Nonlinear Dynamics in the Output of VCSEL under the Modulation of Injection Current

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Abstract
Nonlinear dynamics from Vertical- Cavity Surface- Emitting Laser (VCSEL) are studied theoretically. We consider the modulation of current injected to the laser as the main cause of the usual and chaotic behavior, seen to occurs in the output of the laser. Various types of output is enhanced as a result of the variation of the three control parameters in this system.
Key word: VCSEL, current modulation, square wave output, chaos

الخلاصة
تم في هذا البحث دراسة الحركيات اللاخطية في ليزر شبه الموصل ذي التجويف الشاقولي والباعثة للضوء سطحيا نظريا. اعتبرنا حالة تضمين تيار الحقن لليزر كمسبب أساس لظهور الخرج الاعتيادي والفوضوي في خرج هذا النوع من الليزرات. تمت ملاحظة العديد من أشكال الخرج نتيجة لتغير عوامل السيطرة الثلاث في هذا النظام.
الكلمات المفتاحية : ليزر شبه الموصل ذي التجويف الشاقولي,تضمين تيار, الخرج الفوضوي,خرج الموجة المربعة,الحركيات اللاخطية في ليزر شبه الموصل.

Introduction
Communication and information technologies have become exceedingly important during the last decade and a half. Laser diodes in combination with optical fiber networks are the backbone of modern communication systems like the internet. Semiconductor heterostructures from the basic of both microelectronics components, like transistor , chips and optoelectronic devices, like laser diodes. These structures consist of thin layers of different semiconductor material grown sandwich- like on top of each other on large wafer.

In the case of a laser diode, the light can propagate either parallel or perpendicular to the layered structure. In a vertical- cavity surface- emitting lasers or VCSEL, the light propagates perpendicular to the semiconductor layers. Perpendicular propagation of light in the structure profits from the planar symmetry of the layers on the wafer. This construction allows in particular for integration of many solitary devices into two-dimensional arrays on a single wafer, which can be used for parallel optical data-communication. Integration of lasers into two- dimensional arrays is the optoelectronic equivalent of integration of micro- electronic component into chips. As a result of drastically improved performances, VCSELs have become increasingly popular during last fifteen years.

On the other hand chaos and chaotic behavior have been shown to occurs in all types oflasers(Al- Timimy,2003). Chaotic behavior of semiconductor laser systems was viewed in two different manner, something amazing from fundamental point of view but also as a nuisance to be engineered away in applications. However, since semiconductor lasers were proposed for chaotic communication schemes as is mentioned above

(Mirasso *et al.*,1996) their chaotic properties have turned out to be potentially useful and now demand more attention.

Chaos in semiconductor lasers might occur or enhanced for different situations, viz, two coupled lasers (Thomburg, 1997), optical feedback (Soriano,2003)external- cavity with delayed negative optoelectronic feedback (Li, *et al.*,2006) and feedback with two reflections and self- mixing interferometer (Cheng and Zhang, 2007).

Almost nothing is mentioned in literature that modulation of injection current of VCSELs can cause same types of instabilities or chaos in the output of VCSELs. The message of the present work is to show that chaos can occurs in these lasers for certain choices of control parameters that appear in the rate equations governing the VCSEL.

Theoretical model

The rate equations for the field around the laser cavity E and the polarization P and population N of the laser medium can be approximated by the following set of three first order differential equations (details of its derivation can be found in (Sultan,2009) by one of the present authors

$$\frac{dE}{dt} = -k(1+i\alpha)E + P \ldots (1a)$$

$$\frac{dP}{dt} = -\gamma_{\perp}(1-i\alpha)P + NE \ldots (1b)$$

$$\frac{dN}{dt} = \frac{J}{qd} - \gamma_e N - \frac{i}{\hbar}[(EP^* - E^*P)] \ldots (1c)$$

where E is the electric field, P is the medium polarization, N is the population inversion, k is the cavity decay rate (inverse of photon life time), $\alpha$is the detuning, $\gamma_{\perp}$ is the decay rate of the material polarization, J is the injection current density distribution, q is the electronic charge, d is the active medium thickness,γ_e is the carrier decay rate, $\hbar(=\frac{h}{2\pi})$ is the normalized Planck constant, i is the imaginary number and $P^*(E^*)$ is the complex conjugate of the polarization (electric) field.

