Mathematical Technique for Controlling Time of Designs of cubic Bezier Surface of Three-Dimensional

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Abstract

This paper deals with controlling the time that is needed for the design, in three-dimensional field in graphics. It describes and presents types of some techniques for the representation and interactive of control of the time for the design of surface, based on values of the coefficient of parametric surfaces representation that uses Three-dimensional cubic Bezier surfaces.

A new approach, can be seen in controlling the time needed for generating the design in three dimensions. This new approach uses mathematical technique to control the time that generating the design in three-dimensional. It is a more efficient technique, used in two-dimensional design. The new approach is tested with changing coefficient of parametric surfaces of Bezier. Examples and the result show that this approach is successful in controlling the time.

الخلاصة

للسيطرة على زمن توليد التصميم ثلآثي ألأبعاد في مجال الرسوم . يقدم هذا البحث نماذج لبعض ألأساليب للسيطرة على زمن التصاميم ثلآثي ألأبعاد.بالأعتماد على قيم معاملات المتغيرات لوصف السطوح من الدرجة الثالثة وبألأمكان ملاحظة أسلوبا جديدا في السيطرة على زمن توليد التصاميم ثلاثية ألأبعاد بالأستفادة من سطوح (Bezier).

هذا ألأسلوب الجديد يستخدم تقنية رياضية للسيطرة على زمن التصاميم ثلآثية ألأبعاد والذي هو أكثر كفاءة عند أستخدامه في التصاميم ثلآثية ألأبعاد منه في التصاميم ثنائية ألأبعاد.تم أختبار الطريقة المقترحة من خلال تغيير معاملات منحني Bezier من الدرجة الثالثة ثلاثي ألأبعاد. أثبتت النتائج ان للطريقة المقترحة كفاءة عالية في أمكانية السيطرة على زمن توليد التصميم .

Introduction

Controlling the time for generating the design, is one of the most modern and important branch in three-dimensional field in computer graphics. With the improvement the information technology and especially with the appearance of advanced computer. A mathematic method of progressive, using arithmetic technique for measurement and control of real time of three-dimensional design.

Changing the values of the coefficient of parametric Gallier modified cubic Bezier surfaces, to control the time of generating the design. The time effect by changing the value of the coefficient of parameters generating the design, without changing the design. . Bezier curves are named after Bezier for his work in this field at Renault in the 1960s.Slightly earlier de- Castejalu had already developed mathematically equivalent method of defining Bezier curve.

This work discusses Bezier curves which are a simple kind of Spline. For the sake of concreteness, the emphases is on the special case of original cubic Bezier surfaces, will use this with value of the parameters u and v are between 0 and 1.

Second Gallier modification of cubic Bezier surfaces should allow the user to find a method for controlling generating the time of the design will use this with value of parameters u *and* v. And third developing algorithm of controlling the time of the design by use the coefficient of parameters u *and* v. To find the **optimal time** without changing the controlling points, and the design.

Background

De-Castejalu algorithm is an algorithm which uses a sequence of control points, P_1, P_2, P_3, P_4 to construct a well defined curve $P(u)$ at each value of u from 0 to 1.This provides a way to generate a curve from a set of points. Changing the points will change the curve. *P (u)* is defined as: [Faux 83], [Buss03], [Lengyel 04]. [Jaber 10] [Jaber 05]

$P(u)=(1-u)^3P_1+3(1-u)^2uP_2+3(1-u)u^2P_3+u^3P_4.$ (1)

Eq (1) called original cubic Bezier curve (into the two -dimensional) is dependent on interval *[0, 1]* and uses a sequence of control points $P_1 ... P_{16}$ to define three-dimensional surfaces (Mathematically they are said to be generated from the Cartesian product of two curves). A cubic Bezier surfaces is defined as: [Lengyel 04], [Faux 83], [Watt 00], [Gerald 99], [Lengyel 04], [Klawonn 08].

$$P(u,v)=\sum_{i=0}^{3}\sum_{j=0}^{3}\binom{3}{i}\binom{3}{j}[1-u]^{3-i}[u]^{i}[1-v]^{3-j}[v]^{j}P_{ij} \quad (2)$$

Equation (2) is called original cubic Bezier surfaces.

