# Application of Kalman Estimation Techniques with Fuzzy Logic to a Synchro Digitizer

Waladin K. Sa'id[*] Firas A. Raheem[*] Lina S. Jajo[*]

Received: 21/11/2011
Accepted:

*Abstract:–*
In the quest for digitizing the synchro, this paper proposes a method for processing the synchro format voltages to extract a single reliable shaft angle reading. This eliminates sophisticated and expensive electronic parts and replaces them by software algorithms. Kalman estimation techniques are applied to improve sensor dynamic response, precision and efficiency. Fuzzy logic techniques are used to speed up the estimation process so that the time taken to produce a result is within the time of half a cycle of the excitation frequency (less than 1.25 ms). Wavelet techniques are also used to improve the accuracy much further. The synchro digitizer was simulated using Matlab, and the random noise was taken into effect. Theoretical analysis and experimental data ascertained the above technique.

*Keywords-component; synchro; Kalman estimation technique; fuzzy logic; wavelet filter*

## تقانات مخمن كالمن مع المنطق المضبب على المتزامن الرقمي

**الخلاصة:– من أجل تحويل فولتيات مخرجات المتزامن الى صيغة رقمية، يقترح البحث الحالي طريقة لمعالجة الفولتيات الثلاثة الخارجة من المتزامن للحصول على قراءة منفردة و موثوق بها لزاوية المحور المقاسة. هذه الطريقة سوف تختصر الحاجة الى دوائر ألكترونية معقدة و غالية وذلك بأستخدام التقنيات البرمجية اللازمة. تقنيات كالمان للتخمين أستخدمت وذلك لتحسين الأستجابة الديناميكية للمتحسس، الدقة و الكفاءة. تقنيات المنطق المضبب أستخدمت في بناء مخمن كالمن وذلك لتسريع عملية التخمين لتكون مخرجات المخمن ضمن زمن نصف موجة مصدر تغذية المتزامن (يعني أقل من1.25 ملي ثانية). كذلك أستخدمت تقنية المويجات لتحسين دقة النتائج. تمت برمجة المحاكاة بأستخدام برنامج الماتلاب بحيث أخذت بنظر الأعتبار الضوضاء العشوائية. التحليل النظري و النتائج العملية أثبتت نجاعة هذه الطريقة.**
**الكلمات الرئيسية: المتزامن، تقنية كالمان للتخمين، المنطق المضبب، مرشح المويجات.**

[*] Control& Systems Eng. Dept., University of Technology, Baghdad, Iraq.

## 1. Introduction

These days modern linear and digital integrated circuit technology is used throughout the field of position and motion sensing. Fully integrated solutions augmented with software have resulted in cost-effective solutions to problems which in the past have been solved using expensive electro-mechanical techniques. The synchro device is the most reliable and cost effective electromechanical position-sensing device used for exact angular positioning. It was originally developed for use with analog systems, such as inertial navigation and automatic direction finder [1,2]. Many applications today including, robotics, factory automation, avionics, automotive, machine tools, antennas and others, use digital approaches. Hence digitizing synchro output is a must for up grading these systems for interfacing purposes. A synchro to a digital converter converts three-wire synchro information into a digital format. There are two types of converters or two methods to implement this function, successive approximation and tracking [2].

A synchro digitizer was realized practically based on the method of half cycle synchro signals integration and using the successive approximation concept. The output from the digitizer is three angles for the same synchro shaft angle. These angles are not equal; they vary by as much as 1.2º plus ±0.2º random noise. However, the average of these angles is in error of no more than ±0.3º [3]. In a further development, a multi-layer perceptron acting as an on-line neural network filter was built and applied to the output angles of the synchro digitizer. This neural network was learned using the back-propagation learning algorithm. The measurement error is reduced to ± 0.2º [4].

The digitization process needs some software development, which makes a possible correction to increase the accuracy ratio to produce good results. This paper investigates and develops a software filtering technique based on estimation theory, fuzzy logic and wavelet techniques in order to digitize the synchro format voltages. Kalman filtering provides a tool for obtaining that reliable estimate. Simulation and experimental data are to be used to test and asses the applicability of the filtering concept.

