THE USE OF VIRTUAL REALITY TO DEMONSTRATE TRAJECTORY PLANNING AND CONTROL OF A 3-DOF UNDERACTUATED ROBOT IN A HORIZONTAL PLANE IN REAL-TIME[1]

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Abstract

Real-Time 3D animation and Real-Time Simulation results are reported for a 3R underactuated robot moving in a horizontal plane using Virtual Reality Toolbox™, Real-Time Windows Target™,Real-Time Workshop®, and interfaced to Simulink® under the MATLAB® environment. The reason behind Real-time 3D animation and Real-Time simulation is to provide animated real-time tools in a realistic fashion to demonstrate the effectiveness of the controllers in tracking the desired trajectory and to provide a tool for researchers to test their proposed controllers and observe their behavior using 3-DOF underactuated manipulator in real-time.

Keywords: Virtual-Reality, Underactuated Robot, Nonholonomic Constraint, Real-Time.

استخدام الواقع الافتراضي لعرض تخطيط المسار والسيطرة لذراع روبوت ذي ثلاث مفاصل دوارة تحت الدفعية في المستوى الأفقي وفي الزمن الحقيقي

الخلاصة

يتناول البحث الحالي تسجيل رسوم متحركة ثلاثية الأبعاد وبالزمن الحقيقي وكذلك المحاكاة بالزمنالحقيقي لروبرت ذي ثلاث مفاصل دوارة 3R) تحت الدفعية ويعمل في مستوى أفقي باستعمال لأدوات (Simulink®) وتم تعشيقها مع(Virtual Reality Toolbox™) و(Real-Time Windows Target™) و (Real-Time) Workshop®)و تم تعشيقها مع (Simulink®)الذي يعمل في بيئة (MATLAB®) . إن السبب وراء استخدام الرسوم الثلاثية الأبعاد وبالزمن الحقيقي هو توفير صور متحركة ذات طابع شبيه بالواقع وتتحرك بالزمن الحقيقي لتكون أدوات للباحثين لعرض مدى كفاءة المسيطرات في تتبع المسار المطلوب ومراقبة تصرف الروبوت ذي الثلاث درجات لحرية الحركة وتحت الدفعية في الزمن الحقيقي.

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1- Introduction

Virtual Reality (VR) is a field of study that aims to create a system that provides a synthetic experience for its user(s). The experience is dubbed ‘‘synthetic,’’‘‘illusory,’’ or ‘‘virtual’’ because the sensory stimulation to the user is simulated and generated by the system. The term Virtual Reality is used to describe a computer-generated, highly-realistic artificial world or environment (called a Virtual environment), allowing the user to interact with it in real-time by interfacing some of his actions in the real world back into the virtual environment and providing visual, acoustical and, sometimes, haptic feedback [1]. VR allows people to get the experience of things that would otherwise be very difficult or even impossible to attain in real life. VR may provide invaluable toolsto engineers seeking rapid and inexpensive development for their prototypes.

The aim of this work is to demonstrate trajectory planning and control of a 3-degrees of freedom (3DOF)*underactuated*planar robot with a passive rotational last joint using the Dynamic Feedback Linearization (DFL) method, utilizing the VR toolbox under the MATLAB and Simulink in real time.

Underactuated mechanical systems are mechanical systems with fewer actuators than DOFs [2]. For a conventional robot manipulator, the number of joints is equal to the number of actuators, or actuated joints.Such a fully driven serial mechanism is called a full-actuated system. If the total number of joints is greater than the number of actuators in the mechanism, the system is referred to as an underactuated system. Underactuated mechanical systems may arise from intentional design as in the pendubot [3], and the Acrobot [4].Mobile robot systems are considered to be Underactuated;for example, when a manipulator arm is attached to a mobile platform or an undersea vehicle [5].Alsounderactuation arises due to the mathematical model used for control design as, for example, when joint flexibility is included in the model [6].It is also interesting to note that certain control problems for fully actuated redundant robots are similar to those for underactuated robots [2].

The class of underactuated systems is composed of a variety of mechanical as well as biological systems. A biological system which can be considered underactuated is the human body [7]. When for example; gymnasts perform acrobatic maneuvers on a high bar; they are able to rotate about their wrist by actuating the muscles on their hip and knees. The wrist is therefore a joint kept unactuated, whose displacement can be controlled by the actuation of other joints.

There are a number of advantages to the use of underactuated systems. First, reducing the number of actuators for a robot manipulator will minimize energy consumption, and will be potentially attractive to the applications where energy efficiency is a major concern, such as for space robots [8]. Second, eliminating some actuators will allow more compact design leading to both overall size and total weight reductions. This will ultimately reduce the manufacturing cost and running power. Not only the underactuated system is useful in practice but also the concept is important in analysis of a class of systems that can be considered as virtual underactuated systems. For example, a free-flying space robot system [8] is useful in maintenance tasks in space stations and/or satellites. The concept of underactuated systems provides an approach to modeling dynamic systems with either free bases, or free joints. Some of these mechanisms can be potentially utilized in space and underwater applications.

