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H-infinity Based Full State Feedback Controller Design for Human Swing Leg

Abstract- In this paper, the robustness properties of H-infinity control to produce a dynamic output feedback controller is applied to a human swing leg system. The double pendulum structure is usually used to model this system. The pendulum links will represent the thigh and shank of a human leg. The upper body will be connected to the thigh and then the shank via hip and knee joints. The muscles of thigh and shank are moved by applied two external (servomotor) torques at the hip and knee joints. The mathematical model of the system is developed. The results show that the proposed controller can robustly stabilize the system and achieve a desirable time response specification. The results were obtained by using Matlab program and the achieved time response specifications are raise time $t_r=0.18$ seconds, settling time $t_s=0.25$ seconds and maximum over shoot $M_p=0.03$ for hip joint and $t_r=0.13$ seconds, $t_s=0.21$ seconds and $M_p=0.01$ for knee joint.

Keywords- Human Swing Leg, robust controller, H-infinity control, state feedback controller.

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1. Introduction

The complicated physics of the leg locomotion make it one of the most complex motions in the body of human or in humanoid robots due to its. The walk of the human represents a complex task with some branches of biomechanical minors that must be successfully performed including support body, propulsion forward, and swing leg [1]. Figure 1 shows the structured humanoid robots.



Figure 1: The structured of humanoid robots [2].

The control problem is expressed as a mathematical optimization problem by the control designer to obtain the controller solution. The H-infinity control method is used in the control theory in order to achieve robust performance or stabilization. The H-infinity control method has a significant impact in the development of control systems; the technique is applied on industrial problems. H-infinity control has the advantage over classical control techniques in which are applicable to problems involving multivariable

systems with cross-coupling between channels [3].

For controlling the swing leg system, many researches have been carried out by various control methods [2]. Bazargan-Lari et al. [4] proposed a nonlinear intelligent controller using Adaptive Neural Network control for human swing leg hip and knee joints. The results that have been found for the angular velocity joints represented by a maximum error of about 0.15% and 0.35% for hip and knee joints respectively. Dallali et al. [5] presented a comparison between PID and Linear Quadratic Regulator (LQR) controllers. The PID controller has been used as a basis for quantification of robustness and performance of humanoid robots. A better robustness was obtained from the LQR controller. The test was done on the robot leg from -11.5° to 11.5° and obtained high control action about 30 *N.m*. Reid et al. [6] discussed a control and a potential efficiency of a medical exoskeleton with passive knees for enabling many individuals with paralysis to walk in a natural manner. The dynamics of the passive pendulum of the swing knee was excited and controlled via the behavior of the swing hip to control knee flexion. Paraplegic individuals with minimal muscle spasticity and contractures at the knees may be able to walk with a lighter exoskeleton system with passive knees. Gregg et al. [7] implemented virtual constraints that unify the stance period, coordinate ankle and knee control, and accommodate clinically meaningful conditions on

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a powered prosthetic leg. The saturate prosthesis torques at 80 *N.m* to simulate the torque limit of the experimental prosthesis.

The main goal of this paper is to design a robust H-infinity controller to stabilize the human swing leg system and achieve a desirable tracking.

2. System Mathematical Model

The double pendulum structure can be used to model the human swing leg system. The unconstrained double pendulum schematic diagram is shown in Figure 2. In this figure, θ_1 and θ_2 , represent the rotation angles of the hip and knee joints respectively. The submitted external (servomotors) torques that are responsible for moving the thigh and shank links are denoted by τ_1 and τ_2 [4].

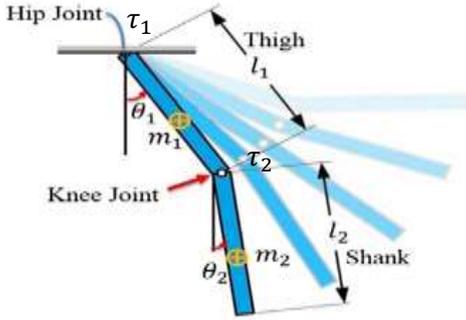


Figure 2: Schematic diagram of a human swing leg [4].

