Statistical Analysis of Rainfall Ecords of Some Iraqi Eteorological Stations

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Abstract

Since rainfall is an integral component in the hydrologic cycle, engineers must be able to quantify rainfall in order to design structures dealing with the collection, conveyance, and storage of runoff. In this paper data of annual rainfall depth (mm) from 1969 to 2009 years for four meteorological stations in Iraq, namely, Baghdad, Najaf, Kerbala, and Diwaniya are analyzed to determine the characteristics of the observed frequency distributions. An attempt is made to fit four of the available theoretical distributions, i.e., the Normal, Log Normal, Log Normal type III, and Gamma distributions. The Chi-Square and Kolmogorov-Smirnov indices, and the graphical goodness of fit tests are applied to compare each of the theoretical distributions and those observed. In the remainder of this paper Intensity-Duration-Frequency (IDF) curves for extreme rainfall values for the four meteorological stations in Iraq with durations of 15, 30, and 60 min. are obtained. Gumbel's extreme value distribution, the Normal and the Log Normal distributions are used to fit the annual extremes rainfall data with 5, 10, 15, and 50 years return periods.

الخلاصة

يعد المطر من المكونات المتممة في الدورة الهيدرولوجية، لذلك يجب على المهندسين أنْ يَكُونوا قادرين على تَحديد كمياته لكي يتم تصميم المنشات التي تتعامل مع تجميع، نقل، وخزن المطر. في هذا البحث تم تحليل بيانات عمق المطر السنوي (مليمتر) من سنة 1969 إلى سنة 2009 لأربع محطات أرصادية في العراق، بغداد، نجف، كربلاء والديوانية، لغرض إيجاد خصائص توزيعات التكرار المُلاحَظِ. أجريت محاولة لمُلاثَمَة أربعة مِنْ التوزيعاتِ النظريةِ المتوفرةِ للبيانات وهي التوزيع الطبيعي، التوزيع اللوغارتمي الطبيعي، التوزيع اللوغارتمي الطبيعي من النوع الثالث و توزيع كاما. تم تطبيق فحص مربّع كاي وفحص كولموكروف- سميرنوف وفحص آخر يعتمد على الطريقة البيانية وذلك لغرض مقارنةِ التوزيعاتِ النظريةِ مع التوزيعات المشاهدة. في الجزء المتبقي من هذا البحث تم الحصول على منحنيات شدة المطر ومدته والتكرار لقيم المطر العليا للمحطاتِ الإرصادية الأربعة لمدة 15، 30، و دقيقة. وتم استعمال توزيع كمبل المتطرّف، التوزيع العلوي المولريةِ مع التوزيعات المشاهدة. في الجزء المتبقي من هذا دقيقة. وتم استعمال توزيع كمبل المتطرّف، التوزيع التوزيع اللوغارتمي الطرية مع التوزيعات المشاهدة. في الجزء المتبقي من هذا دقيقة. وتم استعمال توزيع كمبل المتطرّف، التوزيع الطبيعي والتوزيع الوغارتمي الطريعي لمعرفة مُلائمَة البيانات ولفترات عوده 5، 10، دقيقة. وتم استعمال توزيع كمبل المتطرّف، التوزيع الطبيعي والتوزيع الم العليا لمحطاتِ الإرصادية المرابية ماده 10، 10، و 15، و 50 سنة.

1.Introduction

Rainfall is one of the most important parts of the water resources. It is vitally important to quantify it accurately. Good estimates of mean rainfall are needed as inputs to hydrologic models (ARM *et al.*, 2009). The amount of rainfall received over an area is an important factor in assessing the amount of water available to meet the various demands of agriculture, industry, and other human activities. Therefore, the study of the distribution of rainfall in time and space is very important for the welfare of the national economy. Many applications of rainfall data are enhanced by a knowledge of the actual distribution of rainfall rather than relying on simple summary statistics. There is a large number of studies investigating the use of particular distributions to represent the actual rainfall patterns (Abdullah and Al-Mazroui, 1998). Information on quantiles of extreme rainfall of various durations is needed in the hydraulic design of structures that control storm runoff, such as flood detention reservoirs, sewer systems etc. Such information is usually expressed as a relationship between Intensity-Duration-Frequency (IDF) of extreme rainfall (Bara *et al.*, 2009). Rainfall (IDF) curves are graphically representations of the amount of water that falls

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within a given period of time. These curves are used to help predict when an area will be flooded, or to pinpoint when a certain rainfall rate or a specific volume of flow will recur in the future (Dupont and Allen, 2000).

