

# Simulation and Numerical Analysis in the Steady State Error (S.S.E) Performance of the Variable Structure Control System ( VSC ) at Different Types of Switching Sequences Conditions (S.S.C)

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## Abstract

In this research many tests are accomplished to different types of variable structure controller which operated by sequence timing period. A second and third order control systems are used as a mathematical fundamental to analysis the controller under test. It is found that the proper choosing of arrangement alteration between the structures  $S_1$  and  $S_2$  is achieved by using the proportional or the proportional-integral controllers separately. This will lead to treatment the steady state error and increasing the stability of control system. Also, the efficiency of the control system will be increased depending on some types of switches such as  $R_{ss}$  or  $I_{ss}$  which are defined by  $V_{ss}$  value and exterior disturbances as shown in the design of robot arm and electric lifts.

In this work, we found that depending on the type of switch, the control system may fail in disturbances and a steady state error treatment that is result in a fail in alteration between the two structures  $S_1$  and  $S_2$  of the control system. This will give unsatisfactory response of the control system. Best results are achieved by using intelligent control system where the information is compared by using a feedback system.

## الخلاصة

في هذا البحث تم إجراء اختبارات لأنواع مختلفة من المسيطرات ذات التركيبية المتغيرة التي يمكن السيطرة عليها من خلال الفترة الزمنية المحددة. تم استخدام منظومات السيطرة ذات الدرجة الثانية والثالثة كأساس لوضع الصيغ الرياضية والنظرية لهذا التحليل لدراسة الخصائص الأساسية ومدى تأثيرها في هذه المنظومات. لقد لوحظ من خلال الاختبارات أن الاختيار الصحيح لترتيب التبديل بين التركيبين  $S_1$  و  $S_2$  يتم الحصول عليه من خلال المسيطر التناسبي والمسيطر التناسبي-التكاملي كلاً على إنفراد مما يؤدي إلى معالجة بعض الحالات وبالأخص خطأ الحالة المستقرة وزيادة إستقرار المنظومة وزيادة كفاءتها بالأعتماد على أنواع من المفاتيح المستخدمة مثل ( $R_{ss}$  و  $I_{ss}$ ) والتي تحدد بقيمة الـ ( $V_{ss}$ ) وكذلك نوع الأضطرابات الخارجية وكما هو ملاحظ في تصميم منظومة الذراع الآلي والسيطرة على المصاعد الكهربائية.

بينت نتائج الاختبار حالات محددة لفشل المنظومة ذات التركيبية المتغيرة بالأعتماد على نوع محدد من المفاتيح ولفترة زمنية محددة في معالجة الأضطرابات وعلى وجه الخصوص فيما يتعلق بوجود نسبة حالة الخطأ المستقرة وعدم تحقيق دالة التبديل بين التركيبين  $S_1$  و  $S_2$  لمنظومة السيطرة إضافة إلى حالات توليد إستجابة خطأ غير مقبولة. لقد وجد بأن أفضل النتائج تتحقق عندما تكون هنالك موائمة بين وظائف المنظومة ذات التركيبية المتغيرة ووظائف منظومة السيطرة الذكية التي من خلالها يتم إعادة المعلومات من خلال منظومة تغذية عكسية.

## 1- Introduction:

A control engineer has to decide what to control, how to control and with what accuracy? Each control system has a set of parameters describing its condition.

Theoretically; all the parameters associated with the process or system can be controlled will affect complexity of control. Complexity and accuracy of control will lead to cost of control [M.D.Desai,2008].

For example control of generator voltage , voltage regulation and frequency may be significant for ac generator; so problem is to identity the most or more significant parameter, the accuracy of control required for specific application and the mode of control which may be simple (on/off) or (proportional integral, derivative) control.

The control system may be open loop or closed loop. In open loop control there is no feedback (F.B), control loop is open [Ghosh,1982].

The analysis of variable structure controller (VSC) designed for certain type of reference input, in the presence of different types of switching sequences and disturbances [Roy choudhury, 1986].

A type of switching surface, called moving switching surface (Mss) to improve the robustness of voltage switching surface (Vss), to improve the robustness of Vss. The Mss was designed first to pass arbitrary initial conditions and subsequently to move toward the predetermined switching surface by rotating and shifting [Choi, S.B.cheong, 1988].

The sliding modes of these large algorithm feature abounded control continuously depending on time, with discontinuities in the control derivate, it was also shown that the sliding accuracy is proportional to the square of switching time delay [Utkin, 1973].

Two design methods of switching surfaces are presented which have the interpretation of linear operators, the first based on pole placement, where as the second is based on frequency-shaped quadratic optimal control formulation. The (Vss) model reference adaptive control when only i/p and o/p measurements are available, state variable filters are used in order to obtain differentiator-free controller and when the plant parameters are unknown [Bartolini,1995].

