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The Vortex blobs method simulating the viscous liquid motions

Amjed A.A. Alansari

Al-Kunooze university college, Department of Instrumentation Engineering techniques

Alansary-Abulkareem@kunooz.edu.iq

Abstract

This paper with a title" The vortex method simulating the viscous liquid motions has discussed the born of the blobs and their motion, their collection , the effects of Reynold's Number and it's effects in the hydrodynamics. By using the integration and partial differential equations. And describe the plane motion , using the calculus of probability. We explain the vortex sheet in the mentioned place.Initialy we must prepare the primary equations of the potential vector filed V^t , V_B , V_R^T

Then we must research upon the operator L and the determination of the filed V_A , and the conformal mapping into the canonical region, then by using the analytic function vanishing in the infinity to get Cauchy's integral formula and use it to get Hilbert transforms. We discuss the path of integration ,the boundary properties for the linear circle expressed by plemelij formula and using the application of Harmonic analysis unit reach to final linear integral.

Keywords: Vortex blob, viscous liquid, motion, Reynold's Number

1. Introduction

In the paper the principles of simulations of viscous fluid motion by means of the vortex blobs method have been given. They consist in defining the infinitesimal element for the diffusion process. The determination of the inductive, primary and auxiliary velocity field has also been presented. The regularized equation for the blobs intensity, compensating for the tangent components of the velocity, has also been derived.

" The determination of motion of the viscous liquid at large Reynolds number is one of more difficult issues in hydrodynamics. It seems probable that the vortex blob method, suggested by chorine [1], will allow to execute the simulation of some motion also by means of an average power computer (the simulation here is understood as the Solution of the problem modeling the issue). There is a certain number (available) of publications concerning the use of the method [2],[3],[4],[5],[6], but it seems they don't apply to its formal fundamentals or to mathematical apparatus.

2. The vortex blob method

The plane motion of the viscous liquid described by the vortices equation:

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$$\frac{\partial w}{\partial t}+\frac{\partial}{\partial x}(uv)+\frac{\partial}{\partial y}(vw)=\frac{1}{Re}\Delta w \qquad (1)$$

The sign w designates (the only non-zero) components of velocity and u and v are cartesian components of velocity field obviously, di$\overrightarrow{vv}$ = 0. Equation (1) is the equation of diffusion relation to the moving medium. There is a well-known relationship between the description by means the calculus of probability and the field description of this phenomenon [7] Namely, the homogeneous sufficiently regular Markov process with the strength of probability of $\overrightarrow{or} \rightarrow \vec{r}$ transition in time t denoted by the symbol p($\vec{t}$,$\overrightarrow{ro}$,$\vec{r}$) is called the diffusion process if:

$$\int_{v_z}^{v} P\,(t,\overrightarrow{ro},\vec{r})dxdy=0(t)$$

$$\int_{v_e}^{v} P\,(\vec{r}-\overrightarrow{ro})\;P\,(t,\overrightarrow{ro},\vec{r})dxdy=0(t)\;=\;\vec{v}(\overrightarrow{ro})+0(t)$$

$$\int P\,(\vec{r}-\overrightarrow{ro})\;\times\;(\vec{r}-\overrightarrow{ro})P(t,\overrightarrow{ro},\vec{r})dxdy=\;A(\overrightarrow{ro})+0(t)$$

The sign ⊗ denotes tens or product, A is the symmetric math. ix (positively defined), V_Z-neighbourhood of $\overrightarrow{ro}$. The Vector $\vec{r}$ consists of the components (x,y).