2. FITTING PROBABILITY DISTRIBUTIONS TO THE DATA

There are two methods of fitting theoretical distributions to the data (two general methods of finding point estimation). They are:

2.1.Graphical Fitting

The visualization of the data is generally performed by using probability papers that allow a graphical analysis of the fit provided by the selected model and an understanding of visual information otherwise easily wasted (Erto and Lepore, 2011). Because population parameters that the sample (data) taking from it anonymous, it is difficult to find probability from the theoretical distribution function so, the following approximate equations are used for this purpose:

2.1.1 Blom formula is used to obtain the plotting positions for the Normal, Log Normal, and Log Normal type III distributions (Chow et al., 1988).

 $P_m = \frac{m - 3/8}{n + 1/4}$ (1)

Where : P_m is exceedance probability estimate for the *mth* largest event.

n is number of the observed value.

m is the rank of a value in a list ordered by descending magnitude.

2.1.2 Weibull formula is used to obtain the plotting positions for Gamma distribution (Matalas, 1963).

 $P_m = \frac{m}{n+1} \qquad (2)$

2.2. Mathematical Fitting

With regards to this method, parameters of a statistical model are commonly estimated from a sample with either method of moments estimators, or maximum likelihood estimators. Then the values of parameters are substituted into the chosen probability distribution function to solve it and get the probability (U.S. Army Corps of Engineers, 1994). There are many reasonable probability distributions (Frequency analysis models) which are used in statistical analysis. In this paper four probability distributions are used, i.e., Normal, Log Normal, Log Normal type III, and Gamma distributions. All the parameters of the selected distributions are estimated by the method of moments and maximum likelihood. All statistical distributions and their functions that were analyzed in this paper are shown in Table 1.

Statistical Distributions	Functions
Normal Distribution	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$
Log Normal Distribution	$f(x) = \frac{1}{\sqrt{2\pi} \sigma_y} exp \left[-\frac{1}{2} \left(\frac{y - \mu_y}{\sigma_y}\right)\right]$
Log Normal Type III Distribution	$f(x) = \frac{1}{(x-a)\sqrt{2\pi} \sigma_y} \exp\left[-\frac{[\ln(x-a) - \mu_y]^2}{2\sigma_y^2}\right]$
Gamma Distribution	$f(x) = \frac{1}{F^b \Gamma(b)} x^{b-1} e^{-x/F}$
Extreme Value Type I Distribution	$f(x;d,c) = \frac{1}{c} exp[-(x-d)/c]exp \{-exp$
	$[-(x-d)/c]\}$

Table 1. Statistical distributions and their functions.

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Calculations of probabilities by using method of moments (Station: Najaf) for all the selected distributions are shown in Table 2, and Figure 1 shows frequency curve with respect to the Normal distribution (moments and max. likelihood methods) for Najaf station.

Total Rainfall	Rank	Normal	Log Normal Gamma		LN III
(mm) †	<i>(m)</i>	F(x) %	F(x) %	F(x) %	F(<i>x</i>) %
22.8	1	4.792	0.102	0.878	3.817
30.3	2	6.734	0.748	2.529	5.900
35.9	3	8.544	2.043	4.560	7.921
46.1	4	12.737	7.043	10.142	12.733
48.8	5	14.054	8.974	12.007	14.260
51.4	6	15.406	11.056	13.939	15.829
53.7	7	16.672	13.065	15.752	17.296
54.6	8	17.186	13.890	16.485	17.890
56.0	9	18.004	15.213	17.652	18.837
58.8	10	19.712	17.992	20.073	20.806
64.1	11	23.202	23.634	24.917	24.799
64.2	12	23.271	23.744	25.011	24.878
64.3	13	23.340	23.854	25.105	24.956
65.4	14	24.108	25.073	26.147	25.827
71.4	15	28.527	31.872	31.968	30.786
72.4	16	29.299	33.017	32.954	31.643
75.0	17	31.351	35.991	35.528	33.904
83.8	18	38.703	45.829	44.214	41.817
91.2	19	45.229	53.543	51.279	48.597
91.3	20	45.319	53.643	51.372	48.689
99.6	21	52.788	61.432	58.796	56.167
105.5	22	58.053	66.360	63.669	61.270
109.5	23	61.548	69.410	66.760	64.585
110.7	24	62.579	70.279	67.652	65.553
111.9	25	63.602	71.128	68.528	66.508
112.3	26	63.941	71.406	68.817	66.823
113.6	27	65.035	72.295	69.741	67.837
116.4	28	67.347	74.130	71.667	69.965
117.7	29	68.400	74.946	72.531	70.926
119.9	30	70.146	76.275	73.949	72.511
142.9	31	85.275	86.797	85.636	85.837
145.4	32	86.537	87.627	86.591	86.929
147.6	33	87.585	88.315	87.387	87.836
153.0	34	89.911	89.852	89.171	89.854
159.3	35	92.203	91.395	90.972	91.862
166.5	36	94.312	92.877	92.702	93.745
169.7	37	95.092	93.451	93.372	94.456
170.0	38	95.161	93.503	93.431	94.519
190.7	39	98.335	96.224	96.552	97.621

