# Optimum Design of Land Forming

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## Abstract

This research presents a computerized formulation of an optimum design of land forming considering land slopes in the two opposite directions as the main independent variables .Rosenbrock method of optimization is used in order to minimize the total volume of earth work (i.e cut + fill). The formulation allows considering a large number of design parameters. The least square method is be used to get the starting roles of land slopes, the four -point's method is utilized to calculate the volume of cut and fill.

## الخلاصة :

بينت هذه الدراسة صياغة مبرمجة للحصول على التصميم الأمثل لتشكيل الأراضي باعتبار إن الميول المأخوذة هي باتجاهين متعاكسين يمثلان المتغيرات المستقلة الأساسية في مسألة الامثلية .

وقد تم إجراء الامثلية باستخدام طريقة روزنبروك أن غاية الامثلية هنا هي الحصول على اقل قيمة لدالة الهدف المتمثلة بمجموع الأعمال الترابية (الحفر + الدفن) .

إن صياغة المسألة تسمح بإدخال عدد كبير من الضوابط التصميمية والتي تعرف بالحدود الدنيا والعليا للعناصر التصميمية

وقد استخدمت طريقة اقل المربعات للحصول على قيم ابتدائية للميول التصميمية للدخول في عملية الامثلية .

لقد وجد أن الصياغة المذكورة للمسألة كفوءة في حل مسائل كبيرة نسبياً وبدرجة تقارب ووقت حساب مقبولين وباستخدام الحاسبة الشخصية.

## Introduction:

Land forming design is modified the earth topography to provide a surface with special functional requirement in the field of surface irrigation, there exist several methods that have been developed for land forming design so as to balanced cut and fill to minimize earth work with a specified of fill ratio. These methods include the fixed center methods (least square principle Scaloppi and Willardson (1986) the residual technique Shih and Kriz(1971) and are programming Sowell And Kriz(1973) . Non-rectangular fields have also been conserving by some of the above methods. Shih et al (1989) The relative merits of these and other earlier methods are be given by Scaloppi and Willardson (1986) However all the above methods involve trial and error procedures in the dosing .In (2002) Easa presented a direct method for land grading do sign that eliminate the need for trial and error procedure. However, in practical applications this method is not always appropriate because it is restricted to one-design variable problems only.

Recently many designers and researchers Slaby(1987) are oriented to obtain the optimum design. The sum of cut as the objective for which is linear. Actually the problem can be better formulated considering the volume of cut and fill as the objective for which is non-linear function of the design variables (i.e design slopes in the two main directions and one control point elevation. In (1971) a computer program for land-forming design was pointing by Sun -Fu and Kriz(1971) the method involves trial and error procedure, and no mathematical formulation of optimization carried out.

### The objective of the present work:

In the present work a well known optimization method to find out the optimum design for land forming of a rectangular field with uniform row and cross – row directions was adopted ,the procedure takes into consideration some design constraints like specified cut / fill ratio and allowable slopes . The formulation of the problem in such a mathematical model be solved by Rosenbrock method of optimization is packing into a computer program with field data is presenting for demonstrating the validity of the formulation

**Formulation of the problem:**

**1- Grid systems:**

The rectangular field is divided into a number of grid points with equal spacing (G) in the main directions (i.e row and cross – row directions or x and y directions). An imaginary point (A00) is proposing to be the origin of the grid system (Fig. 1) .However the field boundaries may not be a rectangle, therefore the boundary off sets are be measured and given with other field data.

