axis as shown in Figure 1. The cart is perturbed by a translational disturbance force F . The pendulum is actuated by the control torque N [2].

Depending on the generalized coordinates $[x_c, \dot{x}_c, \theta, \dot{\theta}]^T$ and using Newton 2nd law of

motion, the equations of motion for the ideal TORA system can be expressed by:

$$(M + m)\ddot{x}_c + me(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + kx_c = F \quad (1)$$

$$(I + me^2)\ddot{\theta} + m\ddot{x}_c e \cos\theta = N \quad (2)$$

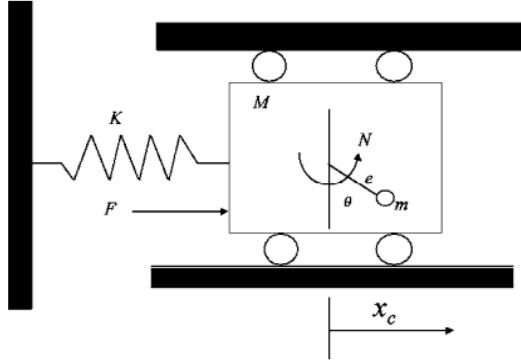


Figure 1: The TORA system schematic structure [3].

Then by introducing the normalized variables:

$$z_1 \triangleq \sqrt{\frac{M+m}{I+me^2}} x_c, \quad \varepsilon \triangleq \frac{me}{\sqrt{(I+me^2)(M+m)}}$$

$$u \triangleq \frac{(M+m)}{k(I+me^2)} N, \quad \tau \triangleq \sqrt{\frac{k}{M+m}} t \quad \text{and} \quad F_d \triangleq \frac{1}{k} \sqrt{\frac{M+m}{I+me^2}} F$$

and rewriting equation (1) and (2) using the normalized variables yields:

$$\dot{z}_1 + z_1 = \varepsilon(\dot{\theta}^2 \sin\theta - \ddot{\theta} \cos\theta) + F_d \quad (3)$$

$$\dot{\theta} = u - \varepsilon \dot{z}_1 \cos\theta \quad (4)$$

where the differentiation is done with respect to τ . By defining:

$$X = (x_1, x_2, x_3, x_4)^T = (z_1, \dot{z}_1, \theta, \dot{\theta})^T$$

The normalized state space representation of the system can be given by:

$$\dot{z}_1 = z_2 \quad (5)$$

$$\dot{z}_2 = \frac{-z_1 + \varepsilon \dot{\theta}_2^2 \sin\theta_1}{1 - \varepsilon^2 \cos^2\theta_1} + \frac{\varepsilon \cos\theta_1}{1 - \varepsilon^2 \cos^2\theta_1} u \quad (6)$$

$$\dot{\theta}_1 = \theta_2 \quad (7)$$

$$\dot{\theta}_2 = \frac{\varepsilon \cos\theta_1 (z_1 - \varepsilon \theta_2^2 \sin\theta_1)}{1 - \varepsilon^2 \cos^2\theta_1} + \frac{1}{1 - \varepsilon^2 \cos^2\theta_1} u \quad (8)$$

where z_1 represents the normalized displacement of the platform from the rest position, $z_2 = \dot{z}_1$, θ_1 is the angle of the arm, $\theta_2 = \dot{\theta}_1$ and u is the control torque applied.

By applying the decoupling algorithm we get [9]:

$$f_1 = \frac{-z_1 + \varepsilon \dot{\theta}_2^2 \sin\theta_1}{1 - \varepsilon^2 \cos^2\theta_1}, \quad g_1 = \frac{\varepsilon \cos\theta_1}{1 - \varepsilon^2 \cos^2\theta_1}, \quad f_2 = \frac{\varepsilon \cos\theta_1 (z_1 - \varepsilon \theta_2^2 \sin\theta_1)}{1 - \varepsilon^2 \cos^2\theta_1}$$

$$\text{and } g_2 = \frac{1}{1 - \varepsilon^2 \cos^2\theta_1}$$

$$\text{then } \frac{g_1}{g_2} = \varepsilon \cos\theta_1$$

$$x_1 = z_1 + \varepsilon \sin\theta_1 \quad (9)$$

$$x_2 = z_2 + \varepsilon \theta_2 \cos\theta_1 \quad (10)$$

$$x_3 = \theta_1 \quad (11)$$

$$x_4 = \theta_2 \quad (12)$$

It is shown that the control goals $z_1 \rightarrow 0, z_2 \rightarrow 0, \theta_1 \rightarrow 0$ and $\theta_2 \rightarrow 0$ are equivalent to $x_i \rightarrow 0, i = 1, 2, 3, 4$.