Gallier Modified De-Castejalu Algorithmic.[Gallier 00]. [Jaber 05].

The modified De-Castejalu algorithm which is an algorithm that uses a sequence of control points $p_i= f(r^{3-i} s^i)$, (for $i= 0, 1, 2,3$) to construct a well defined curve $P(u)$ on a given interval $[r, s]$ for which $r<s$, $u \in [r, s]$, and $(r, s\in Z)$. (where Z is integer number). Let us begin with straight lines (degree one), given any affine frame $[r, s]$ for which $r<s$ for $u \in R$). (where R real number), it can be written uniquely as

$u= [1- \lambda] r+ \lambda s =r+ \lambda [s-r].$

And you can find that:

$\lambda=\dfrac{u-r}{s-r}$, and $1-\lambda=\dfrac{s-u}{s-r}$. Since *F* is affine then

$P(u) = P[(1-\lambda) r+ \lambda s]$

$\therefore P(u) = (1-\lambda) P(r) +\lambda P(s).$ (3)

De Castejalu algorithm uses two control points say *P (r)* and *P (s)* as in Eq (3). Eq (3) is called Gallier modified Bezier curve (two-dimensional space) is dependent on interval *[r, s]*. De Castejalu algorithm at cubic curve is defined as:

$P(u)= (1-\lambda)^3p_0+3 \lambda(1-\lambda)^2 p_1+ 3(1-\lambda)\lambda^2p_2+\lambda^3p_3 .$ (4)

Eq (4) called original Gallier modified cubic Bezier curve (two-dimensional space) is dependent on interval *[r, s]*.

Now let (r_1, s_1) and (r_2, s_2) be two affine frames. Every point *u* can be written as

$u= [1- \lambda] r_1+ \lambda s_1 = r_1+ \lambda [s_1 - r_1].$

And you can find that:

$\lambda=\dfrac{u-r_1}{s_1-r_1}$, and $1-\lambda=\dfrac{s_1-u}{s_1-r_1}$.

Similarly any point *v* can be written as

$v= [1- \beta] r_2+ \beta s_2 = r_2+ \beta [s_2 - r_2].$

And you can find that:

$\beta=\dfrac{v-r_2}{s_2-r_2}$, and $1-\beta=\dfrac{s_2-v}{s_2-r_2}$, where $(r_1< s_1)$ and $(r_2< s_2)$, for $(r_1, s_1, r_2$ *and* $s_2\in Z)$, *u and v.* $\in R$.

De-Castejalu algorithm at cubic surfaces is defined as:

$$\boldsymbol{P(u,v)} = [(1-\lambda)^3 (1-\beta)^3]\boldsymbol{p_1} +3[(1-\lambda)^3 (1-\beta)^2 \beta]\boldsymbol{p_2} + 3[(1-\lambda)^3 (1-\beta) \beta^2]\boldsymbol{p_3} + [(1-\lambda)^3 \beta^3]\boldsymbol{p_4} +3[(1-\lambda)^2 \lambda (1-\beta)^3]\boldsymbol{p_5}+9[(1-\lambda)^2 \lambda (1-\beta)^2 \beta]\boldsymbol{p_6}+ 9[(1-\lambda)^2 \lambda (1-\beta) \beta^2]\boldsymbol{p_7} + 3[(1-\lambda)^2 \lambda \beta^3]\boldsymbol{p_8}+3[(1-\lambda) \lambda^2 (1-\beta)^3]\boldsymbol{p_9} +9[(1-\lambda) \lambda^2 (1-\beta)^2 \beta]\boldsymbol{p_{10}}+9[(1-\lambda) \lambda^2 (1-\beta) \beta^2]\boldsymbol{p_{11}}+ 3[(1-\lambda) \lambda^2 \beta^3]\boldsymbol{p_{12}} +[\lambda^3 (1-\beta)^3]\boldsymbol{p_{13}}+3[\beta \lambda^3 (1-\beta)^2]\boldsymbol{p_{14}} +\boldsymbol{3} [\lambda^3 (1-\beta) \beta^2]\boldsymbol{p_{15}} +[\lambda^3 \beta^3]\boldsymbol{p_{16}} \quad (5)$$