## 2-Theory:-

In this research the field variable structure systems have been maintained at a high level and dictated to sliding mode control which matter effects the interest of control the assists and practically engineers sliding mode control enable efficient control of second order and high-order nonlinear plants operating under uncertainty conditions, which is common for a wide range of modern technological processes [A.Kumar, 2012; A.Rubaai, 2007].

VSC is a viable high speed switching feed back control; for example the feed forward gain is switched between two values or the gains in each feedback path are switched between two value according to some rule, this (VSC) law provides an effective and robust means of controlling linear as well as nonlinear Series Input Series Output (SISO) or Multi Input Multi Output (MIMO) system. VSC is a rather different approach to control; system design in which the controller structure (as a function of states) is changed in order to achieved certain desired objectives [Itkis and Jon Willy, 1978].

The advances in computer technology and high speed switching circuits have made the practical, implementation of VSC a reality and of reasing interest to control engineer so the application area is very large ranging from simple servo system to so phisticated robot arms so it's expected that (VSC) replace the (on-off) servo systems normally used in missiles and aircraft control systems [A. Kumar, 2012].

The VSC offer to the control designer new possibilities for improving the quality of comparison with fixed structures systems, since the wider range of control action, simples a large range of admissible transient processes in the principle of what is called (on-off servo) due to the closer of the VSC.

Let as consider the Characteristics as shown in fig (1-a) which is a relay of a very high sensitivity. A simplified on-off servo system is shown in fig (1-b); it's a nonlinear second-order control system which can be analyzed in phase plane. The nonlinear element of fig (1-a) is usually described by an equation of the form:-

$$F(e) = \text{sgn}(e) \dots \dots \dots (1)$$

Where (e) is the actuating error signal.

The control system shown in Fig (1-b) can be now described mathematically by 2<sup>nd</sup> order nonlinear or differential equation in dimension less-unit and for zero reference input  $r(t)=0$ ; by the form.

$$\frac{d^2x}{d\tau^2} + \frac{dx}{d\tau} + \text{sgn}(x) = 0 \dots\dots\dots (2)$$

Equation (2) can be re-written as a system of two equations.

$$\frac{dy}{d\tau} + y = M; \frac{dx}{d\tau} = y \dots\dots\dots (3)$$

Where  $M = -\text{sgn}(x)$ ; i.e  $M=1$  for  $x < 0$  and  $M=-1$  for  $x > 0$ .

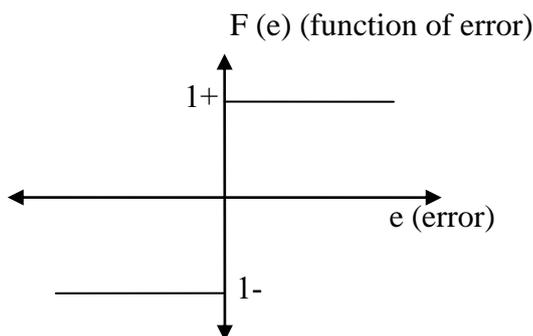


Fig (1-a) symbolic representation of the jump in the servo system

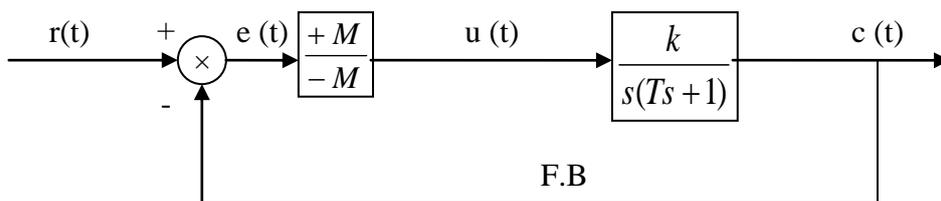


Fig (1-b) A simplified on-off servo system

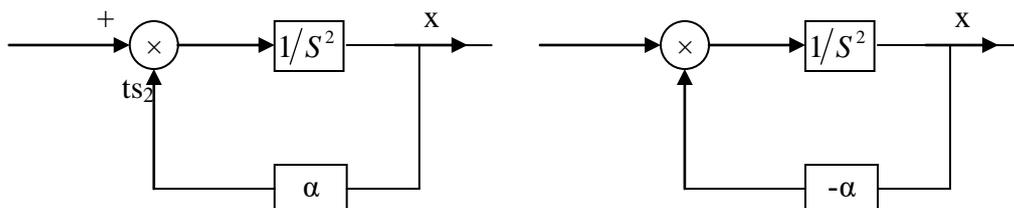
Which are the location that can be Fig (2) is called the phase-plane; x-y (or x-x) for different values of initial conditions as following on integration by under initial conditions such that  $\tau = 0, x = x_0$  and  $y=x(0) = y_0$ ; the equation of phase trajectories are obtained.

$$x = x_0 + y_0 - y + M \ln \frac{y_0 - M}{y - M} \dots\dots\dots (4)$$

Depending on the value of (M) two equations can be obtained;

$$x = x_0 + y_0 - y + \ln \frac{y_0 - 1}{y - 1}; M = 1 \dots\dots\dots (5)$$

$$x = x_0 + y_0 - y - \ln \frac{y_0 + 1}{y + 1}; M = -1 \dots\dots\dots (6)$$



Fig(2) block diagram of two variable of values structures from equation (8).