In the following, The system equations that are obtained using Lagrange's method [2, 4]:

$$\frac{(m_1+3m_2)}{3} l_1^2 \ddot{\theta}_1 + \frac{m_2 l_1 l_2 \dot{\theta}_2}{2} \cos(\theta_1 - \theta_2) + \frac{m_2 l_1 l_2 \dot{\theta}_2^2}{2} \sin(\theta_1 - \theta_2) + \frac{(m_1+2m_2)}{2} g l_1 \sin \theta_1 = \tau_1 \quad (1)$$

$$\frac{m_2}{3} l_2^2 \ddot{\theta}_2 + \frac{m_2 l_1 l_2 \dot{\theta}_1}{2} \cos(\theta_1 - \theta_2) - \frac{m_2 l_1 l_2 \dot{\theta}_1^2}{2} \sin(\theta_1 - \theta_2) + \frac{m_2}{2} g l_2 \sin \theta_2 = \tau_2 \quad (2)$$

where m_1, m_2 are thigh and shank masses, and l_1, l_2 are thigh and shank lengths, respectively.

The swing leg dynamic equations that are modeled as a unconstrained double pendulum, can be rearranged to be [4, 8]:

$$M(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) = \tau \quad (3)$$

where $\theta, \dot{\theta}, \ddot{\theta}$ represent angular positions, angular velocities, and angular accelerations of the joints respectively; $M(\theta)$ denotes the inertia matrix which is 2×2 symmetric positive definite; $C(\theta, \dot{\theta}) \dot{\theta}$ denotes the coriolis and centrifugal torques which is 2×1 vector of satisfying $\dot{M}(\theta) - 2C(\theta, \dot{\theta})$ is a skew-symmetric matrix; $G(\theta)$ denotes the gravitational torques which is 2×1

vector and τ denotes the actuator joint torques which is 2×1 vector of, where

$$M(\theta) = \begin{bmatrix} \frac{(m_1+3m_2)}{3} l_1^2 & \frac{m_2 l_1 l_2}{2} \cos(\theta_1 - \theta_2) \\ \frac{m_2 l_1 l_2}{2} \cos(\theta_1 - \theta_2) & \frac{m_2}{3} l_2^2 \end{bmatrix},$$

$$G(\theta) = \begin{bmatrix} \frac{(m_1+2m_2)}{2} g l_1 \sin \theta_1 \\ \frac{m_2}{2} g l_2 \sin \theta_2 \end{bmatrix}, \quad (4)$$

$$C(\theta, \dot{\theta}) = \begin{bmatrix} 0 & \frac{m_2 l_1 l_2 \dot{\theta}_2}{2} \sin(\theta_1 - \theta_2) \\ \frac{m_2 l_1 l_2 \dot{\theta}_1}{2} \sin(\theta_1 - \theta_2) & 0 \end{bmatrix},$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

Simplifying equations (1) and (2) yields:

$$\ddot{\theta}_1 = \frac{K_4(\tau_1 - K_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - K_3 \sin \theta_1) - K_2 \cos(\theta_1 - \theta_2)(\tau_2 + K_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - K_5 \sin \theta_2)}{(K_1 \times K_4 - K_2^2 \cos^2(\theta_1 - \theta_2))} \quad (5)$$

$$\ddot{\theta}_2 = \frac{K_1(\tau_2 + K_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) - K_5 \sin \theta_2) - K_2 \cos(\theta_1 - \theta_2)(\tau_1 - K_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) - K_3 \sin \theta_1)}{(K_1 \times K_4 - K_2^2 \cos^2(\theta_1 - \theta_2))} \quad (6)$$

where

$$K_1 = \frac{(m_1+3m_2)}{3} l_1^2, K_2 = \frac{m_2 l_1 l_2}{2}, K_3 = \frac{(m_1+2m_2)}{2} g l_1, K_4 = \frac{m_2}{3} l_2^2, K_5 = \frac{m_2}{2} g l_2 \quad (7)$$

Assume the state variables are:

$x_1 = \theta_1$: the upper link angular position.