Following the mathematical trend used in (Sultan, 2009)the rate equation (1) can be converted in terms of photon density S and population N as follows

$$\frac{dS}{dt} = (\Gamma_f G - \gamma_p)S + \Gamma_f \frac{\beta}{\tau_n} N \ldots (2a)$$

$$\frac{dN}{dt} = \frac{J}{qd} - GS - \frac{N}{\tau_n} \ldots (2b)$$

with

$$G = \frac{v_g g(N-N_{tr})}{(1+\varepsilon S)} \ldots (2c)$$

$v_g$is the group velocity, g is the differential gain, Γ_f is the confinement factor,γ_p is the photon decay rate,τ_n is the carrier life time, $\beta$is the spontaneous emission rate, $\varepsilon$is the compression factor and $N_{tr}$is the carrier density at transparency.

The power output P can be calculated using the following relation (Bjerkan, *et al.*,1996)

$$P(t) = \frac{\eta h c V}{2\lambda \Gamma_f \tau_p} S(t) \ldots (3)$$

$\eta$is the efficiency (assumed to equal 1), $h$is the Planck constant, $c$is the velocity of light, $V$is the cavity volume(=$\pi r^2 d$),d is the cavity length,$\lambda$is the laser output wavelength, $S(t)$and $\Gamma_f$have their usual definitions.

To solve the set of equation (2) we used equation (3) and the current density J is written as

$J = \eta_i \, I \, \exp(-\rho^2/r^2)$...(4)

where$\eta_i$is the current efficiency assumed to be ≈1, $r$is the cavity radius and $\rho$is the radius of cylindrical coordinate system (Kh. A. Al- Timimy,2003). I is the current that can be written as follows

$I = I_{dc} + I_{ac}\sin(\omega t)$ ...(5)

whereI_{dc} is the dc part of the injection current and I_{ac} is the ac part. ($\omega = 2\pi f$) is the angular frequency and *f* is the frequency of modulation.

The laser parameters

GaAs VCSEL with heterostructure layer of AlGaAs with $\lambda = 850$ m isconsidered with the parameters given in table (1) used to perform the calculations.

Table (1) Parameters used in the calculation for GaAs VCSEL (Sultan,2009).

Definition	Symbol	Value	Units
Cavity thickness	d	8	nm
Electronic charge	q	1.6 x10^{-19}	C
Differential gain	g	3 x10^{-17}	m^{-1}
Spontaneous recombination factor	β	3 x10^{-5}	-
Group velocity	v_g	7.14 x10^{9}	cm/s
Confinement factor	Γ_f	0.03	-
Photon decay rate	γ_p	5 x10^{11}	sec^{-1}
Gain compression factor	ε	5 x10^{-16}	cm^{3}
Number of carrier at transparency	N_{tr}	1.8 x10^{18}	cm^{-3}
Cavity radius	r	8	µm
Velocity of light	C	3 x10^{8}	m/sec
Boltzmann constant	K	1.38 x10^{-23}	J/K
Carrier life time	τ_n	2.5	ns

Simulation Results and discusion

We numerically calculate the set of equation (2) together with equations (3), (4) and (5) employing the 6^{th} order Runge- Kutta algorithm for variations of the different control parameters viz, current density$I$ (through the variations of $I_{dc}$and I_{ac}) and the frequency of modulation *f*. Figures (1-3) shows examples of the laser output power against time for the variations of I_{dc}, $I_{ac}$and*f*for the values depicted on each figure. The laser output power shows undulation and severe instabilities and chaotic behaviors. For low modulation frequency, the usual output form generated from VCSEL. By increasing the modulation frequency, uniform square wave output occur which switch to various types of square waves as can be seen in fig (s) (1-3) as*f*≥1 GHz and I_{dc}=10 mA.Such behaviors were registered before in different types of semiconductor lasers(Cheng and Zhang, 2007). (Chlouverakis, *et al.*, 2007),(Homayounfar and Adams, 2007).

For the combination (I_{dc} andI_{ac}) chosen and increasing the modulation frequency the usual trends is seen to occur. For*f*=(1– 10) MHz the usual output i.e. without modulation is seen to occur. For*f* =100 MHz the output switches to square wave- output which follows severe transient region. Chaotic output occur when increasing the modulation frequency to higher values accompanied with severe beating frequency. The hight of the transient region increases with increasing of modulation frequency. Different types of square wave output happen to occur in the same output of chosen parameters.

