Design of a Nonlinear Speed Controller for a DC Motor System with Unknown External Torque Based on Backstepping Approach

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Abstract:

A nonlinear PI controller for a system with a mismatched and unknown external disturbance is proposed in the present paper. A Sigmoidal function is proposed to be used in the nonlinear integral element to attenuate the disturbance effect. Formally the controller forces the state to a positively invariant set around the origin. As an application, the nonlinear PI controller is used as a virtual controller for the upper subsystem in the frame of Backstepping control approach for the DC motor system. In this Backstepping controller design the restriction about the disturbance form in the upper system is relaxed. The results show the effectiveness of the nonlinear PI controller to attenuate the effect of unknown and varying disturbance torque and force the angular velocity to follow the desired reference. Finally a reduced order observer is designed to estimate the armature current used in the designed controller for the DC motor system.

Key words: Nonlinear PI, Disturbance Attenuation, Backstepping, Cascade Structure.

تصميم مسيطر لا خطي لمنظومة السيطرة على سرعة محرك التيارالمستمر والذي يتعرض لعزم خارجي غير معروف بالاعتماد على طريقة التراجع الخلفي

الخلاصة:

في هذا البحث ، تم تصميم مسيطر (تناسبي-تكاملي) لا خطي للسيطرة على نظام يتعرض لضوضاء خارجية غير معروفة ، حيث تم تصميم (Sigmoid Function) ليتم أستعمالها كعنصر تكامل لا خطي لتخفيف وازالة تأثير الضوضاء ، وبشكل أساسي فأن المسيطر سوف يجبر الحالة للذهاب إلى مجموعة (Positively Invariant Set) حول نقطة الأصل. كتطبيق ، تم أستعمال المسيطر (التناسبي-التكاملي) كمسيطر وهمي للنظام الفرعي الأعلى في أطار طريقة التراجع الخلفي (Backstepping) للسيطرة على نظام محرك التيار المستمر . في تصميم مسيطر التراجع الخلفي التقييد بشكل الضوضاء في النظام الفرعي الأعلى قد تم تخفيفه ، النتائج أظهرت كفاءة المسيطر (التناسبي-التكاملي)اللا خطي لتخفيف وأزالة تأثير ضوضاء عزم الدوران المتغير وغير المعلوم وإجبار السرعة الدورانية لكي تتبع السرعة المطلوبة . ختاماً تم تصميم مراقب ذو مرتبة متضائلة لكي يخمن تيار الـ(Armature) لكي يتم أستعماله في السيطرة على نظام محرك التيار المستمر.

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1. Introduction

Most control systems will unavoidably encounter disturbances, both internal (pertaining to unknown, nonlinear, time-varying plant dynamics) and external, and the system performance largely depends on how effectively the control system can deal with them [1]. In practice, the DC motor load torque (external disturbance) has an effect on the efficiency of DC motor control systems. The control system of a DC motor will increase the electric current in a DC motor to maintain a desirable speed when the DC motor load torque or disturbance torque is presented [2].

The disturbance torque may threaten overall (system stability), and causes deterioration of tracking performance. Therefore, cancelation the effect of the disturbance is a major design specification of the robust controller.

In general, to deal with disturbance there are two categories: the first is the **Disturbance Rejection**. (V. I. Utkin, 1993[3]), proposed two control strategies for Disturbance Rejection, the first one is to design a sliding mode controller that depends on the angular speed error time derivative under the assumptions that the angular speed and the current of the DC motor can be measured directly and the torque load varies slowly (the torque load $\cong constant$) and to implement the controller. A conventional Luenberger reduced-order observer is designed to estimate the torque load. The second control strategy takes into consideration the case when a DC motor is controlled with a mechanical motion being much slower than an electromagnetic one. In other words, the inductance of the DC motor is very small, so by considering the inductance ≅ zero, the DC motor model is reduced, and, under the assumptions considered in the first strategy, a sliding mode controller is designed depending only on the angular speed error (in contrast to the first one which depends on error time derivative). However, for both cases a Disturbance Observer (DOB) is designed and this implies that the load torque is known, which makes the rejection of the disturbance possible.

(T. Mita and et. al.,1998[4]), analyzed the extended H_∞ controller in view of the disturbance observer (DOB) and showed that it leads to a disturbance-observer-based disturbance-canceling controller, which has an ability to robustly stabilize the closed-loop system automatically.

(J. Srisertpol and C. Khajorntraidet, 2009 [2]) demonstrated a method for estimating the variable torque of DC motors by using a method called "adaptive compensation". The adaptive algorithms used for estimating the variable Torque load is the gradient method and Lyapunov's direct method. The result of the simulation of gradient method and Lyapunov's direct method showed high performance in estimating the DC motor variable torque but the results of the experiment of both methods had low efficiency.

The second category is the **Disturbance Attenuation**. (F. N. Koumboulis and et.al., 2001[5]) demonstrate that the problem of speed control of a permanent magnet DC motor with unknown parameters and unknown external load torque is formulated in the form of an adaptive control system based on the design requirement of asymptotic command following with simultaneous disturbance attenuation. The key point of the disturbance attenuation approach is to identify the system without knowledge, estimation or measurement of the unknown disturbance

(B.-K. Choi and etal.,1999 [6]), proposed a disturbance attenuation method called the model-based disturbance attenuator (MBDA), The MBDA makes the plant perform similarly to the nominal plant, as much as possible, using a compensator. Then, a controller is designed based on the nominal plant.

2. Mathematical model Description

The schematic diagram for DC motor armature Circuit is shown in Fig. (1), where the controller for the armature-DC motor system is the input voltageV_a. The mathematical model that describes the dynamical behavior of the DC motor is given by the following two 1st order differential equations [2]:

$$J\dot{\omega} = -b\omega + K_t i_a - T_L \qquad \textbf{(1-a)}$$

$$V_a = R_a\, i_a + L_a \frac{di_a}{dt} + V_b \qquad \textbf{(1-b)}$$

Where: $V_b = K_b\omega$, is the back emf (V).

Now let $x_1 = \omega - \omega_r$and$x_2 = i_a$, then, the mathematical model in Eqn. (1) can be re-written in state space form as follows:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{b}{J} & \frac{K_t}{J}\\ -\frac{K_b}{L_a} & -\frac{R_a}{L_a}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{L_a}\end{bmatrix}u + \begin{bmatrix}-\frac{b}{J}\\ -\frac{K_b}{L_a}\end{bmatrix}\omega_r + \begin{bmatrix}-\frac{1}{J}\\ 0\end{bmatrix}T_L \qquad \textbf{(2-a)}$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\omega\\ i_a\end{bmatrix} \qquad \textbf{(2-b)}$$

Where:

ω: Angular Velocity (rad/Sec).

ω_r: Desired Angular Velocity (rad/Sec).

i_a: Armature Current (A).

$u = V_a$: control input to the DC motor armature.

T_L: Variable Torque (demand load) (N. m).

$y = \omega$: system output.

Table (1) shows the abbreviations of parameters and their corresponding units used in DC motor state space form [1] (Eqn. (2)).

The load torque in this work is assumed to be variable and unknown and with respect to the DC motor system it is considered as a disturbance. Furthermore, the parameters in Eqn. (1) are assumed to be perfectly known.