$x_2 = \dot{\theta}_1$: the upper link angular velocity.

$x_3 = \theta_2$: the lower link angular position.

$x_4 = \dot{\theta}_2$: the lower link angular velocity

so that

$$\dot{x}_1 = x_2 \quad (8)$$

$$\dot{x}_2 = x_3 \quad (9)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = \frac{K_4(\tau_1 - K_2 x_4^2 \sin(x_1 - x_2) - K_3 \sin(x_1)) - K_2 \cos(x_1 - x_2)(\tau_2 + K_2 x_3^2 \sin(x_1 - x_2) - K_5 \sin(x_2))}{(K_1 \times K_4 - K_2^2 \cos^2(x_1 - x_2))} \quad (10)$$

$$\dot{x}_6 = \frac{K_1(\tau_2 + K_2 x_5^2 \sin(x_1 - x_2) - K_5 \sin(x_2)) - K_2 \cos(x_1 - x_2)(\tau_1 - K_2 x_4^2 \sin(x_1 - x_2) - K_3 \sin(x_1))}{(K_1 \times K_4 - K_2^2 \cos^2(x_1 - x_2))} \quad (11)$$

The outputs are:

$y_1 = \theta_1$: the upper link angular position.

$y_2 = \theta_2$: the lower link angular position.

The inputs are:

$u_1 = \tau_1$: external torque at the upper link actuator.

$u_2 = \tau_2$: external torque at the lower link actuator.

By linearizing equations (8) to (11) using Jacobians' method and with the following initial condition:

$$(x_1, x_3) = (\theta_1, \theta_2) = (10^\circ, 20^\circ), (\dot{x}_1, \dot{x}_2) = (\dot{\theta}_1, \dot{\theta}_2) = (0.3, 0.4) \text{ rad/s and } (\tau_1, \tau_2) = (0.5, 0.5) \text{ N.m.}$$

The resulting state space representation for the system is [9]:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

$$y(t) = Cx(t) + Du(t) \tag{13}$$

where x is a state vector, \dot{x} is a state differential equation, y is the output equation and A, B, C and D are the nominal matrices of the system which obtained as:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -45.8649 & 22.9325 & 0 & 0 \\ 68.7974 & -61.1532 & 0 & 0 \end{bmatrix}, \tag{14}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 56.6706 & -85.0059 \\ -85.0059 & 226.6824 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The parameters of human swing leg system are given in Table 1.

Table 1: The parameters of human swing leg [4].

parameter	value	unit
m_1, m_2	0.1	Kg
l_1, l_2	0.55	m
g	9.81	m/s ²

3. Controller Design

In this paper, the robust H-infinity controller is designed to stabilize and track the human swing leg system with uncertainties. This problem can be defined by the configuration in Figure 3. The system can be seen as 'multi-input, multi-output' MIMO with two inputs and two outputs and it is often referred to as the generalized plant G . The input $d(t)$ is the disturbance, whose effect on the output signal to be minimize. The input $u(t)$ is the

control signal which used to achieve this goal. The output $e(t)$ represents the signal to be minimized and the output $y(t)$ represents the system states which are available for feedback [10, 11].

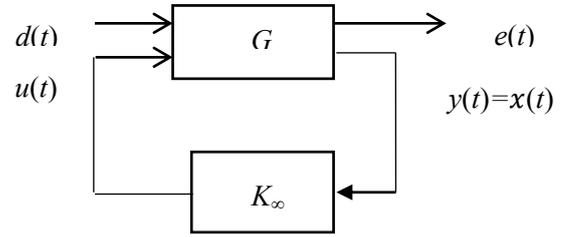


Figure 3: A full state feedback control structure.