## 2. Synchro Analysis Requirements

The stator of a synchro consists of three Y-Connected coils, which are wound with their axes 120° apart. When it is excited with a sinusoidal voltage, the induced voltages across the stator terminals (known as synchro format voltages), will be proportional to the sine of the angle (θ) between the rotor and the stator coil axes (synchro shaft angle) [5]. The synchro format voltages, for an ideal synchro, assume the following general form,

$$V_k = V_{ex} \sin(\Gamma_k)\ ,\ k=1,2,3 \qquad (1)$$

Where, $V_{ex}$ and $\Gamma_k$ are given by $V_m\sin\omega t$ and (θ+120(k-1)), respectively. The three voltages $V_1$, $V_2$, and $V_3$ expressed by equation (1) are the voltages that appear across stator terminals ($S_1$ to $S_3$), ($S_3$ to $S_2$) and ($S_2$ to $S_1$), respectively. $V_m\sin(\omega t)$ is the excitation reference voltage across terminals $R_1$ and $R_2$ (Fig. 1).

The measurement vector is $\mathbf{z} = [V_1\ V_2\ V_3]^T$, and their peak voltages is a maximum when (sinωt) is unity (i.e., at ωt=90°) and are a function of the goal variable θ only. When using equation (1), there is a redundancy in this set of equations since θ can be computed from either ($V_1$,$V_3$), or ($V_2$,$V_1$), or ($V_3$,$V_2$). Using measured voltages ($V_1$,$V_3$), the following is obtained,

$$\theta_{1,3} = \tan^{-1}[\sqrt{3}V_1/(-(V_1+2V_3))] \qquad (2)$$

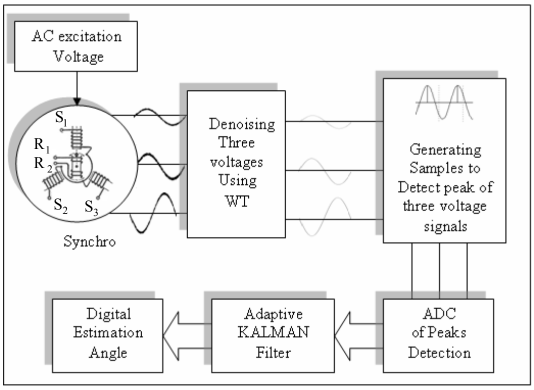


Figure 1 Layout of the synchro digitizer

Using other combinations of equation (1), it is possible to obtain other values of the same synchro shaft angle, i.e. $\theta_{2,1}$ and $\theta_{3,2}$, as shown below;

$$\theta_{2,1} = \tan^{-1}[\sqrt{3}V_1/(V_1+2V_2)] \qquad (3)$$

$$\theta_{3,2} = \tan^{-1}[-\sqrt{3}(V_2+V_3)/(V_2-V_3)] \qquad (4)$$

Theoretically the angles, $\theta_{1,3}$, $\theta_{2,1}$ and $\theta_{3,2}$ are all equal. However, in practice, they differ due to various errors. An experimental set up was assembled using a 12 bit shaft encoder (Fig.2). A 1 MS/s, 12 bit high-speed multifunction card for PCI bus (Advantech type PCI-1712) was used to read the experimental data. Table 1 shows typical peak voltages obtained experimentally. The computed shaft angles $\boldsymbol{\theta}=[\theta_1\ \theta_2\ \theta_3]^T$ are not equal for the same shaft angle. These discrepancies are attributed to random noise in $V_{ex}$, synchro coil non-linearity, etc.

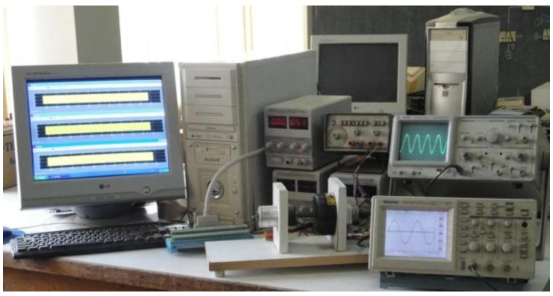

Figure 2 Synchro experimental set up

Hence, the synchro output signals can be distorted (sometimes heavily) due to nonlinearities in the synchro and phase-shift of the transducer, and they often contain noise due to the work environment [6]. A more realistic representation of the voltages that appear across stator terminals is,

$$V_k = (V_m + \beta_k).\sin(\Gamma_k + \alpha_k).\sin(\omega t) + w_k, \quad k=1,2,3 \qquad (5)$$

where, $\beta_k$, $\alpha_k$ and $w_k$ will account for the additive noise, signal distortion, synchro inaccuracies, nonlinearities, measurement errors, multiple interfering signals …etc. $\beta_k$, and $w_k$ are assumed Gaussian distribution random variables with zero mean and standard deviation ($\sigma$).

