EFFICIENT CYCLOSTATIONARY SPECTRUM SENSING USING LOW COMPLEXITY FFT ALGORITHMS

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Abstract- **One of the major problems of cyclostationary spectrum sensing (CSS) system in cognitive radios (CR) are the implementation complexity. One possible way to reduce CSS complexity is to use efficient algorithms for performing Fast Fourier Transformation (FFT). Over the years, a lot of different FFT algorithms have been created. This includes the Split-Radix algorithm., the Fast Hartley Transform (FHT), and slide DFT. This paper investigates the suitable FFT algorithm among the aforementioned techniques, cyclostationary feature detection (CFD)-based spectrum sensing stands out. The methods have been thoroughly compared based on computational time, object size, code size, data dependence (real or complex), and the amount of mathematical operations involved in the computations. Simulation results show that slide FFT is the suitable frequency domain transformation algorithm to use in implementing cyclostationary spectrum sensing in cognitive radios as compared to the other considered algorithms where it provides a significant reduction in FFT stage computation complexity reach to 17% in SRFFT , 70% in FHT and 82% in SDFT while keeping the detection probability at satisfactory level.**

keywords: **Cognitive radio networks, Spectrum sensing, Cyclostationary detector, Computational complexity, Fast Fourier transform**

I. Introduction

Wireless industry is expanding rapidly as a result of the quick development of wireless communication, artificial intelligence, the Internet of Things, and other technologies so the available spectrum is becoming overcrowded[1]. There is a severe underutilization of the useable frequency range, according to recent studies. A crunch in usable spectrum is being caused by the needs of new services in wireless communication. [2]. CR is new area of intelligent wireless communication systems whose parameters of transmission/reception are adjusted in response to the environment. It alleviates the scarcity of spectrum by effective and dynamic utilization of the electromagnetic spectrum[3].

Spectrum sensing (SS) is performed by secondary user's access frequency spectrum dynamically. In order to establish a communication link for the Secondary User (SU) without interfering with the Primary User's (PU) transmission, SS is a crucial component of a CR network that requires the detection of an open spectrum space[4,5]. The three primary approaches for determining the signal presence each have some trade-offs. They are energy detection, matched filtering, and cyclostationary detection[6,7]. In cyclostationary feature detection, Spectrum sensing can be used even with low SNR to effectively detect the presence of primary users even at low SNR levels [8]. When compared to matching filter and energy detection based methods, cyclostationary feature detection (CFD) based spectrum sensing performing better even with negative SNR area (extremely low SNR)[9], This CFD approach is implementable in both the time and frequency

domains; the frequency domain exhibits greater performance and improved reliability, but the computational complexity is higher.

Many works in the literature present different approaches to reducing the complexity of CSS. In[1], To calculate the spectrum sensing method's sensing efficiency considering sensing performance and computing complexity, three efficiency functions are offered. The energy detector, cyclostationary feature detector, covariance matrix detector, and cooperative spectrum detector are all used in the use of a sensing approach to identify idle spectrum. $\ln[2]$ On the basis of FRESH filtering, a quasi-analytical theory of spectrum sensing is developed. The received signal's energy and cyclostationary detection before the detection step can both experience large performance enhancements, as demonstrated. In[8], It is suggested to use a parallel approach to estimate the cyclostationary properties of modulated signals. It made it possible to shorten primary user disturbance duration and the processing time required to extract cyclostationary features. In [10], cyclostationary feature detection (CFD) spectrum sensor that uses less hardware and has a quick response time is presented. However, all above works used classical implementation of FFT and focused on CFD and filtering operations. In [11], suggest a FFT algorithm that skips two steps from the usual five-step FFT approach, reducing its computational complexity and memory needs.In [12], the proposed technique has the ability to reduce the average number of FFTs needed during the acquisition process.

This study suggested less complex cyclostationary detectors through the use of low complexity frequency transform algorithms while ensuring the same detection performance in an AWGN channel. The rest of the paper is structured as follows: In Section II, the feature detection of cyclostationary spectrum sensing is described. In Section III, the FFT implementation algorithms are reviewed and analyzed. Section IV presents the proposed design of Cyclostationary SS using Low Complexity FFT Algorithms. Section V illustrates the obtained simulation results with their evaluation, while Section VI gives the conclusions drawn throughout the work.