A phase trajectory representing a specific process in a system having the block diagram of Fig. (1-b) under some initial conditions will be made up of trajectories for  $M=1$ ; and  $M=-1$  [Stepanenko.Y,2006].

The plotting curve converge towards the origin is called ellipse trajectory source; this is an indication that the system is stable and transient oscillators that takes place in it will gradually be damped out; however the settling time may be large.

Therefore, if the control law  $(Kx)$  is chosen so that  $K$  switches between two different values which change the structure of the closed loop system from one of these forms to another; the trajectories quality different from can be obtained.

$$K = \begin{bmatrix} \alpha - b; 0 < t < t_s; \alpha > 1 \\ -\alpha - b; t < t_s \end{bmatrix} \dots\dots\dots (7)$$

Where the switching sequence times ( $t_s$ ) will be considered later then the closed loop system takes on one of the two structures; where  $\alpha$ : damping ratio of control unit.  $\dot{X} + \alpha X = 0$  ;  $\ddot{X} - \alpha \dot{X} = 0$ ..... (8)

Therefore the closed loop trajectories will take the resulting system is globally an asymptotically stable; even though the two (F.B) systems fig (2) are critically stable and unstable. Now if the initial switching sequence conditions are defined at point (a); i.e in region II; then the control should be  $u_1 = (\alpha-b)x$  until the trajectory inters region (I), at point (b) and at once the control is switched to  $u_2 = (-\alpha-b)x$  until point (c) on line  $X + \sqrt{\alpha} X = 0$  which moves directly to the origin.

In this case these are two moments of switching conditions; then only one moment at switching is required to settle down the system as shown in Fig. (3).

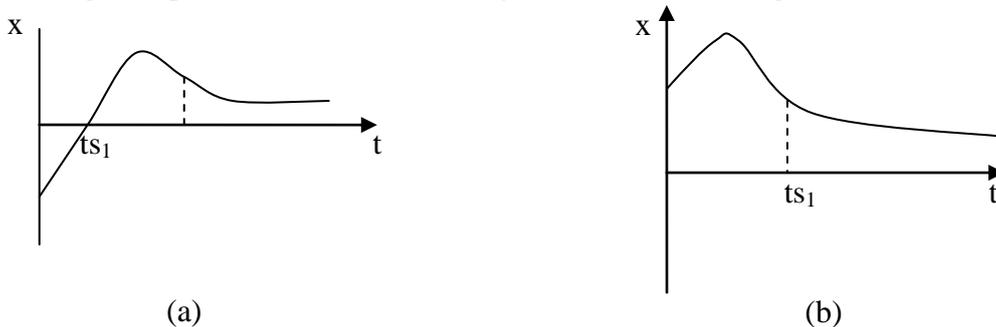


Fig (3) shows two moment of switching conditions with VSC plant

The switching circuit should be designed to realize at each moment the trajectory motion in one of the four regions and hence to switch to the corresponding control law [Young, 1999].

Consider a plant with two accessible states and one control input of the form by numerical method is given by:-

$$u = k_1 x_1 + k_2 x_2 \dots\dots\dots (9)$$

Where the state (F.B) gains  $K_1$  and  $K_2$  take on two possible values say  $\alpha_1$ ;  $\alpha_2$  and  $\beta_1$ ,  $\beta_2$  respectively; the state model:-

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \dots\dots\dots (10)$$

Where  $K(x_1)$  can be equal to  $(-2$  or  $+3)$  according to certain function of  $x_1$ . The considered simplified blocked diagram Fig (4) [Roy choudhury, 1986].