Table 1 Shaft encoder angle, computed shaft angles and synchro voltages.

| $\theta_{ref}$ | 0.559 | 44.786 | 128.130 | 229.293 | 318.79 |
|---|---|---|---|---|---|
| $\theta_1$ | 0.571 | 43.752 | 128.466 | 230.498 | 319.269 |
| $\theta_2$ | 0.578 | 44.466 | 127.432 | 229.348 | 318.994 |
| $\theta_3$ | 0.769 | 44.738 | 127.472 | 229.066 | 319.408 |
| $V_1$ | .017 | 1.014 | 1.2018 | -1.1803 | -1.0079 |
| $V_2$ | 2.338 | 0.3877 | -1.4093 | 0.2875 | 1.5079 |
| $V_3$ | -2.37 | -1.424 | 0.198 | -1.4328 | -0.5098 |

Since $\theta_{1,3} = f(V_1,V_3)$, it is possible to look at the effect of noise on the angle $\theta_{1,3}$ by partially differentiating equation (2) with respect to $V_1$ and $V_2$. Similarly, equations (3) and (4) can be differentiated to obtain;

$$\begin{bmatrix}\Delta\theta_{1,3}\\ \Delta\theta_{2,1}\\ \Delta\theta_{3,2}\end{bmatrix} = \begin{bmatrix}S_{11} & S_{13}\\ S_{21} & S_{22}\\ S_{31} & S_{33}\end{bmatrix}\begin{bmatrix}\Delta V_1 & \Delta V_1 & \Delta V_2\\ \Delta V_3 & \Delta V_2 & \Delta V_3\end{bmatrix} \qquad (6)$$

where,

$$\begin{bmatrix}S_{11} & S_{13}\\ S_{21} & S_{22}\\ S_{31} & S_{33}\end{bmatrix} = \begin{bmatrix}-V_3/D_1 & V_1/D_1\\ V_2/D_2 & -V_1/D_2\\ V_3/D_3 & -V_2/D_3\end{bmatrix},$$

and,

$$\begin{bmatrix}D_1\\ D_2\\ D_3\end{bmatrix} = \frac{2}{\sqrt{3}}\begin{bmatrix}V_1^2 + V_1V_3 + V_3^2\\ V_2^2 + V_2V_1 + V_1^2\\ V_3^2 + V_3V_2 + V_2^2\end{bmatrix}$$

$\Delta\theta_{1,3}$, $\Delta\theta_{2,1}$ and $\Delta\theta_{3,2}$ represent the error in the measurement vector $\mathbf{z}=[V_1\ V_2\ V_3]^T$. Figure 3 shows the sensitivity variations with the measured angle. Of course, these errors are randomly fluctuating about the mean value. Hence, the maximum expected error in $\theta_{1,3}$ is the absolute values of $\pm[|S_{1,1}\Delta V_1|+|S_{1,3}\Delta V_3|]$. Figure 4 shows the maximum possible positive error only for a -10% change in $V_{ex}$

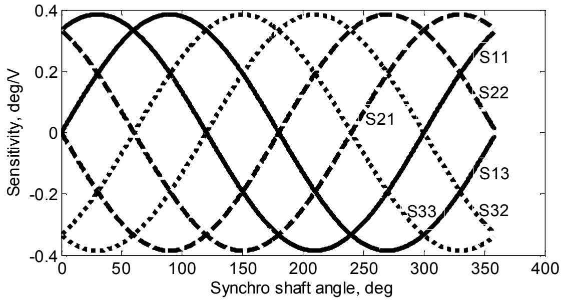


Figure 3 Sensitivities variation with the measured angle

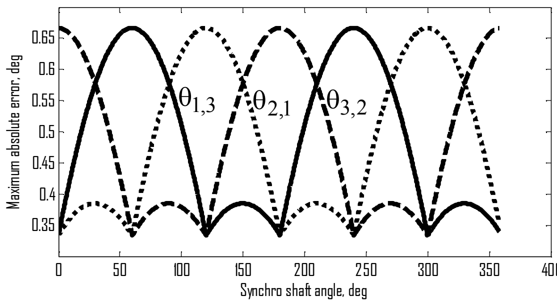


Figure 4 Maximum possible positive error for a -10% change in $V_{ex}$

Figures 3 and 4 clearly show that the noise effect is dependent on the synchro shaft angle. For example, $\theta_{1,3}$ is least sensitive to noise in the range 120-180 degree. The measurement vector $\mathbf{z}=[V_1\ V_2\ V_3]^T$ and the computed $\boldsymbol{\theta}=[\theta_{1,3}\ \theta_{2,1}\ \theta_{3,2}]^T$ therefore should be processed on-line to extract a reliable shaft angle $\theta$ from the three angles ($\theta_{1,3}$, $\theta_{2,1}$ and $\theta_{3,2}$).