Therefore, the full state feedback H-infinity controller will be expressed as [2, 9, 10]:

$$\dot{x}(t) = Ax(t) + B_1u(t) + B_2d(t) \tag{15}$$

$$e(t) = C_1x(t) + D_{11}u(t) \tag{16}$$

$$y(t) = x(t) \tag{17}$$

where $B_1 = B, D_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and

$$C_1 = \begin{bmatrix} 31.6228 & 0 & 0 & 0 \\ 0 & 31.6228 & 0 & 0 \\ 0 & 0 & 0.0316 & 0 \\ 0 & 0 & 0 & 0.0316 \end{bmatrix} \tag{18}$$

B_2 matrix will be taken with two cases:

Case one: without disturbance

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{19}$$

Case two: with disturbance

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{20}$$

The following assumptions are required for the solution of H-infinity problem based on Riccati equation [9]:

1. (A, B_1) and (A, B_2) are stabilizable.

2. (C_2, A) is detectable.

3. $C_1^T D_{11} = 0$ and $D_{11}^T D_{11} = I$.

and the H-infinity norm of the closed-loop transfer function (T_{ed}) should be less than a given value of γ as [12, 13]:

$$\|T_{ed}(s)\|_\infty < \gamma \tag{21}$$

where γ represents an upper bound in the disturbance and uncertainty magnitudes that can be annihilated by the control signal. The control action $u(t)$ responsible for minimizing the cost function $J(t)$ when the disturbance $d(t)$ tries to

maximize it. The physical meanings of the relation compete to each other within infimum (*inf*) and supremum (*sup*). There is trade-off between $u(t)$ and $d(t)$. This problem of *inf-sup* optimization may be expressed by [9, 10]:

$$\inf_u \sup_d J(u, d) < \infty \quad (22)$$

and

$$J(u, d) = \int_0^\infty (e^T e - \gamma^2 d^T d) dt \quad (23)$$

assume that $d(t)$ and $u(t)$ have the following structures [9, 10]:

$$d(t) = K_d x(t) \quad (24)$$

and

$$u(t) = K_c x(t) \quad (25)$$

substituting equations (24) and (25) in (16), yields:

$$e(t) = (C_1 + D_{11} K_c) x(t) \quad (26)$$

therefore

$$e^T e = x^T (C_1^T C_1 + K_c^T K_c) x \quad (27)$$

and

$$d^T d = (K_d x)^T (K_d x) = x^T (K_d^T K_d) x \quad (28)$$

substituting equations (27) and (28) in (23), gives:

$$J(u, d) = \int_0^\infty x^T (C_1^T C_1 + K_c^T K_c - \gamma^2 K_d^T K_d) x dt \quad (29)$$

and substituting equations (24) and (25) in (15), yields:

$$\dot{x} = (A + B_1 K_c + B_2 K_d) x \quad (30)$$

To obtain the Riccati equation, consider:

$$\begin{aligned} d(x^T P x) &= (\dot{x}^T P x + x^T P \dot{x}) dt \\ &= x^T ((A + B_1 K_c + B_2 K_d)^T P + P (A + B_1 K_c + B_2 K_d)) x dt \end{aligned} \quad (31)$$

where P is a positive semidefinite symmetric matrix. Integrating both sides of equation (31) from 0 to ∞ yields [14-16]:

$$\int_0^\infty d(x^T P x) = x^T(\infty) P x(\infty) - x^T(0) P x(0) \quad (32)$$