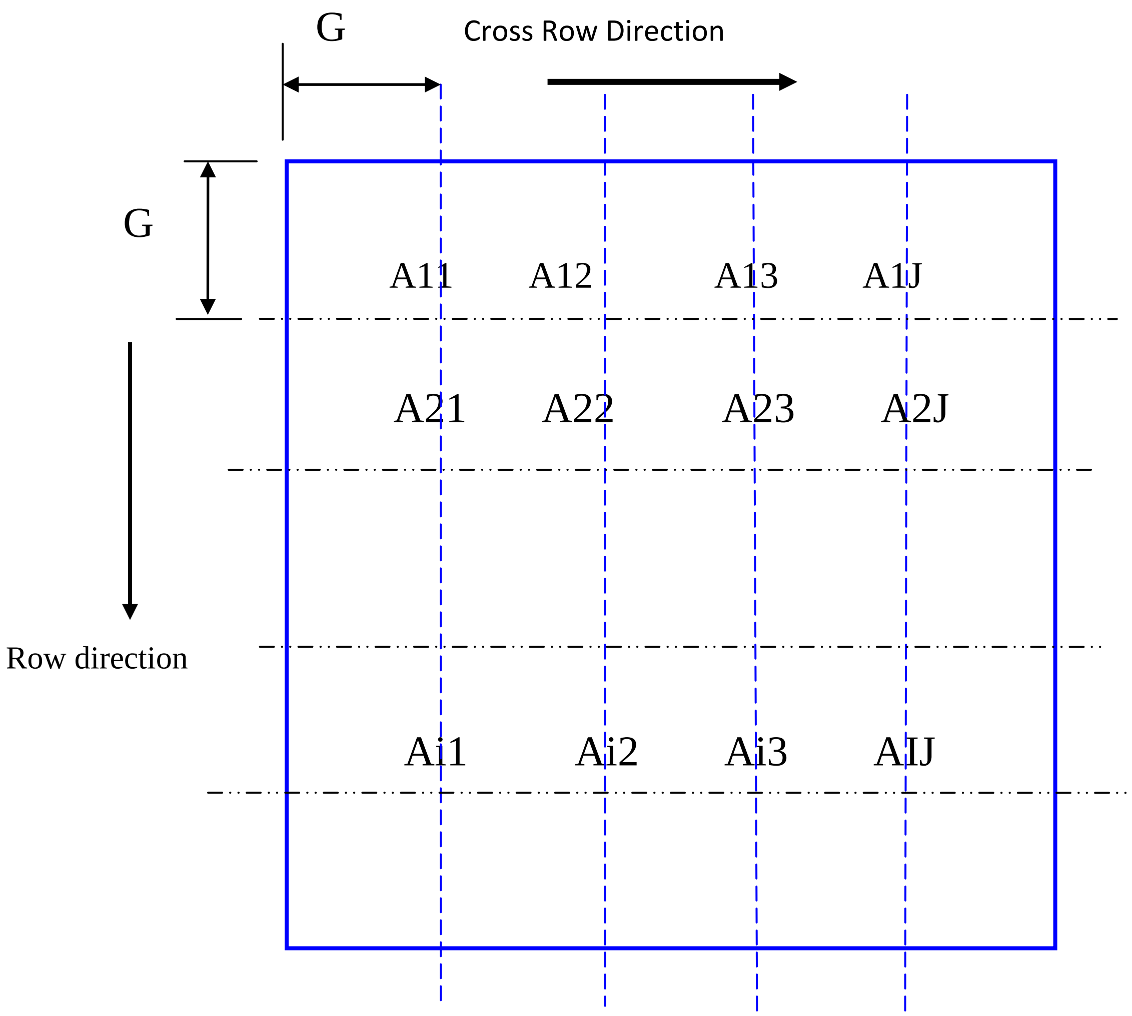



Fig. (1): Grid System

**2- Design parameter:**

All grid point levels (natural ground) are assuming and giving with spacing (G). The most important design variables are the formulation slopes in the two main directions (Sx and Sy ) which are assuming here to be uniform along the field. Other design variables, which directly affects on the design of the centric of the field. However, it is more practical to take the difference between the design level and the natural ground level of the field centric as being a design variable it is called (Ex). Therefore we have three design variable (Sx , Sy and Ex) .

**3- Objective function:**

In land forming design the aim of design to get the best design, which satisfies all the functional requirements of the design cut / fill and with minimum earthwork (i.e minimum volume of cut and fill). According, the choice of total volume of cut and fills, as the objective function of the optimization problem seems to be reasonable in this case.

The volume of cut and fill can be calculated for any set of the design variables (Sx , Sy and Ex) using an appropriate method. In this work, the four- point's method was chose to be use.

The optimization problem can formulated as follow:

Find (x), the set of design variables such that the objective function

Z = f (x) is minimizing ………… (1)

Subject to constraints:

g (x) ≤ 0 ………. (2)

Where:

Z = the total of volume of cut and fill.

g(x) = set of design – constraint functions .

**4- Constraints:**

In general, constraints reflect the design rules functional requirements and so are classing into two groups as follow:

**A-Design constraints:** It is representing to the design equations and restrictions of the design variables.

**B-Side constraints**: It is representing to limitations put on the design variables from the practical point of view such as minimum and maximum values of design slopes.