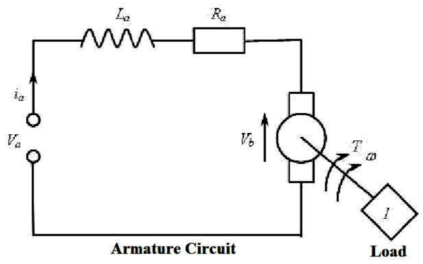

Figure (1): Schematic Diagram for DC Motor Armature Circuit.

3. Statement of the problem

The test to the dynamical model in Eqn. (2) reveals many challenging points. **First**, the matching condition is not satisfied. This is because the disturbance T_L does not lie in the same input channel, i.e., as in Eqn. (1), T_L not lie in Eqn. (1-b) where the input V_a exists. The fail of the matching condition added a great difficulty to the controller design with the unknown and varying disturbance assumption. Later, this problem is clarified when we examine the stability of our controller design in Sec. 6.

Secondly, the system in Eqn. (2) is not a minimum phase system. This property is simply verified by examining the zero dynamics equation after putting $x_2 = 0$ in the $\dot{x}_1$ channel in Eqn. (2). The backstepping approach may be effectively used with this type of system where x_2 appears linearly in $\dot{x}_1$ equation. However, the existence ofunknown variable torque T_L prevents the application of the classical backstepping where the disturbance is disappearing or it may satisfy certain inequality related to the norm ofx_1[7]. This point will be discussed in Sec. 5 and later we will show the ability of relaxing the inequality condition and applying the backstepping approach. This will represent the main contribution of our work. **Finally**, our proposed controller will require the availability of full states for measurements. This is not the case and an observer design is required to estimate the armature current i_a.

4. Reduced order observer design

To design an observer for the DC Motor model states in the presence of an explicit disturbance, a reduced order observer is used here to estimate the armature current i_a only while the angular velocity as described above is the system output. Accordingly the dimension of the reduced order observer is one.

Now the system state x is transformed to the state z via the following transformation:

$$z = T_R x \qquad \textbf{(3)}$$

where $z \in R^1$ and $T_R \in R^{1\times 2}$. Also, the dynamical system of the new state z is given by:

$$\dot{z} = T_R \dot{x} = T_R A x + T_R B u \qquad \textbf{(4)}$$

Since the disturbance (demand load) does not lie in the i_a-channel, then T_R is:

$$T_R = [0 \quad 1] \Rightarrow z = x_2 = i_a \qquad \textbf{(5)}$$

Hence, this yields the$z$ dynamical system as:

$$\dot{z} = -\frac{K_b}{L_a}\omega - \frac{R_a}{L_a}x_2 + \frac{1}{L_a}u$$

$$= -\frac{K_b}{L_a}y - \frac{R_a}{L_a}z + \frac{1}{L_a}u \quad \textbf{(6)}$$

where the output y and the input u are known. Now, let the reduced order observer take the following form [8]:

$$\dot{z_o} = Ez_o + Gu + Py\,, z_o \in R^1 \quad \textbf{(7)}$$

is an estimate to z state. Subtracting Eqn. (4) from (7) to get the error dynamics between the estimate z_o and the state z:

$$\dot{e} = \dot{z_o} - \dot{z} = Ez_o + Gu + Py + \frac{K_b}{L_a}y + \frac{R_a}{L_a}z - \frac{1}{L_a} \quad \textbf{(8)}$$

where $e = z_o - z$. Furthermore by taking:

$$G = T_R B = \frac{1}{L_a}$$

The error dynamical equation in Eqn. (8) becomes:

$$\dot{e} = Ee + Ez + Py + \frac{R_a}{L_a}z + \frac{K_b}{L_a}y$$

$$= Ee + \left(E + \frac{R_a}{L_a}\right)z + \left(P + \frac{K_b}{L_a}\right)y \quad \textbf{(9)}$$

In order for the error e to be minimized to zero, two design conditions should be considered:

1. The selection of the error matrix E should be of negative Eigen value.
2. $\left(E + \frac{R_a}{L_a}\right)z + \left(P + \frac{K_b}{L_a}\right)y = 0$

So from condition (2), the parameters of the observer ($EandP$) can be calculated to satisfy condition (1) as follows:

$$E = -\frac{R_a}{L_a} \text{and} P = -\frac{K_b}{L_a}$$

Finally the observer dynamical system is in the following form:

$$\dot{z_o} = -\frac{R_a}{L_a}z_o + \frac{1}{L_a}u - \frac{K_b}{L_a}y \quad \textbf{(10)}$$

5. Cascade Structure and Backstepping Approach:

The DC motor dynamic model as presented in Eqn. (2) is in the cascade structure form. The cascade form is shown in Fig. (2), where the state ζ is stabilized by the input u which is also regarded as a disturbance to the *z*-subsystem [9]. Moreover, this type of cascade structure is classified as a partial state feedback.

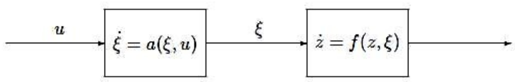

Figure (2): A cascade system [9].

By analogy to the cascade system the current i_a is the input to the ω-subsystem in Eqn. (1-a), while the voltage V_a is the input to the i_a-subsystem in Eqn. (1-b).

For the cascade system the Backstepping, which is originated by Kokotovic et.al in reference [9], is a powerful approach used to design a controller u that will stabilize the overall system dynamics. The first step in this approach is the designing of a virtual controller for the x_1-subsystem ($x_1 = \omega - \omega_r$) [refer to Eqn. (2)]. The virtual controller for the x_1-subsystem is the state x_2 (the current i_a). In the literature (see for example references [7,9]), the x_1-subsystem must satisfy certain structure form. This form may be given by [7]:

$$\dot{x_1} = f(x_1) + g(x_1)x_2 \quad \textbf{(11)}$$

where$f(x_1)$ and $g(x_1)$ are smooth functions and $f(0) = 0$. The x_1-subsystem as given in Eqn. (2) resemble the form in Eqn. (11) but with an additional term. This term is known as a perturbation term and it is given by:

$$\hat{d}(t) = -\frac{b}{J}\omega_r - \frac{1}{J}T_L \quad \textbf{(12)}$$

Eqn. (11) is now rewritten as:

$$\dot{x_1} = f(x_1) + g(x_1)x_2 + \hat{d}(t) \quad \textbf{(13)}$$

where $f(x_1) = -\frac{b}{J}x_1$ and $g(x_1) = \frac{K_t}{J}$ Accordingly, the matching condition is not satisfied as mentioned above. Furthermore, the backstepping can be used to relax the matching condition if $\hat{d}(t)$ satisfies the inequality [7]:

$$|\hat{d}(t)| \le a|x_1|, \;\; a > 0 \quad \textbf{(14)}$$

Again the term $\hat{d}(t)$ in Eqn. (12) does not satisfy the inequality (13) rather it satisfies the following bound:

$$|\hat{d}(t)| \le \frac{b}{J}|\omega_r| + \frac{1}{J}\beta \;\; , |T_L| \le \beta \quad \textbf{(15)}$$

For this situation the best we can do is to design a virtual controller that will bring x_1 to a region ultimately bounded by small bound around the origin (see lemma 9.2 in reference [7]).

A nonlinear PI controller is proposed in this work as a virtual controller that will bring x_1 to a region ultimately bounded by a small number ε. The value of ε is specified by adjusting the parameters of the proposed controller as will be shown in the subsequent section.

