

Robust Deadbeat Controller Design using PSO for Positioning a Permanent Magnet Stepper Motors

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Abstract

This paper presents an application of a robust deadbeat controller for permanent magnet stepper motor. This approach has been considered in order to assure robust stability and performance (disturbance rejection, reference tracking) with the presence of system parameters uncertainty. The Particle Swarm Optimization (PSO) is used to tune the controller parameters by minimizing the cost function subject to H-infinity constraints. It is shown that the designed deadbeat controller presents simple, low order, and robust position control for a permanent magnet stepper motor. A two-phase motor is considered in this paper.

Key Words: Robust Deadbeat Controller, PSO, H-Infinity Constraints

الخلاصة

يقدم البحث تطبيق لمسيطر الـ Deadbeat المتين على منظومة محرك الخطوة. وتم استخدام هذا النوع من المسيطرات لتحقيق استقرارية وأداء للمنظومة بوجود التغيرات وعدم الثبات في معاملات المنظومة. كذلك تم استخدام " Particle Swarm Optimization (PSO) " لغرض ضبط وتنظيم معاملات المسيطر عن طريق تصغير دالة الكلفة والتي تحقق شروط الـ (H-infinity). ويستخدم هذا النوع من المسيطرات لتصميم مسيطر بسيط وذو رتبة واطئة وذات سيطرة متينة على محرك الخطوة. استخدم محرك خطوة ذو طورين في هذا البحث.

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1. Introduction

The stepper motor is an electromagnetic actuator that converts digital pulse inputs to analog shaft motion outputs. It rotates by a specific number of degrees in response to an electrical pulse input; therefore the stepper motor is used in digital control systems. The stepper motors are relatively inexpensive and simple in construction, so they are widely used in our daily life. On the other hand, they are used in practical applications that require incremental motion such as printers, tape drives, hard drives in PC's, machine tools, process control systems, X-Y records, and robotics[1].

The control system is one of the most important elements in stepping motor applications. The control systems of stepping motors are classified into open loop and closed loop schemes. In the open loop control scheme there is no feedback information of position to the controller and therefore it is imperative that the motor must respond correctly to each excitation change. If the excitation changes are made too quickly, the motor is unable to move to the new demanded position and consequently there is a permanent error in the actual position compared to the position expected by the controller. The timing of phase control signals for optimum open-loop performance is reasonably straightforward, if the load parameters remain constant. However, in the applications where the load varies significantly, the timing must be set for the worst conditions (largest load) and the control scheme is then non optimal for all other loads [2, 3]. On the other hand, if high accuracy is needed, the closed loop control scheme is recommended. In closed loop stepping motor systems the instantaneous rotor position is detected via a feedback sensor

and sent to the control unit. The general block diagram of the closed loop scheme is presented in Figure (1).

Two types of stepper motors are widely used, the variable reluctance type and the permanent magnet type. In this paper, the permanent magnet type is used. These types have higher inertia and therefore slower acceleration than variable reluctance types. They also produce more torque per ampere stator current than the variable reluctance [4].

As the controller algorithm is realized by software, there is a possibility of applying different and more sophisticated control algorithms. One of these algorithms is referred to as the deadbeat control algorithm, which ensures an accurate settling of the output signal during a finite, small number of sampling periods [5]. A deadbeat response is a response that proceeds rapidly to the desired level, and holds at that level with minimal overshoot.

In this paper the robust deadbeat controller is designed using PSO to obtain a simple, low order and robust position controller for a permanent magnet stepper motor with the presence of system parameters uncertainty.

2. System Mathematical Model

In this section a mathematical model of the linear motor drive is developed. This linearized model is needed for the robust control technique used in this paper. Basically, the model of the permanent magnet stepping motor consists of two parts, an electrical and a mechanical part. The permanent magnet stepper motor dynamical model includes nonlinearities and contains some physical parameters. The values of physical parameters are not exactly known and can be subjected to

some variations, so the model is not very easy to handle for control synthesis. The model has two phases denoted by A and B . The rotor has $(2Nr)$ magnetic poles, while the stator has a set of identical poles [3, 6].

