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Solution of First Order Complex Differential Equations by complex Sadik Transform Technique

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Article Informations

ABSTRACT

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In this paper, we introduce the complex Sadik integral transform to evaluate the exact solution of first order complex differential equations with constant coefficients . this technique offers a practical and fast strategy for resolving a variety of linear operator problems.

Key Words:

The Complex Sadik transform , Complex Differential Equation , Partial Derivatives.



Introduction

integral transformations are extremely effective solve lots of advanced science engineering problems. Many papers have applied various integral transformations (Laplace, Sumudu , SEE , ... ,etc.) and solved ordinary and partial differential equations and integral operator equations and their applications [3, 4, 5, 6, 7, 10].

A system of partial differential equations with two variables of unknown independent variants can be created from many complex differential equations. Separating actual parties from fictitious ones [8,5].Solving linear ordinary differential equation is one application of the complex Sadik integral transform ,which is employed in many applications and fields of mathematics [8].Due to the nonlinear variables , this technique Cannot be utilized to solve nonlinear differential equations .Never the less nonlinear differential equations can be resolved with the help of a differential transform and the complex Sadik technique.

In this work , we introduce the first order complex differential equations with constant coefficients . By applying Laplace , Aboodh and SEE integral transform were resolved [2 ,9] . Definition and a basic theorem are presented first. Contains numerical examples after that.

Fundamental Definition

Laplace integral transformation and other integral Transformations defined in the time domain where t is greater than or equal zero , including the Sumudu , Elzaki , and SEE I transformations , are analogous to the complex Sadik integral transformation.

Functions of exponential order are defined applying the complex Sadik transformation . We take into account the functions in the set H defined as , [1] :

$$H = \{g(t) : \text{there exists } AM, G_1, G_2 > 0 \text{ such that } |g(t)| < M e^{-iG_j t} \\ , \text{ if } t \in (-1)^j X [0, \infty), \text{ where } j = 1, 2\} \text{ where } i^2 = -1.$$

Where M is a constant must be a finite number for a particular function in the set H , and G_1, G_2 may be finite or infinite. The integral equation then defines the complex Sadik transform indicated by $S_a^c\{g(t)\}$:

$$S_a^c\{g(t)\} = F^c(s^\alpha, \beta) = \frac{1}{s^\beta} \int_0^\infty g(t) e^{-is^\alpha t} dt \quad \dots(1)$$

$$\text{Where } s \in \mathbb{C} , \text{Im}(s^\alpha) < 0 , \beta, \alpha \in \mathcal{R} , i^2 = -1 , \quad [1]$$

The Complex Sadik Transform (CST) for some Basic Fundamental Functions [1 ,8]

In this part , we explain and show some important functions have (CST) :

$$1- S_a^c\{t^m\} = (-i)^{m+1} \frac{m!}{s^{m\alpha+(\alpha+\beta)}} \text{ where } s > 0 \text{ and } m \in \mathbb{N} .$$

$$2- S_a^c\{e^{bt}\} = \frac{-1}{s^\beta} \left[\frac{b}{(s^{2\alpha+b^2})} + \frac{is^\alpha}{(s^{2\alpha+b^2})} \right] \text{ where } s > b , b \text{ is a constant.}$$

$$3- S_a^c\{\sin(bt)\} = \frac{-b}{s^\beta(s^{2\alpha-b^2})} , s > |b| , b \text{ is a constant.}$$

$$4- S_a^c\{\cos(bt)\} = \frac{-is^\alpha}{s^\beta(s^{2\alpha-b^2})} , s > |b| , b \text{ is a constant.}$$

$$5- S_a^c\{\sinh(bt)\} = \frac{-b}{s^\beta(s^{2\alpha+b^2})} , s > 0 , b \text{ is a constant.}$$

$$6- S_a^c\{\cosh(bt)\} = \frac{-is^\alpha}{s^\beta(s^{2\alpha+b^2})} , s > 0 , b \text{ is a constant.}$$