In this paper, section IIdescribes the mathematical model of the passive link dynamics, section III presents motion planning and trajectory control using DFL. Section IV utilizes Simulink software under

MATLAB environment to simulate numerical results of an example where the trajectory motion of the passive link is controlled. Also in section IV a 3D animation is provided for the 3R robot using Virtual Reality Tool box,Real-Time Windows Target,Real-Time Workshop, and interfaced to Simulink all under the MATLAB environment. Finally, the conclusions are provided in section V.

2- Modeling of an Underactuated 3-Link Planar Robot in a horizontal plane

A manipulator with three degrees of freedom in horizontal plane is considered in Fig. 1. The first and second joints are activelycontrolledand are used to control the position of the passive joint in 2D plane. The passive joint is a revolute joint around a vertical axis.

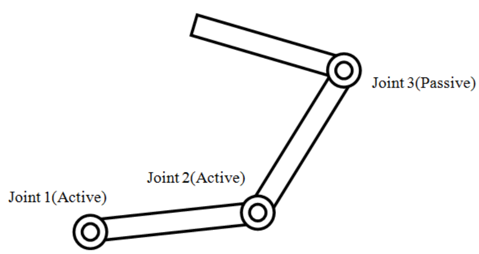

Fig. 1. Three-DOF planar underactuated manipulator.

To simplify the model, the dynamics of the first and second joint are neglected, except that the translational acceleration of the passive joint is assumed to be finite. The work space limit and singularity of the first and second joint are ignored too. The dynamics can be modeled with regard to only the free link as shown in Fig.2, where the generalized coordinates which represent the configuration of the manipulator are (x, y, θ). The equations of motion with respect to the link is written as [9]

$$
\begin{aligned}
f_x &= m\ddot{x} - ml\ddot{\theta}\sin\theta - ml\dot{\theta}^2\cos\theta \\
f_y &= m\ddot{y} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \\
\tau_\theta &= -ml\ddot{x}\sin\theta + ml\ddot{y}\cos\theta + (I_G + ml^2)\ddot{\theta}
\end{aligned} \qquad (1)
$$

Where

m mass of link;

I_G moment of inertia of the link around G;

l distance $|OG|$, between the joint and the center of mass;

(f_x, f_y) translational force at the joint O;

τ_θ torque around the joint O.

As the joint O is passive, $\tau_\theta = 0$. Where $l \neq 0$ and $k \equiv I_G/ml$, isequivalent to the distance of the mechanical property, *center of percussion* (CP) [10], [11]. Center of percussion play an important role in the dynamics of rigid pendulums. In fact, the motion of an oscillating pendulum of a mass m can be described by the equation of motion of a point mass all concentrated in the center of percussion [12]

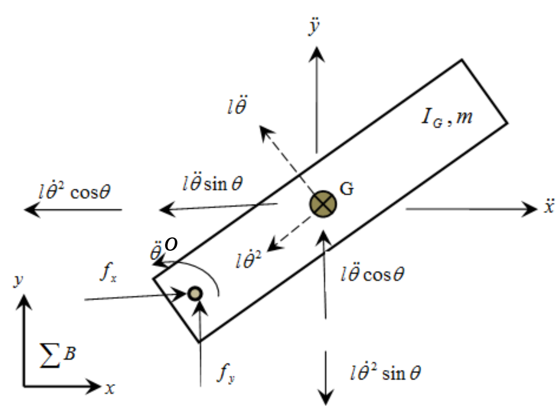

Fig. 2.Acceleration components of the free link.

The constraint on the system is represented in the form ofa 2nd order nonholonomic differential equation as [9]

$$-\ddot{x}\sin\theta+\ddot{y}\cos\theta+k\ddot{\theta}=0 \quad (2)$$

from equation (1),the translational acceleration $(\ddot{x},\ddot{y})$, of the passive joint can be treated as inputs to the system. The state equations of the system is written as [9]

$$\frac{d}{dt}\begin{bmatrix}x\\y\\\theta\\\dot{x}\\\dot{y}\\\dot{\theta}\end{bmatrix}=\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{\theta}\\0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\\1\\0\\\sin\theta/k\end{bmatrix}\ddot{x}+\begin{bmatrix}0\\0\\0\\0\\1\\-\cos\theta/k\end{bmatrix}\ddot{y} \quad (3)$$

It should be noted that the linear approximation of this system is not controllable since no gravity is applied on the passive joint [9].

We may write the dynamic equations in the Euler-Lagrange form for a mechanical system with n degrees of freedom and $m=n-1$ control inputs, denoted by $q\in\Re^n$ the generalized coordinates and by $\tau\in\Re^m$ the control input.The dynamics is written as [13]

$$B(q)\ddot{q}+h(q,\dot{q})+g(q)=F(q)\tau \quad (4)$$

where $B>0$ is the $n\times n$ symmetric inertia matrix, h is the centrifugal and Coriolis vector, $g=(\partial U/\partial q)^T$ is the vector of potential terms, and F is the $n\times m$ input matrix assumed of full rank.