The mechanical part of the permanent magnet stepper motor model can be expressed by [6]:

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + N_r n \Phi_M i_A \sin(N_r \theta) + N_r n \Phi_M i_B \sin(N_r (\theta - \lambda)) + C \text{sign}\left(\frac{d\theta}{dt}\right) + T_L = 0 \quad (1)$$

where J is the moment of rotor inertia (Kg.m^2), D is the viscous damping coefficient (N.m.s.rad^{-1}), C is the coulomb friction coefficient, i_A, i_B are the currents in windings A and B , N_r is the number of the rotor teeth, $n\Phi_M$ is the flux linkage, θ is the rotational angle of the rotor and λ is the tooth pitch in radians and T_L is the load torque [3, 7]. On the other hand, the electrical part of a permanent magnet stepper motor model is described by voltage equations for the stator windings.

$$V - ri_A - L \frac{di_A}{dt} - M \frac{di_B}{dt} + \frac{d}{dt}(n\Phi_M \cos(N_r \theta)) = 0 \quad (2)$$

$$V - ri_B - L \frac{di_B}{dt} - M \frac{di_A}{dt} + \frac{d}{dt}(n\Phi_M \cos(N_r (\theta - \lambda))) = 0 \quad (3)$$

where V is the DC terminal voltage supplied to the stator windings (volt), L denotes the self-inductance of each stator phase (mH), M represents the mutual inductance between phases (mH) and r is stator circuit resistance (ohm). Thus, the complete model of the permanent magnet stepping motor consists of the rotor dynamic equation (1) and differential equations for current equation (2) and (3). Those equations are nonlinear differential equations. Since it is very difficult to deal with nonlinear differential equations

analytically, linearization is needed. Linearization is made with aid of a new variable $\delta\theta$, which represents the deviation of the angle from the equilibrium position. The deviation is a function of time t and it is very small in magnitude. The equilibrium position of the stator [6] is

$$\theta = \frac{\lambda}{2}.$$

When the rotor oscillates about its equilibrium position, the currents in both motor windings will deviate from the stationary value I_o by δi_A and δi_B and the angular rotor position will be expressed by

$$\theta = \frac{\lambda}{2} + \delta\theta.$$

Then the nonlinearities expressed by sine and cosine functions in equations (1), (2) and (3) will be approximated with knowledge of trigonometric identities and when $N_r \delta\theta$ is small angle: $\cos(N_r \delta\theta) \cong 1$ and $\sin(N_r \delta\theta) = N_r \delta\theta$. Then, the linearized model can be expressed by [6, 8]:

$$J \frac{d^2(\delta\theta)}{dt^2} + D \frac{d(\delta\theta)}{dt} + 2N_r^2 n \Phi_M I_o \cos\left(\frac{N_r \lambda}{2}\right) \delta\theta + N_r n \Phi_M \sin\left(\frac{N_r \lambda}{2}\right) (\delta i_A - \delta i_B) + C \text{sign}\left(\frac{d\theta}{dt}\right) = 0 \quad (4)$$

$$r \delta i_A + L \frac{d(\delta i_A)}{dt} + M \frac{d(\delta i_B)}{dt} - N_r n \Phi_M \sin\left(\frac{N_r \lambda}{2}\right) \frac{d(\delta\theta)}{dt} = 0 \quad (5)$$

$$r \delta i_B + L \frac{d(\delta i_B)}{dt} + M \frac{d(\delta i_A)}{dt} + N_r n \Phi_M \sin\left(\frac{N_r \lambda}{2}\right) \frac{d(\delta\theta)}{dt} = 0 \quad (6)$$

where: $\sin\left(\frac{N_r \lambda}{2}\right)$, and $\cos\left(\frac{N_r \lambda}{2}\right)$ are

constants.

