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Entropy Generation Minimization Theoretical Analysis for External Flow Around Horizontal Cylinder at low Reynold Number

Abstract- *The minimization of entropy generation is a helpful method to design optimum thermal system, and find an expression for entropy generation at several ranges of Reynolds number for heat transfer and friction flow irreversibility is a good way to achieve this goal. This study deal with several ranges of Reynolds number that cover 0.1< Re< 40000 divided into four groups. A relation between optimum Reynolds number, entropy generation number, irreversibility distribution ratio, and Began number was obtained. In addition, a relation between the optimum Reynolds number and the duty parameter was obtained for all Reynolds number ranges.*

Keywords-*Entropy generation minimization, external flow, horizontal cylinder.*

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1. Introduction

"Entropy Generation Minimization (EGM) is the method of thermodynamic optimization of real systems that owe their thermodynamic imperfection to heat transfer, fluid flow irreversibilities" [1,3] .Bejan was the first who deal with entropy generation for external flow [1], his study delt with flat plate surface and the optimum Reynolds number was found. Poulikakos and Johnson [2] analyzed the combined heat and mass transfer flow over flat surface and around cylinder. Their study for the cylinder cover the Reynolds number range 40<Re<1000 only. Bejan [3] and Bejan et al. [4] Found the relation between the optimum Reynolds number and duty number for range of Reynolds ($40 < Re < 10^5$). Also Bejan [5] confirm the study of external flow around cylinder and refer to new dimensionless parameter called Bejan number which gave good explanation for the behavior of the flow where the heat or the friction dominate at the same range of Reynolds. In the present study the analysis covered the previous ranges and the low Reynolds number ranges (Re<40).

2. Entropy Generation Analysis

The entropy generation rate in steady state external flow around horizontal cylinder, control volume shown in Figure 1 may be given as follows:

I. Mass balance:

$$\dot m_{in} = \dot m_{out} = \dot m \quad (1)$$

II. Eat balance:

$$\dot m_{in} h_{in} - \dot m_{out}\, h_{out} + \iint q''\, dA = 0 \quad (2)$$

III. Entropy generation balance:

$$\dot S_{gen} = \dot m_{out} s_{out} - \dot m_{in} s_{in} - \iint \frac{q'' dA}{T_s} \quad (3)$$

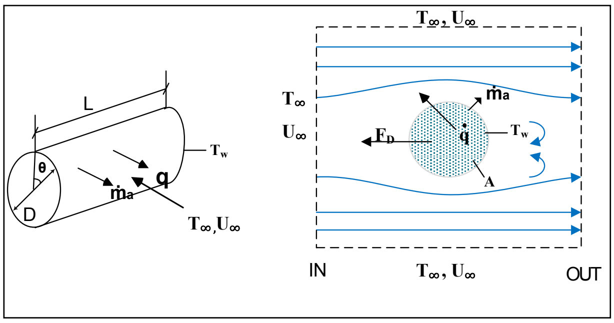

Figure (1) Convective heat transfer in external flow

The Gibbs equation is given as [2]:

$$dh = TdS + vdP \quad (4)$$

Applying Gibbs equation for the control volume in figure (1) to eq.(2) gives:

$$\dot m_{in} h_{in} - \dot m_{out}\, h_{out} = T_\infty(\dot m_{out} s_{out} - \dot m_{in} s_{in}) + \frac{\dot m_{in}}{\rho_\infty}(P_{out} - P_{in}) \quad (5)$$

Following [3,5] it is assumed that the average quantities of the control volume are equal to the free stream quantities.