The Complex Sadik Technique (CST) of Partial Derivative

To create the complex Sadik integral transform of the partial derivatives , we apply the method of integration by parts as follows:

$$\begin{aligned}
S_a^c \left\{ \frac{\partial g(x, t)}{\partial t} \right\} &= \frac{1}{S^\beta} \int_0^\infty \left(\frac{\partial g}{\partial t} \right) e^{-is^\alpha t} dt, \\
&= \lim_{r \rightarrow \infty} \frac{1}{S^\beta} \int_0^r \left(\frac{\partial g}{\partial t} \right) e^{-is^\alpha t} dt, \\
&= \lim_{r \rightarrow \infty} \left[\frac{1}{S^\beta} e^{-is^\alpha t} g(x, t) \right]_0^r + i \frac{1}{S^\beta} s^\alpha \int_0^r e^{-is^\alpha t} g(x, t) dt, \\
S_a^c \left\{ \frac{\partial g}{\partial t} (x, t) \right\} &= is^\alpha S_a^c(x, s) - \frac{1}{S^\beta} g(x, 0). \quad \dots(2)
\end{aligned}$$

Assume that g is a piecewise continuous with exponential order.
Now,

$$\begin{aligned}
S_a^c \left\{ \frac{\partial g}{\partial t} (x, t) \right\} &= \frac{1}{S^\beta} \int_0^\infty \left(\frac{\partial g}{\partial t} (x, t) \right) e^{-iv^\alpha t} dt, \\
&= \frac{\partial}{\partial x} \left[\frac{1}{S^\beta} \int_0^\infty g(x, t) e^{-iv^\alpha t} dt \right] \quad (\text{By applying Leibnitz Rule}).
\end{aligned}$$

$$S_a^c \left\{ \frac{\partial g}{\partial t} (x, t) \right\} = \frac{\partial}{\partial x} [F^c(x, s)] = \frac{d}{dx} [F^c(x, s)].$$

Also, we can find:

$$S_a^c \left\{ \frac{\partial^2 g}{\partial x^2} (x, t) \right\} = \frac{d^2}{dx^2} [F^c(x, s)]$$

To find:

$$S_a^c \left\{ \frac{\partial^2 g}{\partial x^2} (x, t) \right\}.$$

Assume that $f = \frac{\partial g}{\partial t}$, the by applying equation (2), we get:

$$S_a^c \left\{ \frac{\partial^2 g}{\partial x^2} (x, t) \right\} = S_a^c \left\{ \frac{\partial f}{\partial t} (x, t) \right\},$$

$$= is^\alpha S_a^c f(x, t) - \frac{1}{S^\beta} f(x, 0)$$

$$= is^\alpha \left[is^\alpha F^c(x, s) - \frac{1}{S^\beta} g(x, 0) - \frac{1}{S^\beta} \frac{\partial g}{\partial x} (x, 0) \right].$$

then

$$S_a^c \left\{ \frac{\partial^2 g}{\partial t^2} (x, t) \right\} = (is^\alpha)^2 F^c(x, s) - \frac{is^\alpha}{S^\beta} g(x, 0) - \frac{1}{S^\beta} \frac{\partial g}{\partial t} (x, 0).$$

Linearity property of CST , [1]

Let $S_a^c\{g(t)\} = F^c(s)$ and $S_a^c\{h(t)\} = H^c(s)$ then for every A and B are constants:

$$S_a^c\{Ag(t) \pm Bh(t)\} = A S_a^c\{g(t)\} \pm B S_a^c\{h(t)\},$$

$$= AF^c(s) \pm BH^c(s).$$

Complex Derivative , [9]

Let $W = W(Z, \bar{Z})$ be a complex function, where the complex number $Z = x + iy$, and complex function $W(Z, \bar{Z}) = u(x, y) + iv(x, y)$. First order derivatives according to Z and \bar{Z} of $W(Z, \bar{Z})$ are defined by follows:

$$\frac{\partial W}{\partial Z} = W_Z = \frac{1}{2} \left(\frac{\partial W}{\partial x} - i \frac{\partial W}{\partial y} \right) \quad \dots(3)$$

$$\frac{\partial W}{\partial \bar{Z}} = W_{\bar{Z}} = \frac{1}{2} \left(\frac{\partial W}{\partial x} + i \frac{\partial W}{\partial y} \right) \quad \dots(4)$$

Solution of First Order Complex Differential Equations with Constant Coefficients via Complex Sadik Transform