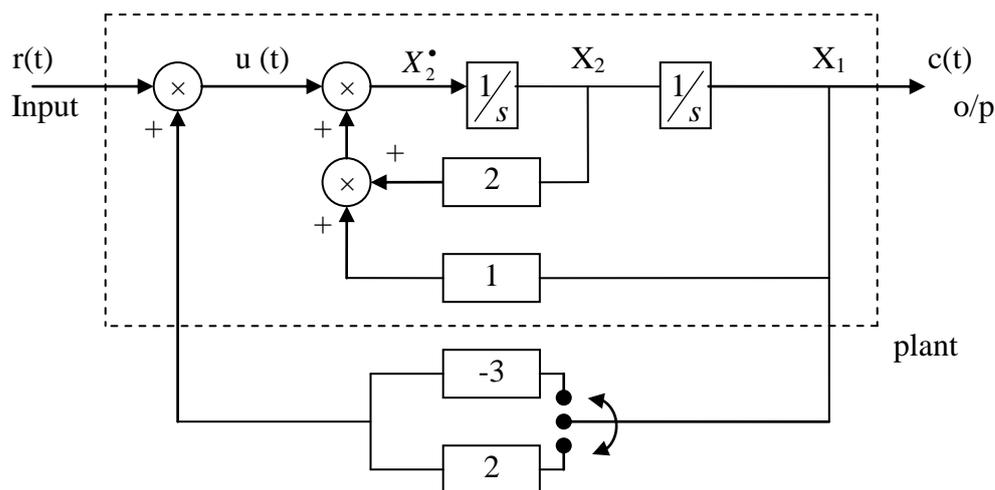


Fig (4) simplified block diagram by linear structure for (VSC) in the plant system.

The main effect of disturbance leads to Steady State Error (S.S.E) in the control system, however this error can be made small by increasing the controller gain providing that the stabilities conditions remain valid. The effect of disturbance can be classified to three cases according to the location of its occurrence; these are: i/p, o/p or in any location of the F.B or feed forward paths for all these cases we can draw the block diagram of Fig (5) [Espana, M.D, 2009].

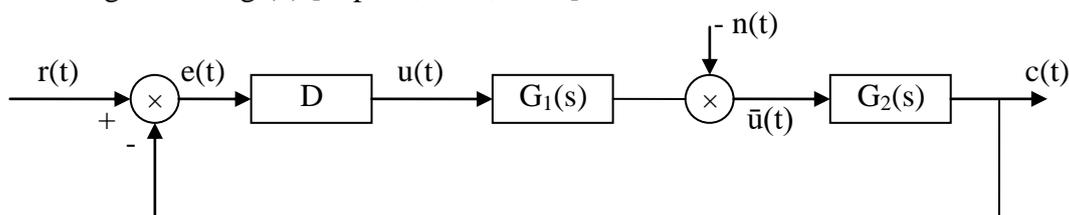


Fig (5) control system under disturbance.

Where D is the controller, n(t) is the disturbance signal and G(s) is the open loop transfer function of the plant; assuming G (s) is divided into two parts G<sub>1</sub>(s) and G<sub>2</sub>(s).  
 $G(s) = G_1(s) G_2(s) \dots\dots\dots (11)$

If n(t) has to be in the i/p of the system then G<sub>1</sub>(s) = 1; at the o/p then G<sub>2</sub>(s)=1. note the -ve of [n(t)] which is used for simplicity of the mathematical derivation that [Banks,S.P,1996]:-

$$\frac{E(s)}{N(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)D(s)} \dots\dots\dots (12)$$

From non-zero steady state error in some applications is not preferable or even permissible. Proportional plus integral amplifiers are the simplest controller cases where the (PI) proportional-integral controllers have the advantage of improving the dynamic response in the steady-state, specifically to obtain (zero S.S.E.). Obviously connecting (PI) controller affects the control system bandwidth (B.W); low gain constant slows the response while higher gain constant may even bring the system to unstable region; then show the blocked diagram in Fig (6).

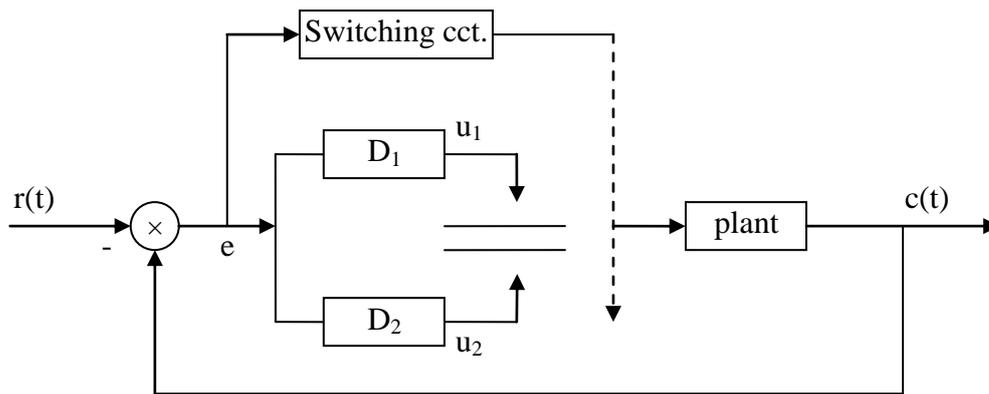


Fig (6) block diagram (p) and (PI) controller for switching circuit with VSS. [Ogata, K, 2005].