II. CYCLOSTATIONARY FEATURE DETECTION

Operations like sampling, filtering, and coding create a signal that can be modeled as a second-order cyclostationary process during the processing of a communication signal. In other words, the modulated signal's mean and autocorrelation function both display periodicity. While the autocorrelation function and the power spectral density are typically used to examine stationary signals, these functions are generalized for the analysis of cyclostationary signals and known as the Cyclic Autocorrelation Function (CAF) and Spectral-Correlation Density Function (SCD) [5].

The CAF is a common tool used in cyclostationary detection to find radio transmissions. Let $d(t)$ and v stand for, respectively, a zero-mean complex continuous signal and lag parameter. Then, the CAF $R^\alpha_d(v)$ is defined as the Fourier coefficient of $d(t)d^*(t+v)$ [6], which can be represented as [13]

$$R^\alpha_d(v) = E\left[d(t)d^*(t+v)e^{-j2\pi\alpha t}\right] \tag{1}$$

$$\approx \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}d(t)d^*(t+v)e^{-j2\pi\alpha t}dt \tag{2}$$

where $E[.], (.)^2$, and a are the expected value, complex conjugate, and the cyclic frequency, respectively. A randomly occurring process with periodic mean and autocorrelation is known as a cyclostationary signal. Cyclostationary results in spectral correlation, which is the temporal correlation of the magnitude and phase of the signal's various frequency components. The spectral correlation density determines the spectral correlation between different frequencies (SCD). Signals are found via cyclostationary detection, which scans the computed SCD or spectral coherence for peaks or shapes (a normalized version of the SCD). Given that stationary noise has no spectral correlation, its theoretical SCD is zero. The spectral correlation density is a two-dimensional function that depicts the correlation between various frequency components as they vary over time. SCD is also known as the cyclic spectrum or the spectral correlation function. Both the frequency f and the cyclic frequency f affect the SCD. The inner product (correlation) between the frequency components at $(f - \alpha/2)$ and $(f + a/2)$ is the SCD output. When $\alpha = 0$ The power spectral density is reduced by the SCD[14]. Formally, the SCD is given by

$$S_x^\alpha(f) = \lim_{T \to \infty} \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{-\Delta t/2}^{\Delta t/2} \frac{1}{T} X_T(t, f + \alpha/2) X_T^*(t, f - \alpha/2) dt \qquad (3)$$

Where X_T is the Fourier transform (finite-time) of the x(u) time-domain input signal; Fourier transform representation over time as [5]:

$$X_T(t, f) = \int_{t-T/2}^{t+T/2} x(u) e^{-j2\pi fu} du \qquad (4)$$

A PU signal can be found in a spectral hole using this differentiating characteristic. In contrast to the latter method, which uses the Fourier transform of the correlation products between spectral and temporal components smoothed across time, the former method employs the Fourier transform of the correlation product between the spectral components.

III. **FFT IMPLEMENTATION ALGORITHMS**

The Fourier transform, H(w), of, h(t), time-domain signal, is given by equation (5), where time is t and the angular frequency is w[8].

$$H(w) = \int_{-\infty}^{\infty} h(t) e^{j2\pi t} dt \qquad (5)$$

The discrete Fourier transform is the digital equivalent of the Fourier transform .it is a bounded length sequence which is more practical than the infinite summation of the Fourier transform[15]. The DFT is defined as

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \qquad (6)$$

The fast Fourier transformations shown here employ a variety of techniques to lower the computational cost of computing the Fourier coefficients. For the efficient computation of the DFT, a sufficient number of FFT algorithms have been earlier developed. The Cooley-Tukey algorithm, which was created in the middle of the 1960 s and led to a flood of works on FFTs, was the first major development[16]. This algorithm reduced the DFT complexity from O(N) to O(N log N) which,

at the time, represented a phenomenal increase in effectiveness[17,18]. This complexity reduction has been attained at various degrees by the algorithms that came after. The Radix-2 algorithm used in the Cooley-Tukey procedure[19]. The Radix-3, Radix-4, and Mixed Radix algorithms were the radix algorithms that were created after that. The Split Radix (SPRAD) algorithm and the Fast Hartley Transform (FHT) method are result of further researches [20]