Hence, equation (30) is stable because the eigen values of $(A + B_1 K_c + B_2 K_d)$ lie in left hand side in s -plane and $x(\infty) = 0$. Therefore

$$\int_0^\infty d(x^T P x) = -x^T(0) P x(0) \quad (33)$$

and P matrix can be determined using the solution of the Riccati equation:

$$P (A + B_1 K_c + B_2 K_d) + (A + B_1 K_c + B_2 K_d)^T P + C_1^T C_1 + K_c^T K_c - \gamma^2 K_d^T K_d = 0 \quad (34)$$

it can be seen that

$$J = x^T(0) P x(0) \quad (35)$$

so that

$$K_c = -B_1^T P \quad (36)$$

and

$$K_d = \frac{1}{\gamma^2} B_2^T P \quad (37)$$

also

$$Q = C_1^T C_1 \quad (38)$$

substituting from equation (36) to (38) in equation (34), yields [17-19]:

$$PA + A^T P + C_1^T C_1 - P(B_1^T B_1 - \frac{1}{\gamma^2} B_2^T B_2) P = 0 \quad (39)$$

The value of γ is selected to be 1.75 and by trial and error Q matrix is selected as:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 1000 & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{bmatrix} \quad (40)$$

the values of P , K_c and K_d are:

$$\begin{aligned} P &= \begin{bmatrix} 48.0723 & 13.5671 & 1.2462 & 0.4679 \\ 13.5671 & 21.1369 & 0.4707 & 0.3148 \\ 1.2462 & 0.4707 & 0.0679 & 0.0274 \\ 0.4679 & 0.3148 & 0.0274 & 0.0132 \end{bmatrix} \\ K_c &= \begin{bmatrix} 30.8539 & -0.0910 & 1.5200 & 0.4274 \\ 0.1189 & 31.3604 & 0.4348 & 0.6701 \end{bmatrix} \\ K_d &= \begin{bmatrix} 0.5597 & 0.2565 & 0.0311 & 0.0133 \\ 0.5597 & 0.2565 & 0.0311 & 0.0133 \end{bmatrix} \end{aligned} \quad (41)$$

4. Results and Discussion

Figure 4 shows the step response of a human swing leg without controllers and represent for hip and knee positions. This system is stable with high oscillation, where the eigen values of the system are $\pm 9.6932i$, $\pm 3.6139i$ and the system is controllable because the system doesn't have singularity.

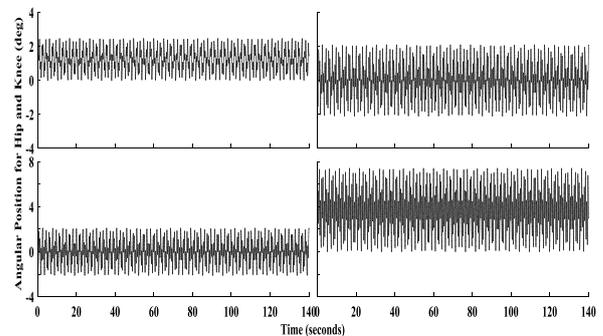


Figure 4: Step response for hip and knee positions for the human swing leg system without controller.

The block diagram and Simulink Matlab for the nonlinear human swing leg system with state

feedback H-infinity controller for achieving desired angular positions (θ_{d1} , θ_{d2}) are shown in Figure 5.

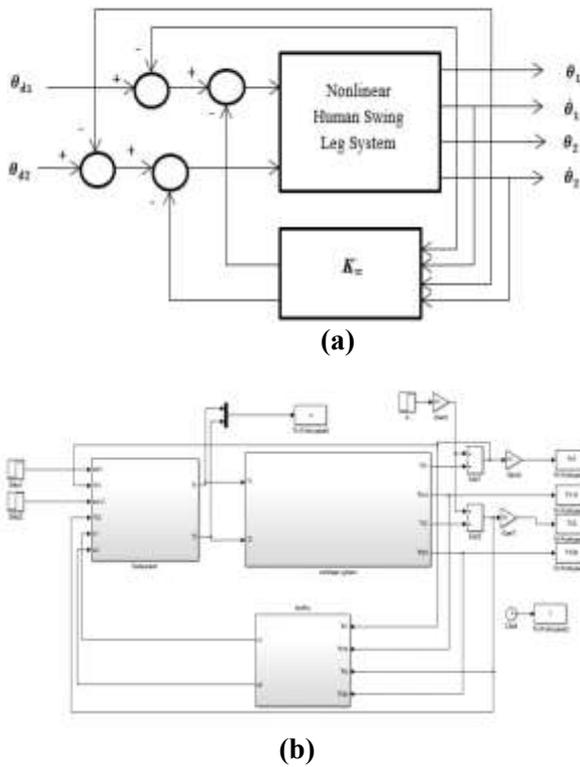


Figure 5: The block diagram and Simulink Matlab for the nonlinear human swing leg system with controller (a) The block diagram (b) The Simulink Matlab.