Substituting the link parameters into equation (4), with $g(q)$=0 (zero gravity), equation (4) is written as

$$\begin{bmatrix}m & 0 & -ml\sin\theta\\0 & m & ml\cos\theta\\-ml\sin\theta & ml\cos\theta & I_G+ml^2\end{bmatrix}\begin{bmatrix}\ddot{x}\\\ddot{y}\\\ddot{\theta}\end{bmatrix}+$$

$$\begin{bmatrix}-ml\dot{\theta}^2\cos\theta\\-ml\dot{\theta}^2\sin\theta\\0\end{bmatrix}=\begin{bmatrix}f_x\\f_y\\0\end{bmatrix} \quad (5)$$

3-<u>Motion Planning and Trajectory Control via Dynamic Feedback Linearization</u>

One effective technique to solve the motion planning and control for 2^{nd} order nonholonomic mechanical system is *Dynamic Feedback Linearization* (DFL). In this methodology, the exact state linearization is based on changing the coordinates of the states and finding a control law so that, in the new coordinates, the closed-loop system is linear and controllable [13],[14]. In robotics, a dynamic feedback has been used for the exact linearization of manipulators with elastic joints and of nonholonomic wheeled mobile robots [15] (and the references therein).

The trajectory motion of the 3^{rd}passive link considered here ware controlled using DFL. Under an appropriate regularity assumption; the robot can be transformed into a fully linear, input-output decoupled system by using a second-order dynamic feedback compensator. As a result of dynamic feedback linearization, each coordinate of theCPis driven independently by an auxiliary input through a chain of integrators. Therefore, it is sufficient to solve an interpolation problem for the CP point to generate a feasible point-to-point trajectory and the associated nominal inputs. As a byproduct of this approach, global exponential tracking of the generated trajectory is guaranteed by adding a linear feedback (in the linearizing coordinates) to the feedforward command. To make the analysis independent from the nature of the first$n-1$ joints, we preliminarily perform a partial linearization of equation (5) via static feedback. The idea is to reduce the dynamics of the active joints to $n-1$ chains of double integrators, so that they can be controlled via acceleration inputs. The partially linearizing static feedback is obtained in the form

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} m - \frac{ml\sin^2\theta}{k} & \frac{ml\sin\theta\cos\theta}{k} \\ \frac{ml\sin\theta\cos\theta}{k} & m - \frac{ml\cos^2\theta}{k} \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

$$-\begin{bmatrix} ml\dot{\theta}^2\cos\theta \\ ml\dot{\theta}^2\sin\theta \end{bmatrix} (6)$$

Putting together equations (5) and (6), the complete closed-loop system becomes

$$\begin{aligned} \ddot{x} &= a_x \\ \ddot{y} &= a_y \qquad (7) \\ \ddot{\theta} &= \frac{\sin\theta}{k}a_x - \frac{\cos\theta}{k}a_y \end{aligned}$$

where, $k = (I_3 + m_3 l_3^2)/m_3 l_3$ is precisely the distance of the CP of the last link from its base. If a uniform mass distribution is assumed, then $k = 2l_3/3$ (l_3 is the length of the third link).Define the Cartesian position of the CP of the last link as output (see Fig 3),

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + k\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} (8)$$

1st and2nd differentiation of equation (8) and substitution of equation (7) yields

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} a_x \\ a_y \end{bmatrix} +$$

$$k\dot{\theta}^2\begin{bmatrix} -\cos\theta \\ -\sin\theta \end{bmatrix} (9)$$

Since the matrix multiplying the acceleration vector (a_x, a_y) is singular, the invertible feedback transformation defined as

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \xi + k\dot{\theta}^2 \\ \sigma_2 \end{bmatrix} (10)$$

where ξ and σ_2 are two auxiliary input variables. As a result of equation (10), equation (9) is written as

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \xi\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} (11)$$

proceeding with differentiation and defining new auxiliary input variables as necessary, 3rd and 4th derivatives become as

$$\begin{bmatrix} \dddot{y}_1 \\ \dddot{y}_2 \end{bmatrix} = R(\theta)\begin{bmatrix} \eta \\ \xi\dot{\theta} \end{bmatrix} (12)$$

where, $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, and avoiding differentiating the input ξ, by adding two integrators on the first channel as

$$\begin{aligned} \dot{\xi} &= \eta \\ \dot{\eta} &= \sigma_1 \end{aligned} \quad (13)$$

with σ_1 the new auxiliary input in place of ξ.

$$\begin{bmatrix} y_1^{[4]} \\ y_2^{[4]} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} (14)$$

with (v_1, v_2) as the new input vector, and the inversion based control is expressed by

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -k/\xi \end{bmatrix} R^T(\theta) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} +$$

$$\begin{bmatrix} \xi\dot{\theta}^2 \\ 2k\dot{\theta}\eta/\xi \end{bmatrix} (15)$$

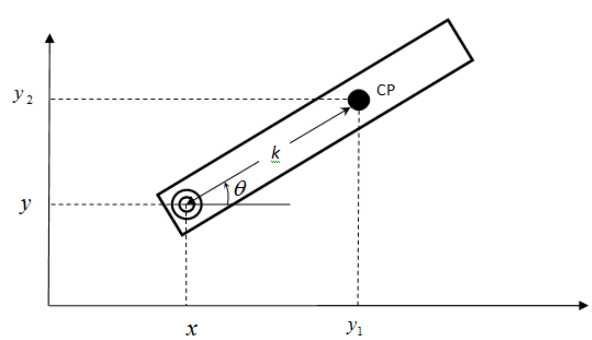

Fig. 3.Cartesian position of the CP of the last link.