## 3. Synchro Format Voltages Processing

Modern estimation theory can be found at the heart of many electronic signal processing systems designed to extract information [7]. The Kalman filter algorithm is to be used to lessen the effect of noise of each peak detection voltage and give an estimate to the most likely synchro angle. It is a powerful tool for obtaining that reliable estimate.

The weighted-Least-Square (WLS) method is adopted for the construction of the estimator because it is a common and effective method. Also it displays a good filtering performance without prior knowledge of the probability density function of the estimates which is required by other methods [8].

Easy and guaranteed estimator design is done if the number of measurement variables $[V_1\ V_2\ V_3]$ is greater than the number of state variables $[\theta]$, as it is in this case. For the nonlinear measurement system, the WLS static state estimator takes the following iterative form [9];

$$\Delta\theta=[[\mathbf{H}]^T[\mathbf{R}_c]^{-1}[\mathbf{H}]]^{-1}[\mathbf{H}]^T[\mathbf{R}_c]^{-1}[\mathbf{V}-\mathbf{V}_e] \quad (4)$$

Where; $\Delta\theta$ is the adjustment after each iterative cycle for any assumed starting $\theta$. The Jacobian matrix **H** is given by;

$$\mathbf{H}=[\partial V_1/\partial\theta\quad \partial V_2/\partial\theta\quad \partial V_3/\partial\theta]^T \qquad (5)$$

Where;

$$\partial V_k/\partial\theta=V_{ex}\cos(\theta+120(k-1)),\ k=1,2,3 \quad (6)$$

The weighted matrix $\mathbf{R}_c$ is a 3×3 diagonal matrix. Its elements are an indication of the accuracy of the related measurements. The $i^{th}$ element of $\mathbf{R}_c$ is taken as $r_{ii}=\sigma_i^2$ [8,10]. The above equations are applied iteratively until the convergence condition is satisfied. The convergence criterion is that $\Delta\theta<10^{-5}$. Figure 5 shows the proposed structure of the synchro estimator.

The wavelet transformer block (shown in Fig.5) performs a correlation analysis; therefore the output is expected to be maximal when the input signal most resembles the mother wavelet [11]. The method of using DWT (discrete wavelet transformer) as denoising tool is based on decomposing the signal into seven levels of wavelet transform by using Daubechies wavelet (db6). Also, a suitable soft threshold value is used where the minimum error is achieved between the detailed coefficients of threshold noisy signal and the original signal. The reconstruction of the signal is achieved by using IDWT (inverse discrete wavelet), as shown in Fig.6 [12].

The data pre-filtering block is used to assimilate and validate the data. It consists of limit checking and simple logic comparisons. This process is important before the estimation process is initiated in order to reject bad data entering the estimation process. This is of particular importance especially when the shaft angle is in the vicinity of $0^o$ and $360^o$.

The task of the θ update block is to alter θ after each iterative process until the convergence condition is met. The time taken for convergence depends on the initial start up of the iterative process. Initial tests showed that some times it takes 700 iterations before Δθ is less than $10^{-5}$. The on-line speed requires the speed of iteration should be less than 1 ms.

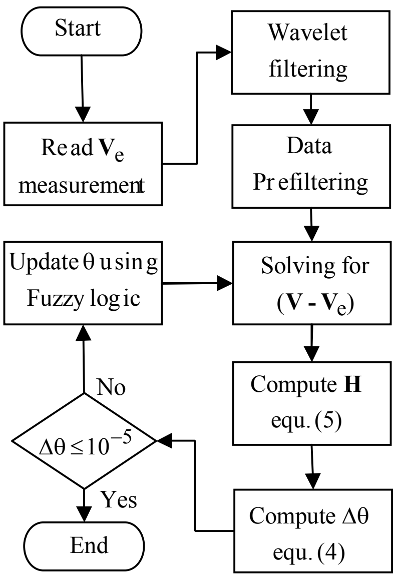


Figure 5 Synchro static state estimator

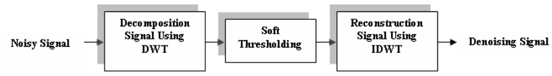


Figure 6 Denoising process using wavelet transformers

The constructed Kalman filter utilizes fuzzy logic reasoning to select Δθ, where rules are put into operation to map inputs and outputs. Expert rules are translated easily by fuzzy logic rules for any angle deviation. One input out of the five triangle membership functions input and one output out of the five membership functions are used [13], as shown in Figs.(8a) and (8b), respectively.