The obtained behavior suggest the possibility of controlling the output shape by the proper choice of the three parameters mentioned above.

Conclusions

In summary, we instigate the nonlinear dynamics generated in the output of VCSEL as a result of modulating the current density injected into the laser system. We find that VCSEL shows the usual behavior under moderate injection current and modulation frequency. Bifurcation cascade, multi- periodic to chaotic oscillations are also obtained, square output is produced too, at higher values of the above mentioned controlled parameters.

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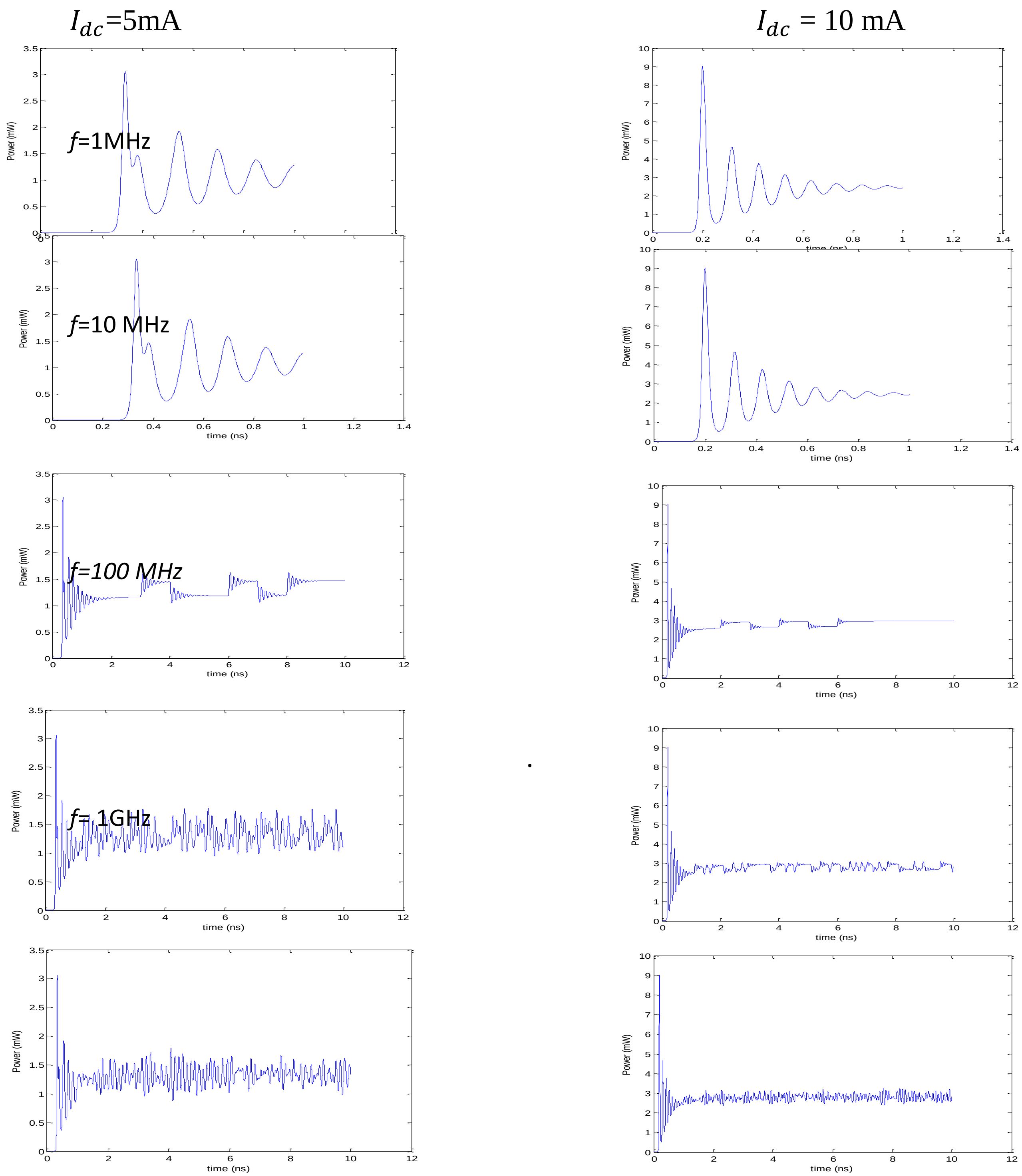

Fig (1) Output power (mW) against time (nsec) for the for the following: I_{ac}=1 mA, Leftcolumn forI_{dc}=5 mA, right column for I_{dc}=10 mA (for the modulation frequency f=1 MHzto10 GHz).