The permanent magnet stepping motor transfer function is derived from equations (4), (5) and (6) with the aid of Laplace transform. The coulomb friction coefficient C is considered to be zero. The resulting form of the transfer function in two-phase excitation is:

$$\frac{\Theta_o}{\Theta_i} = \frac{\frac{r}{L} w_{np}^2}{s^3 + \left(\frac{r}{L_p} + \frac{D}{J}\right) s^2 + \left(\frac{rD}{L_p J} + w_{np}^2 (1 + k_p)\right) s + \left(\frac{r}{L_p}\right) w_{np}^2} = G_p(s) \quad (7)$$

Where: $L_p = L - M$, $w_{np}^2 = \frac{2N^2 r n \Phi_M I_o \cos\left(\frac{N_r \lambda}{2}\right)}{J}$, ,

$$k_p = \frac{n \Phi_M \sin^2\left(\frac{N_r \lambda}{2}\right)}{L_p I_o \cos\left(\frac{N_r \lambda}{2}\right)}$$

Θ_o is the Laplace transform of the actual rotor position, Θ_i represents the Laplace transform of the demanded position and s is the Laplace operator. Table (1) defines the model parameters and summarizes the nominal values of all parameters and their variations used in the controller design [6]. Figure (2) and (3) show the frequency and time response characteristics of the system with all parameters uncertainty ($r, L, M, D, n\Phi_M$). The poor stability and performance of the system is clear.

3. Particle Swarm Optimization (PSO) Algorithm

PSO is a powerful optimization method with high efficiency in comparison to other methods such as Genetic Algorithm (GA) and Harmony Search (HS). The PSO mechanism is initialized with a population of random solutions and searches for optima by updating generations. The potential solutions of PSO are called "particles", which fly through the problem space by following the current optimum particles. Each particle keeps track of its coordinates in the space of the problem, which are associated with the best solution (best fitness) it has achieved so far. The best particle in the population is denoted by (global best), while the best position that has been visited by the current particle is denoted by (local best). The global best individual connects all members of the population to one another. That is, each particle is influenced by every best performance of any member in the entire population. The local best individual is seen as the ability for particles to

remember past personal success. The particle swarm optimization concept involves, at each time step, changing the velocity of each particle towards its global best and local best locations. The particles are manipulated according to the following equations of motion [9, 10]:

$$v_i^{k+1} = w \times v_i^k + c_1 \times rand \times (x_i^b - x_i^k) + c_2 \times rand \times (x_i^g - x_i^k) \quad (8)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (9)$$

where v_i^k is the particle velocity, x_i^k is the current particle position, w is the inertia weight, x_i^b and x_i^g are the best value and the global best value, $rand$ is a random function between 0 and 1, c_1 and c_2 are learning factors. The PSO requires only a few lines of computer code to realize PSO algorithm. Also it is a simple concept, easy to implement, and computationally efficient algorithm [11].

4. Robust Deadbeat Controller

A deadbeat response for linear control systems has the following characteristics [12]:

- i) Zero steady state error.
- ii) Fast response, that is, minimum rise time and settling time.
- iii) Overshoot less than 0.1%.
- iv) Undershoot less than 2%.

When designing a system to obtain a deadbeat response, the closed loop transfer function that meets the deadbeat response is selected and then the controller transfer function is obtained. That is, the designer selects the system parameters to achieve the desired performance. For this reason, the design of a feedback system using deadbeat control leads to predictable system responses [12]. A method for designing robust deadbeat controller subject to H_∞ -norm constraints is

presented in this section. In this method, the PSO algorithm is used to tune the parameters of the closed loop transfer function selected by the deadbeat control algorithm. The tuning is done by minimizing the cost function subject to H_∞ -norm constraints to design a robust deadbeat controller for permanent magnet stepper motor system.