However, most constraints on this problem are from the second group the following constraints are using here:

Sx (min.) ≤ Sx ≤ Sx (max.) ……………….. (3)

Sy (min.) ≤ Sy ≤ Sy (max.) ………………. (4)

Min. value $\leq \frac{cut}{fill} \leq$ max. value ……………… (5)

The upper and lower limits are choosing by designer and given with data of the specific problem.

**5- Optimization method:**

The problem are formulated is constrained optimization problem and can be solved using an appropriate method of optimization. However, the problem is non – linear and so it is classifying as a non – linear programming problem.

The method developed by Rosenbrock (1960) is choosing here. The method has been widely used in solving linear and non – linear optimization problems Slaby (1987) .

Rosenbrock method falls within the numerical category of optimization method. The method uses a direct search technique making use of organization principle of (Gram – Schmidt) in order to give acceleration in both the direction and the step length. It was proving by Slaby (1987) and B everage (2000) to be effective to overcome all the disadvantages of the other search methods. Its coordinate rotation feature allows if to search the optimum solution closely. Although the method has no satisfactory convergence criterion, experience can help a lot in giving a suitable and satisfactory convergence and stability of the solution.

The method was fully explaining many references and need not to repeat here. (Slaby (1987), Rosenbrock (1960), Beverage (2000). Fig (2) shows the flowchart of main Rosenbrock algorithm.

**Solution Procedures:**

**1. Given data:**

The grid system and all points elevations are giving the limits of all the design parameters are also given.

**2. Calculating initial values of the design slopes :**

Several methods are available to calculate uniform slopes which fit a given land topography. The most widely used one is the least square method that was being finding here very effective. In this method the slopes in the x and y directions can calculate:

$$Sx = \frac{\sum_{i=1}^{Ny}\sum_{i=1}^{Nx} HijXij - N\bar{X}\,\bar{H}}{\sum_{i=1}^{Ny}\sum_{i=1}^{Nx}(Xij)^2 - \overline{Nx}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and

$$Sy = \frac{\sum_{i=1}^{Ny}\sum_{i=1}^{Nx} HijYij - N\bar{Y}\,\bar{H}}{\sum_{i=1}^{Ny}\sum_{i=1}^{Nx}(Xij)^2 - \overline{Ny}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where:

N = Nx * Ny (total number of grid point)

Nx = number of grid – points in the x direction

Ny = number of grid – points in the y direction

$\bar{x}$ =location of the centric of the area in the x direction

$\bar{Y}$ = location of the centric of the area in the y direction

$\bar{H}$ = elevation of the centric of the area or average elevations.

X i,j = distance from the Y – axis of the jth station in the ith row .

Y i,j = distance from the X– axis of the jth station in the ith row .

H i,j = ground elevation at the jth station in ith row .

Now if the design surface defined by the two slopes Sx , Sy passes through the centric of the area , the volume of cut will be equal to the volume of fill . However , from the practical point of view , the design should be such that the volume of cut is slightly greater than the volume of fill , (i.e. the cut /fill/ ratio) is greater than unity and is usually taken between 1.1 and 1.5 depending on the type of soil .

To maintain such a ratio the design surface is lowered by small value (Ex) which is considered here as a third design parameter and is obtained during the optimization process.

**3- Calculating the volume of earth works :**

There are several methods to calculate the volumes of cut and fill. The most widely used one's the four – point method and is adopted in this work

It is basing on the assumption that the ground surface between two stations is a smooth plane. Each grid may have either cut at all four corners or fill at all four corners, fill at all four corners or cut at two opposite corners and fill at the other two. The weighted depths of cuts and fills for the grids are equal to the average of the depths at the grid corners,

Therefore, the volumes of cut and fill on an individual grid may be calculation as follows:

$$Vc = \frac{G^2}{4}\left(\frac{Hc^2}{Hc + Hf}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

And

$$Vf = \frac{G^2}{4}\left(\frac{Hf^2}{Hc + Hf}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where:

Vc = volume of cut in an individual grid.

Vf = volume of fill in an individual grid.

Hc = sum of depths of outs at the corners of the grid.

Hf = sum of depths of fill at the corners of the grid.

G = Grid spacing or distance between adjacent stations.

The depths of cut and fill of the grid points can be calculated for any set of values for the de sign variable (Sx , Sy and Ex) as follows :

First, the design elevation for each grid points can calculate:-

$$Ec = (\bar{H} + Ex) + \bar{x} * Sx - \bar{y} * Sy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$$Edij = E0 + Xij * Sx + Yij * Sy \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Where:

E0 = the design elevation of the origin point.