Table 2. Calculations of probabilities by using method of moments (Station: Najaf).

Source: Iraqi Meteorological Office in Baghdad, Iraq.



Figure 1. Frequency curve with respect to the Normal dis. (moments and max. likelihood methods) for Najaf station.

3.GOODNESS OF FIT TESTS

The goodness of fit tests provide objective procedures to determine whether an assumed theoretical distribution provides an adequate description of the observed data, these tests are valid only for rejecting an inadequate model; they cannot prove that the model is correct. Three types of tests applicable to a wide range of distributions are considered in this paper, these are: Chi-square, Kolmogorov-Smirnov indices, and the graphical goodness of fit tests. These tests are applied to all the distributions that are used in this paper.

3.1.Chi-Square Index

The Chi-Squared statistic depends on specifying the number of histogram classes into which the data will be grouped, and there is no rule that gives the correct number to use (Vose, 2010). The Chi-square test statistic is computed from the relationship:

where O_i is the observed and E_i is the expected number of observation in the *ith* class interval (based on the probability distribution being tested). The expected numbers are calculated by multiplying the expected relative frequency by the total number of observation (**Barkotulla** *et al.*, 2009). This statistic is distributed as a χ^2 random variable with k-p-1 degrees of freedom (k is the number of class intervals and p is the number of parameters estimated by sample data). The hypothesis that the data are from a population with the specified distribution is accepted if χ^2 is lower than the chisquare percent point function with k-p-1 degrees of freedom v and a significance level of **a** (expressed as 1-a as confidence level, typically, 95% is chosen as the confidence limit). The value of χ^2 is determined from published χ^2 tables with v degrees of

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freedom at the 5% level of significance (**Ricci, 2005**). Results of Chi-Square index are shown in Table 3.

Theo		The	Normal Dis.		Log Normal Dis.		Gamma Dis.		LN III Dis.	
Stations v		Theo. Chi Sa	Obs. C	Chi-Sq.	Obs. (Chi-Sq.	Obs. C	Chi-Sq.	Obs. C	Chi-Sq.
	•	CIII-54.	M.	M.L.	М.	M.L.	M.	M.L.	М.	M.L.
Najaf	5	11.0710	10.8860	10.8860	9.4765	7.0481	7.3796	6.7515	9.1656	9.2072
Kerbala	5	11.0710	24.6932	24.6932	9.7515	7.3054	6.8894	5.6213	7.3269	7.9623
Diwaniya	7	14.0670	10.5244	10.5244	8.1002	7.1581	7.0827	6.9159	7.0699	7.7395
Baghdad	7	14.0670	20.9912	20.9912	7.4778	6.8071	7.1983	7.2000	7.0315	7.9188

Table 3. Chi-Square index for the stations that are used in the paper.

3.2.Kolmogorov-Smirnov Index

Kolmogorov-Smirnov (K-S) goodness of fit test is based on a statistic that measures the deviation of the observed cumulative histogram from the hypothesized cumulative distribution function (Soong, 2004). Results of K-S index are shown in Table 4.

Table 4. The values of Kolmogorov-Smirnov index for all the stations and with confidence level equal 95%.

				Obs. D.							
Stations	n	Theo.	Normal Dis.		Normal Dis. Log normal Dis.		Gamma Dis.		LN III		
		D .	M.	M.L.	М.	M.L.	M.	M.L.	M.	M.L.	
Najaf	39	0.213	0.12239	0.12239	0.10824	0.09231	0.09750	0.07492	0.10070	0.12159	
Kerbala	39	0.213	0.07571	0.07571	0.12476	0.12759	0.09802	0.10453	0.08130	0.07487	
Diwaniya	39	0.213	0.16732	0.16732	0.10253	0.11242	0.12552	0.12785	0.12540	0.11814	
Baghdad	39	0.213	0.11973	0.11973	0.07506	0.06626	0.06494	0.06493	0.06364	0.06882	

3.3. The Index of Goodness of Fit based on the Probability-Rainfall Plot

This index depends on the deviations of points that represent the sample from the theoretical frequency curve on the plot of relationship between probability and rainfall. This index represents the average absolute value of the deviations. The theoretical distribution which has the smallest value for this index represents the more suitable distribution for the data (Alkhafagiy, 1995). Results of this index are shown in Table 5.