Eq (5) is called original Gallier modified cubic Bezier surfaces. Substitution of

$\lambda = \frac{u-r_1}{s_1-r_1}$ and $\beta = \frac{v-r_2}{s_2-r_2}$ in (5) gives:

$$P(u,v) = [\frac{s_1-u}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}]^3]p_1 + 3[[\frac{s_1-u}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}]^2 \frac{v-r_2}{s_2-r_2}]p_2$$

$$+ 3[[\frac{s_1-u}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}] [\frac{v-r_2}{s_2-r_2}]^2]p_3 + [[\frac{s_1-u}{s_1-r_1}]^3 [\frac{v-r_2}{s_2-r_2}]^3]p_4 + 3[[\frac{s_1-u}{s_1-r_1}$$

$$]^2 \frac{u-r_1}{s_1-r_1} [\frac{s_2-v}{s_2-r_2}]^3]p_5 + 9[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{s_2-v}{s_2-r_2}]^2 [\frac{s_2-v}{s_2-r_2}]] p_6$$

$$+ 9[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{s_2-v}{s_2-r_2}] [\frac{v-r_2}{s_2-r_2}]^2] p_7 + 3[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{v-r_2}{s_2-r_2}]^3] p_8$$

$$+3[[\frac{s_1-u}{s_1-r_1}] [\frac{s_1-u}{s_1-r_1}]^2 [\frac{s_2-v}{s_2-r_2}]^3]p_9 + 9[[\frac{s_1-u}{s_1-r_1}] [\frac{u-r_1}{s_1-r_1}]^2 [\frac{s_2-v}{s_2-r_2}]^2$$

$$\frac{v-r_2}{s_2-r_2}]p_{10} + 9[[\frac{s_1-u}{s_1-r_1}] [\frac{u-r_1}{s_1-r_1}]^2 [\frac{s_2-v}{s_2-r_2}][\frac{v-r_2}{s_2-r_2}]^2]p_{11} + 3[\frac{s_1-u}{s_1-r_1}] [\frac{u-r_1}{s_1-r_1}]^2$$

$$[\frac{v-r_2}{s_2-r_2}]^3]p_{12} + [[\frac{u-r_1}{s_1-r_1}]^3 [\frac{v-r_2}{s_2-r_2}]^3]p_{13} + 3[[\frac{v-r_2}{s_2-r_2}] [\frac{u-r_1}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}]^2]p_{14}$$

$$+3 [[\frac{u-r_1}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}][\frac{v-r_2}{s_2-r_2}]^2]p_{15} + [[\frac{u-r_1}{s_1-r_1}]^3 [\frac{v-r_2}{s_2-r_2}]^3]p_{16} . \qquad (6)$$

Eq (6) called Gallier modified cubic Bezier surfaces, is dependent on $[r_1, s_1]$ *and* $[r_2, s_2]$. [Gallier 00].

<u>Note I</u> The coefficients of the control points in the Bezier surfaces in equations (5 or6) are called Bernstein Polynomials given as:

$$[(1-\lambda)+\lambda]^3[(1-\beta)+\beta]^3 = [\frac{s_1-u}{s_1-r_1} + \frac{u-r_1}{s_1-r_1}]^3 \quad [\frac{s_2-v}{s_2-r_2} + \frac{v-r_2}{s_2-r_2}]^3 =$$

$$[\frac{s_1-u}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}]^3] + 3[[\frac{s_1-u}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}]^2 \frac{v-r_2}{s_2-r_2}] + 3[[\frac{s_1-u}{s_1-r_1}]^3$$

$$[\frac{s_2-v}{s_2-r_2}] [\frac{v-r_2}{s_2-r_2}]^2] + [[\frac{s_1-u}{s_1-r_1}]^3 [\frac{v-r_2}{s_2-r_2}]^3] + 3[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{s_2-v}{s_2-r_2}]^3]$$

$$+9[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{s_2-v}{s_2-r_2}]^2 [\frac{s_2-v}{s_2-r_2}]] + 9[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{s_2-v}{s_2-r_2}]$$