Figures 6 and 7 represent the time response after applying the designed H-infinity controller to the system in cases of stabilization and tracking. Figure 6 shows that the achieved settling time is (0.3 seconds) with no over shoot. Further, the proposed controller has achieved an accepted control actions that can avoid the saturation problem.

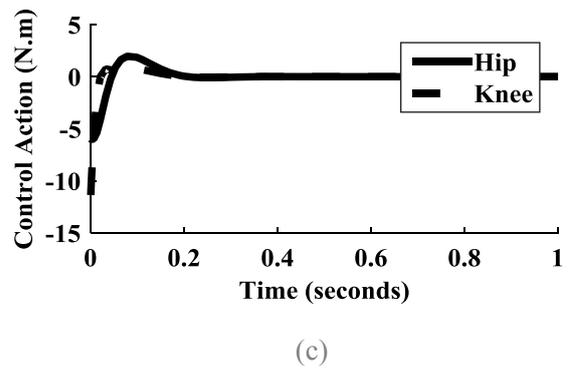
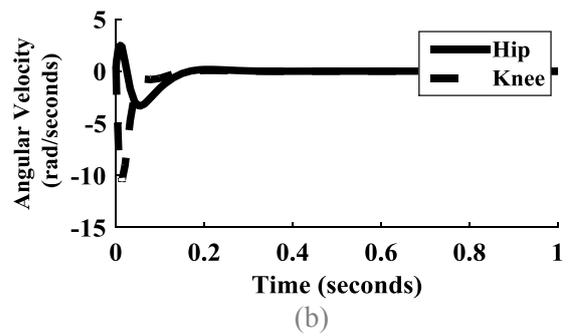
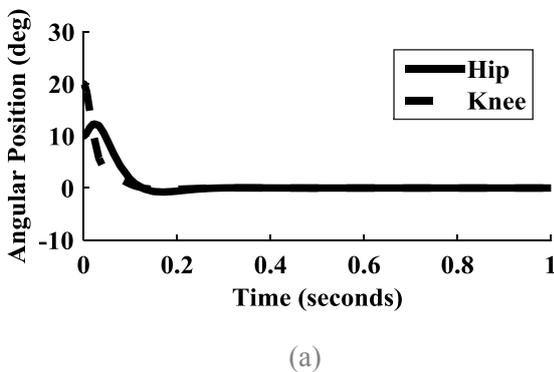
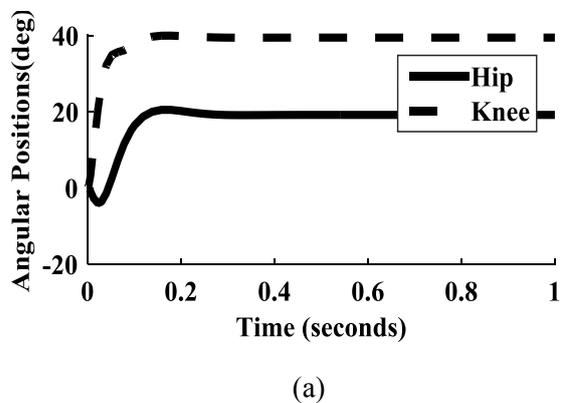
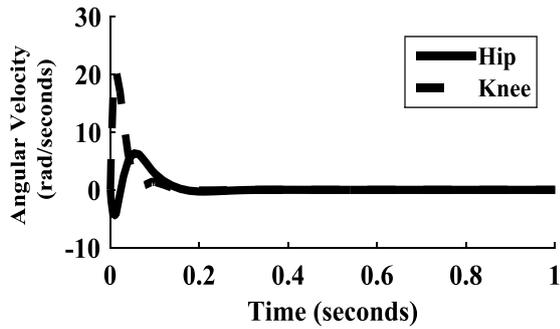