The final step in the controller design based on the Backstepping approach is the design of the actual controller u. The work of the controller u is to force x_2 to follow a desired reference; i.e., to regulate the error function defined by

$$e = x_2 - v \quad \textbf{(16)}$$

to the origin. The reference $v$in Eqn. (16) is the virtual controller designed in the first step. So, the controller u in the backstepping approach is utilized to force the state x_2 after a certain period of time to behave as a virtual control that will control the state x_1. However, during this period of time x_2 acts as a disturbance to the x_1 -subsystem. A hidden danger that mayarise during this period is the intricate **peaking phenomenon** [10]. To avoid the peaking phenomenon, a growth restriction on the interconnected term in the system dynamics in Eqn. (2) must be fulfilled. The stability of the system dynamics using the proposed controller will be analyzed in the following section.

6. Design of a Nonlinear PI Controller and The Stability Analysis

As illustrated in the previous section, the virtual controller is designed first. This controller is the desired reference for the second step in calculating the actual controller$u$. Hence, let us first rewrite the

x_1 -subsystem, replacing x_2 by v as the virtual controller, in the following form:

$$\dot{x}_1 = -\frac{b}{J}x_1 + \frac{K_t}{J}v - \frac{b}{J}\omega_r - \frac{1}{J}T_L \qquad \textbf{(17)}$$

Now, the nonlinear PI virtual controller considered here is as follows:

$$v = \frac{J}{K_t}\left(\frac{b}{J}\omega_r + \frac{b}{J}x_1 - k_p x_1 - k_i \int_0^t S(x_1)\,d\tau\right) \quad \textbf{(18)}$$

The $S(x_1)$ is the sigmoidal function [is a mathematical function having an S-shaped, in general it is real-valued and differentiable [11] that is used instead of x_1. In this work the arc tan function is used as a sigmoidal function i.e.

$$S(x_1) = tan^{-1}(\mu x_1)\ ,\ \mu > 1 \qquad \textbf{(19)}$$

Accordingly, Eqn. (17) becomes

$$\dot{x}_1 = -k_p x_1 - k_i \int_0^t tan^{-1}(\mu x_1)\,d\tau - \frac{1}{J}T_L$$

Since the disturbance load is a bounded quantity, but does not satisfy the inequality (14), the virtual controller will be able only to bring x_1 to a region near the origin as discussed in the previous section, i.e., a steady state error will exist. This error will be influenced by the selection of the proposed controller parameters k_p, k_i and μ.

The second design step is the design of the actual controller u. The error is defined as in Eqn. (16) ($e = x_2 - v$), and the task of the controller, in this stage, is to regulate the error to zero. This task is also accomplished by means of a nonlinear PI controller given by:

$$u = L_a\left(a_1(x_1 + \omega_r) + a_2 x_2 + a_3 tan^{-1}(\mu x_1) - K_{pp}e - K_{ii}\int_0^t tan^{-1}(\gamma e)\,d\tau\right) \qquad \textbf{(20)}$$

where:

$$a_1 = \frac{K_b}{L_a} - \frac{b^2}{JK_t} + \frac{bk_p}{K_t}$$

$a_2 = \frac{R_a}{L_a} + \frac{b}{J} - k_p$ and;

$$a_3 = -\frac{Jk_i}{K_t}$$

The error dynamics $\dot{e}$ accordingly is

$$\dot{e} = -k_{pp}e - k_{ii}\int_0^t tan^{-1}(\gamma e)\,d\tau + \frac{1}{K_t}\left(\frac{b}{J} - k_p\right)T_L$$

Again the steady state error is influenced directly by the choice of the parameters for the control law in Eqn. (20). The derivation of the controller u in Eqn. (20) is found in appendix A.

In the above controller design, the virtual controller is designed to control the x_1 -subsystem where the state x_2 (the armature current) is the virtual controller. This represents the first step. In the second step the actual controller is designed based on regulating the difference error ($e = x_2 - v$) to zero in a finite period of time. Therefore, after this period the error goes to zero and therefore the state x_2 will be equal to the virtual controller v. Moreover, our analysis about the stability and the effectiveness of the virtual controller is significant also after this period of time. Thus, we are interested in studying the stability of the interconnected system during this period of time, i.e., the error $e \neq 0$. In fact, we want to show that the state x_1 in the upper subsystem (x_1-subsystem) will not escape to infinity during this period. This type of behavior is due to the peaking phenomenon which is a fundamental structural obstacle not only to the solution of global, but also semi global stabilization problem [9].

For these reasons, the task of global stabilization of the cascade system (Eqn. (2)) does not only require the stability assumption about the subsystems, but it also imposes a severe linear growth restriction on the **interconnected term**[9]. To find the interconnected term for the cascade system in Eqn. (2), Eqn. (17) is rewritten after considering the error Eqn. (16) as follows:

$$\dot{x}_1 = -k_p x_1 - k_i \int_0^t S(x_1)\, d\tau - \frac{1}{J} T_L + \frac{K_t}{J} e \quad \textbf{(21)}$$

The term $\psi(x_1, e) = \frac{K_t}{J} e$ in Eqn. (21) is known as the interconnected term. For global stabilization, in addition to stability and stabilizability of the subsystems, the interconnected term must have a linear growth in x_1 (theorem 4.7 and assumption 4.5 in reference [9]). Since the interconnected term $\psi(x_1, e)$ satisfy the linear growth assumption:

$$|\psi(x_1, e)| \leq \frac{K_t}{J} |e|$$

then the cascade system in Eqn. (2) with the proposed controller (Eqn. (20)), is a globally stable system. With this result, the validity of the controller formula as proposed in Eqn. (20) is proved.

7. Discussion of Simulations Results:

This section is dedicated to demonstrating the simulation results of the control of speed of DC motor using Backstepping control methodology with nonlinear PI control scheme. Also all the simulation results for the designed control algorithm are proven using MATLAB Rev. (14.8) (2009-a). In addition the current estimation is included in these simulations where it is regarded as an additional state augmented with the system model in Eqn. (2).

Three disturbance torque types are taken in the simulations test. The first disturbance type which acts on system dynamicsis when the demand torque is unknown but constant quantity. The demand torque is taken as a step form,

$$T_L = 0.1 \quad \textbf{(22)}$$

while, the second form is taken as a sinusoidal form (shown in Fig. (3)),

$$T_L(t) = 0.1 \sin(0.1t) \quad \textbf{(23)}$$

The final test is taken as a variable step as shown in Fig.(4). Figure (5) shows the simulation result for the estimated angular velocity of the DC motor, it can be seen that the proposed Backstepping based nonlinear PI controller succeed to bring the motor velocity to its desired level ($104.72\ rad/sec$ ($1000\ RPM$)), although the velocity response is dropped into about $(-20\ rad/sec)$ (the negative sign indicates direction reversal). Actually this can be justified to the fact that the system under Backstepping control responds first to the negative step torque load input $(-\frac{1}{J}T_L)$.This behavior reveals the nature of the controller when it is based on Backstepping approach.

Figure (6), shows the simulation results of the armature current state, it can be noted that the armature current is about $(2\ Amp)$ to maintain the angular velocity to its desired level under step torque load.The simulation result for the armature Voltage V_a is presented in Fig. (7) where a 12 Volts is needed to overcome the effect of applied torque. Figure (8) shows the time history of the error function (Eqn. (16)) while Fig. (9) shows the virtual controller (Eqn. (18)) compared with estimated armature current. Both figures show that the Backstepping controller enforces the estimated armature current to mimic the virtual controller in about (1.4 Sec.).