$$[\frac{v-r_2}{s_2-r_2}]^2] + 3[[\frac{s_1-u}{s_1-r_1}]^2 \frac{u-r_1}{s_1-r_1} [\frac{v-r_2}{s_2-r_2}]^3] + 3[[\frac{s_1-u}{s_1-r_1}] [\frac{s_1-u}{s_1-r_1}]^2 [\frac{s_2-v}{s_2-r_2}]^3]$$

$$+9[[\frac{s_1-u}{s_1-r_1}] [\frac{u-r_1}{s_1-r_1}]^2 [\frac{s_2-v}{s_2-r_2}]^2 \frac{v-r_2}{s_2-r_2}] + 9[[\frac{s_1-u}{s_1-r_1}] [\frac{u-r_1}{s_1-r_1}]^2$$

$$[\frac{s_2-v}{s_2-r_2}][\frac{v-r_2}{s_2-r_2}]^2] + 3[\frac{s_1-u}{s_1-r_1}] [\frac{u-r_1}{s_1-r_1}]^2 [\frac{v-r_2}{s_2-r_2}]^3] + [[\frac{u-r_1}{s_1-r_1}]^3 [\frac{v-r_2}{s_2-r_2}$$

$$]^3] + 3[[\frac{v-r_2}{s_2-r_2}] [\frac{u-r_1}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}]^2] + 3[[\frac{u-r_1}{s_1-r_1}]^3 [\frac{s_2-v}{s_2-r_2}][\frac{v-r_2}{s_2-r_2}]^2]$$

$$+[[\frac{u-r_1}{s_1-r_1}]^3 [\frac{v-r_2}{s_2-r_2}]^3 = 1. \qquad (7)$$

It should be noted the cubic Bernstein polynomials are the terms that are obtained by expanding the expression and then collecting terms in the various powers of

$\frac{s_1-u}{s_1-r_1}$, $\frac{u-r_1}{s_1-r_1}$, $\frac{s_2-v}{s_2-r_2}$ and $\frac{v-r_2}{s_2-r_2}$.

This equation immediately yields an important property of these polynomials. They are added to unity at every *u and v*, mathematically

$$\sum_{i=0}^{3}\sum_{j=0}^{3}\binom{3}{i}\binom{3}{j}[1-\lambda]^{3-i}[\lambda]^{i}[1-\beta]^{3-j}[\beta]^{j} =$$

$$\sum_{i=0}^{3}\sum_{j=0}^{3}\binom{3}{i}\binom{3}{j}[\frac{s_1-u}{s_1-r_1}]^{3-i}[\frac{u-r_1}{s_1-r_1}]^{i}[\frac{s_2-v}{s_2-r_2}]^{3-j}[\frac{v-r_2}{s_2-r_2}]^{j} = 1, \quad (8)$$

where

$\binom{3}{i}=\frac{3!}{i!(3-i)!}$, $\binom{3}{j}=\frac{3!}{j!(3-j)!}$, which is called Bernstein polynomial of degree three [Gallier 00].

These control points play a major role in the de-Castejalu algorithm and its extensions. The polynomial design defined *P* passes through the two point's p_0 and p_{16}, but not through the other control points. For r_1 =0, s_1 =1, and r_2 =0 ,s_2 =1 Eqs (5 or 6) becomes: -

$$P(u, v) = \sum_{i=0}^{3}\sum_{j=0}^{3}\binom{3}{i}\binom{3}{j}[1-u]^{3-i}[u]^{i}[1-v]^{3-j}[v]^{j} P_{ij} \quad (9)$$

Eq (9) called original cubic Bezier surfaces is dependent on frame *[0, 1]* [Faux 83], [Watt 00], [Gerald 99], [Buss03] ,[Klawonn 08]. It is identical to equation (2).[where ***(*** $p_{00}, p_{01}, p_{02}, p_{03}, p_{10}, p_{11}, p_{12}, p_{13}, p_{20}, p_{21}, p_{22}, p_{23}, p_{30}, p_{31}, p_{32}, p_{33}$***) =(*** $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}$***).***

Note II

Given a set of control points [Gerald 99]