Since

$$\begin{aligned} \dot{x}_2 &= \dot{z}_2 + \varepsilon \dot{\theta}_2 \cos\theta_1 - \varepsilon \theta_2^2 \sin\theta_1 \\ &= \frac{-z_1 + \varepsilon \dot{\theta}_2^2 \sin\theta_1}{1 - \varepsilon^2 \cos^2\theta_1} + \frac{\varepsilon \cos\theta_1}{1 - \varepsilon^2 \cos^2\theta_1} u \\ &\quad + \varepsilon \left(\frac{\varepsilon \cos\theta_1 (z_1 - \varepsilon \theta_2^2 \sin\theta_1)}{1 - \varepsilon^2 \cos^2\theta_1} \right. \\ &\quad \left. + \frac{1}{1 - \varepsilon^2 \cos^2\theta_1} u \right) \cos\theta_1 - \varepsilon \theta_2^2 \sin\theta_1 \\ &= \frac{-z_1 + \varepsilon \dot{\theta}_2^2 \sin\theta_1}{1 - \varepsilon^2 \cos^2\theta_1} \\ &\quad + \frac{\varepsilon^2 \cos\theta_1 (z_1 - \varepsilon \theta_2^2 \sin\theta_1)}{1 - \varepsilon^2 \cos^2\theta_1} \\ &\quad - \varepsilon \theta_2^2 \sin\theta_1 \\ &= \frac{-z_1 (1 - \varepsilon^2 \cos\theta_1) + \varepsilon \dot{\theta}_2^2 \sin\theta_1 (1 - \varepsilon^2 \cos\theta_1)}{1 - \varepsilon^2 \cos^2\theta_1} \\ &\quad - \varepsilon \theta_2^2 \sin\theta_1 \end{aligned}$$

$$= -z_1 = -x_1 + \varepsilon \sin x_3 \quad (13)$$

$$\text{Let } v = \frac{\varepsilon \cos\theta_1 (z_1 - \varepsilon \theta_2^2 \sin\theta_1)}{1 - \varepsilon^2 \cos^2\theta_1} - \frac{1}{1 - \varepsilon^2 \cos^2\theta_1} u$$

$$\text{i.e. } u = \varepsilon \cos\theta_1 (z_1 - \varepsilon \theta_2^2 \sin\theta_1) - (1 - \varepsilon^2 \cos^2\theta_1) v \quad (14)$$

From equation (13) $z_1 = x_1 - \varepsilon \sin x_3$ then:

$$N = \varepsilon \cos x_3 (x_1 (1 + x_4^2) \varepsilon \sin x_3) - (1 - \varepsilon^2 \cos^2 x_3) v \quad (15)$$

From the above analysis, the system equations can be decoupled as:

$$\dot{x}_1 = x_2 \quad (16)$$

$$\dot{x}_2 = -x_1 + \varepsilon \sin x_3 + 11\varepsilon x_3 \quad (17)$$

$$\dot{x}_3 = x_4 \quad (18)$$

$$\dot{x}_4 = u \quad (19)$$

The Jacobian linearization method is used to obtain the TORA system linearized model of the TORA system. The resulted model is:

$$\frac{dx(t)}{dt} = f(x(t), u) \quad (20)$$

where:

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (21)$$

The Jacobian equation is applied for the system with equilibrium points (X_0, U_0) defined as:

$$X_0 = (x_{10}, x_{20}, x_{30}, x_{40}) = (1, 0, 0, 0) \quad (22)$$

$$U_0 = 0 \quad (23)$$

Then, the final model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 12\varepsilon & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} [u] \quad (24)$$

$$\begin{bmatrix} x_1(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + D[u] \quad (25)$$

3 Controller Design

TORA system model can be expressed as a LTI by:

$$\dot{x}(t) = Ax(t) + B_1 d(t) + B_2 u(t) \quad (26)$$

$$e(t) = C_1 x(t) + D_{12} u(t) \quad (27)$$

$$y(t) = x(t) \quad (28)$$

To represent the system uncertainty the uncertain TORA model can be described by:

$$\dot{x}(t) = Ax(t) + \Delta Ax + B_1 u(t) + \Delta B_1 u(t) + B_2 u(t) + \Delta B_2 u(t) + d(t) \quad (29)$$

where $x(t) \in R^n$ is the state vector, $e(t) \in R^h$ is the output to be controlled vector, $y(t) \in R^r$ is the output, $u(t) \in R^m$ is the control signal vector, $d(t) \in R^t$ the disturbance, A, B_2 state space system matrix, B_1 disturbance matrix, $\Delta A, \Delta B_1, \Delta B_2$ explain the matched uncertainty in A, B_1, B_2 and C_1, D_{12} represent controller design matrix.