In order to examine the designed observer (Eqn. (10)) potential, the previous test is repeated but with perturbed initial conditions ($\omega(0) = 52.36\ \mathrm{rad/Sec}$ (500 RPM), $i_a(0) = 1\ Amp$).The time history of angular velocity is shown in Fig. (10),where it is regulated to the desired velocity within the same performance shown in the first test. Figure (11) is dedicated to give a comparative view to the armature current i_a with the estimated armature current z_o , where the observer succeeds to coincide with the real armature current in about less than 5 $msec$. This indicates the ability of the designed observer to estimate the armature current under the presence of unknown torque load; and provide the controller with the required measurement for the armature current to deliver the control action needed to handle the system to the desired speed. The time history of the error function (Eqn. (16))is shown in Fig. (13), while Fig. (14) shows the virtual controller (Eqn. (18)) compared with estimated armature current. Again both figures show that the Backstepping controller is able to bring the estimated armature current to mimic the virtual controller in about (1.4 Sec.).

To explore the potential and capability of the proposed Backstepping controller, the simulation is repeated but with the second type (sinusoidal form Eqn. (23), shown in Fig. (3)). Figure (15) illustrates the time history of the angular velocity of the DC motor. It can be noticed that although the system undergoes a variable demand load, the designed Backstepping controller was efficient enough to maintain the angular velocity to the desired level in about (0.1 Sec.).The estimated armature current is shown in Fig. (16), while the armature voltage time history is shown in Fig. (17), both the armature current and voltage fluctuate in the same sense with the demand torque load (the negative sign indicates direction reversal).

The main job of the armature voltage is to force the armature current to follow the virtual controller. This is shown in Fig. (18), where the error function (Eqn. (16)) between the (estimated) armature current and the virtual controller v surface is plotted.In Fig.(19),the (estimated) armature current and the virtual controller v time history is plotted. It is clear from the above two aforementioned figures that the controller succeeds to minimize the error between the current and the virtual controller to the origin in about 0.06 Sec. i.e., the control voltage causes the armature current to act like the virtual controller. Before this time, the error between the armature current and the virtual controller is acting like an additional disturbance applied to the system. This can be more clarified in Fig. (19).

The sinusoidal fluctuation nature of the armature current response reveals the activity of the proposed controller to deal with a sinusoidal demand (disturbance) load. This form of the armature current is mainly to eliminate the effect of the demand (disturbance) load. In all cases the current, as a virtual controller, can't eliminate a variable disturbance torque like in Eqn. (23), rather than the controller attenuate its effect and suppress it to a minimum level.

Finally, the third test is applied using the third type of Torque load with variable step values (shown in Fig. (4)). Figure (20) shows the simulation results for the angular velocity for the DC motor. It can be noticed that the system undergoes transient responses in times (10, 20,30, and 40sec), this is the same case discussed in the first test where the system with Backstepping controller needs to overcome the effect of the demand load $(-\frac{1}{J}T_L)$ at these times (namely at the first instants of the change in demand load). However the Backstepping controller succeeds to regulate the angular velocity to its desired value at every step change. Figure (21) shows the time history of the estimated armature current while Fig. (22) is dedicated to show the armature voltage. The profile of both estimated current and voltage revealed the Backstepping approach in attenuating the demand load disturbance. Figure(23) shows the error function behavior under variable step demand load, while Fig. (24) shows the comparison between estimated armature and virtual controller. Again it can be seen that at every step changing the controller enforces the estimated armature current to act like the virtual controller.

8. Conclusions:

In this paper, we have proposed a nonlinear PI controller for a system having mismatched unknown and varying external disturbance. The arc tan function has been used as the nonlinear function to the error in the integral element instead of the linear error function. The controller design was designed based on Backstepping concept where the current was regarded as a virtual controller that will attenuate the disturbance effect and regulate the error to a small region around the error origin named as positively invariant set. After that the actual controller (the voltage) was designed to regulate the error between the current and the virtual controller to a nearly zero value. To perform the proposed controller law a reduced order observer was designed to estimate the current which appears in the control law. The effectiveness of the proposed nonlinear control to a DC motor was proved via the simulation results with a current observer and the angular velocity as the output. In these simulation results the control action forces the current to follow the virtual controller (desired signal), then the current will regulate the error between the angular velocity and the reference one to a very small value in the presence of a different disturbance unknown types. Finally the Backstepping nature in the controller design was clarified in the figures plotted for the angular velocity, where the angular velocity responds to the disturbance torque until the current is equal to the virtual controller and then force the angular velocity to rise and follow the desired reference.

Appendix (A): Derivation of Eqn. (20):

To derive the control formula in Eqn. (20), the error function e is differentiating as:

$$\frac{de}{dt} = \frac{dx_2}{dt} - \frac{dv}{dt}$$

$$= -\frac{K_b}{L_a}x_1 - \frac{R_a}{L_a}x_2 + \frac{1}{L_a}u - \frac{K_b}{L_a}\omega_r - \frac{J}{K_t}\left\{\left(\frac{b}{J} - k_p\right)\left(-\frac{b}{J}x_1 + \frac{K_t}{J}x_2 - \frac{b}{J}\omega_r - \frac{1}{J}T_L\right) - k_i tan^{-1}(\mu x_1)\right\}$$

$$= -k_{pp}e - k_{ii}\int_0^t tan^{-1}(\gamma e)\, d\tau + \frac{1}{K_t}\left(\frac{b}{J} - k_p\right)T_L$$

Where k_{pp} and k_{ii} are the desired proportional and integral parameters. As a result, the control u is equal to:

$$u = L_a\left\{\left(\frac{K_b}{L_a} - \frac{b^2}{JK_t} + \frac{bk_p}{K_t}\right)(x_1 + \omega_r) + \left(\frac{R_a}{L_a} + \frac{b}{J} - k_p\right)x_2 - \frac{Jk_i}{K_t}tan^{-1}(\mu x_1)\right\}$$

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Table (1): DC Motor Parameters Definition and their Corresponding Units [1].

Para.	Definition	Value	Units
J	Moment of inertia.	2.0069×10^{-5}	$kg.m^2/rad$
K_t	Torque constant.	0.052	$N.m/A$
K_b	Electromotive force constant.	0.057	$V.s/rad$
b	Linear approximation of viscous friction.	3.3677×10^{-5}	$N.m.s/rad$
R_a	Resistance	2.9981	Ω
L_a	Inductance	2.0864×10^{-3}	H

Table (2): Controller Parameters Values.

Para.	Value
k_p	120
k_i	1000
K_{pp}	100
K_{ii}	100
μ	500
γ	500

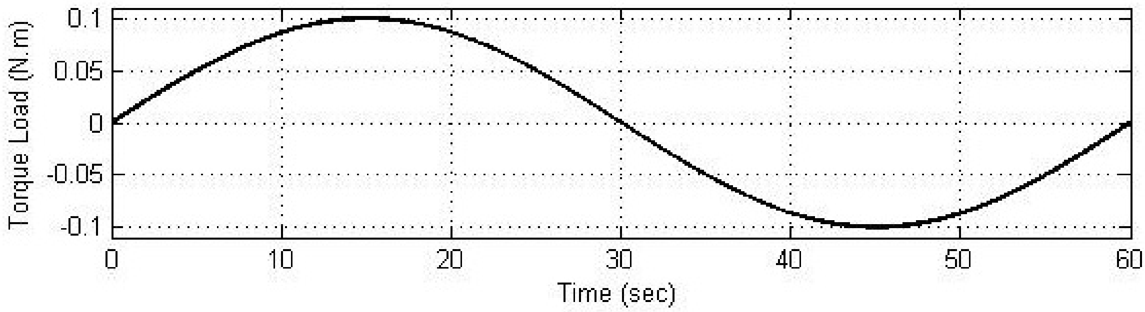


Figure (3): Variable Sinusoidal Torque Load.