A. FAST HARTLEY TRANSFORM FHT

The DHT's core kernel is real, in contrast to the DFT's complex exponential kernel, which is the primary distinction between the previously stated DFT calculations and the DHT. The DHT coefficient is represented by the following formula using the input data points: [8]

$$X(k) = \sum_{n=1}^{N-1} x(n) . \left[\cos\left(\frac{2\pi nk}{N}\right) + \sin\left(\frac{2\pi nk}{N}\right) \right] \tag{7}$$

Where $X(k)$ is the DHT coefficient of the $x(n)$ input signal. As a result, real multiplications in a DHT take the place of complex multiplications in a DFT. Each complex multiplication in the summation for complex data needs to do four of real multiplications and two of real additions in DFT[21]. This calculation for the DHT only requires two of real multiplications and one of real addition. To translate the DHT output into the common DFT coefficients, It is necessary to do a low-cost coefficient translation from the Hartley domain to the Fourier domain. [22].

B. SPLIT-RADIX ALGORITHM (SRFFT)

Standard radix-2 algorithms are built on the fast synthesis two DFTs with a half-length, while radix-4 algorithms are built on the quick synthesis of four DFTs with quarter-lengths. One half-length DFT and two quarter-length DFTs are synthesized to form the foundation of the SRFFT algorithm[23]. This is allowed because in the Radix-2 algorithm computations, the evenindexed values can be computed independently of the odd-indexed ones. The odd-numbered points are calculated via the SRFFT algorithm using the Radix-4 algorithm. Consequently, the Npoint DFT is split into two N/4-point DFTs and one N/2-point DFT[22] as shown in Fig. 1 . The basic definition of the DFT is:

With $W = e^{-j2\pi/N}$

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + x\left(n + \frac{N}{2}\right) \right] . W^{4kn} \tag{8}$$

for the even index terms

$$X(4k+3) = \sum_{n=0}^{\frac{N}{4}-1} [g(n) + jf(n)]W^{3n} \cdot W^{4kn} \tag{9}$$

and

$$X(4k+1) = \sum_{n=0}^{\frac{N}{4}-1} [g(n) - jf(n)]W^{n} \cdot W^{4kn} \tag{10}$$

for the odd index terms Where

$$g(n) = x(n) - x\left(n + \frac{N}{2}\right) \quad \text{and} \tag{11}$$

$$f(n) = x\left(n + \frac{N}{4}\right) - x\left(n + \frac{3N}{2}\right) \tag{12}$$
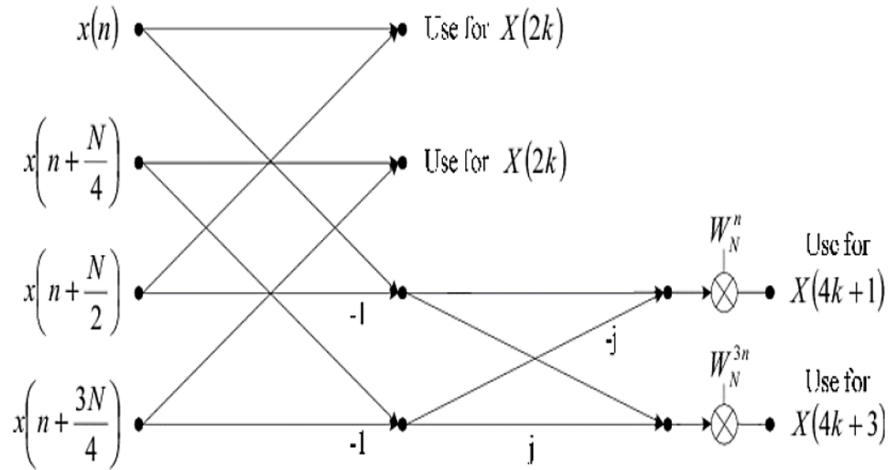

Figure 1: Split-Radix butterfly structure[24]

An N-point DFT is formed by successive usage of decompositions. In this case, the computations are treated as a radix-2 method without the need for any additional mid-level DFT calculations. Analysis of the SRFFT algorithm's butterfly structures reveals about $(4N * \log_2 N)$ are needed in comparison to $(4.25N * \log_2 N)$ for radix-4 and $(5N * \log_2 N)$ for radix-2 algorithms.