The concentration of p satisfies the simple (second) Kolmogorov equation:

$$\frac{\partial P(t,\overrightarrow{ro},\vec{r})}{\partial t}+\frac{\partial}{\partial x^i}\left[v^i(t,\overrightarrow{r})\,P(t,\overrightarrow{ro},\vec{r})\right]=\frac{1}{2}\times\frac{\partial^2}{\partial x^i\,\partial x^j}(A^{ij}(t,\overrightarrow{r})\times P(t,\overrightarrow{ro},\overrightarrow{r}) \qquad (2)$$

we will introduce the function Γ ($\overrightarrow{ro}$) and subsequantly the velocity w:

$$W(t,\overrightarrow{r})=\int_{-\infty}^{\infty}\Gamma(\overrightarrow{ro})\;P(t,\overrightarrow{ro},\vec{r})\;dx_o dy_o$$

it is easy to notice that w fluids equation (1) with appropriate $\vec{V}$ and A: $A^{ij}=\frac{2}{Re}$ and $A^{iK}=0$ $\forall i \neq k$ Then the diffusion process determines the infinitesimal element $\Delta\,\vec{r}$

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$$\Delta \vec{r} = \vec{V}\,(t,\overrightarrow{r})\,\Delta t + A(t,\overrightarrow{r})\,\Delta\omega$$

in which $\Delta\vec{w}$ is the vector of increase of wiener process. [8] In the considered case in the presence of spherical symmetry of A, we obtain the stochastic differential equation:

$$\overrightarrow{\mathrm{dr}} = \vec{V}\; dt + \mathrm{d}\vec{R} \qquad (3)$$

With the vector $\mathrm{d}\vec{R}$ of random components with distribution of mean zero values and variances 2dt / Re. The physical interpretation of the given description is the following: the diffusion in relation to the continuous medium moving with the velocity of diffusing substance particle. The vector describes the motion of each particle:

$$\vec{r}\,(t+\Delta t) = \vec{R}_f(t+\Delta t) + \vec{R}_R \qquad (4)$$

where, $\vec{R}_F$ follows from the movement of the continues medium:

$$\frac{\mathrm{d}\vec{R}_f}{dt} = \vec{V} \qquad\qquad \vec{R}_{f/t} = \vec{r}(\mathrm{t})$$

and $\vec{R}_R$ is the random component with previously given properties. The moment Δt means the time between samplings. The vorticity w is the equivalent of concerntration of diffusing Substance. After integrating the strength with respect to the coordinates of the particle the mass is obtained. This Calculation yields to the following concertation for vorticity:

$$\int \mathrm{wdxdy} \;=\; \int_{-\infty}^{\infty} \left(\vec{n},\overline{rot},\vec{v}\right)dxdy = \int \vec{v}\,dt = \Gamma$$

Chorine [1], on the basis of the given interpretation, introduced to the structure called vortex blobs, while are the counterparts of particles of the diffusing substance. They are the carries of the velocity and are insignificantly small with respect to the sizes in the scale characteristic for the re- going of motion and move in the presented way. The distribution of the vortices inside blobs in a secondary matter (Similarly to the distribution of mass in the molecule of the diffusing substance). The essential thing is that each blob induces velocity field defined - for the distances considerably surpassing its dimensions by the formulae:

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$$\left.\begin{aligned} u_1 &= \frac{\Gamma i}{2\pi} \times \frac{y_1 - y_i}{{r_{1i}}^2} = -\frac{\Gamma i}{2\pi} \times \frac{\partial}{\partial x_1} \arctan \frac{y_1 - y_i}{x_1 - x_i} \\ v_1 &= \frac{\Gamma i}{2\pi} \times \frac{x_1 - x_i}{{r_{1i}}^2} = -\frac{\Gamma i}{2\pi} \times \frac{\partial}{\partial y_1} \arctan \frac{y_1 - y_i}{x_1 - x_i} \end{aligned}\right\} \quad \text{------------} \; 5$$

This fact is the result of simplification of the formulae for the potential:

$$\varphi(\mathrm{A}) = -\frac{1}{2\pi} \int \int \mathrm{w}\,(\mathrm{B}) \mathrm{arc\,tan} \frac{\mathrm{y_A} - \mathrm{y_B}}{\mathrm{X_A} - \mathrm{X_B}}\, \mathrm{ds_B}$$

at the distance r_{Ai} between the point A and the middle " i " of the considered blob surpassing considerably it's diameter. If $\mathrm{Y_{Ai}}$ is comparable to the smaller than this quarter. If r_{Ai} is comparable to the smaller than this quantity, then the formulae lose this significance, one should write:

$$\begin{aligned} \mathrm{u}_A &= -\frac{\Gamma \mathrm{i}}{2\pi}\, \mathrm{x}(\, x_A - x_i), y_A - Y_\mathrm{i} \\ \mathrm{v}_A &= -\frac{\Gamma \mathrm{i}}{2\pi}\, \mathrm{y}(\, x_A - x_i), y_A - Y_\mathrm{i} \end{aligned} \quad \text{------------} \; 6$$

functions X and Y pass continuously into the ones use previously and fulfil the equation of continuity:

$$x_{xA} - y_{yA} = 0$$

furthermore, they are connected with vorticity w:

$$y_{\S} - y_{\eta} = \mathrm{w}(\S\ ,\eta)$$

$$\S = x_A - x_i\ , \eta = (y_A\ , y_i)$$

for §, ɳ embodied inside the A considered blob. Chorine [1] suggested.

$$\left.\begin{aligned} \mathrm{x} &= \frac{\eta}{\sigma r} \\ \mathrm{y} &= \frac{\S}{\sigma r} \end{aligned}\right\} = \quad r^2 = \S^2 + \eta^2 \leq \sigma^2 \quad \text{------------} \; 7$$

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defining the blobs are circles with radius (different for a particular blobs)

3. The velocity field

They will present the velocity field $\vec{v}$ as the sum of a few components

$$\vec{v} = \overrightarrow{v_p} + \overrightarrow{v_o} + +\vec{v}_w + \vec{v}_A \qquad (8)$$

which mean in succession:

$\overrightarrow{v_p}$ the velocity of the potential flow

$\overrightarrow{v_o}$ the velocity induced by the set of blobs previously formed and the ones already existing in the flow

$\vec{v}_w$ the velocity induced by the set of blobs originating at the present moment on the walls of the substance (or other surface) and keeping contact with them.

$\vec{v}_A$ the velocity of the auxiliary potential flow

If follows from the given specification, we assumed the formation of new blobs on the boundary of substance being flowed around (or the known surface of the type of boundary of the immiscible fluid). The proof for this seems to be a well-known fact of the vortices formation (vortex sheet) in the mentioned places.

Let us consider the boundary conditions imposed on the velocity field. Let us assume that walls of substances which the fluid move with velocity $\overrightarrow{v_B}$. It is vector function of time and the boundary point of region. Due the existence of viscous one should write:

$$\overrightarrow{v_{1B}} = \overrightarrow{v_B} \qquad (9)$$

Expressing Eq. (8) in the components tangent and normal to the boundary we will obtain:

$$V_P{}^1 + V_0{}^1 + V_W{}^1 + V_A{}^1 = V_B{}^1 \qquad (10)$$
$$V_O{}^n + V_w{}^n + V_A{}^n = V_B{}^n$$

In this way we have defined $\overrightarrow{vp}$. It is apotential velocity field satisfying the condition:

$$\vec{n} \, . \overrightarrow{v_p} = 0 \qquad (11)$$

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The determination of such field is the subject of classical aerodynamics (or hydrodynamics) and is commonly known. The potential part of (V_A) may be found, if one of the boundary values (${V_A}^1$ or ${V_A}^n$) is defined.

At the same time the boundary value of remaining components may be determined. Let us denote this operation by the symbol L: (${V_A}^n \rightarrow {V_A}^1$)

$$ {V_A}^1 = L\,({V_A}^n) \qquad (12) $$

it will turn out, L is a linear integral operator with singular Kernel on this basis of the second of the Eq. (10) it may be written:

$$ {V_n}^1 = L({V_A}^n) - L\,({V_0}^n) - L\,({V_W}^n) \qquad (13) $$

and then substituted into the first of Eq (10). If $\overrightarrow{Vp}$ is Known (determined previously), $\vec{V}_B$ the set vector function and results from the position and circulation of blobs not originating at the present moment, then the equation:

$$ {V_B}^1 = {V_P}^1 + {V_W}^1 + {V_0}^1 + L({V_B}^n) - L\,({V_0}^n) - L\,({V_W}^n) \qquad (14) $$

is an Integral equation with represent to the density of the circulation defining $\overrightarrow{(V_W)}$through particular, presently formed bolbs.