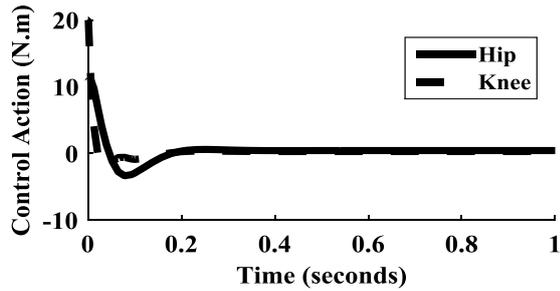
Figure 6: State trajectories and control actions for the nonlinear human swing leg system with initial conditions $\theta_1 = 10^\circ$ and $\theta_2 = 20^\circ$. (a) position (b) velocity (c) control action.

The ability of the controlled system to track specific trajectories has been shown in Figure 7. The achieved time response specifications are $t_r=0.18$ seconds, $t_s=0.25$ seconds and $M_p=0.03$ for hip joint and $t_r=0.13$ seconds, $t_s=0.21$ seconds and $M_p=0.01$ for knee joint. From this figure, it can be noticed that the states of the system with full state feedback H-infinity controller are stable and reaches to the equilibrium point. Also, It is shown that, low control effort is required using full state feedback H-infinity controller.





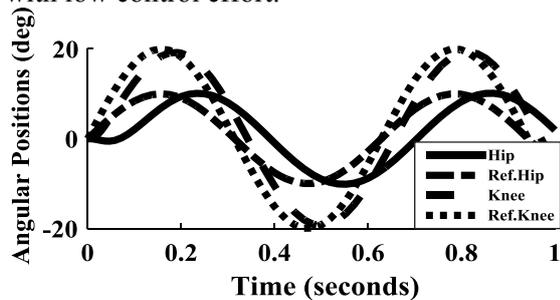
(b)



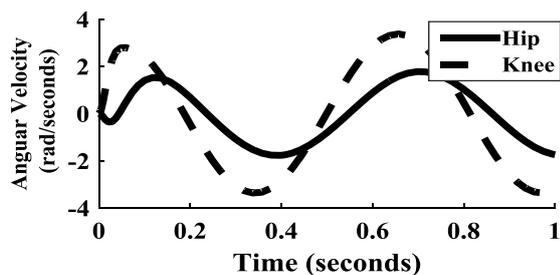
(c)

Figure 7: Time response for the nonlinear human swing leg system with $\theta_{d1} = 20^\circ$ and $\theta_{d2} = 40^\circ$. (a) position (b) velocity (c) control action.

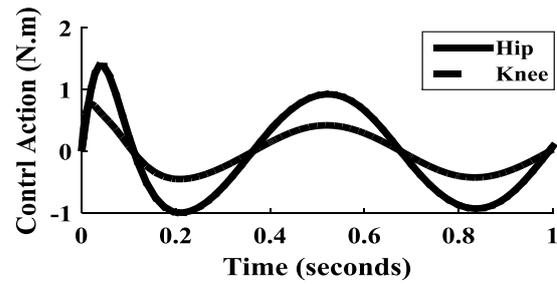
The response of the system to sinusoidal signal which represents the ability of the system to track desired trajectories is shown in Figure 8. It is shown that, the full state feedback controller can force the system to track the desired trajectory with low control effort.