C. Slide DFT (SDFT)

The DFT shifting theory or circular shift concept is the fundamental idea behind the SDFT [8] If the DFT of a windowed time-domain sequences(finite-length) is $X(k)$, then the DFT of that sequence, whereas one sample is circularly shifted, is

$$X(k)e^{j2\pi k/N}$$

. Thus, the original (unshifted) spectral elements are multiplied by $e^{j2\pi k/N}$, where k is the DFT bin of interest, to provide the spectral components of a shifted time sequence. This process is expressed by [25]

$$S_k(n) = S_k(n-1)e^{j2\pi k/N} - x(n-N) + x(n) \tag{13}$$

Where $S_k(n)$ is the new spectral component and $S_k(n-1)$ is the previous spectral component. SDFT first setting the window size to N of the input data and apply the standard DFT algorithm such as FFT to the windowed data then add

new samples to the window and remove the oldest samples to maintain the window size to N. SDFT continuously updated by subtract the contribution of oldest samples and add the contribution of newly added samples until all samples is being processed. SDFT Fig. 2 illustrate the difference between DFT and SDFT processing plan
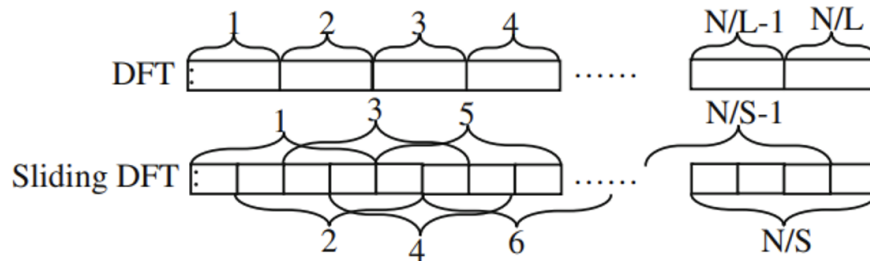

Figure 2: DFT and SDFT data processing plan[26]

The SDFT needed two of real additions plus one of complex multiplication for each sample output. The cost of each subsequent computations for the sliding DFT N-point output is then $O(N)$ as opposed to $O(N^2)$ for the DFT and $O(N \log_2 N)$ for the FFT. According to the rapidly expanding computational capabilities in integrated circuit design, SDFT is now an important topic for analyses of time-frequency spectrum.[25]

IV. CYCLOSTATIONARY SS USING LOW COMPLEXITY FFT ALGORITHMS

The flowchart of proposed cyclostationary sensing based the use of low complexity frequency transform methods is shown in Fig. 3.The major purpose of the recommended sensing system is to minimize the complexity technique of cyclostationary while keeping a good ratio of detection of P_d in comparison with traditional FFT in terms of Computational complexity. The three low complexity frequency transform methods explained in section III (FHT, SRFFT, and SDFT) are used to achieve the complexity reduction of CSS system.