Combining Eqs.(1) with Eqs.(5) and substituting into eq.(2) yields:

$$\dot S_{gen} = \iint q'' \left(\frac{1}{T_\infty} - \frac{1}{T_s}\right) dA - \dot m_{in} \frac{P_{out} - P_{in}}{\rho_\infty T_\infty} \quad (6)$$

The drag force and the flow rate may be given as:

$$F = A_{cross}(P_{in} - P_{out}) \quad \text{and}$$

$$\dot m = A_{cross}\, \rho_\infty U_\infty \quad (7)$$

Substituting Eq. (7) in Eqs. (6) gives:

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$$\dot{S}_{gen} = \iint q^{"} \left(\frac{1}{T_\infty} - \frac{1}{T_s}\right) dA + \frac{U_\infty F}{T_\infty} \quad (8)$$

Assume the $|T_s - T_\infty|$ is smaller than absolute T_s or T_∞, Eqs.(8) can be expand in Taylor series and by neglected the terms of second and higher orders[2,3] :

$$\dot{S}_{gen} = \frac{1}{T_\infty^2} \iint q^{"} (T_s - T_\infty) dA + \frac{U_\infty F}{T_\infty} \quad (9)$$

The first term in right hand represent the entropy generation due to heat and the second the entropy due to friction.

The drag force per unit length can be defined [6,8]:

$$F/_l = \int_0^\pi \frac{1}{2} \rho_\infty U_\infty^2 C_{f,x}\, r d\theta = \frac{1}{2} \rho_\infty U_\infty^2 \frac{D}{2} \int_0^\pi C_{f,x} d\theta \quad (10)$$

The temperature related to fluxes may be given as:

$$T_s - T_\infty = \frac{q^{"}}{\alpha} \quad (11)$$

Substituting Eqs. (10) and (11) into eq. (9):

$$\frac{\dot{S}_{gen}}{l} = \frac{q^{"2}}{T_\infty^2}\, 2 \frac{D}{2} \int_0^\pi \frac{d\theta}{\alpha} + \frac{U_\infty^3 \rho_\infty}{2T_\infty} \frac{D}{2} 2 \int_0^\pi C_{f,x}\, d\theta \quad (12)$$

Where $\alpha = a \mathrm{Re}_D^m \mathrm{Pr}^n \frac{k_\infty}{D}$; and $\int_0^\pi C_{f,x}\, d\theta = C_f$ following references [6,8]

$$C_f = b\, \mathrm{Re}_D^{-y} \quad (13)$$

Substituting Eqs. (13) and (11) into (12) yields

$$\frac{\dot{S}_{gen}}{l} = \frac{q^{"2}}{T_\infty^2} \frac{D^2\, Re_D^{-m} Pr^{-n}}{a\, k_\infty} \int_0^\pi d\theta + \frac{U_\infty^3 \rho_\infty D}{2T_\infty}\, bRe_D^{-y}$$

$$\frac{\dot{S}_{gen}}{l} = \frac{q^{"2}}{T_\infty^2} \frac{D^2 Re_D^{-m} Pr^{-n}}{a\, k_\infty} \pi + \frac{U_\infty^3 \rho_\infty D}{2T_\infty}\, bRe_D^{-y} \quad (14)$$

Where $q' = \frac{q}{l} = q^{"} \pi D \rightarrow q^{"} = \frac{q'}{\pi D} \rightarrow$

$q^{"2} = \frac{q'^2}{\pi^2 D^2}$, and $Re_D = \frac{\rho U D}{\mu} \rightarrow$

$U = \frac{Re_D\, \mu}{\rho D}$

Substituting into Eqs. (14) gives:

$$\frac{\dot{S}_{gen}}{l} = \frac{1}{a\pi} \frac{q'^2}{T_\infty^2} \frac{Pr^{-n}}{k_\infty} Re_D^{-m} + \frac{b}{2} \frac{U_\infty^2 \mu_\infty}{T_\infty} Re_D^{1-y} \quad (15)$$

Equation (15) is the general form for entropy generation for external flow around cylinder, where the constant a, n, and m are given in table (1) following the references [7,8]:

Table (1) constant for equation (15):