under the regularity assumption, the matrix multiplying the inputs is nonsingular or, equivalently, that $\xi \neq 0$. The initialization of the compensator state at time t = 0, i.e., $(\xi(0),\eta(0))$, is arbitrary. As a byproduct of the linearization, a new set of state coordinates can be defined consisting of the output function and its derivatives up to the 3rd order (i.e. $y_1, y_2, \dot{y}_1, \dot{y}_2, \ddot{y}_1, \ddot{y}_2, \dddot{y}_1, \dddot{y}_2$). The inverse transformation from these linearizing coordinates are written as $\theta = ATAN\ 2\{sign(\xi)\ddot{y}_2, sign(\xi)\ddot{y}_1\}$ (16)

$$\xi = \ddot{y}_1 \cos\theta + \ddot{y}_2 \sin\theta \quad (17)$$

$$\begin{bmatrix}\eta\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta/\xi & \cos\theta/\xi\end{bmatrix}\begin{bmatrix}\dddot{y}_1\\ \dddot{y}_2\end{bmatrix} \quad (18)$$

$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}y_1\\ y_2\end{bmatrix} - k\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} \quad (19)$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}\dot{y}_1\\ \dot{y}_2\end{bmatrix} - k\dot{\theta}\begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix} \quad (20)$$

The problem of trajectory planning can be formulated as an *interpolation* problem using smooth parametric functions $y_1(r)$ and $y_2(r)$, with a timing law $r = r(t)$. For simplicity, one can directly generate trajectories $y_1(t)$ and $y_2(t)$. In particular, assume that at time t = 0 the robot starts from a generic state $(q_s, \dot{q}_s) = (x_s, y_s, \theta_s, \dot{x}_s, \dot{y}_s, \dot{\theta}_s)$ to reach a goal state $(q_g, \dot{q}_g) = (x_g, y_g, \theta_g, \dot{x}_g, \dot{y}_g, \dot{\theta}_g)$ at time t = T.The appropriate boundary conditions for the new state variables, i.e., y_1, y_2 and their derivatives up to the third order are,

at time t=0

$$\begin{bmatrix}y_1(0)\\ \dot{y}_1(0)\\ \ddot{y}_1(0)\\ \dddot{y}_1(0)\end{bmatrix} = \begin{bmatrix}y_{1s}\\ \dot{y}_{1s}\\ \ddot{y}_{1s}\\ \dddot{y}_{1s}\end{bmatrix}, \begin{bmatrix}y_2(0)\\ \dot{y}_2(0)\\ \ddot{y}_2(0)\\ \dddot{y}_2(0)\end{bmatrix} = \begin{bmatrix}y_{2s}\\ \dot{y}_{2s}\\ \ddot{y}_{2s}\\ \dddot{y}_{2s}\end{bmatrix}$$

at time t=T

$$\begin{bmatrix}y_1(T)\\ \dot{y}_1(T)\\ \ddot{y}_1(T)\\ \dddot{y}_1(T)\end{bmatrix} = \begin{bmatrix}y_{1g}\\ \dot{y}_{1g}\\ \ddot{y}_{1g}\\ \dddot{y}_{1g}\end{bmatrix}, \begin{bmatrix}y_2(T)\\ \dot{y}_2(T)\\ \ddot{y}_2(T)\\ \dddot{y}_2(T)\end{bmatrix} = \begin{bmatrix}y_{2g}\\ \dot{y}_{2g}\\ \ddot{y}_{2g}\\ \dddot{y}_{2g}\end{bmatrix}$$

a straightforward solution to the interpolation problem is to generate trajectories as polynomials of seventh degree:

$$y_i(t) = \sum_{j=0}^{7} a_{ij} t^j, \qquad i = 1,2, (21)$$

An expression for the coefficients a_j where j=0,…,3,is straight forwardand for j=4,…,7 may be written in matrix form as

$$\begin{bmatrix}a_4\\ a_5\\ a_6\\ a_7\end{bmatrix} = \begin{bmatrix}T^4 & T^5 & T^6 & T^7\\ 4T^3 & 5T^4 & 6T^5 & 7T^6\\ 12T^2 & 20T^3 & 30T^4 & 42T^5\\ 24T & 60T^2 & 120T^3 & 210T^4\end{bmatrix}^{-1} \times$$

$$\begin{bmatrix} y_g - y_s - T\dot{y}_s - \frac{T^2}{2}\ddot{y}_s - \frac{T^3}{6}\dddot{y}_s\\ \dot{y}_g - \dot{y}_s - T\ddot{y}_s - \frac{T^2}{2}\dddot{y}_s\\ \ddot{y}_g - \ddot{y}_s - T\dddot{y}_s\\ \dddot{y}_g - \dddot{y}_s \end{bmatrix} (22)$$

the open-loop commands that realize this trajectory are

$$v_i(t) = 840a_{i7}t^3 + 360a_{i6}t^2 + 120a_{i5}t + 24a_{i4}, \quad i = 1,2 \quad (23)$$

The selection of initial and final compensator states (ξ_s, η_s) and (ξ_g, η_g) affects the boundary conditions, and thus the generated motion inside the chosen class of interpolating functions. In particular, the compensator states should be chosen so as to avoid the singularity $\xi = 0$ during the motion. The problem of *tracking* the generated trajectories will now be discussed. The feedforward commands resulting from a trajectory planning algorithm yield the desired robot reconfiguration only in nominal conditions, i.e., initial state matched with the desired reference trajectory and absence of disturbances during motion. Feedback control must be used to alleviate the effects of an initial state error and of different kinds of perturbations. The linearizing controller and the feedback control is shown in Fig. 10.