4. The primary equation

The equation (14) may be essentially simplified on the basis of the operator L properties. It assigns the component tangent to the boundary line to the normal component of the potential vector field. If a certain potential vector field $\vec{V}$defines the components tangent $\overrightarrow{vt}$ and normal v^non the boundary of the region, then the following relation occurs. on the

$$ V^t = L\,(v^n) $$

This fact is obvious on the basis of univocal determination of derivatives while solving the Numerical problem, we will make use of the given property represented in some components of the Sum (8) as

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potential and no-potential. We will denote the former with index 1 and the later with index 2 Applying the described distribution to V_w and V_o we will obtain:

$$V_B{}^t = V_P{}^t + V_W{}^t + V_{20}{}^t + L(V_B{}^n) - L\,(V_{1w}{}^n) - L\,(V_{2W}{}^n) - L\,(V_{10}{}^n) - L\,(V_{20}{}^n)$$

after reduction of terms of type $(V_{20}{}^n - L\,(V_{10}{}^n)$ we will have:

$$V_R{}^t = V_P{}^t + V_{2w}{}^t + V_{20}{}^t + L\,(V_{2w}{}^n) - L\,(V_{10}{}^n) \qquad (15)$$

We will call it the primary equation, on the basis of properties of blobs we infer that primary equation is the result for not potential interactions, therefore local, occurring within the limits of the blob structure. we will also add them at according to Eq. (8) the velocity field is$\overrightarrow{V_P}, \overrightarrow{V_P}$ result of the motion of non-viscous field $(\bar{v}_{p\,,}\bar{v}_A)$ as well as the existance of the components induced by the blobs of different kind. The component $\overrightarrow{V_O}$follows from the existance. of the blobs formed previously carried by the fluid turbulence stimulation"). Such field has the arbitrary components on the wall. The reduction of the tangent component to the set value takes place due to the existence of the potential component (the component of "close range") in the interaction of blobs being in contact with the wall. The normal Component induced both by the old generation blobs as well presently formed ones is reduced by the normal component of the auxiliary potential field $\overrightarrow{V_1}$. However, eliminating the normal component, in accordance with the properties of the potential field, ascertain regent component is being introduced. This fact leads to the integral equation (15) unknown is the distribution of circulation in the group of blobs being in Contact with the wall. Let us also notice that due to the identity of velocities of the wall and the fluid the only means of boundary-type blobs displacement is the random walk. Thus (Since the velocities of the wall and fluid are equal) it is also the only way for blob to cross the wall. According to the diffusive analog the "elastic collision" of the rigid bodies or elimination of a given blob on the Conventional (immaterial) boundary of the region should be reduced, if such boundary occurs.

5. The operator L and the determination of the field $\overrightarrow{V_A}$?

The operator L and the potential field $\overrightarrow{V_A}$ may be evidently det- ermined in two product cases, namely, when the conformal mapping of the region of motion into the upper half-plane or the outside of the circle is known. Let us denote the Cartesian components of the field $\overrightarrow{V_A}$ by symbols u and v. The quantity u-iv is an analytic function. If the conformal mapping into the canonical region (the half-plane, the outside of the circle) is designates by the symbol ξ=ξ (z),

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then:

$$(\mathrm{u}-\mathrm{iv})(\xi)=(\mathrm{u}-\mathrm{iv})\frac{\mathrm{dz}}{\mathrm{d}\xi} \qquad (16)$$

is the analytic function of the variable. The first case is present in fig. (1) Let us notice that:

$$\frac{\mathrm{dz}}{\mathrm{d}\xi}=\left|\frac{\mathrm{dz}}{\mathrm{d}\xi}\right|_{\eta}=0$$