Range of Re	Nusselt Number
0.4 – 4	$Nu=0.989\, Re_D^{0.33} Pr^{1/3}$
4 – 40	$Nu=0.911\, Re_D^{0.385} Pr^{1/3}$
40 – 4000	$Nu=0.683\, Re_D^{0.466} Pr^{1/3}$
4000 – 40000	$Nu=0.193\, Re_D^{0.618} Pr^{1/3}$

The drag coefficient as a function of Reynold number is obtained by curve fitting for the range similar approximately to the range of Nusselt number above depending on the map from reference [9], figure 2 :

Range of Re	Drag coefficient	Standard Error	Correlation Coefficient
0.1 – 4	$C_f=10.17\, Re_D^{-0.784}$	0.7703330	0.9996
4 – 40	$C_f=6.634\, Re_D^{-0.302}$	0.0811095	0.9958
40 – 4000	$C_f=4.681\, Re_D^{-0.2}$	0.0763557	0.9965
4000 – 40000	$C_f=1.1$		

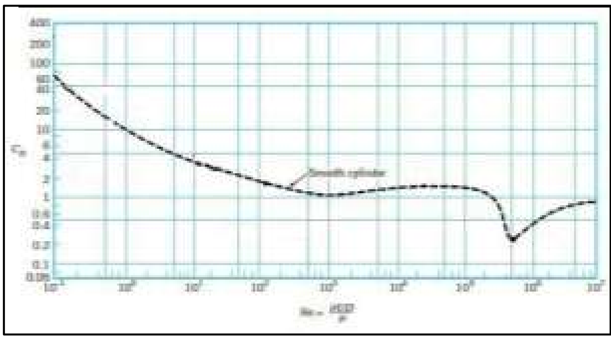

Table 2: drag coefficient correlations

Figure 2: The relation between Drag force coefficient Cd and Reynolds number. Reference [9].

Equation (15) will be rewritten for four ranges of Reynolds number as follow:

$0.1 < Re_D < 4$

$$\frac{\dot{S}_{gen}}{l} = 0.322 \frac{q'^2}{T_\infty^2} \frac{Pr^{-\frac{1}{3}}}{k} Re_D^{-0.33} + 5.085 \frac{\mu_\infty U_\infty^2}{T_\infty} Re_D^{0.216} \quad (16)$$

$4 < Re_D < 40$

$$\frac{\dot{S}_{gen}}{l} = 0.349 \frac{q'^2}{T_\infty^2} \frac{Pr^{-\frac{1}{3}}}{k} Re_D^{-0.385} + 3.317 \frac{\mu_\infty U_\infty^2}{T_\infty} Re_D^{0.698} \quad (17)$$

$40 < Re_D < 4000$

$$\frac{\dot{S}_{gen}}{l} = 0.466 \frac{q'^2}{T_\infty^2} \frac{Pr^{-\frac{1}{3}}}{k} Re_D^{-0.466} + 2.3405 \frac{\mu_\infty U_\infty^2}{T_\infty} Re_D^{0.8} \quad (18)$$

$4000 < Re_D < 40000$

$$\frac{\dot{S}_{gen}}{l} = 1.649 \frac{q'^2}{T_\infty^2} \frac{Pr^{-\frac{1}{3}}}{k} Re_D^{-0.618} + 0.55 \frac{\mu_\infty U_\infty^2}{T_\infty} Re_D \quad (19)$$

The optimum Reynolds number ($Re_{D\,opt}$) was found by differentiation the Eqs. (16), (17), (18), and (19) with respecting to Reynolds number and equating the results to zero as follow:

Range of Re	Re optimum	
0.4 – 4	$Re_{Dopt} = 0.139\, \beta^{\frac{1}{0.546}}$	(20)
4 – 40	$Re_{Dopt} = 0.722\, \beta^{\frac{1}{1.083}}$	(21)