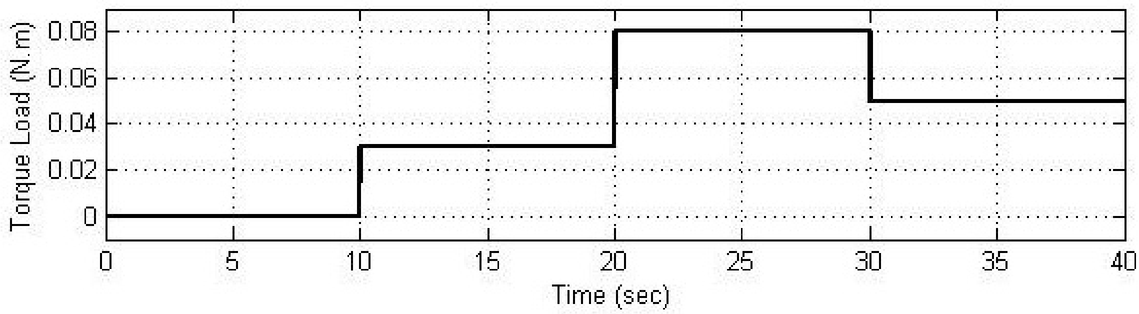


Figure (4): Variable Step Torque Load.

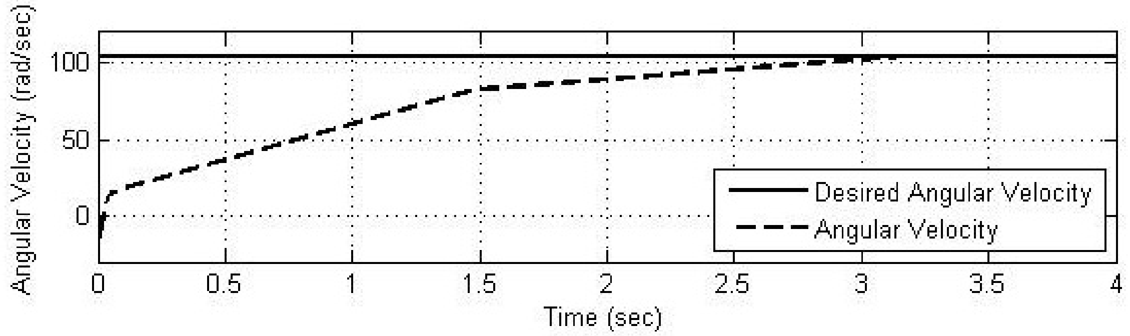


Figure (5): Simulation Results of Angular Velocity ωunder aStep Torque Load.

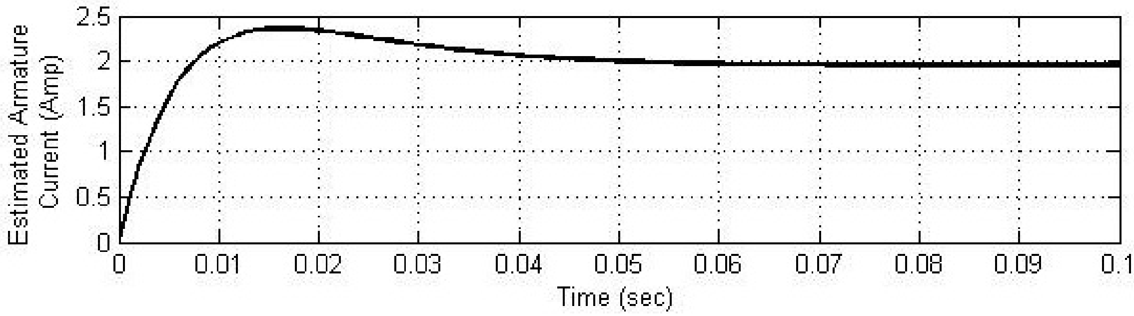


Figure (6): Simulation Results of Estimated Armature Current $z_o$under Step Torque Load.

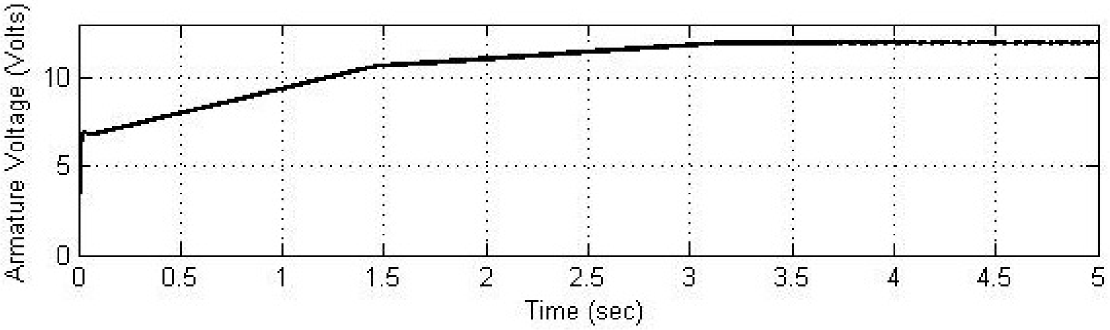


Figure (7): Armature VoltageV_a underStep Torque Load.

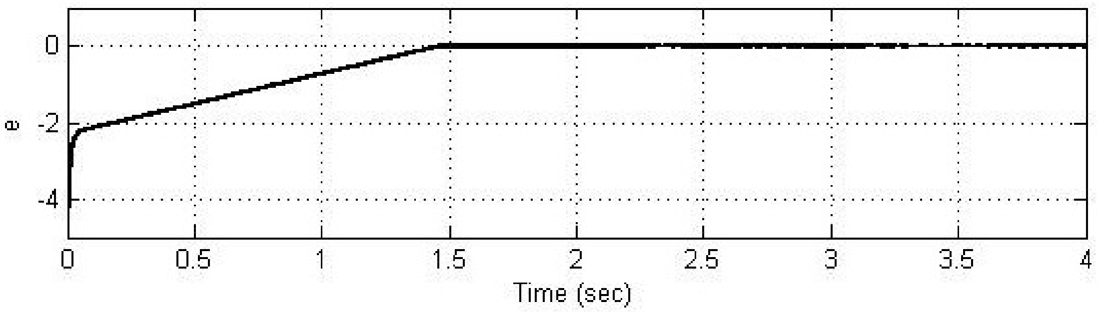


Figure (8): Simulation Results of Error Functionunder Step Torque Load.