In this section , we introduce the following important theorem with examples:

Theorem(7.1): let a, b and c be real numbers , $G(Z, \bar{Z})$ is a polynomial of Z and \bar{Z} and $W = u + iv$ is a complex function then the real $Re(W)$ and imaginary parts $Im(W)$ of solution of:

$$a \frac{\partial W}{\partial Z} + b \frac{\partial W}{\partial \bar{Z}} + CW = G(Z, \bar{Z}) , \quad \dots(5)$$

and $W(x, 0) = 0$,

are

$$u = F^{c-1} \left\{ \frac{(a+b) \frac{\partial}{\partial x} [2F_3^c(x,s) + (a-b) \frac{1}{s^\beta} v(x,0)] + 2c [(2F_3^c(x,s) + (a-b) \frac{1}{s^\beta} v(x,0)) - (a-b)is^\alpha [2F_4^c(x,s) + (b-a) \frac{1}{s^\beta} u(x,0)]]}{\Delta} - \frac{(a-b)is^\alpha [2F_4^c(x,s) + (b-a) \frac{1}{s^\beta} u(x,0)]}{\Delta} \right\}.$$

$$V = F^{c-1} \left\{ \frac{(a+b) \frac{\partial}{\partial x} [2F_4^c(x,s) + (b-a) \frac{1}{s^\beta} u(x,0)] + 2c [(2F_4^c(x,s) + (b-a) \frac{1}{s^\beta} u(x,0)) - (b-a)is^\alpha [2F_3^c(x,s) + (a-b) \frac{1}{s^\beta} v(x,0)]]}{\Delta} - \frac{(b-a)is^\alpha [2F_3^c(x,s) + (a-b) \frac{1}{s^\beta} v(x,0)]}{\Delta} \right\}.$$

$$\text{Where } \Delta = \begin{vmatrix} (a+b)D + 2C & (a-b)is^\alpha \\ (a-b)is^\alpha & (a+b)D + 2C \end{vmatrix} = ((a+b)D + 2C + (is^\alpha (a-b)))^2$$

And $F_1^c, F_2^c, F_3^c, F_4^c$ are integral transform of u, v, G_1, G_2 respectively.

Proof

We use equations (3) and (4) in equation (5), we have

$$\frac{a}{2} \left(\frac{\partial W}{\partial x} - i \frac{\partial W}{\partial y} \right) + \frac{b}{2} \left(\frac{\partial W}{\partial x} + i \frac{\partial W}{\partial y} \right) + cW = G_1(Z, \bar{Z}) + i G_2(Z, \bar{Z}) \quad \dots(6)$$

If we choose $W = u + iv$ in equation (6), following equation is obtained:

$$a \left(\frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + i \frac{\partial v}{\partial y} \right) + b \left(\frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} - i \frac{\partial v}{\partial y} \right) + 2CW$$

$$= 2 G_1(x, y) + i G_2(x, y) \quad \dots(7)$$

If equation (7) is separated to real and imaginary parts, then the following system is obtained:

$$(a+b) \frac{\partial u}{\partial x} + (a-b) \frac{\partial v}{\partial y} + 2Cu = 2 G_1(x, y) , \quad \dots(8)$$

$$(a+b) \frac{\partial v}{\partial x} + (b-a) \frac{\partial u}{\partial y} + 2Cv = 2 G_2(x, y) . \quad \dots(9)$$

By using complex Sadik Transform (CST) for above equations, then we get the following equations:

$$(a+b) \frac{\partial F_1^c}{\partial x} + (a-b)(is^\alpha F_2^c - \frac{1}{s^\beta} v(x,0)) + 2CF_1^c = 2CF^c , \quad \dots(10)$$

$$(a+b) \frac{\partial F_2^c}{\partial x} + (a-b)(is^\alpha F_1^c - \frac{1}{s^\beta} u(x,0)) + 2CF_2^c = 2CF_4^c . \quad \dots(11)$$