Link	Length (m)	Mass (kg)	K (CP) in (m)
1	1.5	Notspecified	not required
2	1.5	Not specified	not required
3	1	1	2/3*

* Uniform mass distribution is assumed

Table 1.3R robot parameters.

4-Simulation and 3D Animation Results

The robot parameters that will be considered are presented in table 1.Initial states of the passive link are assumed to be as

$$x_{start} = 0.5m \qquad \dot{x}_{start} = 0 \text{ m/s}$$
$$y_{start} = 1m \quad , \qquad \dot{y}_{start} = 0 \text{ m/s} \quad (24)$$
$$\phi_{start} = 90^{\circ} \qquad \dot{\phi}_{start} = 0 \text{ rad/s}$$

The goal states are planned to be as

$$x_{goal} = 0.5m \qquad \dot{x}_{goal} = 0 \text{ m/s}$$
$$y_{goal} = 1m \quad , \qquad \dot{y}_{goal} = 0 \text{ m/s} \quad (25)$$
$$\phi_{goal} = 0^{\circ} \qquad \dot{\phi}_{goal} = 0 \text{ rad/s}$$

The controller states are assumed to be as

$$\xi_{start} = -0.1 \text{ m/s}^2 \quad (26)$$

$$\xi_{goal} = -0.1 \text{ m/s}^2$$

See Fig. 4.Trajectory time is T=10 seconds.

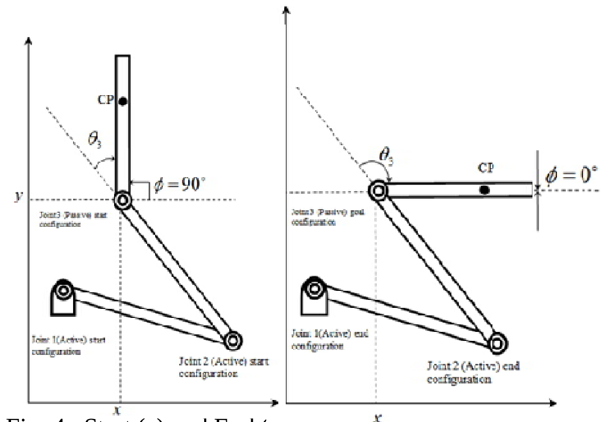

Fig. 4. Start (a) and End (b) configuration for the 3R planar underactuated robot.

A series of experimental tests were carried out and Figs. 5 to 9 summarize the results. Figure 5 shows the reset-to-reset path of the CP of the 3rd link. The figure also presents the path of the passive joint necessary to achieve this target.Figure 6 displays the evolution of the CP in terms of y_1 and y_2 and Fig.7 indicates the high-quality performance of the controller. The required torque to achieve this endis shown in Fig. 8. Figure 9 displays thestroboscopic motion of the 3R robot.Figures 11 and 12 are snap shots of the animated results with their real time stamps [16].The builder environment of the robot model that was developedfor this purpose isshown in Fig. 13.

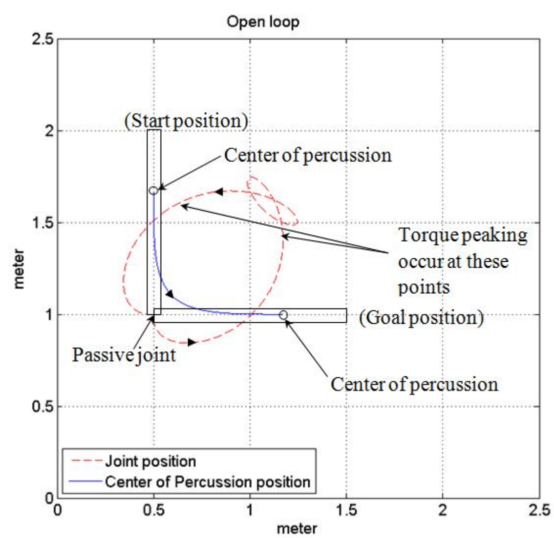

Fig. 5.Rest-to-rest planning for the third link.

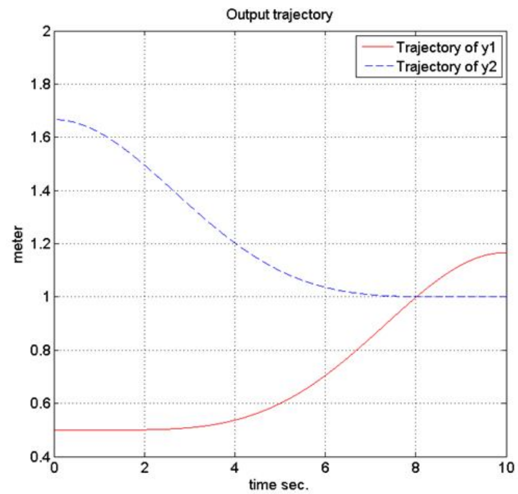

Fig. 6.Time evolution of CP.