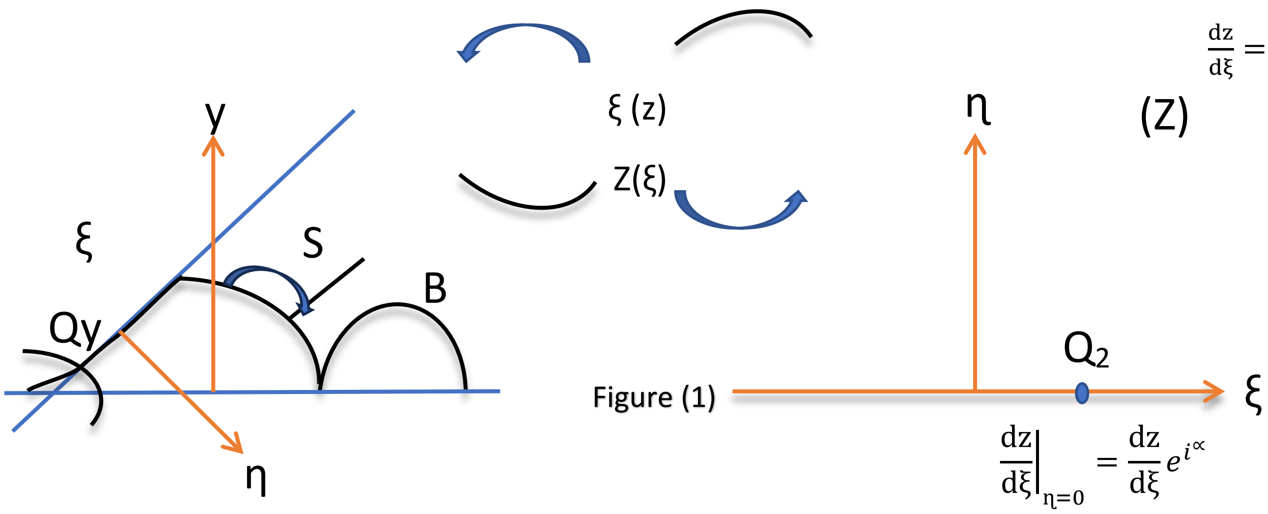

Figure (1)

$$\left.\frac{\mathrm{dz}}{\mathrm{d}\xi}\right|_{\eta=0}=\frac{\mathrm{dz}}{\mathrm{d}\xi}e^{i\propto}$$

where ∝ denotes the angle between the axis ox and the tangent to the boundary of the region in the plane Z. On this line B, which constitutes this boundary, it is:

$$V^{t}-iV^{n}=(u-iv)|_{B^{e^{i\propto}}}=(u-iv)|_{B}\frac{\left.\frac{\mathrm{dz}}{\mathrm{d}\xi}\right|_{\eta=0}}{\frac{\mathrm{dz}}{\mathrm{d}\xi}}$$

Thus, on basis of Eq. (16) we obtain:

$$V^{t}-iv^{n}\frac{\mathrm{dz}}{\mathrm{d}\xi}=(V-iv)_{\eta=o}=(u-iv)|_{B}(\xi) \qquad (17)$$

The analytic function vanishing in the infinity (U-iV→ 0 for ξ → ∞) and having the boundary values $(U-iv)|_{B}$ may be present by Cauchy's integral formula:

$$(U-iv)(\xi)=\frac{1}{2\pi\mathrm{i}}\oint\frac{(U-iv)(\xi)}{(\xi)-(\xi)_{o}}$$

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- in which they are of integration in a straight line ŋ=0 and semicircle with the great radius and the center in the origin of coordinates. on the basis of Plemelij formula [9] for$(\xi) \rightarrow (\xi)_o \in$ (real axis) and the disappearing of U-iv for $(\xi \rightarrow \infty)$. we obtain:

$$(U - iv)(\xi_O) = \frac{1}{\mathrm{i}\pi} v.p = \int_{-\infty}^{\infty} \frac{(U - iv)d\,\xi}{\xi - \xi_o}$$

separating the real part from imaginary one will get Hilbert transforms:

$$\left.\begin{aligned} U\,(\S_O) &= -\frac{1}{\mathrm{i}\pi} v.p = \int_{-\infty}^{\infty} \frac{V(\xi)d\,(\xi)}{\xi - \xi_o} \\ U\,(\xi_O) &= \frac{1}{\mathrm{i}\pi} V.P = \int_{-\infty}^{\infty} \frac{U(\xi)d\,(\xi)}{\xi - \xi_o} \end{aligned}\right\} \qquad (18)$$

connecting the boundary values of the potential field. Using Eq-(17) we determine, with the help of S=S(ξ) transfer motion, the function V(ξ) and then calculate, which allows the determination of V^t

$$V^t = -\frac{1}{\pi}\left(\frac{\mathrm{d}2}{\mathrm{d}(\xi_O)}\right)^{-1} V.P \int_{-\infty}^{\infty} \frac{v^n \frac{\mathrm{ds}}{\mathrm{d}\xi} d\,(\xi)}{\xi - \xi_o} \qquad (19)$$