Table 5. The values of goodness of fit by using graphical figures for the relationship between rainfall & probability.

Stations	Normal Dis.		Log Nor	mal Dis.	LN	III	Gamma Dis.	
	M.	M. L.	M.	M. L.	M.	M. L.	M.	M. L.
Najaf	0.0891	0.0891	0.1172	0.1047	0.1070	0.1030	***	***
Kerbala	0.0646	0.0646	0.1055	0.1433	0.0826	0.0832	***	***
Diwaniya	0.0653	0.0653	0.0579	0.0559	0.0650	0.0670	***	***
Baghdad	0.0702	0.0702	0.0520	0.0477	0.0865	0.0857	***	***

4.ESTABLISHMENT OF RAINFALL (IDF) CURVES

Rainfall intensity at different durations and frequencies are presented as Intensity-Duration-Frequency (IDF) curves which are widely needed for planning, designing, and operating water resource projects, this is to protect the project against flooding and to use the flood water for agriculture use by collecting it in reservoirs (Ghahraman and Hosseini, 2005).

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4.1.Methodology

The intensity-duration-frequency analysis starts by gathering records of different durations. After the data is gathered, annual extremes are extracted from the record for each duration. The annual extreme data is then fit to a probability distribution to estimate rainfall quantities. In this study Gumbel extreme value distribution, the Normal and Log Normal distributions are used to fit the annual extreme rainfall data.

Gumbel Distribution

The form of Gumbel probability distribution may be written as follows:

 $x_t = \mu + K_t \sigma \qquad (4)$

where x_t represents the magnitude of the T-year event, μ and σ are the mean and standard deviation of the annual maximum series, and K_t is a frequency factor depending on the return period T or probability of non exceedance P_T which can be calculated from generated uniform random numbers $0 < P_{\tau} < 1$, that is $T = 1/(1 - P_{\tau})$ (Saf, 2005). The frequency factor is applicable to many probability distributions used in hydrologic frequency analysis, for the Gumbel distribution K_t is obtained from Equation 5 (Prodanovic and Simonovic, 2007).

 $K_t = \frac{-\sqrt{6}}{\pi} \left[0.5772 + ln \left(ln \left[\frac{T}{T-1} \right] \right) \right]$(5)

Tables from 6 to 9 show the values of intensities by using Gumbel distribution for all the selected stations. Figures from 2 to 5 show IDF curves for all the stations that are used in this paper for Gumbel distribution.

Normal Distribution

The value of frequency factor that depends on return period for the Normal

P= Exceedance probability

When p > 0.5; 1-p is substituted for p in Equation 7.

Alternatively the frequency factor is computed by using tables. This gives the value of Kt depends on skew coefficient (Cs=0) with different return periods. When the value of K_t is computed then it is substituted in Equation 4 to obtain the value of extreme rainfall intensity (Barkotulla et al., 2009).

Log Normal Distribution

The value of frequency factor for the Log Normal distribution is computed by the same way as the Normal distribution, but the value of extreme rainfall intensity depends on the logarithm of the data. Then this value is used in Equation 8 to obtain the value of extreme rainfall intensity.

 $y_t = \bar{y} + K_t S_y \tag{8}$

Where y_t is represents the magnitude of the *T*-year event, \bar{y} and S_y are the mean and standard deviation of the annual maximum series for the logarithms of the data.