Edij = Design elevation of point at jth station on ith row.

Then the depth of cut or fill is been calculated from:

Dij = Hij – Edij ………………………… (12)

If the sign of the Dij is (+) then it is out and is denoted (cij) and if it is ( – ) then it is fill and is denoted (Fij) .

## 4- Calculating the volume of cut and fill from boundaries:

If the area is not rectangular or any boundary is not straight , the volume of earthwork is calculate for a rectangular area as described before and then an additional volume is calculated from the boundaries as follows Fig. (3).

Let b1 and b2 be the two boundary offsets from the two points Aij , Aij+1 . Now the grid is completing by adding two grid points A1,j+1 with assumed an assumed elevation identical to the elevation of points Aij, Ai j+1 . The volume of cut and fill for this grid is calculated in the same way as described in the previous step and is denoted (Vc and VF) the actual volumes of cut and fill from the boundary strip is calculated from :

$$Vcb = \frac{[(b1 + b2)/2] * G}{G^2} * Vc \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$Vfb = \frac{[(b1 + b2)/2] * G}{G^2} * Vf \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

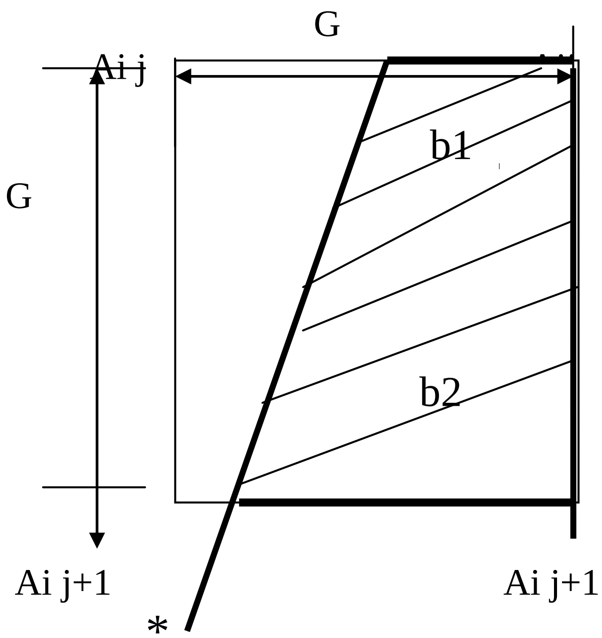



Fig .(3) Boundary line

Where :
Vcb = volume of cut from boundary strip.
Vfb = volume of fill from boundary strip.

The total volume of cut and fill is the summation of all grid volumes in addition to the volumes from the boundary strips.

**5- Optimization Process :**

After choosing the initial values for the design variables. Rosenbrock method of optimization requires that the initial values must satisfy all the constraints and does not lie within any boundary zone; so the initial value must be checked and revised if necessary to get a feasible starting point. Then Rosenbrock procedure will start and search for the optimum values of the design variables.

The search is stopping when a certain convergence criterion reached. Then following criterion is been chosen:

$$\left|\frac{Z^{k+1} - Z^{k}}{Z^{k}}\right| \leq e^{1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

or

$$|X^{k+1} - X^{k}| \leq e^{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Where:
$Z^k$ = the value of the objective function at the kth stage.
$Z^{k+1}$ = the value of the objective function at the k+1th stage.
$X^k$ = value of the design variables at the kth stage.
$X^{k+1}$ = value of the design variables at the k+1th stage.
$e^1$, $e^2$ = specified convergence limits which are being chosen here after several trails on different examples.
$e^1 = 10^{-5}$ and $e^2 = 10^{-7}$ were found to be suitable for good convergence.

## The computer program:

The formulation of the problem as explained above is translating into a computer program and Fig. (4) Show a flowchart of the program.