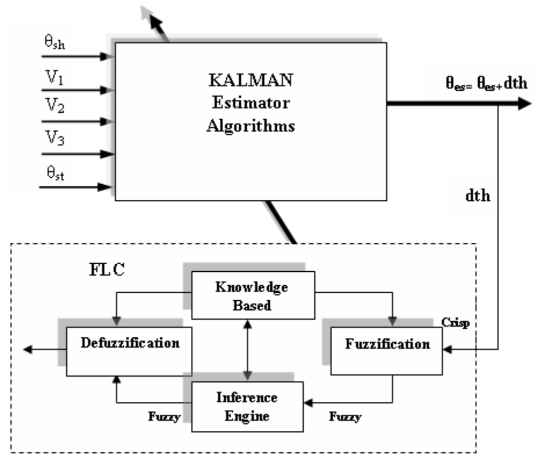


Figure 7 Adaptive Kalman estimator filter using FLC

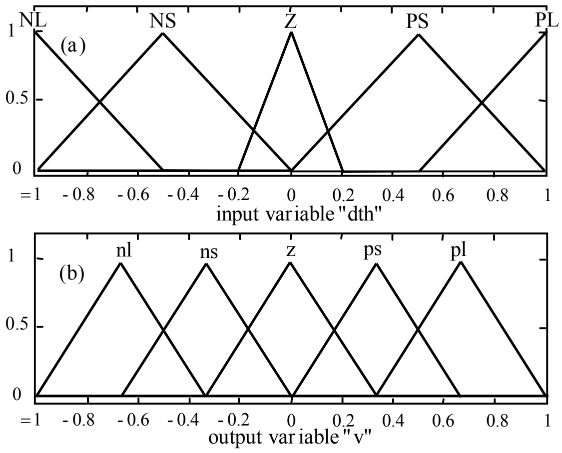


Figure 8 Membership functions ((a) input and (b) output) used in FLC to adapt the Kalman filter estimator

## 4. Simulation

The synchro digitizer shown in Fig.1 was simulated using the Matlab simulink setup shown in Figs. 9 and 10. For any given shaft angle, the raw measurements representing the synchro format voltages are generated by adding an appropriate noise to each measurement, as shown in Fig.9. In real life systems, however, these measurements are actual synchro coil readings. The function of the peak detection block is to make use of the excitation voltage to detect the peaks of the synchro voltages (Fig.1).

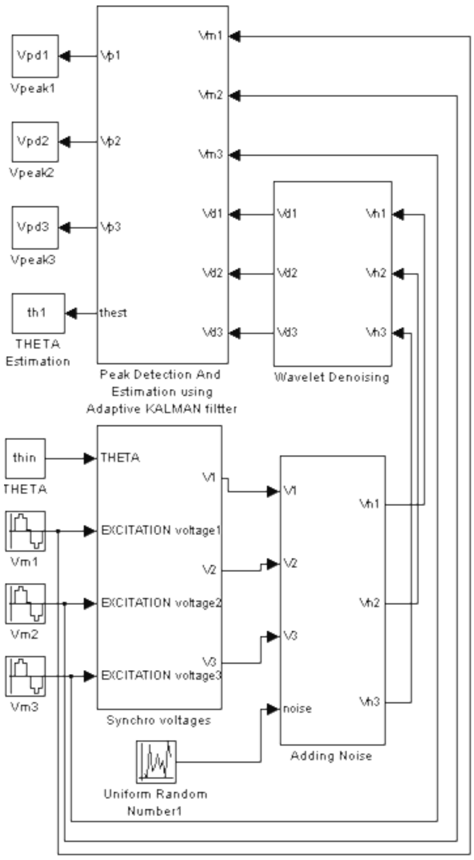


Figure 9 Matlab simulink synchro simulator

Figure 9 shows the wavelet filtering and the estimation blocks. Figure 10 shows the simulink setup block of the DWT and the IDWT shown in Fig.6. The denoising is based on decomposing the signal into seven levels of wavelet transform by using the Daubechies wavelet (db6) together with a suitable soft threshold value.