$p_i=(x_i, y_i, z_i)$ for $i=0, 1,2,3$.

treat the coordinates of each point as a three-component vector.

$$P_i=\begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix}.$$

The set of points, in parametric form is

$$P(u, v) = \begin{pmatrix} x(u,v) \\ y(u,v) \\ z(u,v) \end{pmatrix} \qquad r_1 \le u \le s_1 \text{ and } r_2 \le v \le s_2$$

Developing Gallier modified cubic Bezier surfaces

The arithmetic technique can be used for purpose for measurement and controlling the time of design by using Gallier modified cubic Bezier surfaces. Mathematically one can purpose that $r_1 =, r_2$ *and* $s_1 = s_2$, in Eqs (5 or 6).Since Mathematically they are said to be generated from the Cartesian product of two curves, and gives controlling the time of generating the design, and the optimal time of design. Now by changing the coefficient (r_1 or s_1 or r_1 and s_1), can see the following effect of the time of the designs

Effect on the Time If Differences between Values of s_1 and r_1 are fixed

In this case the value of parameters r_1 = fixed, and s_1 =fixed. If the difference between the values of the coefficient of parameters s_1 and r_1 is fixed in Eq (5 or 6) gives, the time of design does not change, without changing any of the control points, and the design See Fig 1.Table (1).

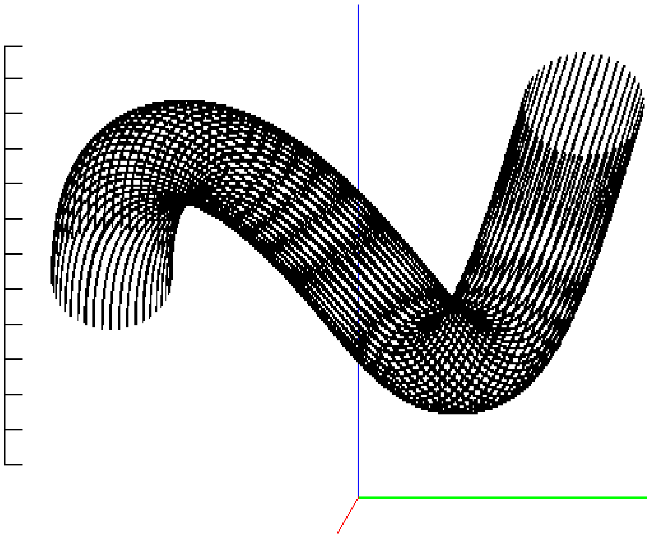

Fig. 1. Bezier Surfaces of three-dimensional. When $r_1$and $s_1$are fixed, the difference between the values of s_1 and r_1 is fixed without changing any of the control points, [r_1 =0, and s_1=10, 6 sec].

Effect on the Time If Varies s_1 and r_1 is Fixed

This case is dependent on Gallier modified cubic Bezier surfaces. Study the case where the values of the coefficient of parameters r_1 is fixed, and $s_1$varied, when s_1 is taken to decreases, then the time decreases. When s_1 is taken to increase, then the time increases with respect to difference between s_1 and r_1 issuch that (s_1 > r_1), without changing any of the control points, and the design See Fig 2,3 . Table (2).

Table2. Difference values of between s_1 and r_1 varies

r_1	s_1	$\lvert s_1 - r_1 \rvert$	Time(second)
0	1	1	1
0	10	10	6
0	20	20	13
0	30	30	19
0	40	40	25
0	50	50	31
0	60	60	38
0	70	70	44
0	80	80	51
0	90	90	56
0	100	100	63