### 3- Simulation, results and calculations:

Consider the VSS fig (6), first, one may argue to switch similar controller, i.e both ( $D_1$ ) and ( $D_2$ ) are of (PI) type. In this case the VSC becomes of type one system all the time and the VSS design procedure is conventional. But since the system order is increased then an additional interial element is added, which decreases the response speed in the first part of the response, while this is preferable in the 2<sup>nd</sup> part due to the damping increment. It will be more advantageous to ( $S_1$ ) a proportional controller (P) with high gain constant without stability considerations and (PI) with significant small gain ( $g$ ) constant in the ( $S_2$ ). Such switching sequence condition (S.S.C) will ensure good dynamic  $c/c_s$  and zero (S.S.E). Consider VSS 2<sup>nd</sup> order system of zero type with (P) and (PI) controller as shown in Fig (6).

$$\left. \begin{aligned} u_1 &= c_1 e(t) \\ u_2 &= c_2 e(t) + c_3 \int e(t) dt \end{aligned} \right\} \dots\dots\dots (13)$$

At certain switching moments ( $t_s$ ); it is required that  $S(t_s)=0$ ; a numerical method should be used to obtain the value of this switching moment ( $t_s$ ), the application of (Vss) as mentioned Fig (7) and Fig (8) for 2<sup>nd</sup> order plant.

Fig (7) depicts the response of type one control system for both step and ramp inputs. Unlike for conventional control, Vss with (PI) and controller gives zero steady state error for both i/ps. Fig (8) shows the response of type zero control system; when Vss using (P) and (PI) controllers is implemented again zero steady state error is profit.

In the adaptive scheme offers the control in the sense that a proper sequence of connecting (P) and (PI) structures has to be performed and hence connecting the right switch circuit for conventional control when the system suffers from disturbance.

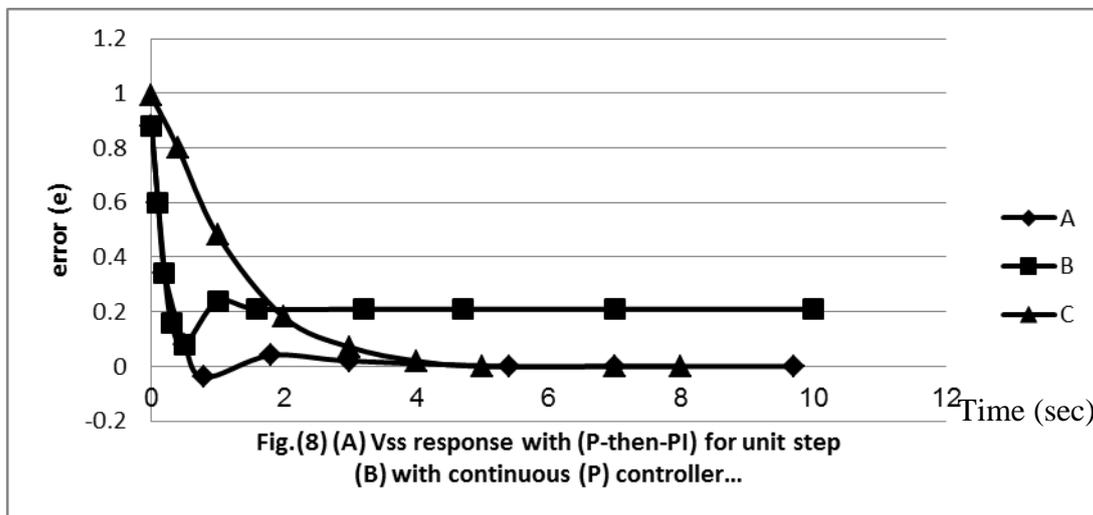
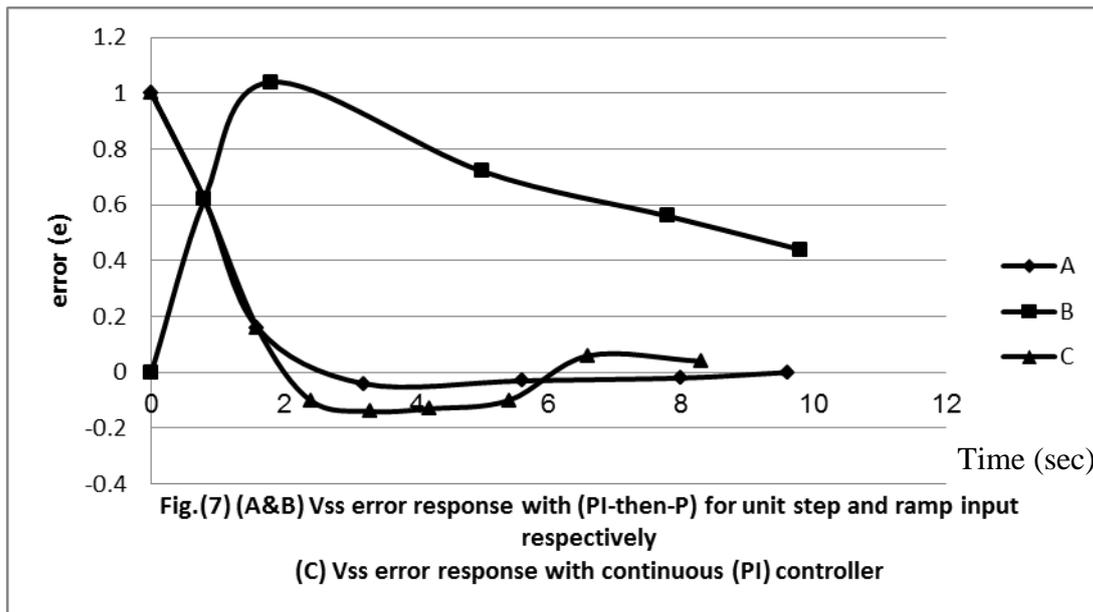