(a)



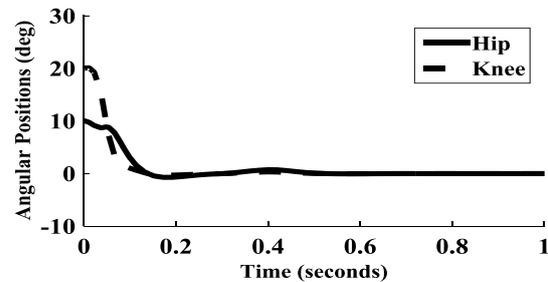
(b)



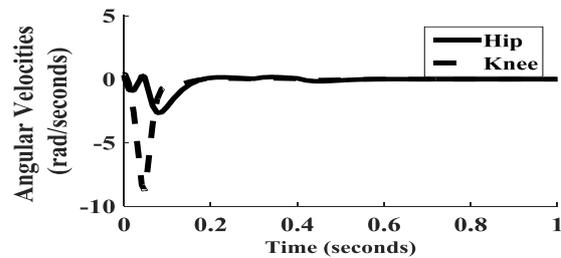
(c)

Figure 8: Time response of the nonlinear human swing leg system (a) position (b) velocity (c) control action.

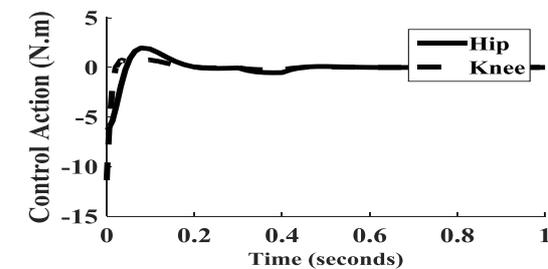
To test the robustness of the controlled system, two tests are given. The first test is for the controlled system when a disturbance is applied. The effect of the disturbance can be shown in Figure 9. The applied disturbance is 10% from the reference input and it was applied at $t=0.3$ seconds. It shows that the proposed controller can effectively rejects the disturbance.



(a)



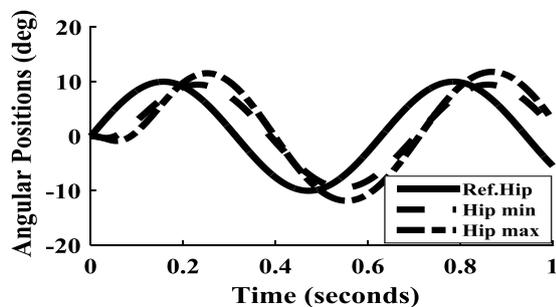
(b)



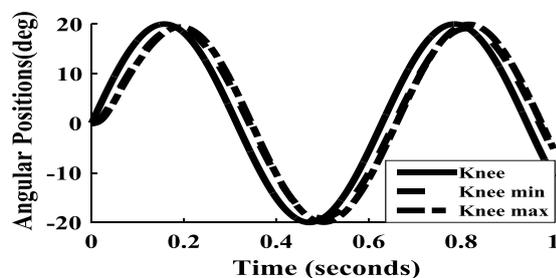
(c)

Figure 9: Disturbance properties of the nonlinear human swing leg system with initial condition $\theta_1 = 10^\circ$ and $\theta_2 = 30^\circ$. (a) position (b) velocity (c) control action.

The second test is for the controlled system with $\pm 20\%$ variation in system parameters. It is obvious from Figure 10 that the proposed controller has a high ability to compensate the system parameters variation and achieve a more desirable time response.



(a)



(b)

Figure 10: Time response of the nonlinear human swing leg system with $\pm 20\%$ variation in system parameters. (a) hip position (b) knee position.

5. Conclusions

In this paper, the design of full state feedback H-infinity controller for a human swing leg system has been presented. The controller was designed using linearized model and then applied to nonlinear model. It was found that a desirable robustness in stability and performance can be achieved using the proposed controller. The effectiveness of the proposed controller has been examined by considering 20% perturbation in system parameters. It was found that the controller achieved the required robustness. The advantage of H-infinity controller over classical control techniques is readily applicable to problems involving multivariable systems. Further, it can be concluded that H-infinity controller can effectively overcome the cross-coupling between channels.

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