We have obtained the operator L: $V^n \rightarrow V^t$. The field $\overrightarrow{V_1}$ is defined by means of the Cauchy's integral formulae (part of them) and the conformal mapping:

$$(U - iv) = \frac{1}{2\mathrm{i}\pi} \frac{\mathrm{d}\xi}{\mathrm{dZ}} = \int_{-\infty}^{\infty} \frac{(U - iv)\,(\xi)\,d\xi}{\xi - \xi_o} \qquad (20)$$

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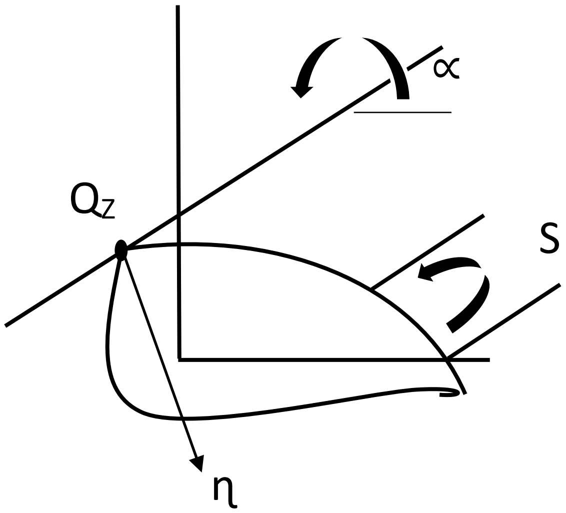
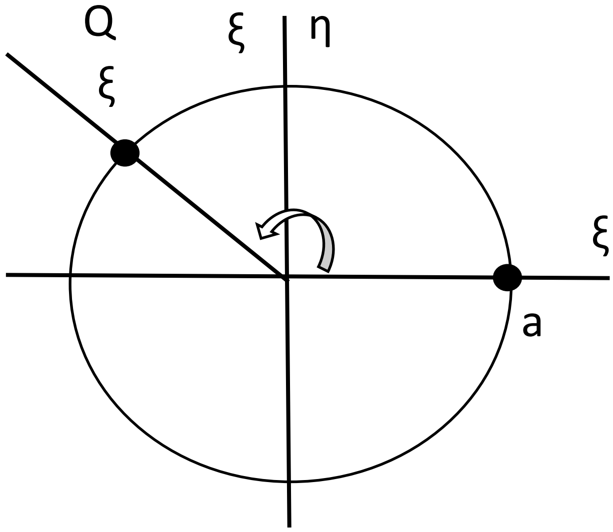


figure (2)



because U follow from Hilbert transformation (18). The second of mentioned case is more difficult fig. (2). The derivative of the conformal mapping can be expressed as:

$$\frac{dZ}{d\xi} = \left(1 - \frac{a}{\xi}\right) = \exp\{f(\xi)\} \qquad (21)$$

As a result, the following relation occurs:

$$\frac{dS}{ad\theta} = 2\sin\frac{a}{2}\exp\{p(\theta)\} \qquad (22)$$

we have assumed the existence of the cusp with common n tangent in the point z (a). If in Eq. (21) the factor $(1\text{-}\frac{a}{\xi}\ )^{n}$ with $1 \geq \beta \geq 0$ will be applied, then the angle between" the tangents in the point of run-off is contained within the range [o, π]. Let us remind of a well-known formula

$$\left.\frac{d2}{d\xi}\right|_{|\xi|=\propto} = \frac{d2}{do}\left.\frac{do}{d\xi}\right|_{|\xi|=\propto} = \frac{ds}{ad\theta}e^{i\propto}\frac{i}{ie^{i\theta}}$$

in which $\propto =$ arc tan $\frac{dy}{dx}$ dy and the defination of the analytic function (u-iv) (ξ) given beforehand we can write $(V^t - iV^n)_B = (u - iv)^{ei^x}{}_B = (v_r - iv_\theta)|_{|\xi|=\propto}\frac{1}{\frac{ds}{ad\theta}}$