## Application and results:

To show the validity of the explained formulation several examples were solved one of them is be present here:

Grid spacing = 100 m

Number of points in x – direction (Nx) = 5

Number of points in y – direction (Ny) = 4

The ground elevations of all grid points are been shown in the Fig. (5) The boundary offsets are as follows:

**1-** Distance in (m) from the first row direction to the field boundary = 50.0 , 50.0 , 50.0 , 50.0 .

**2-** Distance in (m) from the last row direction to the field boundary = 3.0 , 10.0 , 20.0, 30.0 .

**3**-Distance in (m) from the first cross -row direction to the field boundary = 50.0 , 52.0 , 77.0 , 71.0 , 60.0 .

**4-**Distance in (m) from the last cross – row direction to the field boundary = 80.0, 90.0, 70.0, 50.0 .

The maximum and minimum values for the design parameters are being chosen as follows:

$- 0.5 \leq Sx$ and $Sy \leq -0.001$

$1.1 \leq$ (cut / fill) ratio $\leq 1.5$

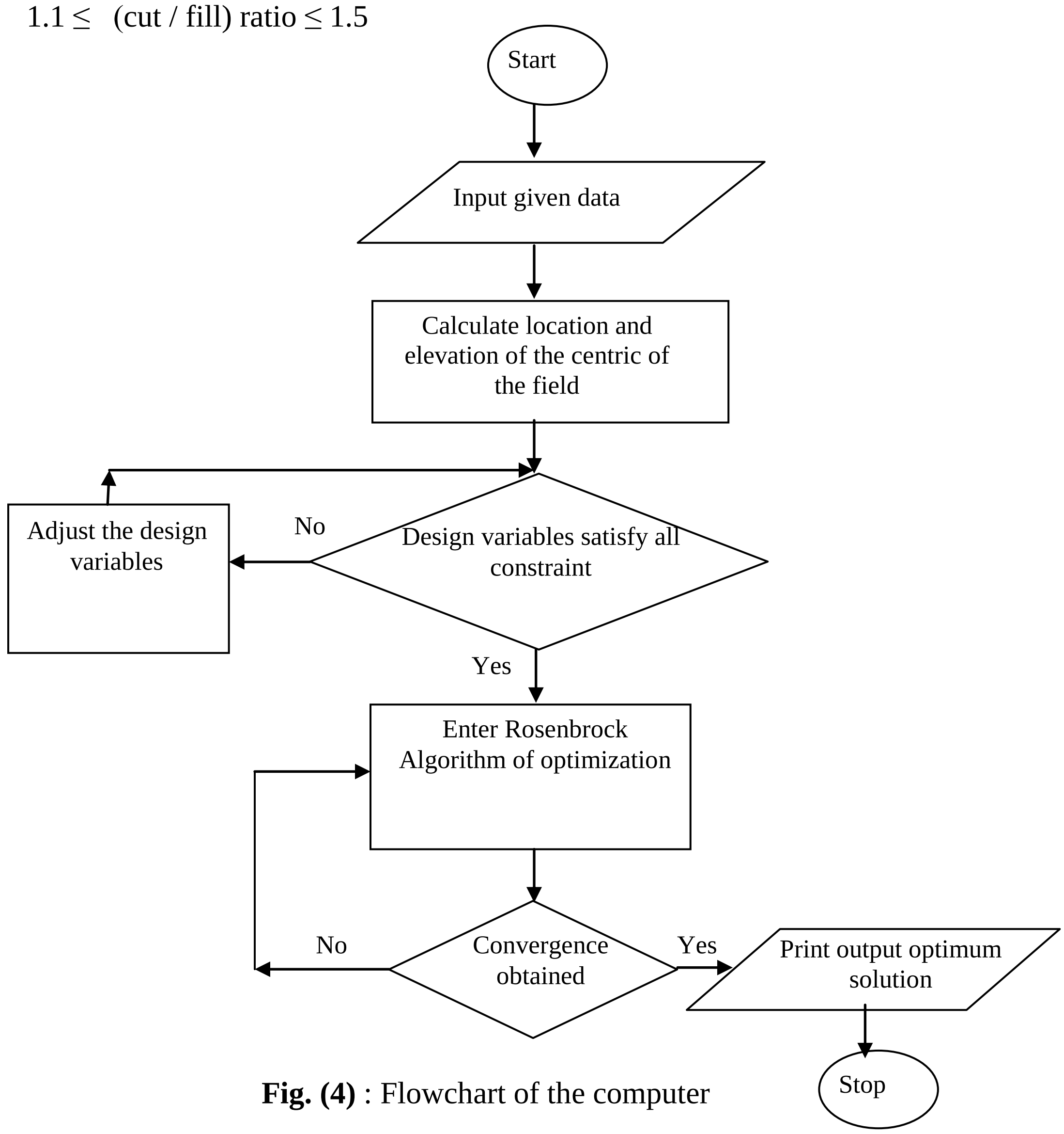



**Fig. (4)** : Flowchart of the computer

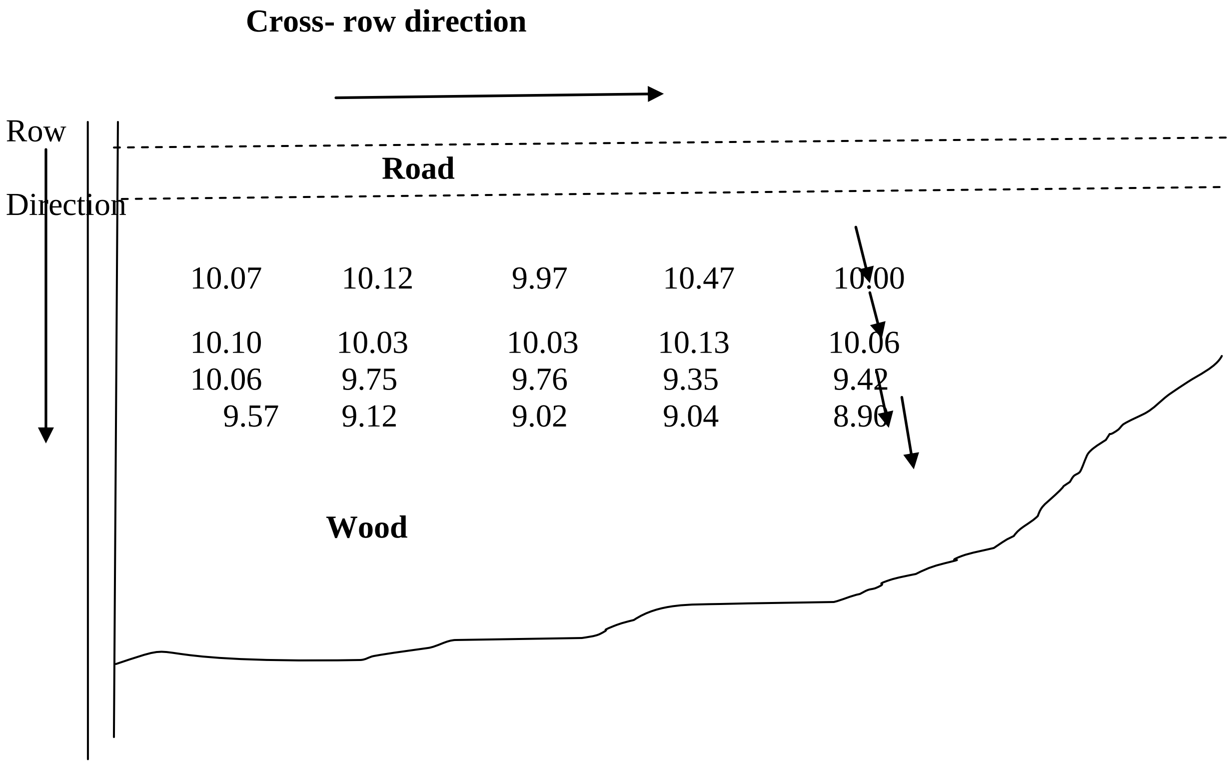



**Fig (5)**: Example of field data showing elevation and boundaries

**The optimum design results from the computer output shown in table (1)**

initial values of Design variables :
Sx = -0.07 , Sy = -0.33 , Ex = -0.002
Initial Step Size = 0.002
Final Optimum Values Of Design Variables :
Sx = -0.0803 , Sy = -0.382 , Ex = -0.0162
Final optimum volume of (cut/fill) = 28978.9
Final cut/fill ratio = 1.1
Final design elevation of Grid points :

| | | | | |
|---|---|---|---|---|
| 10.4712 | 10.3910 | 10.3107 | 10.2304 | 10.1501 |
| 10.0890 | 10.0087 | 09.9284 | 09.8181 | 09.7678 |
| 09.7067 | 09.6264 | 09.5461 | 09.4659 | 09.3856 |
| 09.3244 | 09.2442 | 09.1639 | 09.0836 | 09.0033 |