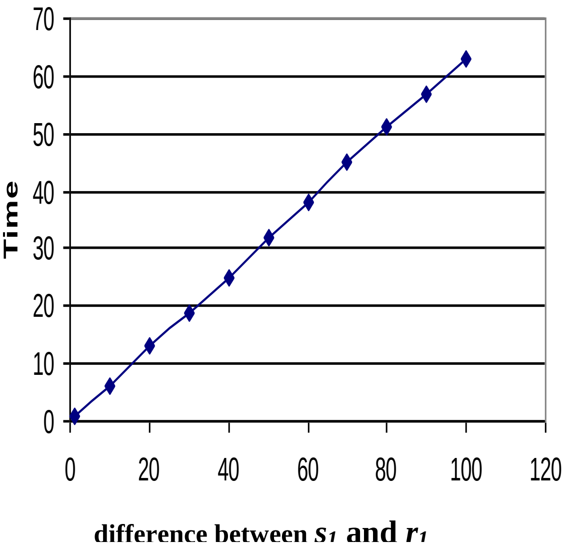

Fig. 2. The relationship between the difference between s_1 and r_1 and the Time

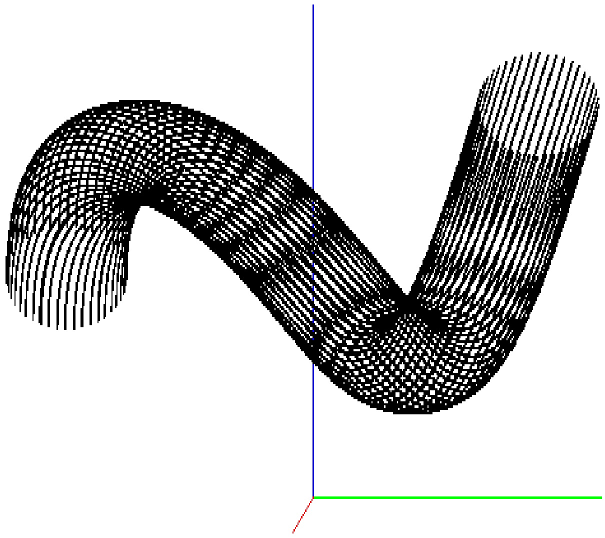

Fig. 3. Bezier Surfaces of three-dimensional, When values of $r_1$is fixed, and s_1 varies without changing any of the control points, [r_1 =0, and s_1 =*50*, 32 sec].

Effect on the Time If r_1Varying and s_1 Fixed

This case is dependent on Gallier modified cubic Bezier surfaces. Study the type when the values of s_1 is taken fixed, and r_1 varied, when r_1 is taken to decrease, then the time decreases. When you take $r_1$to increase, then the time increases with respect to difference between s_1 and $r_1$such that ($r_1 < s_1$), without changing any of the control points, and the design See Fig 4,5 . Table (3).

Table3. The values of r_1Varies and s_1 Fixed

r_1	s_1	$\lvert s_1 - r_1 \rvert$	Time(second)
99	100	1	1
90	100	10	6
80	100	20	13
70	100	30	19
60	100	40	25
50	100	50	31
40	100	60	38
30	100	70	44
20	100	80	51
10	100	90	56
0	100	100	63