In this paper the designed controller is used with the plant so that the following conditions must be achieved [13]:

- i) The nominal closed loop system is asymptotically stable.
- ii) The robust stability performance satisfies the following equation:

$$\|W_T T\|_\infty < 1 \quad (10)$$

- iii) The disturbance attenuation performance satisfies the following equation:

$$\|W_p S\|_\infty < 1 \quad (11)$$

where W_T is the plant multiplicative uncertainty function and this function has been obtained by converting the structured (parametric) uncertainty into unstructured uncertainty. The obtained model of unstructured uncertainty is:

$$W_T(s) = \frac{0.0513s^3 + 12627s^2 + 1.376 \times 10^6 s + 6.567 \times 10^9}{s^3 + 58010s^2 + 2.758 \times 10^7 s + 6.484 \times 10^9} \quad (12)$$

T is the complementary sensitivity function of the system, which is defined as:

$$T(s) = G_c(s)G_p(s)(1 + G_c(s)G_p(s))^{-1} \quad (13)$$

W_p is a stable weighting function and it has been selected as:

$$W_p(s) = \frac{\beta(\alpha s^2 + 2\zeta_1 w_c \sqrt{\alpha} s + w_c^2)}{(\beta s^2 + 2\zeta_2 w_c \sqrt{\beta} s + w_c^2)} \quad (14)$$

where β is the d.c. gain of the function which controls the disturbance rejection,

α is the high frequency gain which controls the response peak overshoot, w_c is the function crossover frequency, ζ_1 and ζ_2 are the damping ratios of crossover frequency. S is the sensitivity function of the system, which is defined as:

$$S(s) = (1 + G_c(s)G_p(s))^{-1} \quad (15)$$

$G_p(s)$ is the nominal plant. Equations (10) and (11) can be combined in one equation to be simultaneously satisfied, the combined equation is:

$$\|W_T T + W_p S\|_\infty < 1 \quad (16)$$

To achieve robust performance and to improve the time and frequency response characteristics of the system, a combination of H_∞ -norm specifications, time domain specifications represented by the performance index (ITAE) and frequency domain specifications represented by the system gain and phase margins has been used. The cost function can be expressed as:

$$h(x) = \|W_T T + W_p S\|_\infty + \int_0^{t_f} |e(t)| dt + (GM)^{-1} + (PM)^{-1} \quad (17)$$

where GM and PM are the system gain and phase margins and they are obtained from the resulting loop transmission, $L(s) = G_c(s)G_p(s)$ and t_f is selected to be the settling time of the system. The PSO algorithm is used to minimize the cost function in equation (17) to achieve the robustness requirements using deadbeat controller.

To determine the coefficients that yield the suboptimal and robust deadbeat response for the third order transfer function of the permanent magnet stepper motor system, the following third order closed loop transfer function has been selected:

$$T(s) = \frac{\phi^3}{s^3 + b_1\phi s^2 + b_2\phi^2 s + \phi^3} \quad (18)$$

The coefficients ϕ , b_1 , b_2 have been assigned the values using PSO algorithm to meet the requirements of robust deadbeat response by minimizing the cost function $h(x)$ in equation (17). In this case x is a vector of the parameters to be obtained and it can be expressed as:

$$x = [\phi \ b_1 \ b_2 \ \beta \ \alpha \ \zeta_1 \ \zeta_2 \ w_c] \quad (19)$$

The parameters used for carrying out the design of robust deadbeat controller using PSO are population size equal to 10, inertia weight factor $w = 2$, $c_1 = 2$, $c_2 = 2$, maximum iteration is set to 10 and finally, the number of function evaluations is 100. On the other hand, setting the number of iterations to 10 result in obtaining the best optimal cost function, where increasing the number of iteration did not improve the convergence of the PSO algorithm significantly. Then, the transfer function of the deadbeat controller can be obtained by:

$$G_c(s) = \frac{1}{G_p(s)} \frac{T(s)}{1 - T(s)} \quad (20)$$

Figure (4) shows the block diagram of the designed deadbeat controller. The obtained controller, $G_c(s)$ and corresponding prefilter, $F(s)$ and performance weighting function, $W_p(s)$ are:

$$G_c(s) = \frac{0.2612s^2 + 22.62s + 1.222 \times 10^5}{s(s + 515.8)} \quad (21)$$

$$W_p(s) = \frac{106.1s^2 + 1.214 \times 10^5 s + 5.684 \times 10^5}{724.7s^2 + 1.496 \times 10^4 s + 3.921 \times 10^4} \quad (22)$$

$$F(s) = \frac{1}{0.0035s + 1} \quad (23)$$

The PSO steps for obtaining the optimal parameters of the robust deadbeat

controller and performance weighting function can be summarized as:

1. Define the system model $G_p(s)$.
2. Define the structure of $W_p(s)$.
3. Define the structure of $G_c(s)$, $F(s)$.
4. Initialize the individuals of the population randomly in the search space.
5. For each initial x of the population, where x is the vector of the parameters to be optimised, determine the cost function in equation (17).
6. Compare each value of equation (17) with its personal best x_i . The best value among the x_i is denoted as x_i^g .
7. Update the velocity of each individual x according to (9).
8. Update the position of each individual x according to (8).
9. If the number of iterations reaches the maximum, then go to step 10, otherwise, go to step 5.
10. Calculate equation (16).
11. If the criterion in equation (16) is satisfied, then go to step 12, otherwise "No solution exists in the given search domain", then go to step 3.
12. The latest x_i^g is the optimal controller parameter.

5. Results and Discussion

The ability of the designed controller to meet the specified closed loop performance is demonstrated in this section. Figure (5) shows the resulting sensitivity function and it is clear that the sensitivity function lies below the inverse of performance weighting function, W_p .

This means that the performance criterion was satisfied. Figure (6) shows the resulting complementary sensitivity function compared with the multiplicative uncertainty function. From this figure, it can be seen that the complementary sensitivity has magnitudes less than the magnitudes of the inverse of the multiplicative uncertainty function, W_T for all range of frequencies. This means that the performance criterion in equation (16) has been satisfied for the designed deadbeat controller and the robust stability and performance for the system have been achieved.

Figure (7) shows the frequency responses of the uncertain system when the designed controller is applied, where a gain margin and phase margin of 29 dB and 66.4° respectively have been obtained. This means that the system is always stable and the variation in the system model parameters has not affected. Therefore, the robust stability has been satisfied for the system using the designed deadbeat controller.

The closed loop time response of the uncertain system with the designed deadbeat controller is shown in Figure (8). It can be seen that the controller can effectively track the reference and compensate the uncertain system with assuring high control performance of the system. Table (2) compares the performances of the classical Quantitative Feedback Theory (QFT) in [6] and the robust deadbeat controller designed in this paper. The table clearly shows the superiority of the robust deadbeat controller in terms of time and frequency response specifications. Moreover, it was found that the resulting controller by the robust deadbeat controller method using

PSO is lower order than that obtained using QFT controller in [6].

5. Conclusion

An application of the robust deadbeat controller to a permanent magnet two phase stepping motor has been presented. It was shown that the robust stability was satisfied by applying the deadbeat controller with the presence of system parameters uncertainty. Also it was clear that the obtained controller is simple, robust and lower order and the achieved time response specifications are very desirable for this system.

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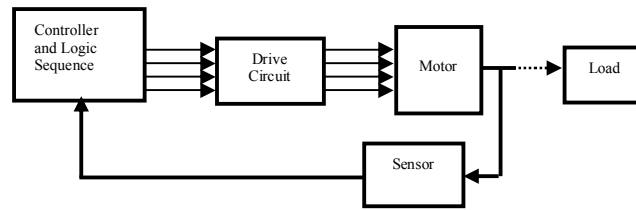


Figure 1: Block diagram of closed loop stepping motor.

Table (1): The nominal model parameters and their range.