If equations (10) and (11) is regulate and is applied Cramer rule, then

equations are obtained:

$$(a+b) \frac{\partial F_1^c}{\partial x} + (a-b)is^\alpha F_2^c + 2CF_1^c = 2CF_3^c + (a-b) \frac{1}{s^\beta} v(x,0),$$

$$(a+b) \frac{\partial F_2^c}{\partial x} + (b-a)is^\alpha F_1^c + 2CF_2^c = 2CF_4^c + (b-a) \frac{1}{s^\beta} u(x,0),$$

$$F_1^c = \frac{\begin{vmatrix} 2F_3^c + (a-b)\frac{1}{S^\beta}v(x,0) & (a-b)is^\alpha \\ 2F_4^c + (b-a)\frac{1}{S^\beta}u(x,0) & (b-a)D+2C \end{vmatrix}}{\Delta} \quad \dots(12)$$

And

$$F_2^c = \frac{\begin{vmatrix} (a+b)D+2C & 2F_3^c + (a-b)\frac{1}{S^\beta}v(x,0) \\ (b-a)is^\alpha & 2F_4^c + (b-a)\frac{1}{S^\beta}u(x,0) \end{vmatrix}}{\Delta} ,$$

$$F_2^c = \frac{(a+b)\frac{\partial}{\partial x} [2F_4^c(x,s) + (b-a)\frac{1}{S^\beta}u(x,0)] + 2C [2F_4^c(x,s) + (b-a)\frac{1}{S^\beta}u(x,0)]}{\Delta} - \frac{(a-b)is^\alpha [2F_4^c(x,s) + (b-a)\frac{1}{S^\beta}u(x,0)]}{\Delta} \quad \dots(13)$$

Following are obtained form inverse CST of equations (12)and (13):

$$u(x,s) = F^{c-1} \left\{ \frac{(a+b)\frac{\partial}{\partial x} [2F_3^c(x,s) + (a-b)\frac{1}{S^\beta}v(x,0)] + 2C [2F_3^c(x,s) + (a-b)\frac{1}{S^\beta}v(x,0)]}{\Delta} - \frac{(a-b)is^\alpha [2F_4^c(x,s) + (b-a)\frac{1}{S^\beta}u(x,0)]}{\Delta} \right\} , \quad \dots(14)$$

$$v(x,y) F^{c-1} \left\{ \frac{(a+b)\frac{\partial}{\partial x} [F_4^c(x,s) + (b-a)\frac{1}{S^\beta}u(x,0)] + 2C [2F_4^c(x,s) + (b-a)\frac{1}{S^\beta}u(x,0)]}{\Delta} - \frac{(b-a)is^\alpha [2F_3^c(x,s) + (a-b)\frac{1}{S^\beta}v(x,0)]}{\Delta} \right\} . \quad \dots (15)$$

Example (7.1): The complex differential equation

$$3\frac{\partial W}{\partial Z} + \frac{\partial W}{\partial \bar{Z}} = 0 ,$$

with condition $W(x,0) = x^2$.

Coefficients of the above equation are $a = 3$, $b = 1$, $c = 0$

And $(x,y) = 0$. Form the theorem (7.1), we have :

$$\Delta = (a+b)D + 2c)^2 + (is^\alpha (b-a))^2 = 16D^2 + 4(is^\alpha)^2 .$$

And

$$u(x,y) = F^{c-1} \left\{ \frac{-2is^\alpha (\frac{1}{S^\beta} x^2)}{16D^2 + 4(is^\alpha)^2} = F^{c-1} \left\{ \frac{i \frac{S^\alpha}{S^\beta} x^2}{4D^2 + (is^\alpha)^2} \right\} ,$$

$$u(x,y) = F^{c-1} \left\{ \frac{\frac{1}{S^\beta} x^2}{is^\alpha [(\frac{2D}{is^\alpha})^2 + 1]} = F^{c-1} \left\{ \frac{S^{-\beta}}{is^\alpha} \left[1 - \left(\frac{2D}{is^\alpha} \right)^2 \right] x^2 \right\} ,$$

$$u(x,y) = F^{c-1} \left\{ \frac{S^{-\beta}}{is^\alpha} \left[x^2 - \frac{8}{(is^\alpha)^2} \right] \right\} ,$$

$$u(x, y) = x^2 F^{c-1} \left\{ \frac{s^{-\beta}}{is} \right\} - 8 F^{c-1} \left\{ \frac{s^{-\beta}}{(is^\alpha)^3} \right\},$$

$$u(x, y) = x^2 F^{c-1} \left\{ \frac{(-i)s^{-\beta}}{s^\alpha} \right\} - 8 F^{c-1} \left\{ \frac{(-i)^3 s^{-\beta}}{s^{3\alpha}} \right\} = x^2 - 4y^2$$