The obtaining of a scalar control law is the main objective of this work. The control law is illustrated by:

$$u = u_{sw} + u_{eq} \quad (30)$$

where u_{sw} is the corrective control used to compensate the deviations from the sliding plane. u_{eq} is the equivalent control or the robust control which guarantees that the rate of change of the sliding plane is equal to zero to stay on the sliding surface.

The first part of the control law is achieved by unity sliding design, and the second one will be performed using H_2 state feedback controller. A unity sliding mode control approach for the system is presented with the

uncertainty and disturbance with the TORA system model [10].

From equation (29) which includes the terms that represent the parametric uncertainty and disturbance and the matrix A is not full rank, the Singular Value Decomposition (SVD) technique is used to obtain a full rank matrix then with a controllable pair (A, B) yields:

$$A_o = A - BK \quad (31)$$

where K is selected such that the matrix A_o is Hurwitz with the desired characteristic roots. then equation (29) becomes:

$$\dot{x}(t) = A_o x(t) + \Delta Ax + B_1 u_o(t) + \Delta B_1 u_o(t) + B_2 u_o(t) + \Delta B_2 + d(t) \quad (32)$$

Now, let the uncertainty in matrices A and B can be written as:

$$\Delta A = BA_\delta \text{ and } \Delta B = BB_\delta$$

Then the bracket $\{\Delta Ax + \Delta Bu + d(t)\}$ can be written as:

$$\Delta Ax + \Delta B_1 u + \Delta B_2 u + d(t) = B\{A_\delta x + B_\delta u + \delta(t)\} = B\{(A_\delta - B_\delta K)x + B_\delta u_o + \delta(t)\} \quad (33)$$

Assuming that the uncertainty (A_δ, B_δ) and the external disturbance $(\delta(t))$ are bounded, then we can get:

$$\|(A_\delta - B_\delta K)x + B_\delta u_o + \delta(t)\| \leq \alpha \|x\| + \beta \|u_o\| + \varepsilon \quad (34)$$

$$u = -\gamma(\|x\|) \frac{s}{\|s\|} - Kx \quad (35)$$

where

$$S = B^T 2Px \text{ and } \gamma(\|x\|) = \frac{1}{(1-\beta)} \{\alpha \|x\| + \varepsilon + k\}, \quad k > 0$$

So, the sliding mode control law basically is:

$$u_{sw} = -\gamma(\|x\|) \frac{s}{\|s\|} \quad (36)$$

The H_2 control objective is to obtain an optimal controller that minimizes a quadratic performance index (the H_2 norm) of the system. Also, this controller offers a way of combining the design criteria of quadratic performance and disturbance rejection [11].

For the control system in equations (26),(27) and (28), it is required that the matrix A is of full rank, the pairs (A, B_1) and (A, B_2) are required to be stabilizable, (C_1, A) is required to be detectable and it is required that all the system state measurements are possible.

Figure 2 presents the block diagram of the full state feedback H_2 control. Assume that:

$$M = \begin{bmatrix} A & B_1 & B_2 \\ C & 0 & D_{12} \\ I & 0 & 0 \end{bmatrix} \quad (37)$$

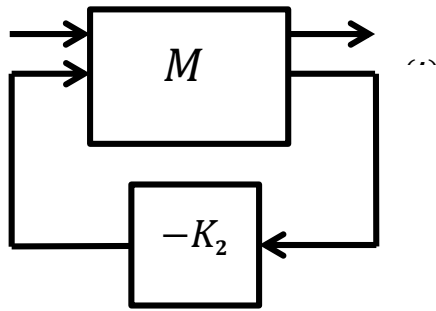


Figure 2: H₂ control structure [11].