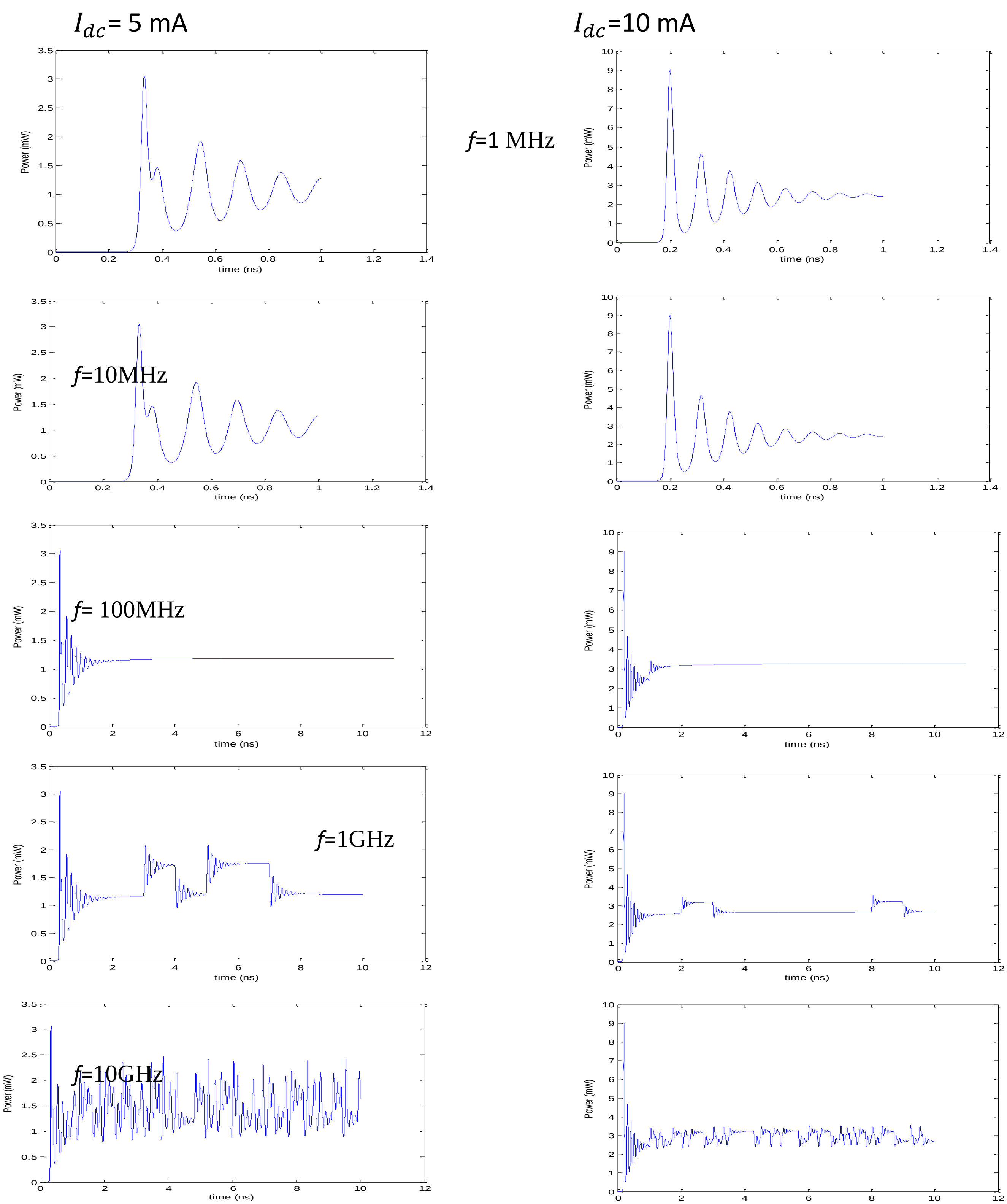

Fig (2) Output power (mW) against time (nsec) for the for the following: I_{ac}=2 mA, Left columnforI_{dc}=5 mA, right column for I_{dc}=10 mA (for the modulation frequency f=1 MHzto10 GHz).