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and then

$$\left.\begin{aligned} V_r|_{|\xi|=\propto} &= -V^n\frac{ds}{ad\theta} \\ V_\theta|_{|\xi|=\propto} &= V^t\frac{ds}{ad\theta} \end{aligned}\right\} \quad (23)$$

we will introduce the new analytic function f(ξ)

$$f(\xi) = \frac{\xi}{a}(U - iV) \quad (24)$$

for $|\S| = a$ it determinates the boundary values V_r -iV_O Then in turn, for$|\xi| \rightarrow \infty(u - iv)$ what not necessarily concerns the function F. Assuming that $F\infty = \lim_{|\S|\rightarrow\infty} F(\xi)$,we can write

$$f(\xi) = F_\infty + F_*(\xi)\,, F_\infty = \mu + iv \quad (25)$$

with the function

$$f_*(\xi) = \frac{1}{2\pi i}\oint\frac{f_*(\gamma)d\gamma}{\gamma - \xi}$$

The path of integration is shown in Figure (3). The boundary property for the linear circle, expressed by plemelij formulae, and disappearing of f_* for (γ) ⟶ ∞ lead to the equation:

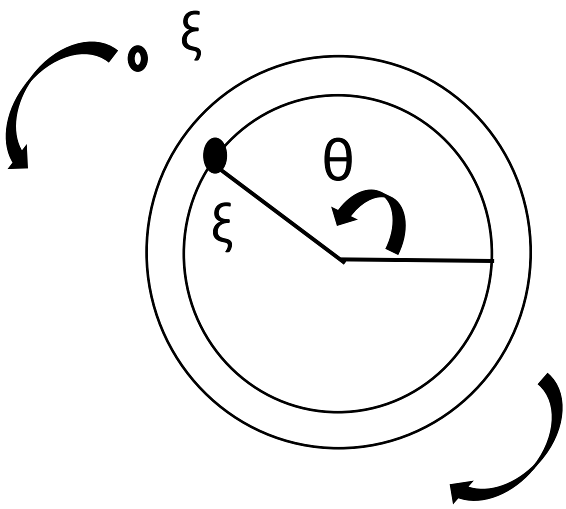

Fig-(3)

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$$F_*(\theta) = \frac{i}{2\pi} V.P \int_0^{2\pi} F_*(\varphi) ctg \frac{\varphi - \theta}{2} d\varphi - \frac{1}{2} \int_0^{2\pi} f_* \varphi d\varphi \qquad (26)$$

Substituting Re$f_*(\theta)$ =v$_r$ $(\theta) - \mu$ $and\ Im\ F_*(\theta) = -V_\theta \theta - V$ In eq (26)

And making use of the integral V.P = $\int_0^{2\pi} ctg \frac{\varphi-\theta}{2} d\varphi$ disappearances, we

$$\left.\begin{aligned} V_r(\theta) &= \frac{1}{2\pi} V.P \int_0^{2\pi} V_\theta(\varphi) Ctg \frac{\varphi - \theta}{2} d\varphi + 2\mu - \int_0^{2\pi} V\mu(\varphi) d\varphi \\ V_\theta(\theta) &= -\frac{1}{2\pi} V.P \int_0^{2\pi} V_r(\varphi) Ctg \frac{\varphi - \theta}{2} d\varphi + 2V - \int_0^{2\pi} V(\varphi) d\varphi \end{aligned}\right\} \qquad (2)$$

Integrating with respect to θ we determine μ and V:

$$\left.\begin{aligned} V &= \frac{1}{2\pi} \int_0^{2\pi} \mu r(\varphi) d\varphi \\ \mu &= -\frac{1}{2\pi} \int_0^{2\pi} V\theta(\varphi) d\varphi \end{aligned}\right\} \qquad (28)$$