40 – 4000 $$Re_{Dopt} = 1.824\, \beta^{\frac{1}{1.266}} \quad (22)$$

4000 – 40000 $$Re_{Dopt} = 14.64\, \beta^{\frac{1}{1.618}} \quad (23)$$

Where β is the duty parameter found by A.Bejan [1,3]:

$$\beta = \frac{q'^2}{U_\infty^2 k_\infty \mu_\infty T_\infty Pr^n}$$

- The other important parameter is the irreversibility distribution ratio $\emptyset$ which defines the fluid friction contribution to heat transfer irreversibility.
- *Be* This parameter is defined by[5] as a Bejan number :

$$Be = \frac{1}{1+\emptyset}$$

"When Be=1 is the limit at which the heat transfer irreversibility dominates, Be=0 is the opposite limit at which friction irreversibility is dominated by fluid effects, and Be=1/2 is the case which the heat transfer and fluid friction entropy generation rates are equal".

- Now to introduce the entropy generation number *Ns*, substitute Re_{opt} in entropy generation Eqs. (16), (17), (18), (19) yield new set of eqs. for $\dot{S}_{gen.min}$, the ratio of $\dot{S}_{gen}$ to $\dot{S}_{gen.min}$ called entropy generation number *Ns*, and the form of the equations becomes as follow according to the range of Reynolds number:

$$N_{s1} = \frac{\dot{S}_{gen}}{\dot{S}_{gen.min}} = \left(\frac{Re_D}{Re_{Dopt}}\right)^{-0.33} + \left(\frac{Re_D}{Re_{Dopt}}\right)^{0.216} \quad (24)$$

$$N_{s2} = \frac{\dot{S}_{gen}}{\dot{S}_{gen.min}} = \left(\frac{Re_D}{Re_{Dopt}}\right)^{-0.385} + \left(\frac{Re_D}{Re_{Dopt}}\right)^{0.698} \quad (25)$$

$$N_{s3} = \frac{\dot{S}_{gen}}{\dot{S}_{gen.min}} = \left(\frac{Re_D}{Re_{Dopt}}\right)^{-0.466} + \left(\frac{Re_D}{Re_{Dopt}}\right)^{0.8} \quad (26)$$

$$N_{s4} = \frac{\dot{S}_{gen}}{\dot{S}_{gen.min}} = \left(\frac{Re_D}{Re_{Dopt}}\right)^{-0.618} + \left(\frac{Re_D}{Re_{Dopt}}\right) \quad (27)$$

The forms above leads to introduce N_s =2 at $\left(\frac{Re_D}{Re_{Dopt}}\right) = 1$, but the effect of heat and friction is simultaneous done which mean that the entropy generation number must be N_s =1 at $\left(\frac{Re_D}{Re_{Dopt}}\right) =$ 1,this fact lead A. Began [3,5] to divide the effect of heat and friction by 2/3 and 1/3 respectively.

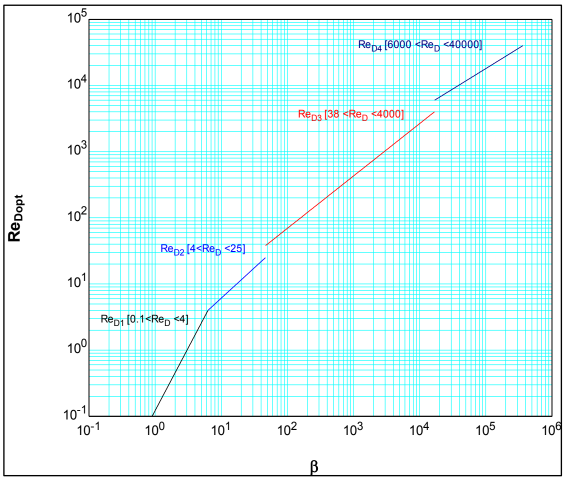

Figure (3) The relation between optimum Reynolds number and duty parameter

3. Results and Discussion

The relation between optimum Reynolds number and the duty parameter is shown in figure (3) for four ranges of Reynolds number. The relation is proportionally increases, and the increase is more for high Reynolds number than the lower one.