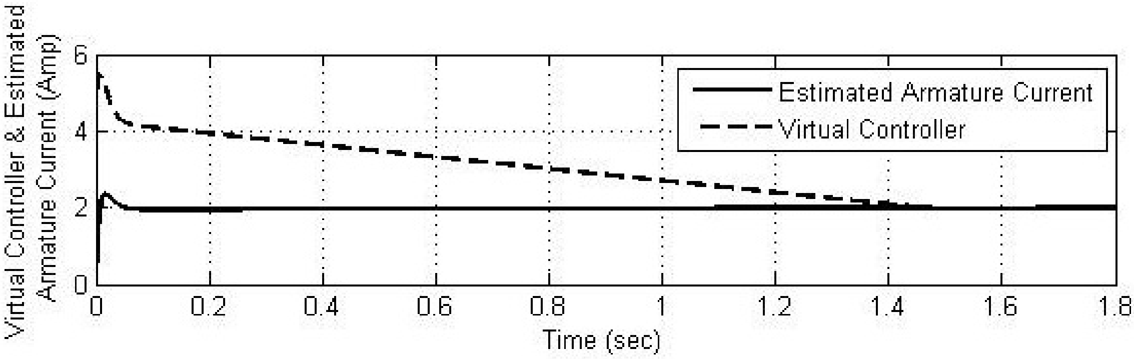


Figure (9): Simulation Results of Virtual Controller $v$and Estimated Armature Current.

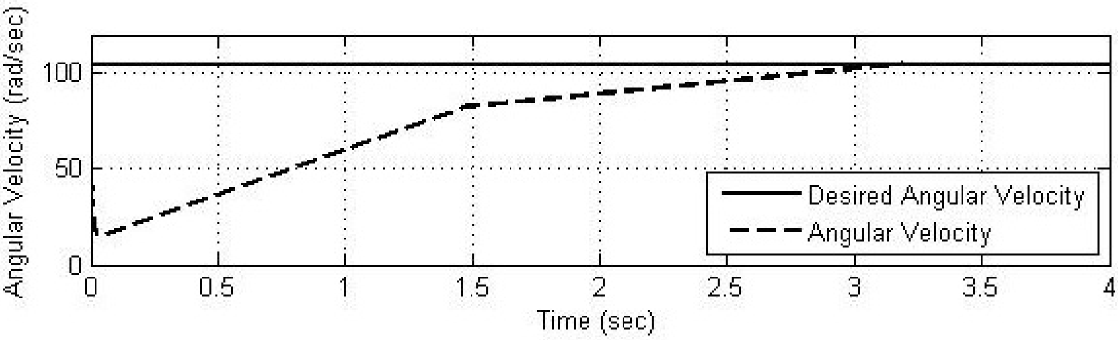


Figure (10): Simulation Results of Angular Velocity ω under a Step Torque Load with a perturbed initial conditions

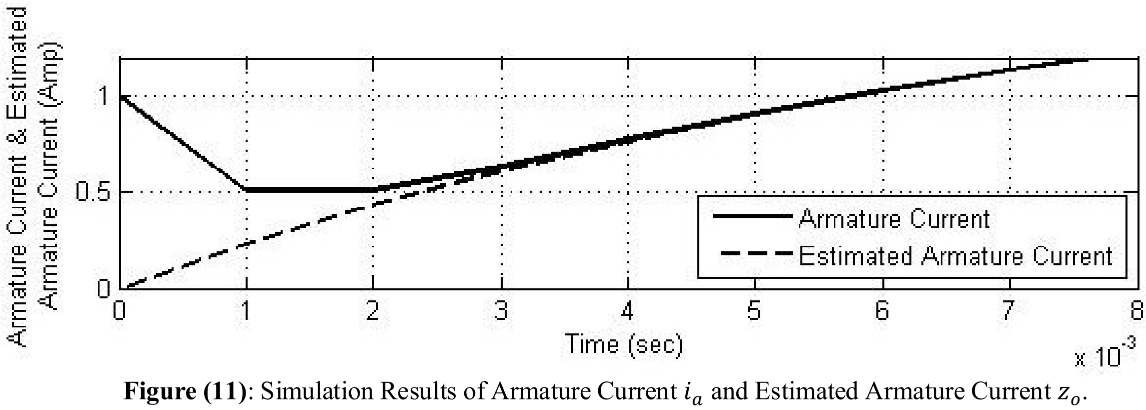


Figure (11): Simulation Results of Armature Current i_a and Estimated Armature Current z_o.

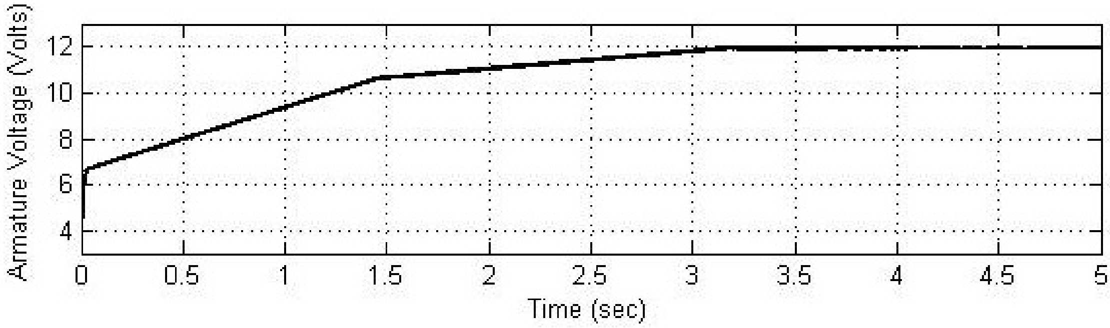


Figure (12): Armature Velocity Va Under Step Torque Load with perturbed Initial Conditions.

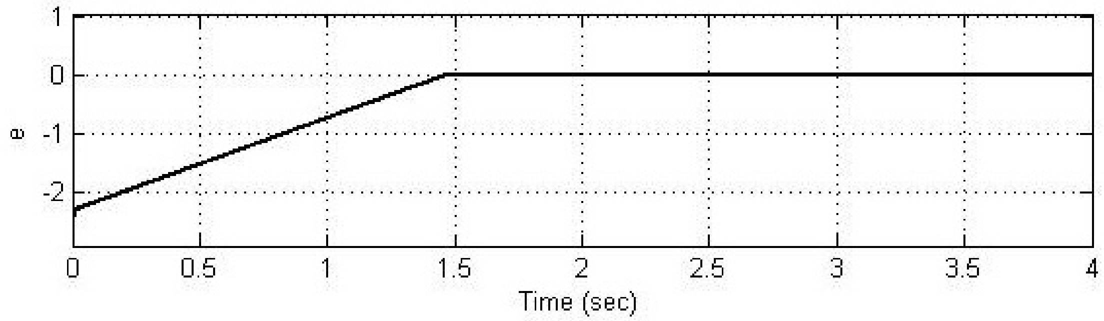


Figure (13): Simulation Results of Error Function under Step Torque Load with perturbed Initial Conditions.

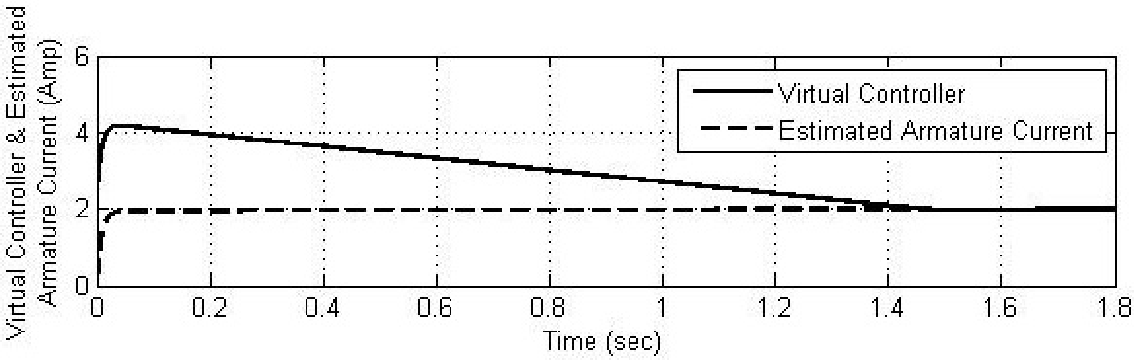


Figure (14): Simulation Results of Virtual Controller v and Estimated Armature Currentwith perturbed Initi Conditions.