where $w(t)$ is the exogenous inputs (set point $r(t)$ and disturbance input $d(t)$). The system error for a white noise input can be represented using H₂ norm by:

$$\|T_{ed}\|_{H_2}^2 = E(e^T(t)e(t)) \quad (38)$$

where T_{ed} illustrates the total transfer function of the system to be controlled from $d(t)$ to $e(t)$, then from equation (38), we can obtain: $e^T(t)e(t) = x(t)^T Q_f x(t) +$

$$2x(t)^T N_f u(t) + u(t)^T R_f u(t) \quad (39)$$

The minimization of $\|T_{ed}\|_{H_2}^2$ is equivalent to the stochastic regulator problem solution by setting: $Q_f = C_1^T C_1$, $N_f = C_1^T D_{12}$ and $R_f = D_{12}^T D_{12}$ then, the cost function to be minimized is [11]:

$$E(e^T(t)e(t)) = J(x(t), u(t)) = \int_0^\infty [x(t)^T Q_f x(t) + 2x(t)^T N_f u(t) + u(t)^T R_f u(t)] dt \quad (40)$$

Consequently:

$$J = \int_{t_0}^{t_f} [x(t)^T Q_f x(t) + 2x(t)^T N_f u(t) + u(t)^T R_f u(t)] dt \quad (41)$$

where $Q_f \in Q_f^{n \times n}$ is a symmetric positive semidefinite state weighting matrix, $N_f \in N_f^{n \times n}$ is a symmetric positive semidefinite state and control weighting matrix and $R_f \in R_f^{n \times n}$ is a control weighting matrix which is required to be symmetric positive definite. The optimal control action is:

$$u_{eq} = -K x(t) \quad (42)$$

where

$$K = R_f^{-1} (B_1^T P + N_f^T) \quad (43)$$

The Riccati equation is:

$$(A - B_2 R_f^{-1} N_f^T)^T P + P(A - B_2 R_f^{-1} N_f^T) - P B_2 R_f^{-1} B_1^T P + Q_f - N_f R_f^{-1} N_f^T = 0 \quad (44)$$

where K is represents the state feedback gain matrix.

4 Results and Discussion

Figure 3 shows the time response of the system before applying the proposed controller. It shows that the system is oscillatory and the design of a controller is required. The response of the system using sliding mode control and H₂ sliding mode controller is shown in Figure 4. It is shown from sliding mode control results that the system position reaches the steady state within 7 sec. and the pendulum angle deviates between -46° and 30°. The time response specifications of the system using H₂ sliding mode controller are: position settling time equal to 5 sec. and the pendulum angle deviates between 0° and 6.3°. The resulting control signal can be shown in Figure 5. As can be seen, the control signal of sliding mode control deviates between -116 N.m and 45 N.m and for H₂ sliding mode controller, the control signal deviates between 1.5 N.m and -0.5 N.m.

From this result, it is clear that the superiority of the proposed H₂ sliding mode controller in that it can achieve a more improved time response in comparison to that obtained using sliding mode controller. Moreover, it can be seen that a low control effort has been achieved as shown in Figure 5.

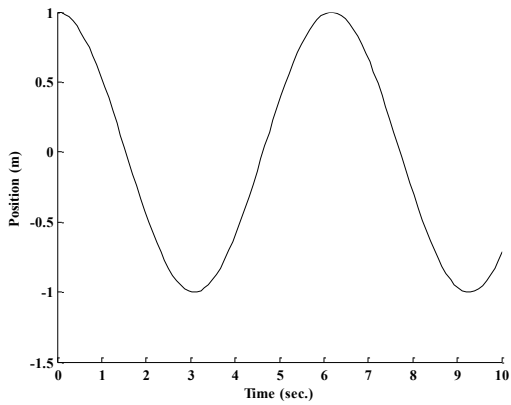
The resulting state feedback gain matrix which represents the H₂ control part is:

$$K_2 = [0.5964 \quad 46.2602 \quad 101.3292 \quad 18.7792]$$

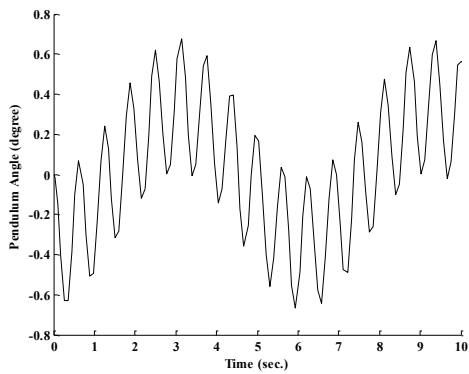
For robustness test of a change of ±50% in system parameters is considered. Figure 6 shows the output response of the system with parameters uncertainty. Figure 8 shows the output response of the system with disturbance of 1 N.m at the 4.5 second. As is seen, the proposed can compensate for the change in system parameter and achieve the desired performance.