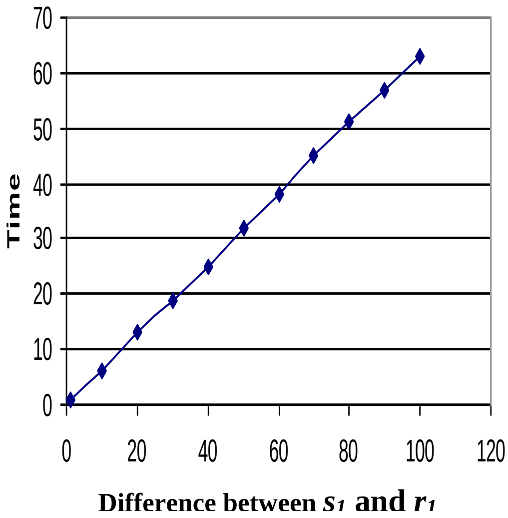

Fig. 4. The relationship between the difference between s_1 and r_1 and the Time

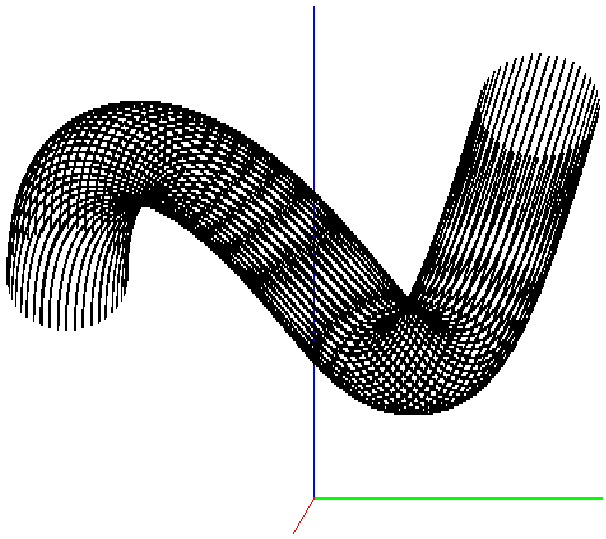

Fig. 5. Bezier Surfaces of three-dimensional. When the values of $s_1$is fixed, and r_1 varies without changing any of the control points, [r_1 =90, and s_1 =*100*, 6 sec].

Effect on the Time If the values of $s_1$and r_1 Varies

This case is dependent on Gallier modified cubic Bezier surfaces. Study the type when the values the values of r_1 and $s_1$vary, when you take $s_1$or r_1 to decrease, then the time decreases. When s_1 or r_1 is taken to increase, then the time increases with respect to difference between s_1 and r_1 such that ($r_1 < s_1$). It is found in this case that the time of design can be moved. By changing the controlling coefficient parameters in Eqs (5 or 6), in only parts of r_1 and s_1 the time of design can be moved without change in any of the control points, and the design. See Fig 6,7. Table (4).

Table4. If the values of $s_1$and r_1 Varies

r_1	s_1	$\lvert s_1 - r_1 \rvert$	Time(second)
-1	0	1	1
-10	0	10	6
-10	10	20	13
10	20	30	19
-30	10	40	25
10	40	50	31
10	70	60	38
10	80	70	44
-30	50	80	51
-60	30	90	56
-50	50	100	63

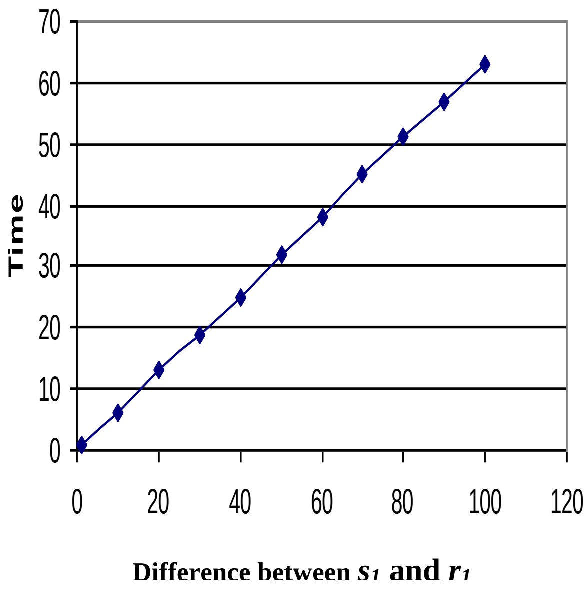

Fig. 6. The relationship between the difference between the values of s_1 and r_1 and the Time

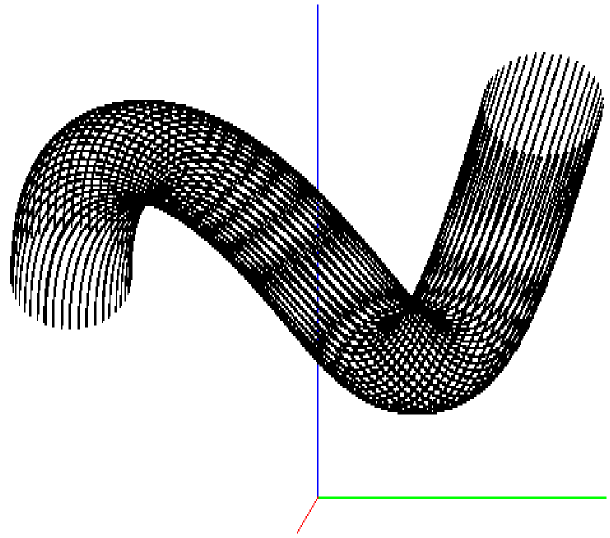

Fig. 7. Bezier Surfaces of three-dimensional, when the values of s_1 and r_1 vary, without changing any of the control points, [r_1 =30, and s_1 =40, 6 sec].