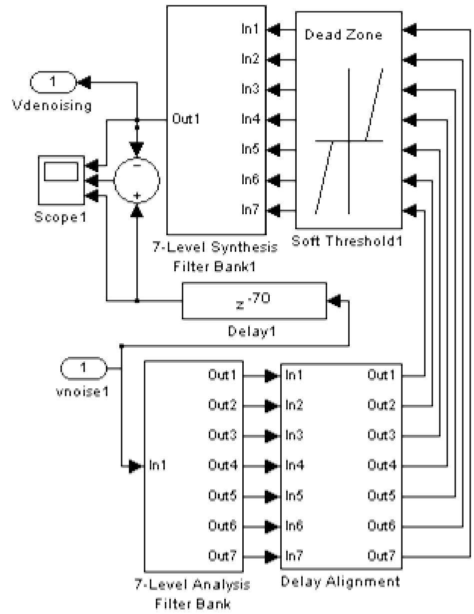


Figure 10 Daubechies based DWT and IDWT

## 5. Results and Discussion

A series of simulation tests were carried out to test the developed Kalman estimator. Typical simulation results are shown in Figs. 11 and 12 where the added random noise is ±3.5% of $V_{ex}$. Noise generates different peak voltages for the same shaft angle. This leads to errors in the estimated angle. Wavelet filtering smoothes out the estimated shaft angle reading from about $1^o$ to $0.15^o$.

For experimental purposes the simulink set up shown in Fig.9 was used, now the lower two blocks are replaced by the real synchro. Now the inputs to the wavelet denosing block are the outputs of the PCI-1712 card. A PCI-1712 data acquisition block is installed in the Matlab's data acquisition tool box. The data in tables 2 and 3 are extracted using the Matlab simulink setup. Different shaft angles generate different peak voltages [$V_1$ $V_2$ $V_3$] which are used to verify the proposed method. The tables demonstrate the effectiveness of the fuzzy logic based Kalman filter algorithm in speeding up the estimator iteration process where the number of iterations is reduced to 2-5 iterations.

Table 2 Number of iterations for various synchro shaft angles with percentage of error for each value of angle.

| $\theta_{shaft}$ | $\theta_{est}$ | $\theta_{est}$ wavelet | No. of iterations |
|---|---|---|---|
| 0.559 | 16.100 | 7.513 | 257 |
| 44.786 | -1.060 | 0.236 | 202 |
| 89.659 | 0.494 | 0.090 | 249 |
| 128.130 | -0.881 | 0.156 | 255 |
| 229.290 | 0.167 | 0.099 | 196 |
| 318.790 | 0.290 | 0.063 | 238 |

Table 3 Number of iterations for various synchro shaft angles with fuzzy logic based convergence with percentage of error for each value of angle..

| $\theta_{shaft}$ | $\theta_{est}$ | $\theta_{est}$ wavelet | No. of fuzzy iterations |
|---|---|---|---|
| 0.559 | 13.416 | 2.504 | 4 |
| 44.786 | -0.460 | -0.129 | 4 |
| 89.659 | 0.111 | -0.016 | 5 |
| 128.130 | -0.304 | -0.071 | 3 |
| 229.290 | 0.082 | 0.033 | 2 |
| 318.790 | 0.078 | 0.004 | 4 |

Figure 13 shows typical experimental synchro voltages at 44.787° shaft angle. The estimated shaft angle is shown in Fig. 14 with and without wavelet filtering. The absolute error variation is reduced from 0.6° to 0.15° over a period of 40 cycles, as shown in Fig. 15a.

Figures 15b to 15f confirm the effectiveness of the wavelet filtering where the absolute error variation is within 0.15°. It is interesting to note that at shaft angle of 359.916° (Fig. 15f), the angle fluctuates between 359.6° and 360.2° (or 0.2°). One solution is to base the final output on the domain of the values of $\theta_{1,3}$, $\theta_{2,1}$ and $\theta_{3,2}$. Any two values lying in the same domain is a most likely choice. The output can also be based on the repeatability of the angles $[\theta_{1,3}\ \theta_{2,1}\ \theta_{3,2}]^T$ after each cycle.