Duration (min)	Intensity (mm/hr)							
	5 year	10 year	15 year	50 year				
15	39.8	48.8	53.9	68.7				
30	22.3	28.7	32.1	42.1				
60	13.9	17.5	19.5	25.3				
Table 7. Res	sults for Najaf	station (Gum	bel distributio	on).				
Duration (min)		Intensity	(mm/hr)					
Duration (mm.)	5 year	10 year	15 year	50 year				
15	56.0	70.4	78.6	102.2				
30	34.4	43.1	47.9	62.0				
60	19.6	24.0	26.4	33.5				
Table 8. Results f	or Diwaniya st	ation (Gumbe	el distribution	l).				
Duration (min)	Intensity (mm/hr)							
Duration (mm.)	5 year	10 year	15 year	50 year				
15	37.7	47.1	52.3	67.6				
30	25.9	32.7	36.5	47.7				
60	15.9	19.8	21.9	28.2				
Table 9. Resu	lts for Baghda	d station (Gui	mbel distribut	tion).				
Duration (min)		Intensity (mm/hr)						
	5 year	10 year	15 year	50 year				
15	41.5	52.9	59.4	78.0				
30	26.8	34.1	38.3	50.2				
60	16.8	21.1	23.5	30.6				

Table 6. Results for Kerbala station (Gumbel distribution).



Figure 2. IDF Curve for Kerbala station (Gumbel distribution).

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Figure 3. IDF Curve for Najaf station (Gumbel distribution).



Figure 4. IDF Curve for Diwaniya station (Gumbel distribution).



Figure 5. IDF Curve for Baghdad station (Gumbel distribution).

5.CONCLUSION

In this paper four probability distributions are used, i.e., Normal, Log Normal, log Normal type III, and Gamma distributions. These distributions are applied to the data of rainfall depth for four selected stations in Iraq, namely, Baghdad, Najaf, Kerbala, and Diwaniya. Moments and maximum likelihood methods are used to estimate the parameters of the selected distributions. Graphical method depending on two approximate equations (Blom, and Weibull) for the plotting positions on the probability paper is also applied to check the suitability of the distributions to the data, three goodness of fit test are used: The Chi-Square, Kolmogorov-Smirnov, and the graphical goodness of fit tests.

The results of Chi-Square index are as follows:

- 1. The Normal distribution is acceptable for Najaf and Diwaniya stations by using both the moments and maximum likelihood methods except Baghdad and Kerbala stations.
- 2. The Log Normal distribution is acceptable for all the stations by using the two methods of estimation (moments and maximum likelihood).
- 3. The Log Normal type III distribution is acceptable for all the stations by using the moments and maximum likelihood methods.
- 4.Gamma distribution is acceptable for all the stations by using the two methods (moments and maximum likelihood).

The results of the Kolomogrov-Smirnov index show that all the stations are acceptable for the four distributions by using the two methods (moments and maximum likelihood).

The results of the test based on the plot represent the relationship between probability and rainfall shows that the Normal distribution is suitable for Najaf and Kerbala stations by using moments and maximum likelihood methods, but the Log Normal distribution using maximum likelihood method is suitable for Diwaniya and Baghdad stations. The results for the three statistical tests are shown in Table 10.

	Chi-Square Index		K-S	Index	D Index		
Stations	Success	Estimation	Success	Estimation	Success	Estimation	
	Dis.	method	Dis.	method	Dis.	method	
Najaf	Gamma	(M.L.)	Gamma	(M.L.)	Normal	(M.+M.L.)	
Kerbala	Gamma	(M.L.)	LN (III)	(M.L.)	Normal	(M.+M.L.)	
Diwaniya	Gamma	(M.L.)	LN	(M.)	L. N.	(M.L.)	
Baghdad	L.N.	(M.L.)	LN (III)	(M.)	L. N.	(M.L.)	

Table 10. Summary of results for the three indices.

The (IDF) curves for the four stations in Iraq (Baghdad, Najaf, Kerbala, and Diwaniya) are constructed by using Gumbel extreme value distribution, the Normal, and Log Normal distributions. By plotting the histograms and the probability curves, the results show that for Najaf and Diwaniya stations (15, 30, and 60 min. durations) the Gumbel extreme value distribution is better than the Normal and the Log Normal distributions since the sum of the squared differences between the histogram and the probability curve (Σ (difference)²) for the Gumbel extreme value distribution is minimum, but for 15, 30, and 60 min. durations for Baghdad station and 30, 60 min. durations for Kerbala station the Log Normal distribution is better than the other distributions, for 15 min. duration Kerbala station the Normal distribution is better.

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LIST OF SYMBOLS

μ	:Mean.
σ	:Standard deviation.
f(x)	:Density function for a distribution.
x	:Independent variable.
У	:Independent variable for logarithm.
<i>b, F</i>	:Gamma distribution parameters.
Γ	:Gamma function.
<i>c</i> , <i>d</i>	:Extreme value type I distribution parameters.
а	:Log Normal type III distribution parameter.
χ^2	:Chi-square test.
K_t	:Frequency factor.
Т	:Return period.
t	:Duration.