Optimal Time of Design

The Optimal Time of design generation, by using the values of the coefficient of parameters r_1 or s_1 or r_1 and s_1 in Gallier modified cubic Bezier surfaces. If the difference between parameters r_1 and s_1 is one unit.

The explanation of the above result is shown in the following cases, figures and tables obtained by using Gallier modified cubic Bezier surfaces as in following example.

This case is dependent on Gallier modified cubic Bezier surfaces. Study the **optimal time**, when the difference between parameters s_1 and r_1 is one unit, without changing any of the control points, and the design .See Fig 8. Table (5).

Table5 .Difference between the values of the coefficient of parameters $s_1$and r_1 is one unit.

r_1	s_1	$\lvert s_1 - r_1 \rvert$	Time(second)
0	1	1	1
-1	0	1	1
10	11	1	1
50	51	1	1
100	101	1	1
200	201	1	1
0	1	1	1
-1	0	1	1
10	11	1	1
50	51	1	1

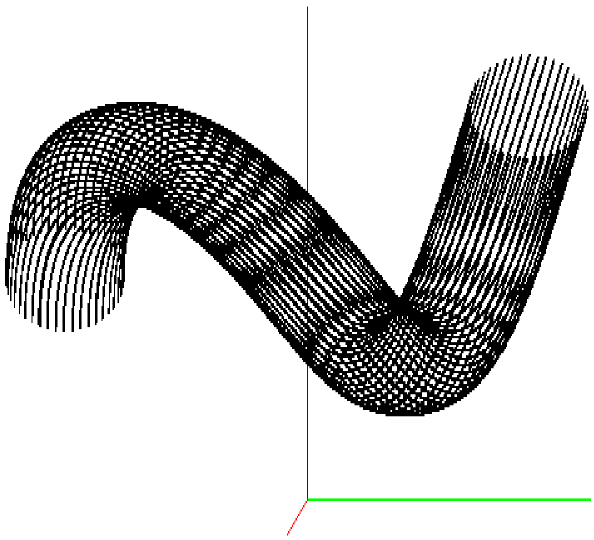

Fig. 8 Cubic Bezier of three-dimensional, when difference between parameters $s_1$and r_1 is one unit without changing any of the control points, [r_1 =0, and s_1 = 1, 1 sec].

Conclusions

From this work the following points can be regarding technique for controlling the time:

♦The design of three-dimensional surfaces mathematically is said to be generated from the Cartesian product of two curves. The design in three-dimensional is made more efficiently with technique, which is used in two-dimension.

♦ Controlling of the time of, design, by using Gallier modify cubic Bezier surfaces.
It can be mathematically proposed that $r_1 = r_2$ and $s_1 = s_2$, in Eqs (5 or 6).

♦ Controlling of the time of, design, by changing the values of the coefficient of the parameters (r_1 or s_1 or r_1 and s_1

♦At changing the values of the coefficient of the parameters (r_1 or s_1 or r_1 and s_1), the time decreases, or increases, with respect to difference between s_1 and r_1, decreases, or increase respectively. See Fig 2 ,4and 6. Table (1, 2, and 3).

♦The Optimal Time of the design generation is when the difference between the values of the coefficient of the parameters r_1 and s_1 is one unit.

♦ Our propose for controlling the time of the design is made by using de- Castejalu algorithm, which can nest nicely when the design uses the value of r_1 and s_1 in a control polygon of the design. But it do not nest nicely when they are outside the polygon. Can be considered to be the polygon formed by placing an elastic band a round the control points. This follows from the fact that the basis function sum to unity for all u and v. See Eqs (7 and 8).

♦Simple error in the time may come from cumulative parts of second because of repetition of graphical three-dimensional which are used in design. This effect can be easily overcome by using advanced computer.

♦ The time in all above types or points is controlled without changing the controlling points, of the design.

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