**Table (1)**: Summary of computer results for example (1)

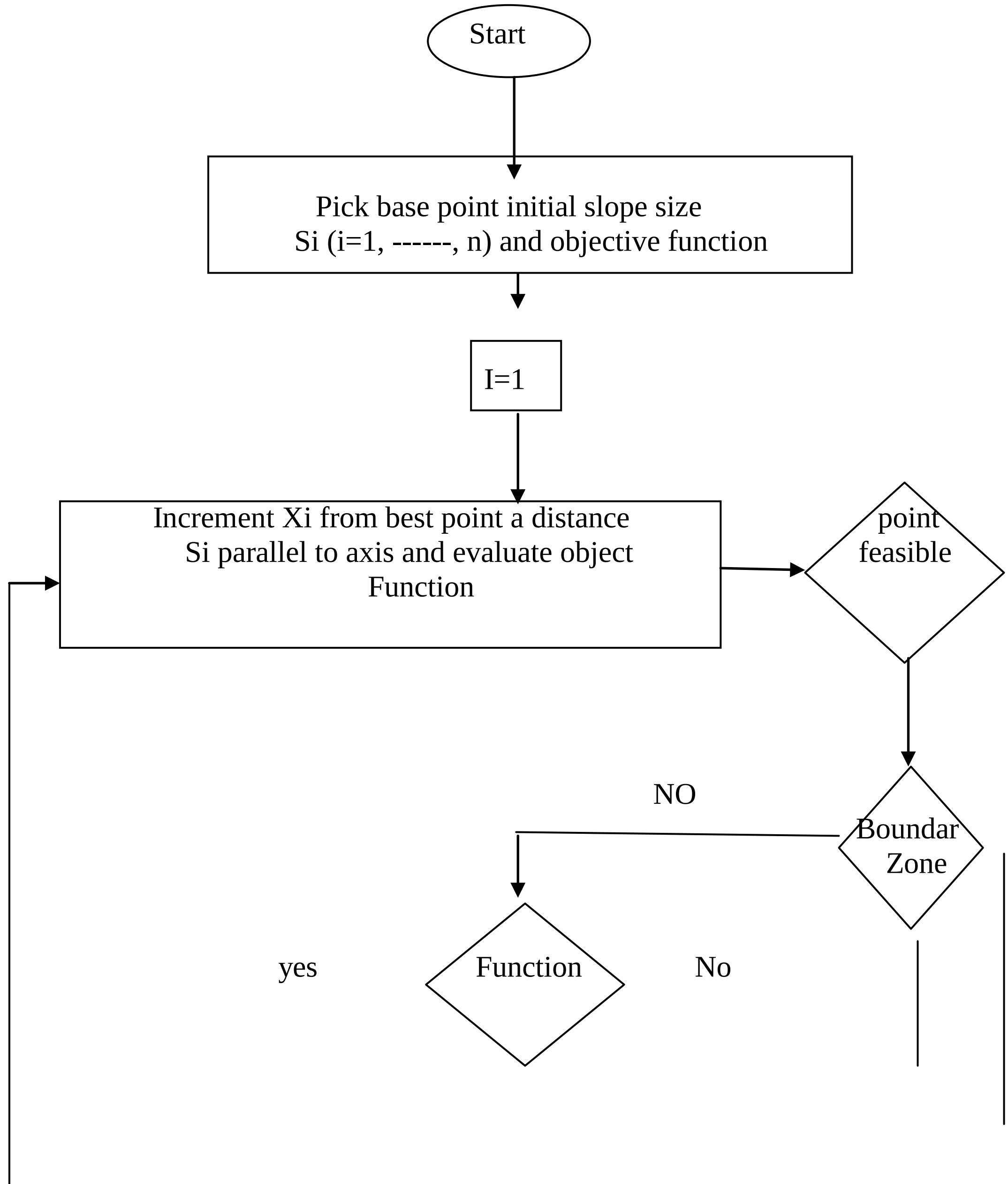
**Discussion and calculations:**

A- The formulation presented in this paper gives an easy method of optimum land forming design, which saves time and effort for designer. The use of the least square method in obtaining the initial values for the design variables proved to useful in saving time , in the test examples solved using the program , the optimum solution was found to be very near to the starting points exert in cases where the nature of the ground did not fit with the allowable design slopes .

B- Although the use of the four – point method in calculation the value of earthwork is sufficiently accurate for the purpose of a work .Other methods can be use to obtain more precise results, such as the field grid area method developed by Shih and Kriz (1971) it may give better solutions. Since the optimization process, depend on the elevation of the objective function (or the volume earthwork).

C- The Rosenbrock method of optimization proved to successful attaining the optimum time however as any bother numerical method, it can only reach a local optimum but not a global one – there is no spirited method to check whether the global optimum or only the local one has been reached . One can only begin the search operation at a number of initial base points and it the same result be accepted with a certain degree of confidence depending on the number of iterations at tented . However, in this work, it was notice and after many trail on several examples, there existed more than one optimum for each the objective functions obtained when starting with initial values of design variables give by the least square method. Even so the problem need a more detailed sensitivity any convergence analysis.