In order to further show the effectiveness of the proposed H₂ sliding mode controller, a comparison with previously done works has been considered. Table 1 compares the results of the proposed controller and Decoupled Self-tuning Signed-Distance Fuzzy Sliding Mode Controller (DSSFSC) which was done by Hung *et al* in 2007 [4]. The table clearly shows that the benefits of the proposed

controller over the controllers done previously.

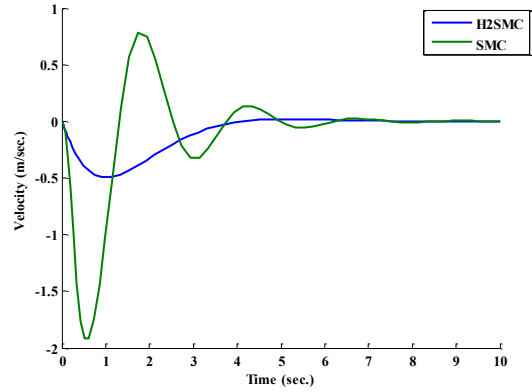


(a)

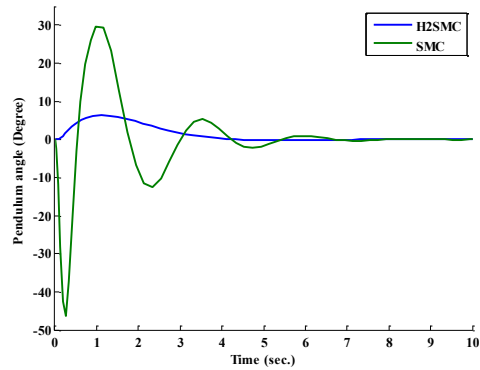


(b)

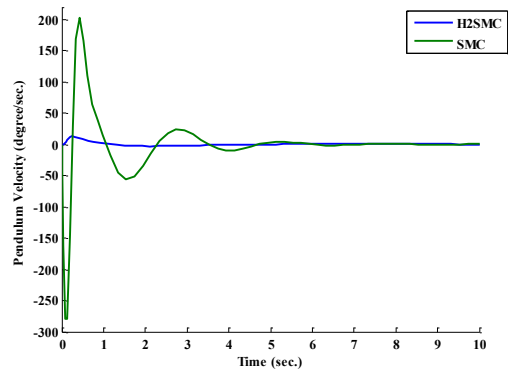
Figure 3: System response without controller (a) position (b) pendulum angle.



(b)

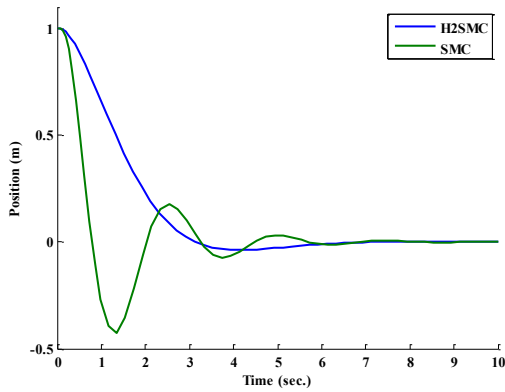


(c)



(d)

Figure 4: Nonlinear system response with sliding mode controller and H₂ sliding mode controller (a) position (b) velocity (c) pendulum angle, (d) pendulum velocity.



(a)

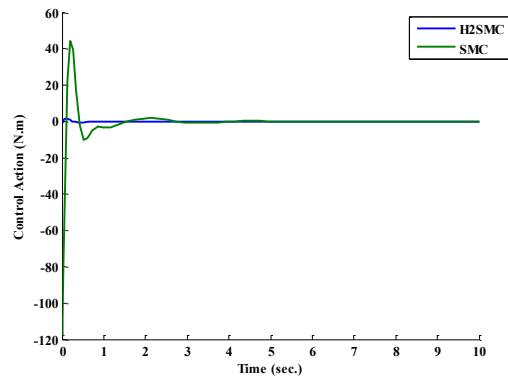
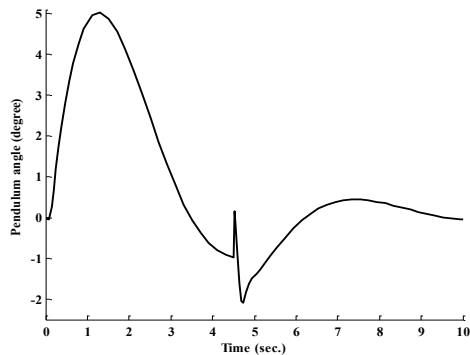
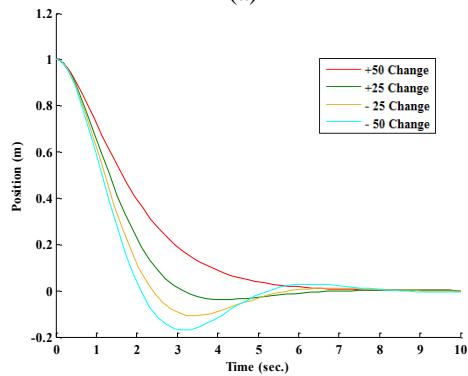