The approach to the proposed CSS system is: first, The PU signal which is a QPSK input signal is firstly processed through the AWGN channel. After that, the PU signal is detected by using the cyclostationary detector. The PU signal samples are subjected to an autocorrelation function. Then, by applying one of the reduced complexity FFT methods convert the result into frequency domain determine the cyclic spectrum (CS) and comparing it to the preset threshold. The PU is present according to the results of the local sensing if it is higher than the threshold; else, the PU is absent.
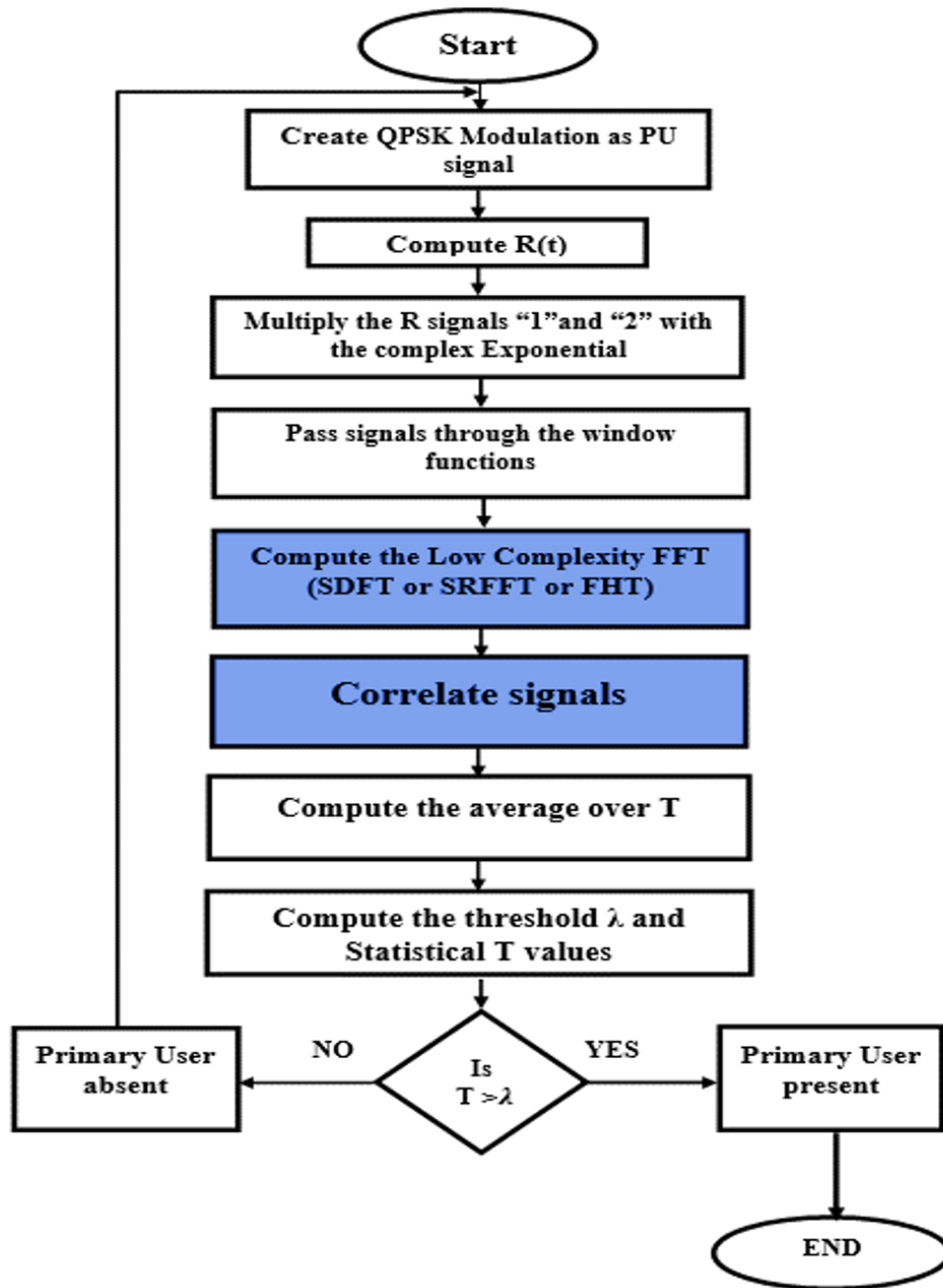
Figure 3: The proposed low complexity cyclostationary spectrum sensing scheme

V. SIMULATION RESULTS AND DISCUSSION

MATLAB $2022b$ installed on core i7 Laptop is used to perform simulations of the cyclostationary spectrum sensor. The arithmetic complexity has the biggest impact on how effectively the algorithms work. It is typically stated as a count of actual additions and multiplications. This is not a very good benchmark for general-purpose computers, and other aspects

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must also be taken into account. For applications with limited memory, for example, the issue of memory utilization is important. The following equation illustrates the cyclostationary approach complexity in term of method computations using conventional and proposed methods [27]:

$$CS = CS_{\text{auto}} + CS_{\text{freq}} \tag{14}$$

Where CS is the total computational complexity of the cyclostationary technique, CS_{auto} is the computational complexity through the autocorrelation process, and CS_{freq} is the computational complexity of conversion to the frequency domain.