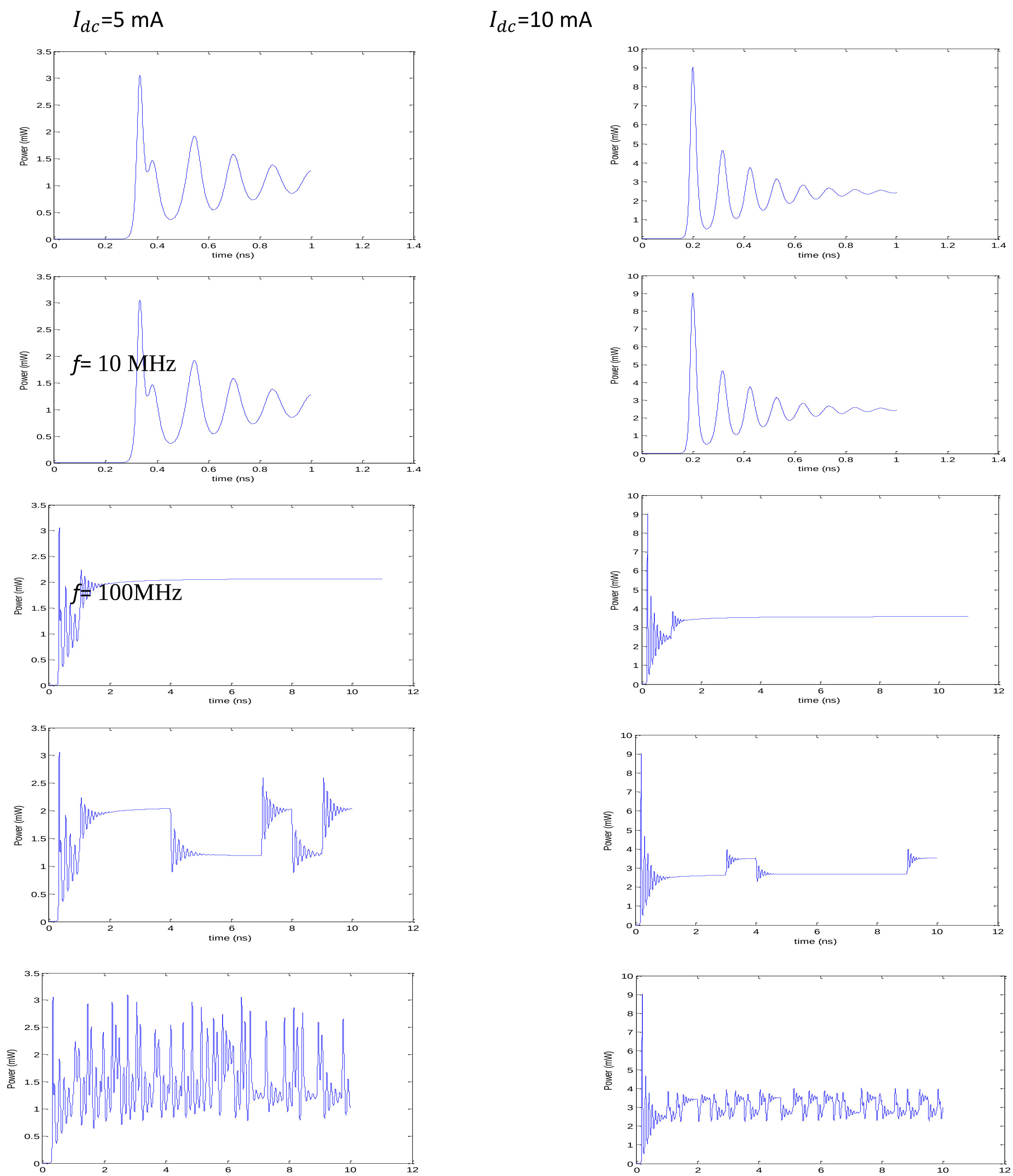

Fig (3) Output power (mW) against time (nsec) for the for the following: I_{ac}=3 mA,Left column forI_{dc}=5 mA, right column for I_{dc}=10 mA (for the modulation frequency f=1 MHzto 10 GHz).