D-The uniform slope type of design is using in the formulation to simplify the calculation procedure. However, other types of design with more than uniform slope in each row and cross row direction can be using and inserting in the program to give a more sophisticated design.



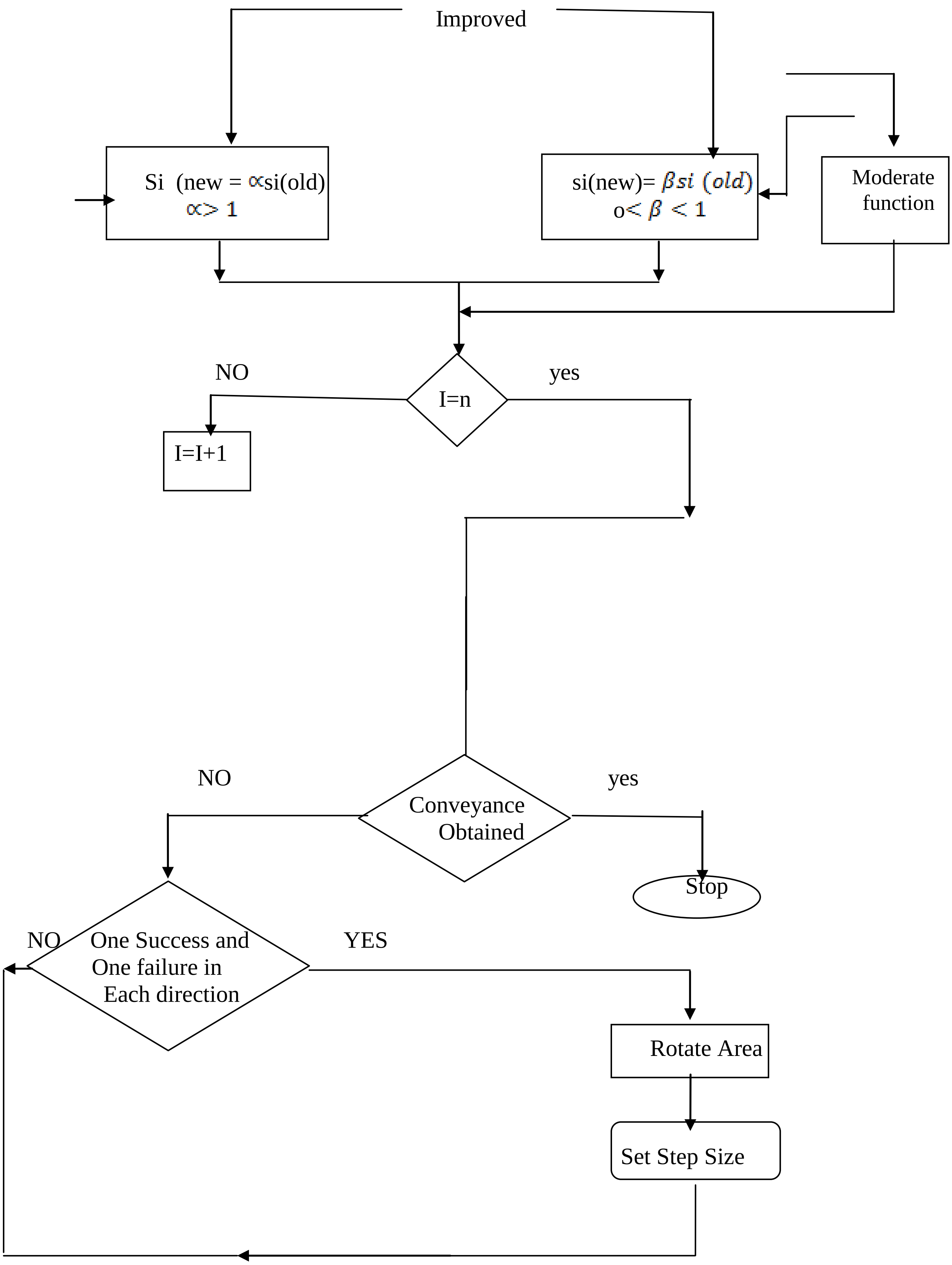



Fig.(2) Flow Chart showing Rosenbrock method

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