And on the other hand:

$$v(x, y) = F^{c-1} \left\{ 4 \frac{\partial}{\partial x} \left[\frac{-2is^\alpha x^2}{16D^2 + 4(is^\alpha)^2} \right] \right\},$$

$$= -16 F^{c-1} \left\{ \frac{s^{-\beta} x}{4[4D^2 + (is^\alpha)^2]} \right\},$$

$$= -4 F^{c-1} \left\{ \frac{s^{-\beta} x}{(is^\alpha)^2 \left[\left(\frac{2D}{is^\alpha} \right)^2 + 1 \right]} \right\},$$

$$= -4 F^{c-1} \left\{ \frac{s^{-\beta}}{(is^\alpha)^2} \left[1 - \left(\frac{2D}{is^\alpha} \right)^2 \right] x \right\},$$

$$= -4 F^{c-1} \left\{ \frac{x s^{-\beta}}{(is^\alpha)^2} \right\},$$

$$\text{Then } v(x, y) = -4x F^{c-1} \left\{ \frac{(-i)^2}{s^{2\alpha+\beta}} \right\} = -4xy.$$

So we obtain

$$w = u + iv = x^2 - 4y^2 - 4ixy.$$

Example (7.2) Consider the following complex differential equation :

$$\frac{\partial w}{\partial z} + 2 \frac{\partial w}{\partial \bar{z}} = Z,$$

$$\text{With condition } W(x, 0) = x$$

Coefficients of the above equation are $a = 1$, $b = 2$, $c = 0$

And $(x, y) = x + iy$. Form the theorem (7.1), we have :

$$\Delta = (a + b)D + 2C)^2 + (is^\alpha (b - a))^2 = 9D^2 + (is^\alpha)^2.$$

And

$$u(x, y) = F^{c-1} \left\{ \frac{3 \frac{\partial}{\partial x} \left[\frac{2x(-is^\beta)}{s^\alpha} \right] + is^\alpha \left[\frac{2(-i)^2 s^\beta}{s^{2\alpha}} + 2s^{-\beta} x \right]}{9D^2 + (is^\alpha)^2} \right\},$$

After simple computations, we get:

$$u(x, y) = F^{c-1} \left\{ \frac{8(-i)^3 s^{-\beta}}{s^{3\alpha}} \right\} + x F^{c-1} \left\{ \frac{(-i)s^{-\beta}}{s^\alpha} \right\},$$

$$u(x, y) = 4y^2 + x$$

And on the other hand:

$$v(x,y) = F^{c^{-1}} \left\{ \frac{3 \frac{\partial}{\partial x} \left[\frac{2(-i)^2 s^{-\beta}}{s^{2\alpha}} + p(x)x \right] + is^\alpha \left[\frac{2(-i)S^\beta x}{s^\alpha} \right]}{9D^2 + (is^\alpha)^2} \right\},$$

$$= F^{c^{-1}} \left\{ \frac{3s^{-\beta} - 2s^{-\beta}x}{(is^\alpha)^2 [(3D)^2 + 1]} \right\}, \text{ After simple computations, we get:}$$

$$v(x,y) = (3 - 2x)y.$$

So, we obtain

$$w = 4y^2 + i(3 - 2x)y.$$

Conclusion

In this work, we can use the complex Sadik Transform (CST) to solve the 1st order complex differential equations with constant coefficients by introducing two examples that explain the interest of the complex Sadik integral (CST) of reaching to accurate solution.

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حل المعادلات التفاضلية المعقدة من الرتبة الأولى باستخدام تقنية تحويل صاوق المعقد

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الملخص:

في هذه الورقة، نقدم تحويل صاوق التكامل المعقد لأيجاد الحل الدقيق للمعادلات التفاضلية المعقدة من الرتبة الأولى ذات المعاملات الثابتة. تقدم هذه التقنية استراتيجية عملية وسريعة لحل مجموعة متنوعة من مشاكل المؤثر الخطي.