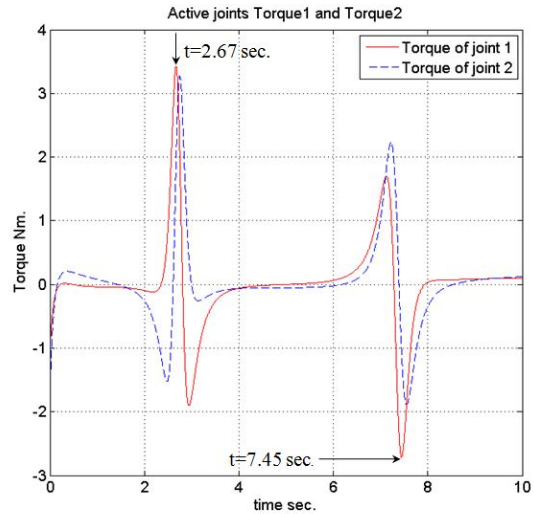

Fig. 8.Torques of the active joints, 1 and 2.

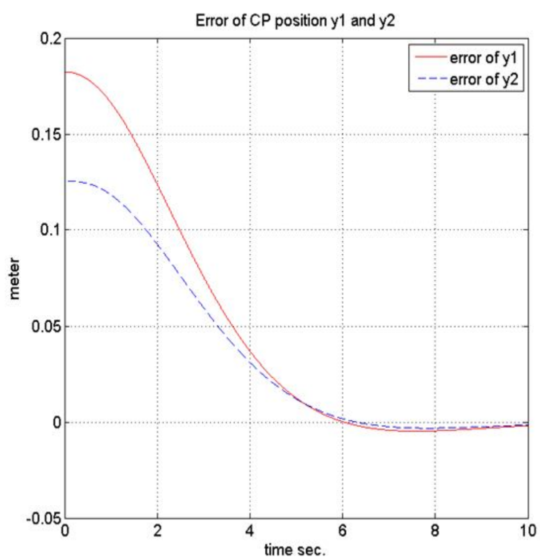

Fig. 7.Trajectory tracking error.

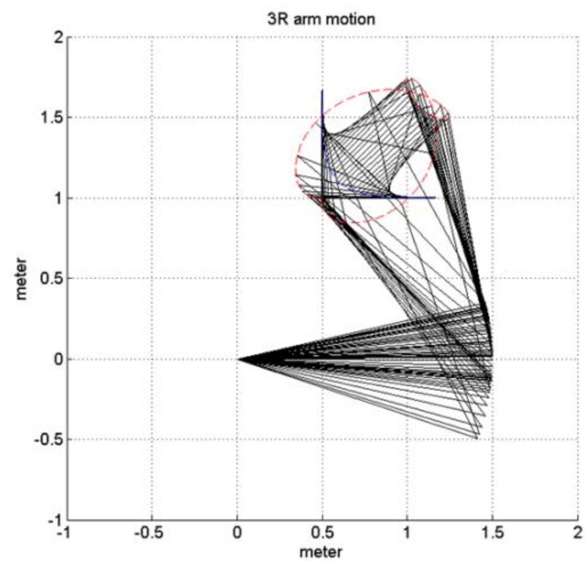

Fig. 9.Stroboscopic motion of the 3R robot.

5-Conclusion

An interactive tool for analysis was presented and exemplified on the trajectory planning and control in VR environment for a 3-DOF underactuated robot. VR was chosen to create a simulation because it was faster, cheaper, and safer than actually programming the real robot in real time. Results of using this methodology for training before interaction with a physical robot shows that the use of the virtual environment for learning to control a robotic device provides sufficient training to allow a user to become more effective in implementing a new task in a novel situation.