PARAMETER	MINIMUM VALUE	NOMINAL VALUE	MAXIMUM VALUE
Stator resistance (r) ohm	29.7	33	36.3
Self inductance (L) mH	4.86	5.4	5.94
Matual inductance (M) mH	0.36	0.4	0.44
Rotor inertia (J) $g.cm^2$		0.16	
Number of rotor teeth (N_r)		6	
Viscous friction (D) (N.m.s/rad)* 10^{-5}	1.215	1.35	1.485
Tooth pitch (λ) rad		$\pi/12$	
Stationary current (I_o) Amper		0.15	
Flux linkage ($n\Phi_M$) $T.m^2 * 10^{-3}$	1.08	1.2	1.32

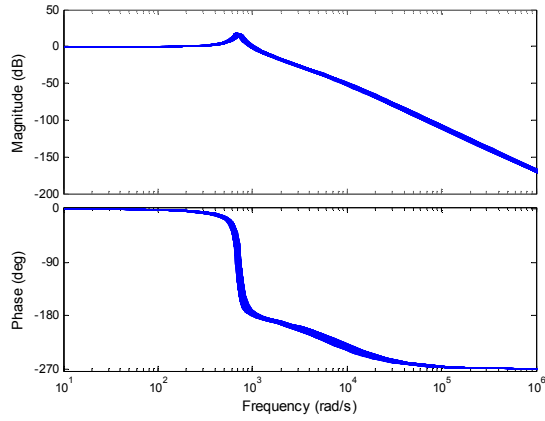


Figure 2: Frequency response characteristics of the uncertain system without the controller.

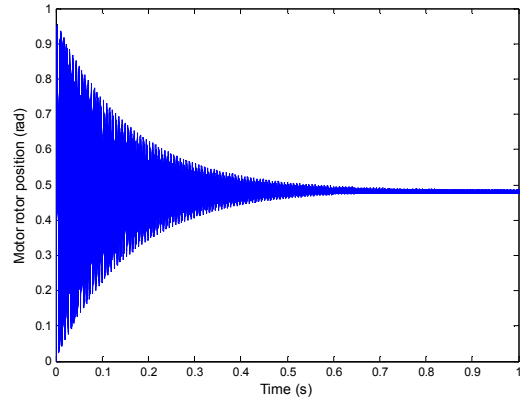


Figure 3: Closed loop time response characteristics of the uncertain system without the controller.

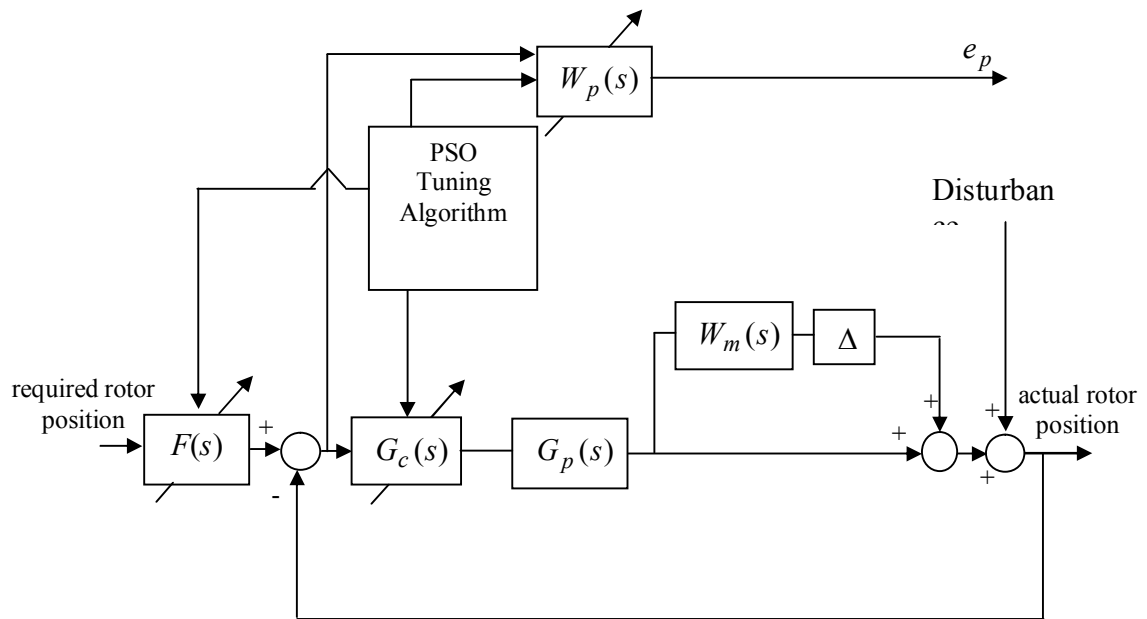


Figure 4: Block diagram of the designed robust deadbeat controller using PSO for permanent magnet stepper motor system.