$$CS_{\text{auto}} = \text{No. of real multiplications} + \text{No. of real additions}$$
$$= 4Ns + 4Ns - 2 = 8Ns - 2 \tag{15}$$

Table I summarizes the computational complexity expressions of the used algorithms and the complexity reduction ratio, while Fig. 4 shows numerical computational complexities versus number of samples. It can be seen in both table and figure that SDFT and FHT achieves significant reduction in computational complexity compared to traditional FFT, while SRFFT achieves only a slight reduction.

TABLE I
Computational complexity of FFT algorithms

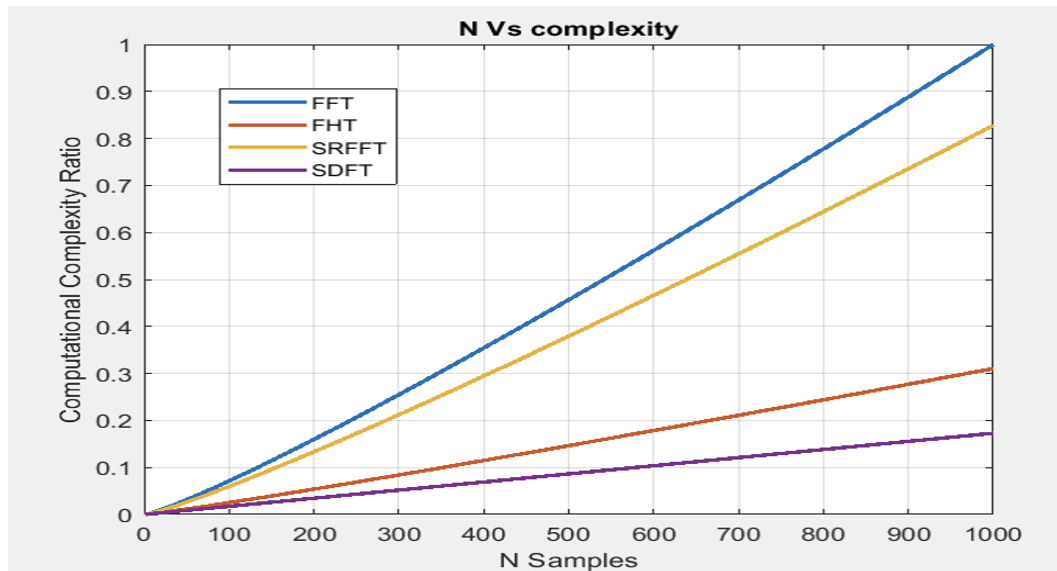| Algorithm	Computational complexity equation	Complexity reduction ratio
FFT	$CS1 = 8Ns - 2 + O\left(5Ns\log_2(Ns)\right)$ [22]	–
SRFFT	$CS1 = 8Ns - 2 + O\left(4Ns\log_2(Ns)\right)$ [22]	17%
FHT	$CS1 = 8Ns - 2 + O\left(Ns\log_2(Ns)\right)$ [28]	70%
Slide DFT	$CS3 = 8Ns - 2 + O(Ns)$ [26]	82%

Figure 4: The performance curves of a various number of samples in relation to computation complexity ratio

Table II displays how various methods' computation times vary as a function of the FFT by executing the algorithms in MATLAB by using MATLAB instructions to display the elapsed time of each algorithm . However, the amount of RAM and cache being used has an effect on the performance. The effect is more significant for higher-order FFTs, when cache misses are more frequent, as would be expected. Additionally important is how well the CPUs perform with floating point operations compared to integer ones. Table III displayed the memory usage when each algorithm for a 1024-point by executing the algorithms in MATLAB by using MATLAB instructions to display the memory used with each algorithm. Table IV show a performance comparison of proposed algorithms with previous implemented algorithms. The table shows that the low-complexity FFT algorithms, which require performing a smaller number of computations, also have low memory storage compared to other algorithms with higher complexity.

TABLE II
Computation time (in milliseconds), of various algorithms.