This behavior is due to the type of flow around the cylinder in each range of Reynolds number as shown in figure (4).

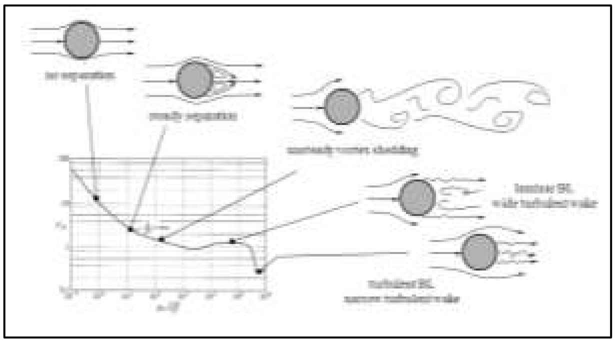

Figure (4) Types of flow pattern around cylinder. Reference [9].

At low Reynolds number as in ranges Re_{D1} and Re_{D2}, the heat transfer behavior is natural convection and the first range is of laminar naturel convection and the second for turbulent natural convection. There is a gap between the second range and the third range due to the mixed region between the natural and force convection. Another gap between the third and fourth ranges which is another transition region. These gaps are eliminated and did not covered by the correlations.

The Entropy generation number N_s via the ratio of Reynolds to optimum Reynolds or the ratio of diameter to optimum diameter is shown in figure (5) for all Reynolds ranges.

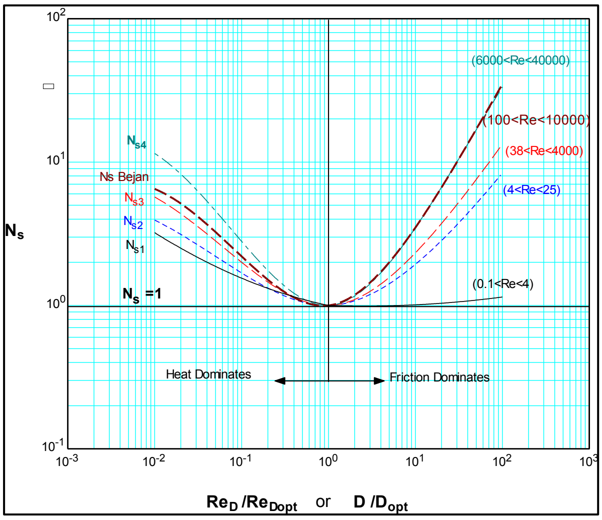

Figure (5) The relation between entropy generation number and Re_D/Re_{Dopt}.

The shape of the curves are positive second order parabolic. The magnitude of Ns =1 at the ratio of Reynolds=1, and left direction from this number is the area of the transfer effect while the right is the effect of friction flow, so the optimum is at N_s =1. Bejan [3,5]result is identical to the highest range of Reynolds number at friction part and lower in the heat part. At lower Reynolds region the effect of entropy generation is constant and very low and that may be due to the effect of the natural convection. The figure gave a very good picture can be used to compare the any research's results in these working ranges.

The irreversibility distribution ratio $\emptyset$ shown in figure (6) which illustrated the optimum magnitude of $\emptyset_{opt}$ = 0.5 and the effect of heat and friction flow for all ranges of Reynolds number.