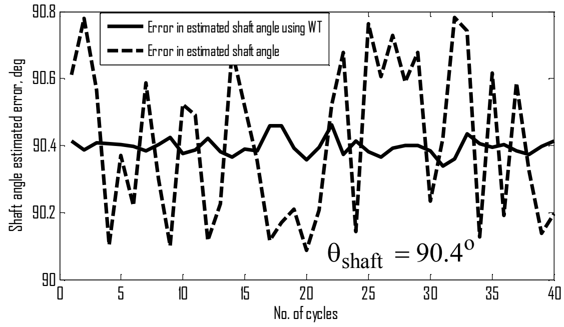


Figure 11 Simulated synchro output error with ±3.5% of $V_{ex}$ noise

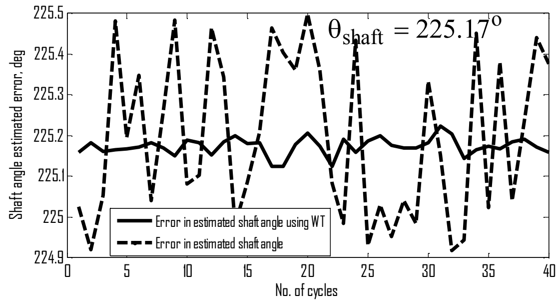


Figure 12 Simulated synchro output error with ±3.5% of $V_{ex}$ noise

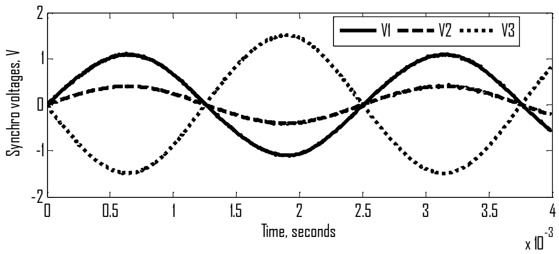


Figure 13 Synchro out voltages at $\theta_{shaft}$=44.787°

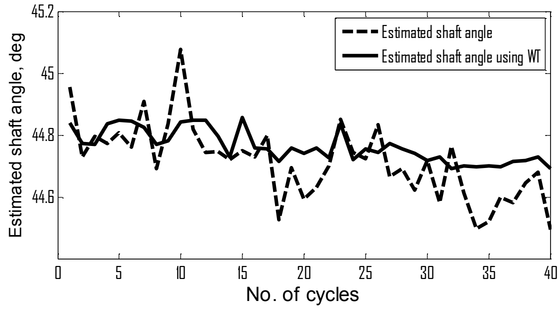


Figure 14 Synchro shaft angle at $\theta_{shaft}$=44.787°

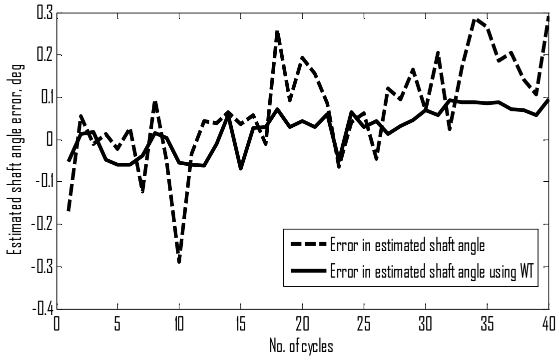


Figure 15a) Error in synchro shaft angle at $\theta_{shaft}$=44.78$^o$

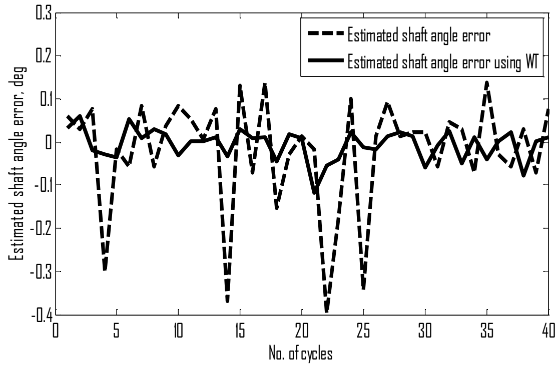


Figure 15d) Error in synchro shaft angle at $\theta_{shaft}$=269.810$^o$

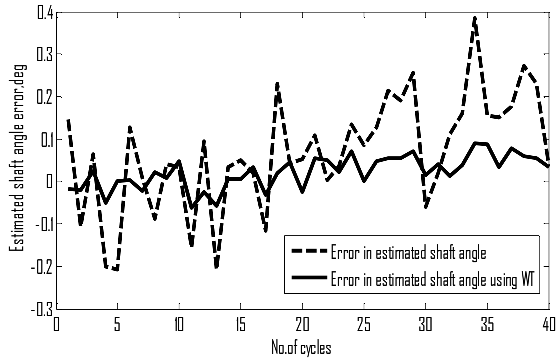


Figure 15b) Error in synchro shaft angle at $\theta_{shaft}$=128.130$^o$

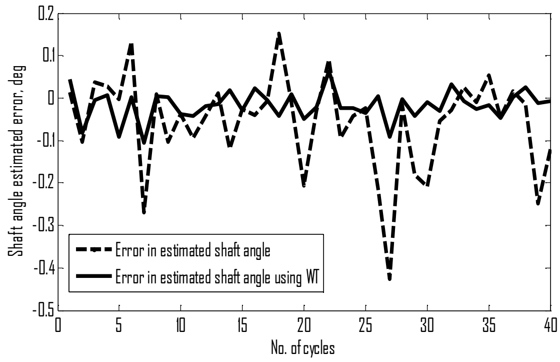


Figure 15e) Error in synchro shaft angle at $\theta_{shaft}$=318.790$^o$

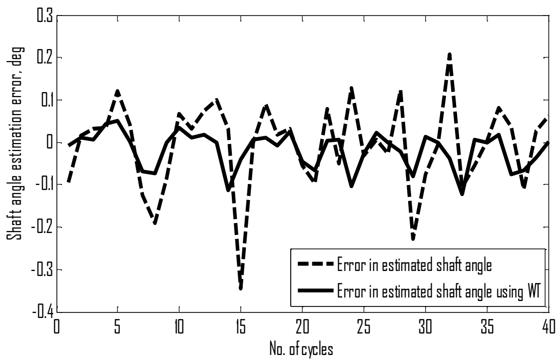


Figure 15c) Error in synchro shaft angle at $\theta_{shaft}$=229.293$^o$

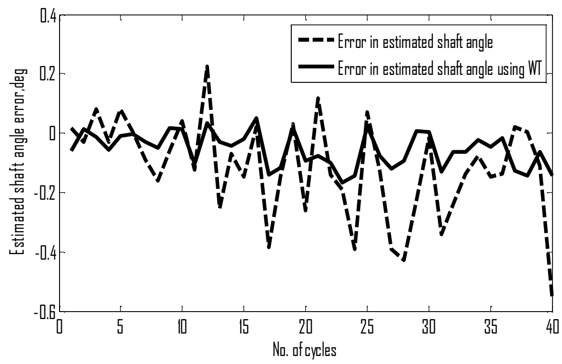


Figure 15f) Error in synchro shaft angle at $\theta_{shaft}$=359.916$^o$

6. **Conclusions**

The synchro format voltages were exploited to evaluate the shaft angle. The simulation work involved additive noise generation, filtering and estimation. Fuzzy logic was applied successfully to render the software suitable for on line estimation. The experimental application of the DWT and IDWT improved the instrument's output. The method is based on decomposing the signal into seven wavelet transform and selecting threshold value by finding minimum error of denoising and original wavelet sub signal. The proposed software processing may be improved further if experimental data is used in assessing the weighted matrix ($\mathbf{R}_c$) to include all possible noise changes that affect the measurements of the synchro format voltages as dictated by the sensor sensitivity.