Fig (9) shows a proposal for such control then principle is to use a system model and controlled system is applied the o/p/s from both channel are subtracted to generate another actuating signal  $\hat{e}(t)$ . the latter acts on an additional switching cct  $SC_3$ . when the controlled system doesn't suffer from any kind of disturbance then  $\hat{e}(t)$  will be zero and the purpose of  $SC_3$  will be to connect the switching cct  $SC_3$  which is designed according to control mode,  $\hat{e}(t)$  will be no longer zero and hence  $SC_2$  is now connected.

A general mathematical model of such adaptive scheme is moderate benefits for synthesis purpose; the switching circuit.  $SC_1$  and  $SC_2$  can be designed separately. The third switch circuit. Is no more than a logical computer, however for certain case, a simulation a logarithm is easy to be performing.

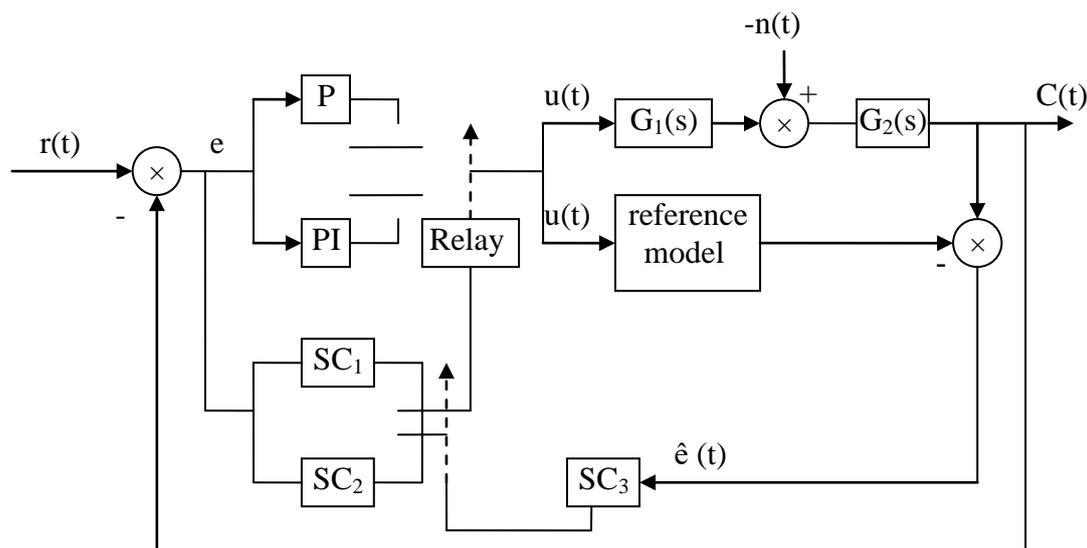


Fig (9) adaptive VSS using (P&amp;PI) controller

The simulation results are obtained through the following two cases.

- 1/ Application to second order system using derived analytical formula.
- 2/ simulation for higher order system using numerical analysis programs.

The insensitivity to parameter variations in Vss, is a consequence of the existence of sliding regime. So the quality of the control design is related to the switching function accuracy and switching conditions selections; which leads to validate the conditions of the existence of sliding regime.

Therefore there is need to introduce some other forms of switching conditions in order to a capability of switching occurrence mostly and to improve the robustness of Vss.

Below are the switching conditions which have been investigated in the simulation cases.

$$SW_1 = S(t) \cdot \text{SGN}(e)$$

$$SW_2 = S(t) \cdot \text{SGN}(\bar{e})$$

$$SW_3 = (\bar{e} \text{ equal } 0) \text{ single switching occurrence.}$$

$$SW_4 = S(t) \cdot \text{SGN}(e) \text{ single switching occurrence.}$$

These kinds of switching conditions imply two possibilities of switching moment occurrence. So far, our interest has been confined to the existence of sliding regime in a Vss with in the proposed mode of switching sequence; which will be called inverse switching sequence (Iss) in the face of the regular switching sequence (Rss).