Figure 5: The resulting control signal using sliding mode control and H₂ sliding mode controller for the nonlinear system.

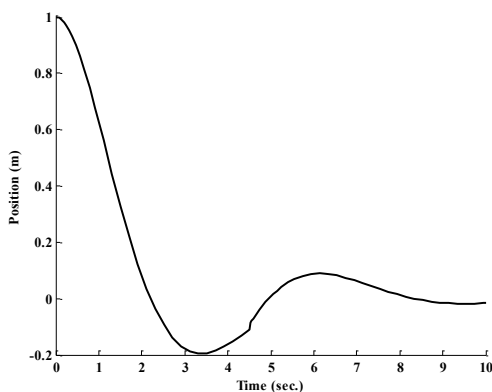


(a)

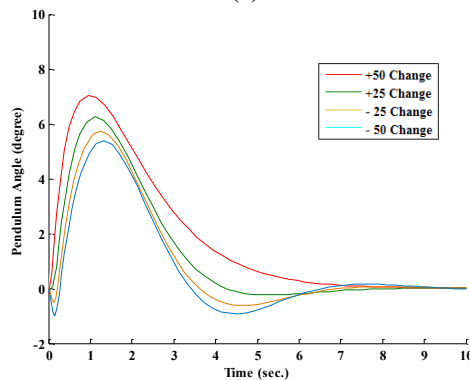


(b)

Figure 6: Nonlinear controlled system response with parameters uncertainty using H₂ sliding mode controller (a) position (b) pendulum angle.



(a)



(b)

Figure 7: Nonlinear controlled system response with disturbance using H₂ sliding mode controller (a) position (b) pendulum angle.

Table 1: Comparison between DSSFSMC, optimal sliding mode control and H₂ sliding mode controller.

Controller	System position		Pendulum angle		
	t_s (sec.)	t_r (sec.)	t_s (sec.)	t_r (sec.)	Dev. (degree)
DSSFSMC	19	1.36	62	1.2	100.33 to -84.3
Optimal SMC	4.9	0.46	5.5	0.38	-45.33 to 30.3
H ₂ SMC	2.7	0.7	3.9	0.2	0 to 6.3

5 Conclusion

In this paper, the design of H₂ sliding mode control for TORA system has been presented. It was shown that the combination between H₂ control and sliding mode control has achieved a performance better than if only one of them is used. Also, it was found that the proposed controller can compensate the model uncertainties to improve the system performance significantly. A comparison with previous works has been considered to illustrate the efficiency of the proposed controller.

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تصميم مسيطر H₂ المدمج مع مسيطر النمط الانزلاقي للسيطرة على منظومة TORA

مصطفى جبار كاظم
هندسة السيطرة والنظم
الجامعة التكنولوجية

حازم ابراهيم علي
هندسة السيطرة والنظم
الجامعة التكنولوجية

الخلاصة

في هذا البحث تم اقتراح مسيطر النمط الانزلاقي الامثل للسيطرة على منظومة TORA والتي تعتبر منظومة ثنائية درجة الحرية ومنظومة تحت التحكم. ويعتبر التداخل اللاخطي الموجود في هذه المنظومة واحدا من اهم المشاكل التي تواجه المسيطر المصمم. ولغرض تحسين اكثر لاداء المسيطر الانزلاقي النمط تم دمجه مع المسيطر H₂ ليعطي مواصفات اداء للمنظومة افضل مما لو تم استخدام المسيطر انزلاقي النمط فقط. النتائج المستحصلة تبين كفاءة وقوة المسيطر المقترح في توفير الاستقرار بوجود التغيرات المحتملة في معاملات المنظومة وبوجود المعوقات.