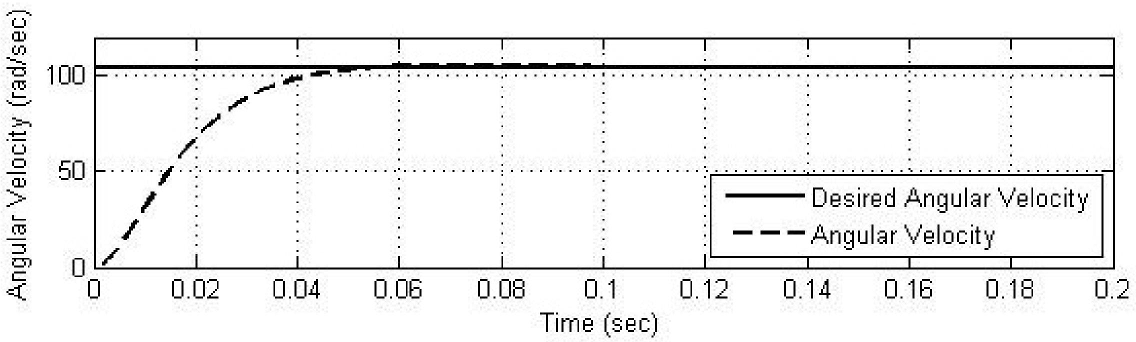


Figure (15): Simulation Results of Angular Velocity ωunder a Sinusoidal Torque Load.

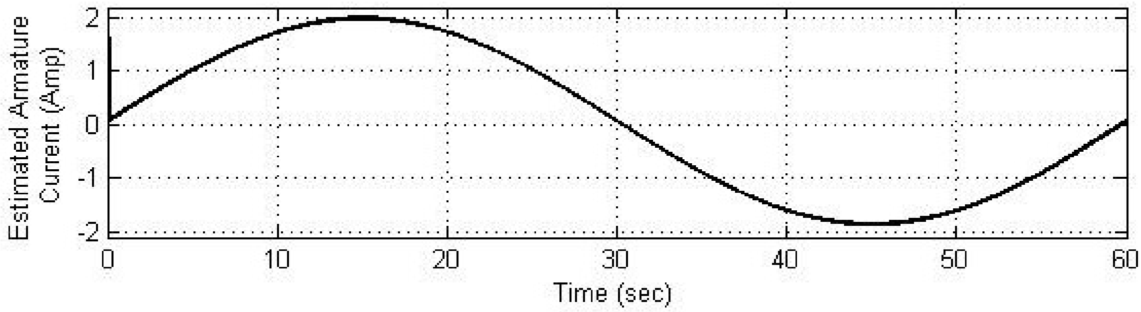


Figure (16): Simulation Results of Estimated Armature Current $z_o$underSinusoidal Torque Load.

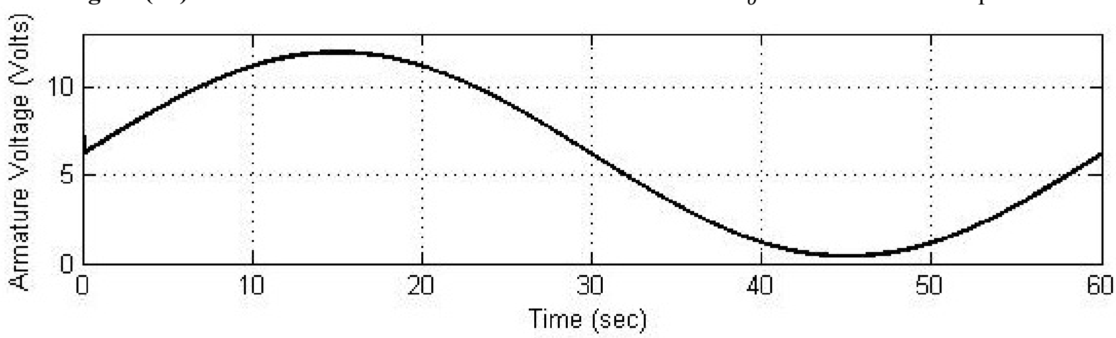


Figure (17): Simulation Results of Armature Voltage$V_a$underSinusoidal Torque Load.

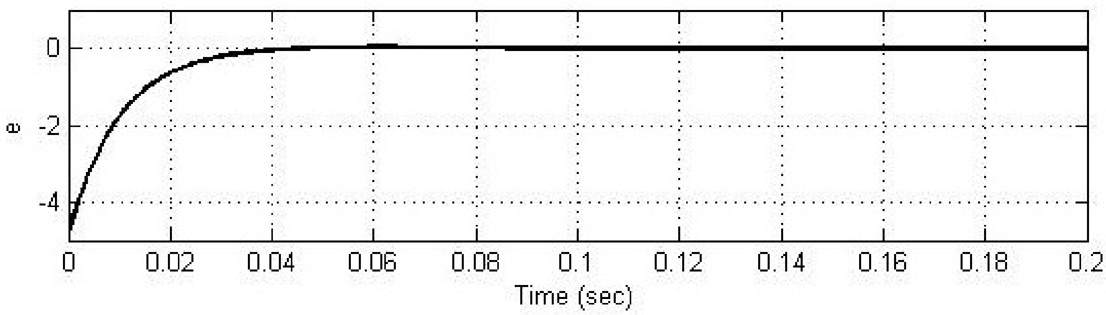


Figure (18): Simulation Results of Error Function under Sinusoidal Torque Load.

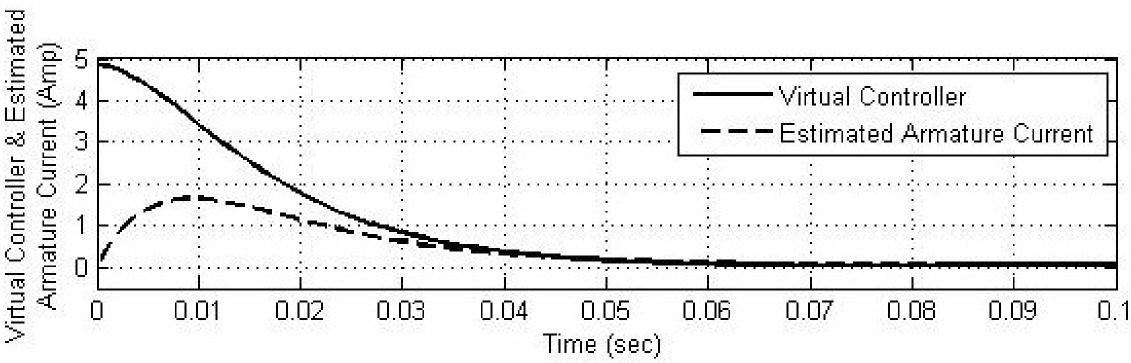
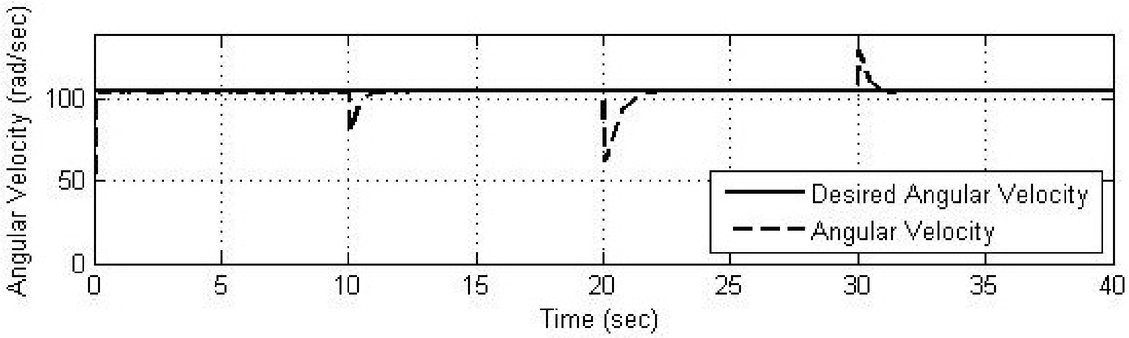