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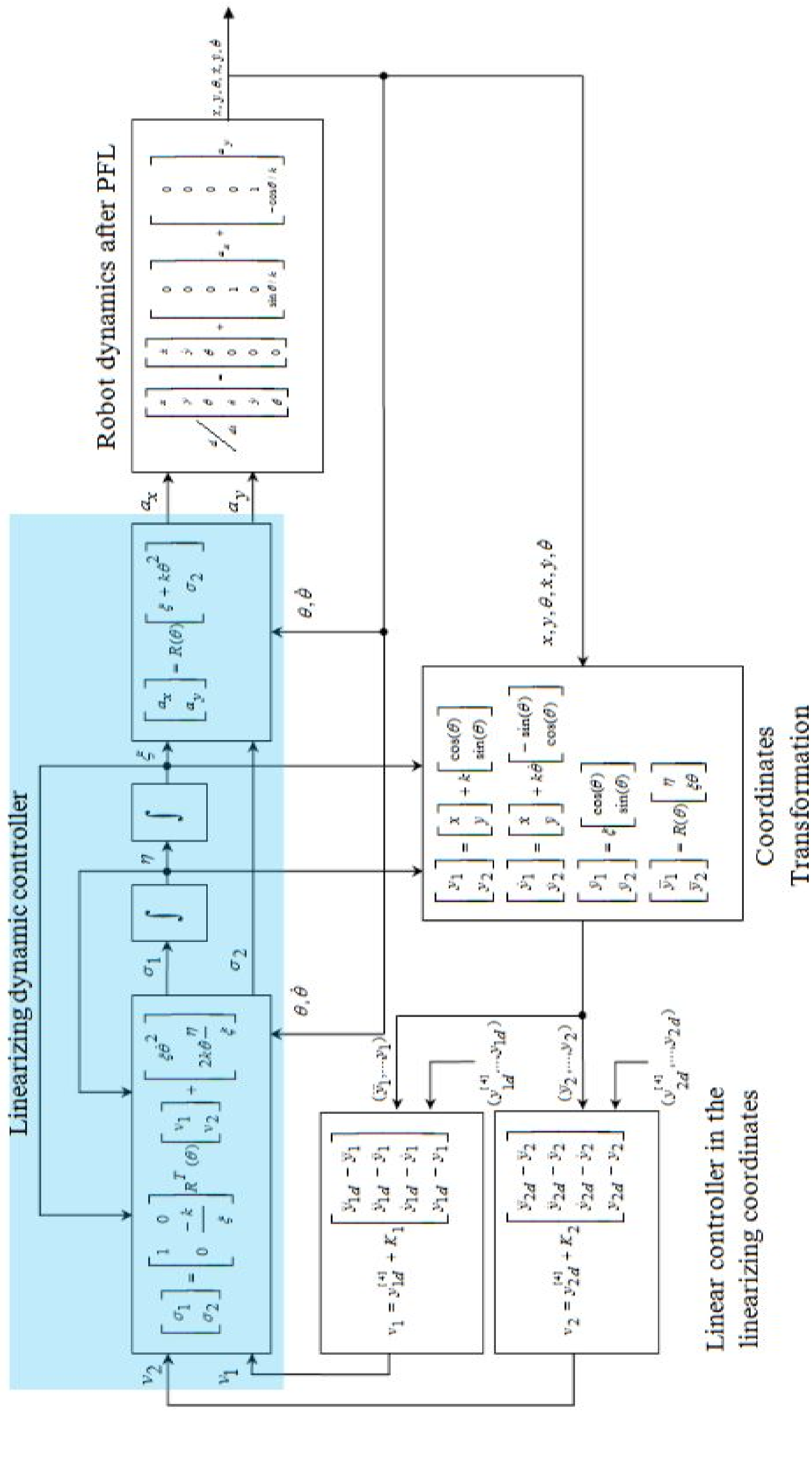

Fig. 10. Linearizing dynamic controller and tracking controller acting on the robot dynamics.