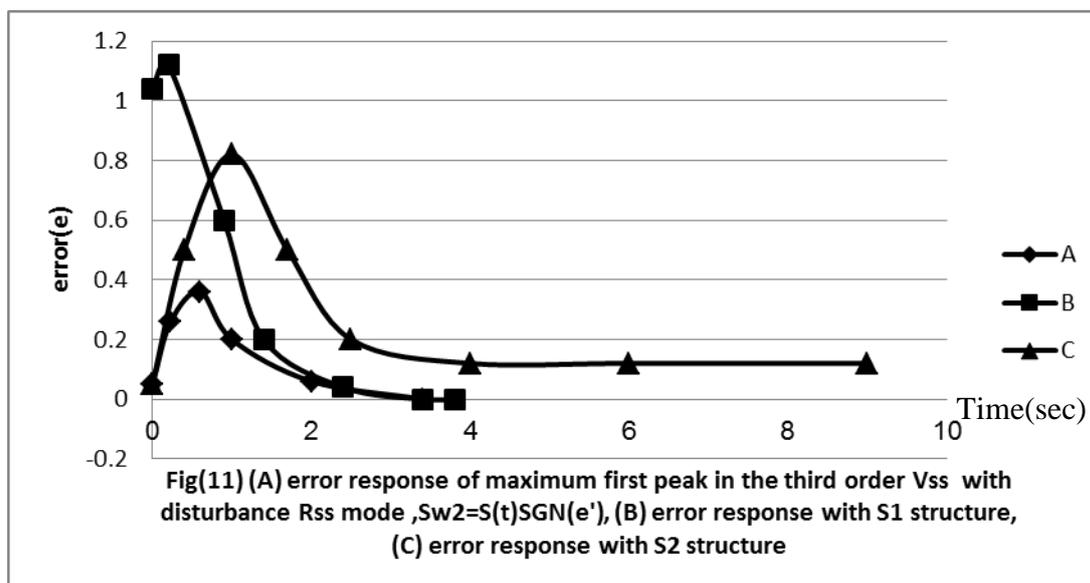
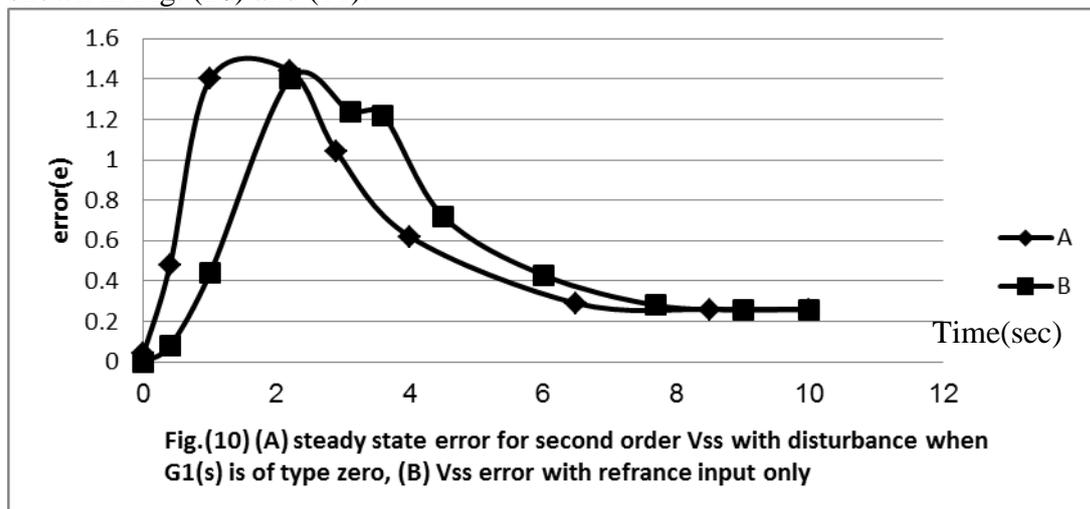
This type of switching sequence Iss, has been proposed in order to enhance the robustness with respect to external disturbances. Therefore in Rss the under damped structure preceded the over damped structure.

The simulation of numerical examples structures and properties are shown in table (1).

Table (1) simulation examples structures and properties for (2-switches)in Iss-Rss model.

System	Properties	Under damped $S_1$	Over damped $S_2$
2-nd order system $\frac{1}{s(s+1)}$	$\frac{E(s)}{R(s)}$	$\frac{s^2 + 0.8s}{s^2 + 0.8s + 1}$	$\frac{s^2 + 6s}{s^2 + 6s + 1}$
	c/cs roots	$-0.43 \mp j0.861$	$-0.637, -4.243$
	Controller parameters	$K_0=1, K_1=0.2$	$K_0= 3.4, K_1= -4.5$
3-rd order system $\frac{1}{s(s+1)(s+2)}$	$\frac{E(s)}{R(s)}$	$\frac{s^3 + 8.25s^2 + 20.2s}{s^3 + 8.25s^2 + 20.2s + 18.2}$	$\frac{s^3 + 2s^2 + 3.5s}{s^3 + 2s^2 + 3.5s + 7}$
	c/cs roots	$-2 \mp j2.3$	$-2.4; 2.8; -3.2$
	Controller parameters	$K_0=8, K_1=-3, K_2=1$	$K_0= 16.32, K_1= -20.2, K_2= -4.24$