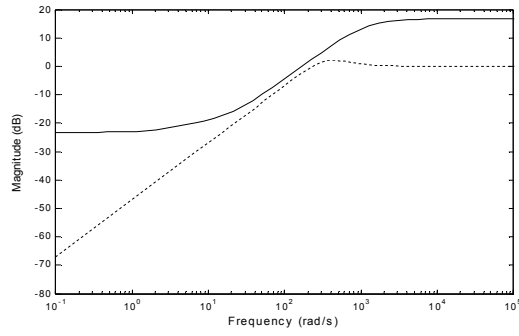


Figure 5: The resulting sensitivity function, S (dotted line) compared with performance weighting function, W_p (solid line).

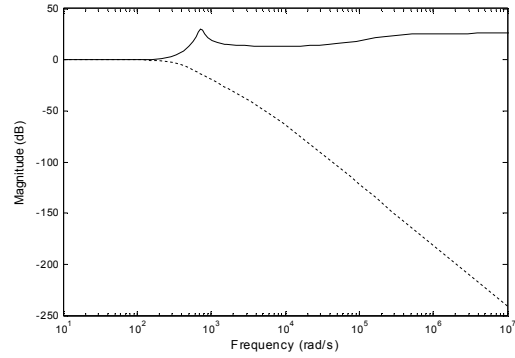


Figure 6: The resulting complementary sensitivity function, T (dotted line) compared with multiplicative uncertainty, W_T (solid line).

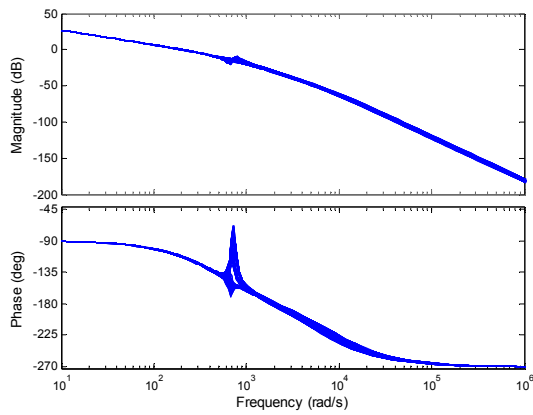


Figure 7: Frequency response characteristics of the uncertain system with controller.

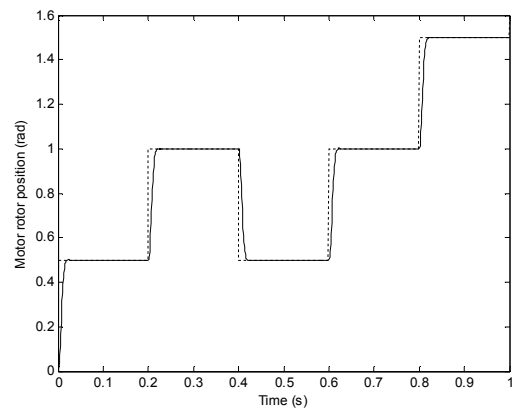


Figure 8: Closed loop time response characteristics of the uncertain system with controller. Reference (dotted line), Output (solid line).

Table (2): Comparison between QFT and Robust deadbeat controllers.

Controller/Specifications	Rise time (s)	Settling time (s)	Gain margin (dB)	Phase margin (degree)
QFT	0.04	0.015	5.1	47
Deadbeat	0.02	0.01	29	66.4