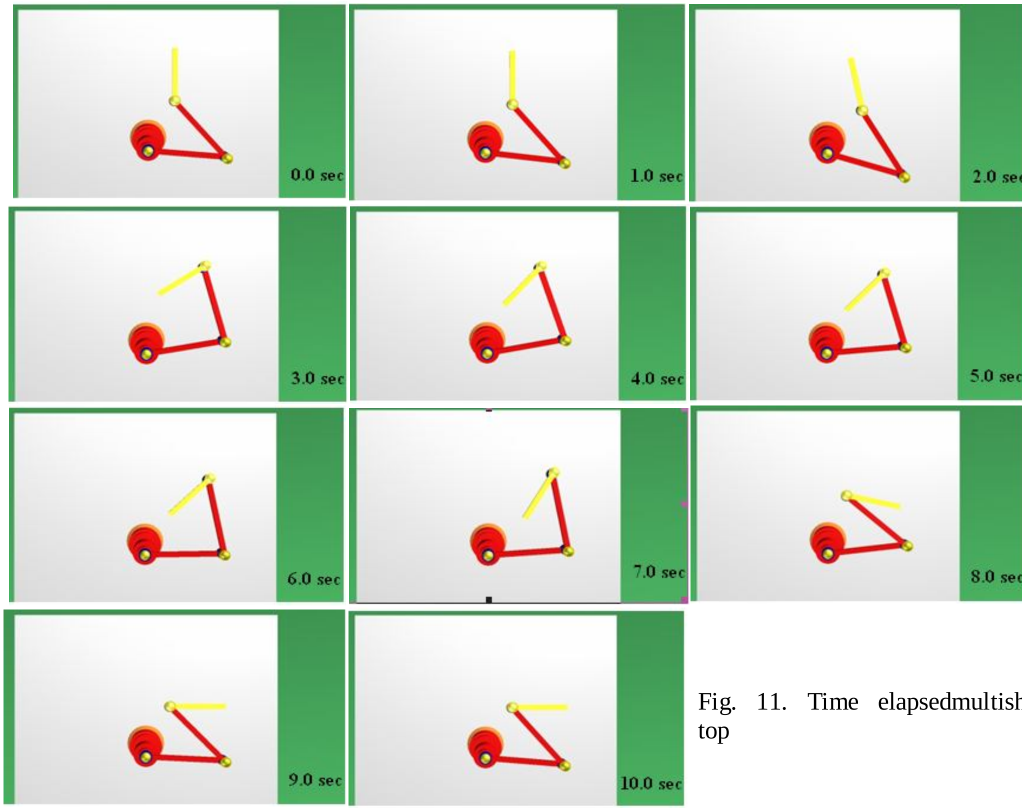

Fig. 11. Time elapsedmultishot top

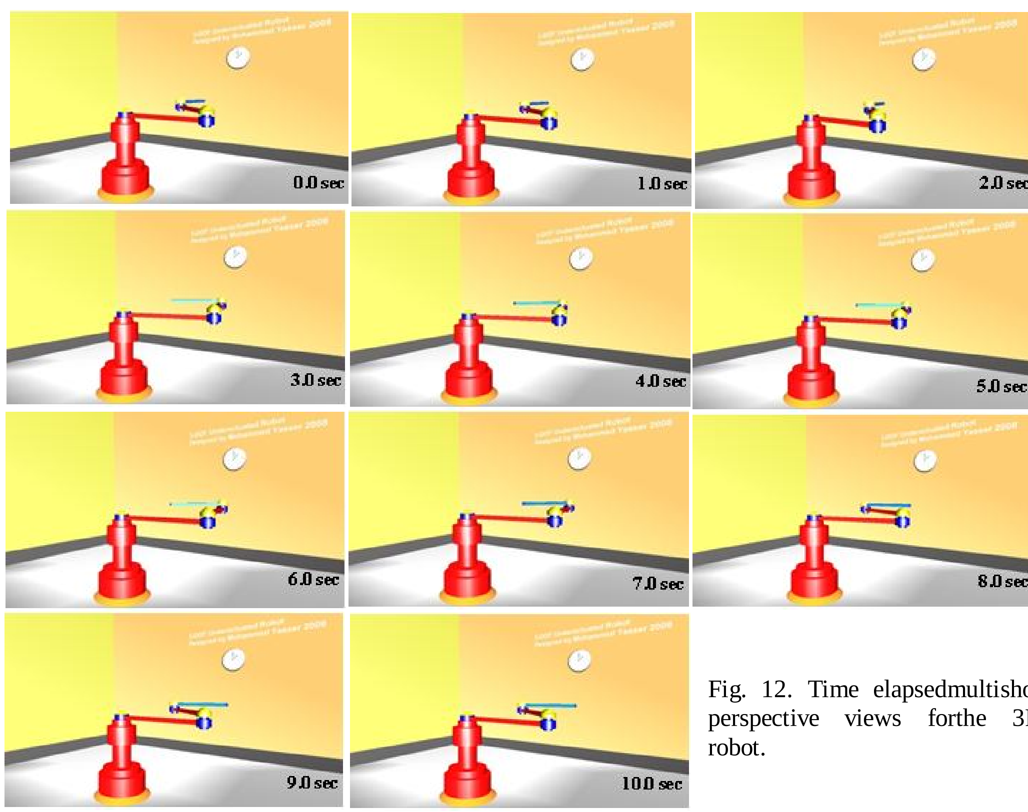

Fig. 12. Time elapsedmultishot perspective views forthe 3R robot.

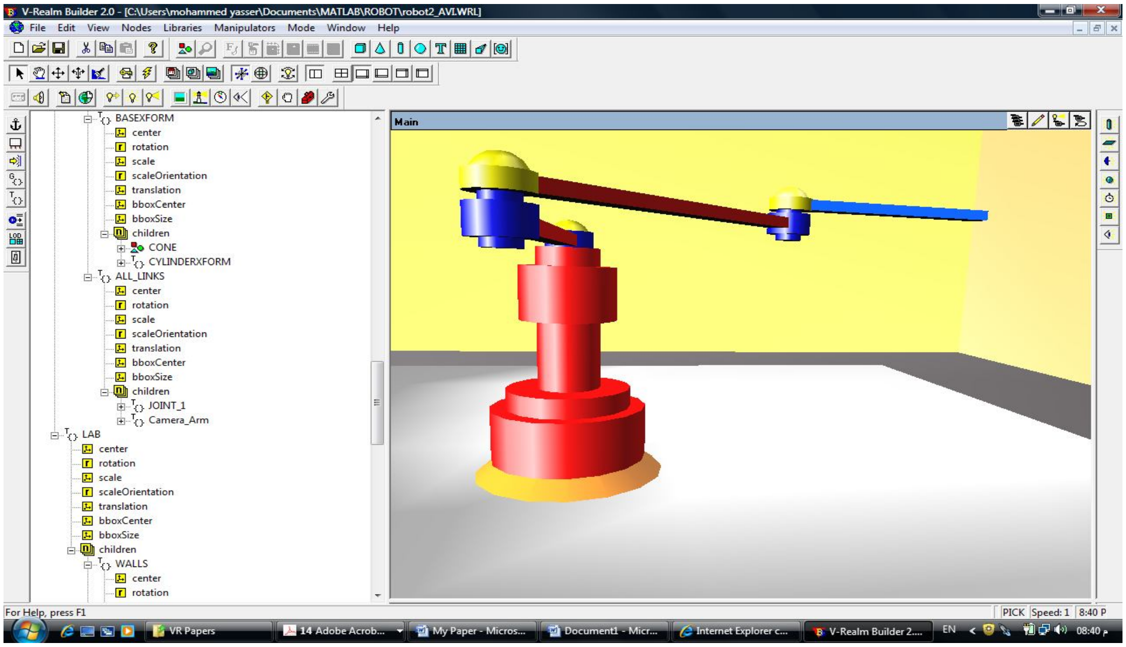

Fig. 13.Vrealm builder environment.