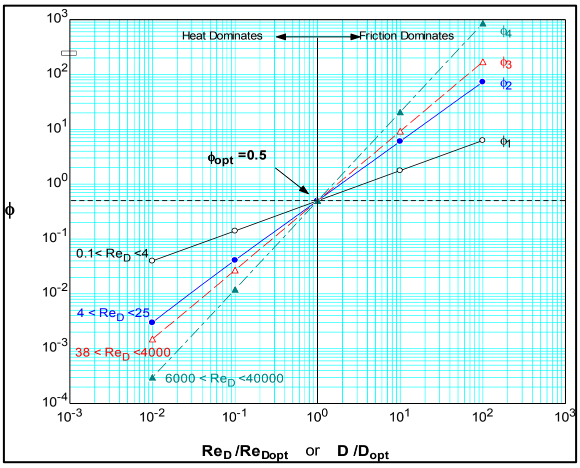

Figure (6) The relation between Irreversibility distribution number and Re_D/Re_{Dopt}.

The optimum is at the ratio Re_D/Re_{Dopt}=1 where the effect of the heat and friction flow in entropy generation is equal and a minimum.

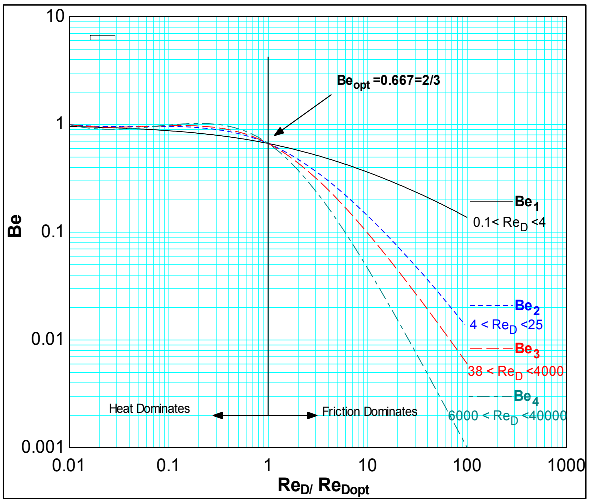

Figure (7) The relation between Bejan number and Re_D/Re_{Dopt}.

Figure (7) shows the relation between Bejan number (Be) and Re_D/Re_{Dopt}, and the heat effect part with the friction flow part. The optimum Be_{opt} =0.667, which confirm the Bejan suggestion[3] for the percent effect of the heat to the friction flow. In the range of heat all flow act near Be=1 while in friction part the flow carry on to zero as given in the definition of Bejan number.

4.Conclusion:

According to the results the important conclusion points are: the effect of entropy generation number at all Reynolds number via Re_D/Re_{Dopt} is same except at low Reynolds range, the reason is the using of Reynolds Number with force convection put actually the natural convection is the dominator, and the effect of the radiation which is an important factor at low ranges.

The optimum design for thermal system was taken at N_s =1and Re_D/Re_{Dopt}=1, where $\emptyset_{opt}$ = 0.5, and Be_{opt} =0.667.

Four empirical relations were found to compare the actual system behavior with the analytical design.

Nomenclature

A: Area (m^2)
Be: Bejan number
C_f: Drag force coefficient
$C_{f,x}$:Local Drag force coefficient
D: Cylinder diameter (m)
F: Drag force (N)
H: Enthalpy (kJ/kg)
k: Thermal conductivity (W/m.C)
L: Length (m)
$\dot{m}$:Mass flow rate(kg/sec)

Ns: Entropy generation number
Nu: Nusselt number
P: Pressure (Pa)
Pr: Prandtl number
q: Heat flux (W)
$\dot{q}$: Heat flux per length (W/m)
$\ddot{q}$: Heat flux per area (W/m^2)
Re_D : Reynolds number around cylinder
Re_{Dopt} :Optimum Reynolds number
r : Radius (m)
S_{gen} : Entropy generation rate (W/K)
T : Temperature (K)
U : Velocity(m/sec)

Greek letters

β :Duty number
∅: Irreversibility distribution number
μ :Dynamic viscosity (m^2/sec)
θ: Angle (degree)
ρ: Density (kg/m^3)
α: Heat transfer coefficient (W/m^2.C)

Subscript

in: Inlet
out: Outlet
∞: free stream
s : Surface
x : Local
cross: Cross section

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