Algorithm	FFT Order			
	256	**512**	**1024**	**2048**
FFT	0.578	0.582	0.688	0.677
SRFFT	0.547	0.555	0.674	0.619
FHT	0.535	0.541	0.547	0.583
Slide DFT	0.515	0.523	0.533	0.566

Fig. 5 shows the performance curves of the used algorithms in regards to the probability of signal detected of P_d versus E_b/N_0 in the AWGN and Rayleigh fading channel, compared to the conventional approach. It can be noticed in this figure that the low complexity algorithms have some degradation in detection probability which about 0.02 in SRFT and about 0.01 in FHT with Improved probability of detection in SDFT in compared with of traditional FFT. This clearly proves

TABLE III
Memory usage (in MB) in computing a 1024-point FFT.

Algorithm	Memory usage
FFT	2744
SRFFT	2781
FHT	2683
Slide DFT	2665

TABLE IV
Performance of The Proposed FFTs Compared With Previous Implementations

Algorithms	[9]	[10]	Proposed algorithms		
			SRFFT	FHT	Slide-DFT
Complexity reduction	-	40%	17%	70%	82%
Memory	4096	-	2781	2683	2665
Time	0.3 s	1 ms	0.674 ms	0.547 ms	0.533 ms

that the complexity of cyclostationary spectrum sensor can be reduced using the proper algorithm for frequency domain transformation.
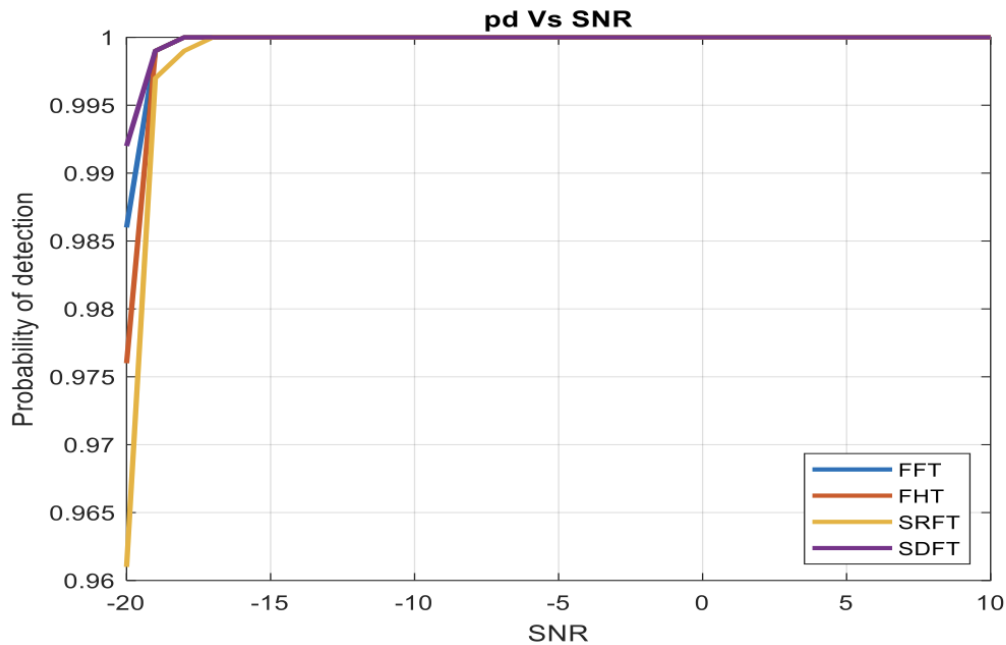

Figure 5: The performance curves of traditional and proposed detection methods over AWGN channel.

VI. CONCLUSION

In this paper, the use of low complexity FFT algorithms is proposed to reduce the complexity of cyclostationary spectrum sensing technique and investigate its impact on the detection performance. The simulation's analysis indicates that the proposed scheme is quite effective when using Slide DFT in terms of the time of execution and memory. The Slide DFT

is the fastest algorithm on all platforms with a reasonable dynamic memory requirement. The use of different frequency transform algorithms does not affect the system's performance in regards to the probability of a signal being detected compared with traditional algorithms.

CONFLICTS OF INTEREST

The author declares no conflict of interest.

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