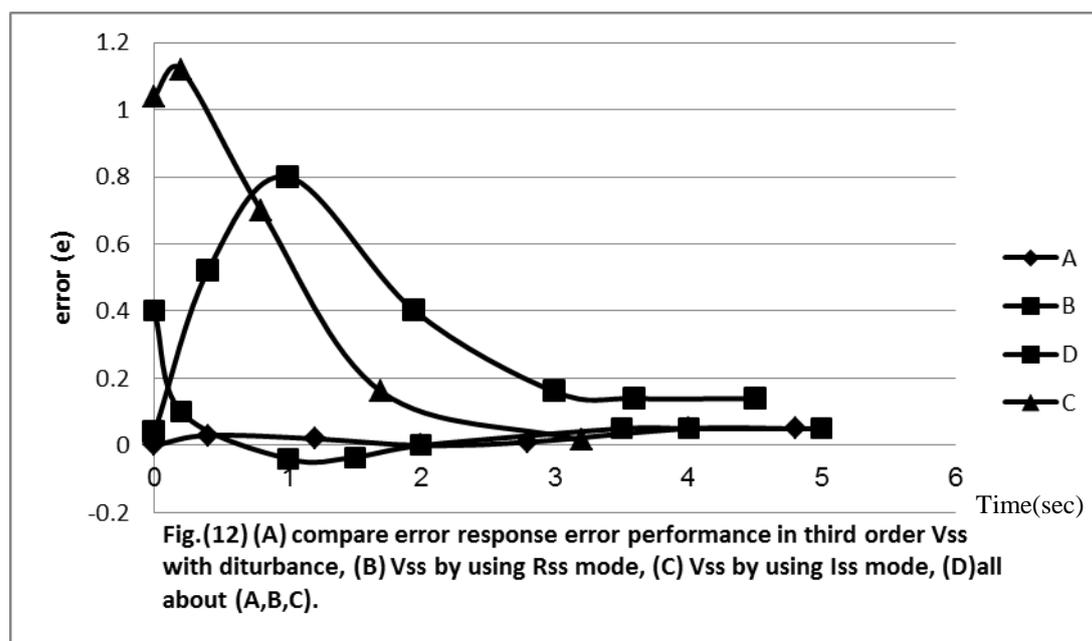
The error response possesses the highest maximum peak by using Rss mode by using switching conditions ( $SW_2$ ) after that by using ( $SW_1$ ) for 2<sup>nd</sup> and 3<sup>rd</sup> order as shown in Fig. (10) and (11).



The error response almost possesses the lowest maximum peak, by using ISS mode the switching conditions ( $SW_2$ ) and the ( $SW_1$ ) shown in Fig (11).

In order to evaluate the efficiency of different types of switching conditions on the VSS behavior, it is interesting to make performance comparison. The analysis of the simulation results leads to the following conclusion:

The switching condition  $SW_2 = S(t) \text{SGN}(\bar{e})$  gives VSS among all other switching conditions the best performance, when  $G_1(S)$  is of type one and the switching sequence is Iss mode Fig (12).



This switching condition ( $SW_2$ ) represents the best, according to these rules, while it becomes the worst in other cases, because of non-switching occurrence and oscillatory condition.

Therefore the proper selection of the switching condition and the switching sequence mode, lead to improved system robustness with respect to external disturbances even if there was no overall performance recovery.

All possible formulation of switching conditions are tested for both sequence mode of switching Iss and Rss as shown in table(2).

Also in a variety of this formulation it is noted that although the existence of switching is achieved the response is not free of steady state error or it is of oscillatory unacceptable nature.

Table (2) the switching timing ( $t_s$ ) for different switching conditions (Iss), (Rss) and effected the response.

Switching timing (msec.)	Switching conditions mode	Types of structures work	Av. (S.S.E)
$t_1=0.7 ; t_2=1.03$ $t_3=3.46 ; t_4=5.80$	Iss	$S_1$	0.22
$t_1=1 ; t_2= 3.25$ $t_3=6.24 ; t_4=7.32$	Rss	$S_2$	0.737

#### 4- Conclusions

The points below summarize the conclusions:-

- 1- The system deep analysis and mathematical formulation for Rss and Iss of disturbance effect at (o/p) state F.B; and the solution for successful switching circuit algorithm.
- 2- From switching sequences conditions if the first connected (S1) has complex conjugate roots then at ( $t_m$ ) the error signal is as that with the case ( $r(t) \neq 0$ ) and hence the same principle can be used to switch as with control i/p signal.
- 3- Investigating (Vss) ability to withstand stochastic disturbance by using model reference adaptive system control activity to eliminate by VSC in the effectiveness (S.S.E) response.

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