Figure (19): Simulation Results of Virtual Controller v and Estimated Armature Current.



Figure(20): Simulation Results of Angular Velocity ω under a Variable Step Torque Load.

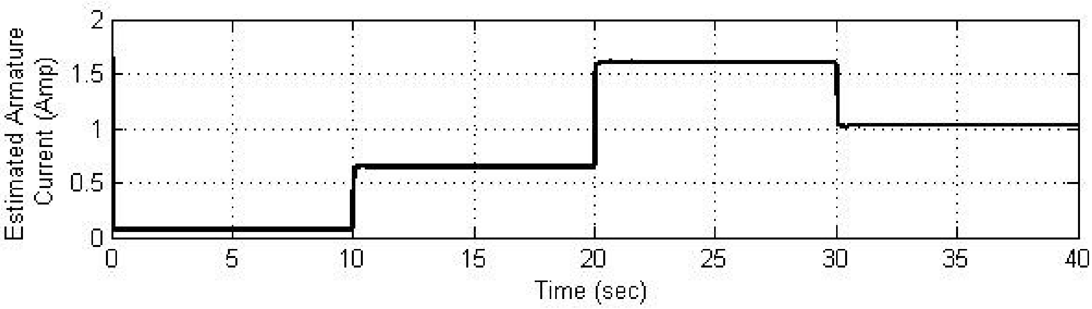


Figure (21): Simulation Results of Estimated Armature Current $z_o$underVariable Step Torque Load.

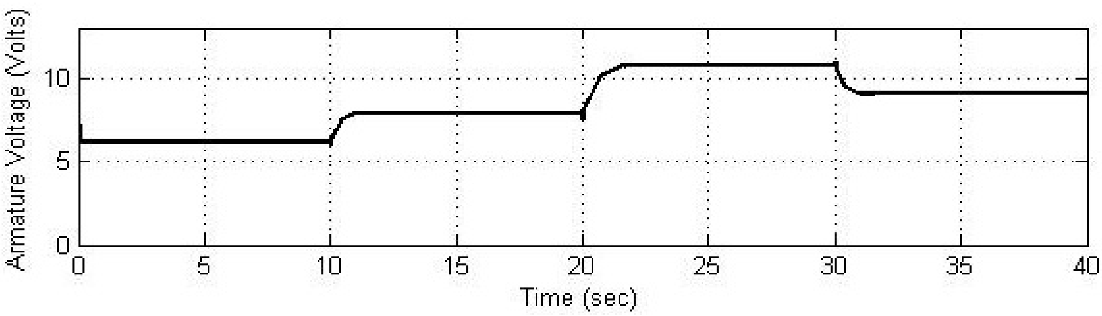


Figure (22): Simulation Results of Armature Voltage V_a under a Variable Step Torque Load.

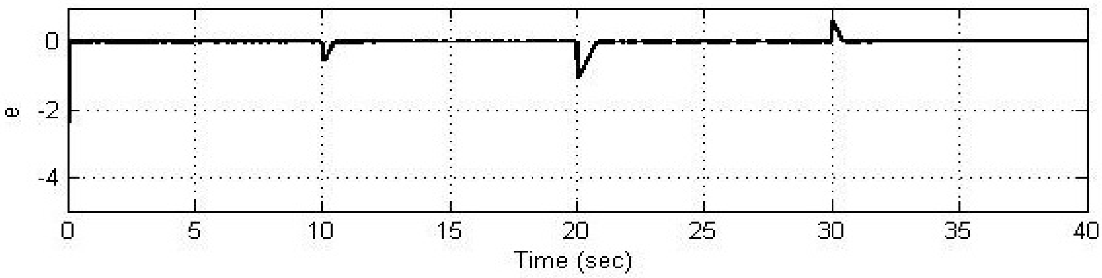


Figure (23): Simulation Results of Error Function under Variable Step Torque Load.

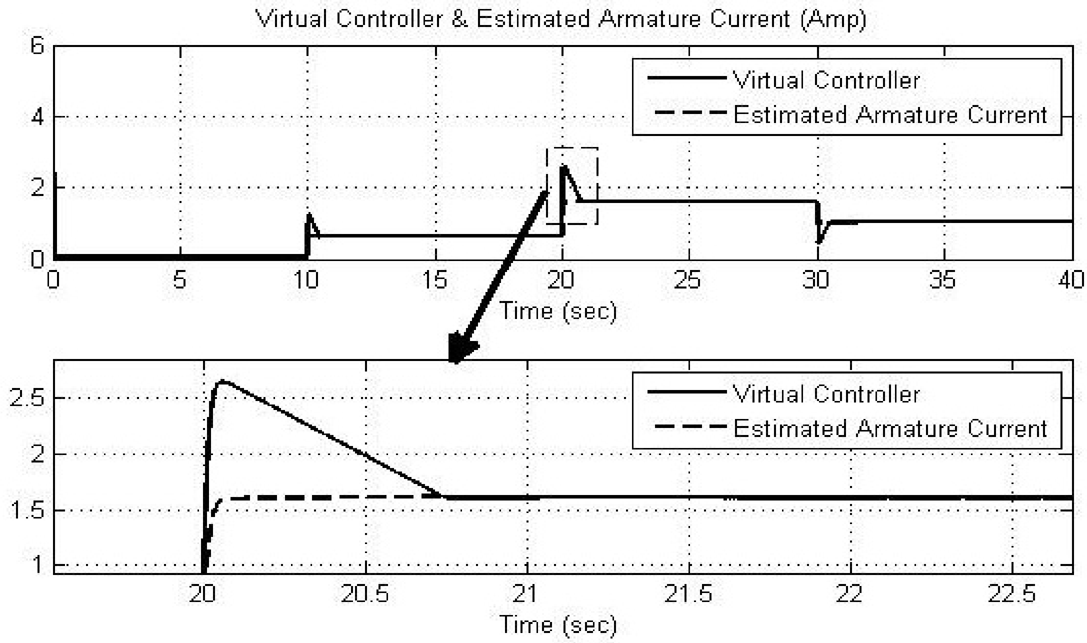


Figure (24): Simulation Results of Virtual Controller v and Estimated Armature Current.