***References***

[1] E.O. Doebelin, " Measurement Systems Application and Design ", McGraw-Hill, 1990.

[2] Synchro/Resolver Conversion Handbook, Data Device Corporation, 4th edition, 1994.

[3] W.K. Sa'id, and F. A. R. Al-Muttalibi, "Design and implementation of successive approximation type synchro digitizer", IJCCCE, Vol.4, No.1, 2004.

[4] W.K. Sa'id, and F. A. R. Al-Muttalibi, "Neural Networks using for measurement of angular position in Synchro-Digitizer Systems", VIII International Scientific-Practical Materials conference, Southern-Russia State Technical Univ. (SRSTU), Novocherkassk City, pp. 71-79, 2007.

[5] G. S. Boyes, "Synchro And Resolver Conversion", Memory Devices, 1980.

[6] B. Loriferne "Analogue-Digital and Digital-Analogue Conversion ", Heyden & Son, 1982.

[7] S. M. Kay, "Fundamentals of Statistical Signal Processing: Estimation Theory", Prentice-Hall, 1993.

[8] K. L. Lo, Z. M. Song, E. Marchand, and A. Pinkerton "Development of a static state estimator for a power station boiler-Part II: Estimation Algorithm and Bad Data Processing", Electric Power System Research, Vol. 18, pp. 191-203, 1990.

[9] A.J. Wood, and B. F. Wollenberg. "Power generation, operation and control", John Wiley & Sons Inc., 1984.

[10] G. M. Clarke, "Statistics and experimental design, Edward Arnold, UK,2nd edition, 1980.

[11] J. C. Goswami, and A. K. Chan, Fundamentals of Wavelets, Theory, Algorithms, and Applications, John Wiley & Sons, Inc.,1999.

[12] Mikhled Alfaouri and Khaled Daqrouq, " ECG Signal Denoising By Wavelet Transform Thresholding", American Journal of Applied Sciences ISSN 1546-9239, Volume 5; Issue 3; Start page: 276; 2008.

[13] W. Greg, and G. Bishop ,"An Introduction to the Kalman Filter", course, University of North Carolina at Chapel Hill, Department of Computer Science, Chapel Hill, NC 27599-3175, Copyright 2001 by ACM.