Because $\frac{ds}{ad\varphi} = 2 \sin \frac{\varphi}{2} \exp\big(p(\varphi)\big)$

And we get:

$$\int_0^{2\pi} V^n(\varphi) \cos \frac{\varphi}{2} \exp\{p(\varphi)\} d\varphi$$

Equation (27) can be simplified. After reduction of constants 2μ and 2V, we will get:

$$V_r(\theta) = \frac{1}{2\pi i} V.P \int_0^{2\pi} V\theta(\varphi) Ctg \frac{\varphi - \theta}{2} + \frac{1}{2\pi} \int_0^{2\pi} V_r(\varphi) d\varphi$$

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$$V_\theta(\theta) = -\frac{1}{2\pi i} V.P \int_0^{2\pi} Vr(\varphi) ctg\, \frac{\varphi - \theta}{2} d\varphi + \frac{1}{2\pi} \int_0^{2\pi} V_\theta(\varphi) d\varphi$$

It is necessary to use Kutta-Zukowsky hypothesis: V(θ) = 0. This fact will allow to determine the constant (the constant μ follows from the determination of $V_r(\theta)$ coming from V^n and($\frac{ds}{ad\theta}$)

$$0 = -\frac{1}{2\pi i} V.P \int_0^{2\pi} V\gamma(\varphi) ctg\, \frac{\varphi}{2} d\varphi - V$$

we will also add that, since $\frac{ds}{ad\theta}$ Vanishes for θ=0," ade this integrall is not singular and following relation occurs:

$$V.P \int_0^{2\pi} Vr(\varphi) Ctg\, \frac{\varphi}{2} d\varphi = \int_0^{2\pi} V^n \frac{ds}{ad\varphi}\, Ctg\, \frac{\varphi}{2} d\varphi$$

Substitution of the calculated value (the second of Eq. (28) into the second equation (27-a) and the conformal mapping $\xi \rightarrow Z$ allows to define the operator:

$$L: V^t = -\frac{1}{2\pi} \left(\frac{ds}{ad\theta}\right)^{-1} \{V.P \int_0^{2\pi} V^n \frac{ds}{ad\varphi} Ctg\, \frac{\varphi - \theta}{2} d\varphi \; - \int_0^{2\pi} V^n \frac{ds}{ad\varphi}\, Ctg\, \frac{\varphi}{2} d\varphi \qquad (29)$$

forming subtraction and using Eq. (22) will obtain:

$$V^t = \frac{1}{2\pi i}\, e^{-p(\theta)}\, V.P \int_0^{2\pi} V^n\, \frac{e^{p(\theta)}}{\sin\frac{\varphi-\theta}{2}} d\varphi \qquad (30)$$

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The next issue is the determination of the velocity field V_A. we execute it on the basis of the definition:

$$F_* = -\frac{1}{2\pi}\int_{(\gamma)=a}^{2\pi} \frac{F_*(\gamma)d\gamma}{\gamma-\xi} d\varphi \cdot$$

and the trans formation $F_* \to F$ as well as the conformal ma- piping $\xi \to Z$

$$u - iv = \left(\frac{dz}{d\xi}\right)^{-1}\left[-\frac{1}{2\pi}\frac{a}{\xi}F_*(\xi) + \frac{aF_\infty}{\xi}\right] \cdot$$

finally, let us notice that the operation $V_r \to V\theta$ resulting from the separation of the real and imaginary parts of the function $(F_*)(\xi$Vanishing in the infinity) may be also preformed using the conjugate of the function F_* by the Series:

$$F_*(\xi) = \sum_{K=1}^{\infty}(A_K + iB_K)\ (\frac{a}{\xi})^k \cdot$$

and the application of the harmonic analysis

6-Discretization of the primary equation

Equation (15) defining in circulations of the blob structures formed at the present moment is in force on the line of which acicula or areal straight line are the conformal images.

n particular cases it may be a part of mentioned remaining part not the condition of stickiness is then. introduced but so called “Slip “

we will introduce the partition of this line by means of the nodes determined by the length arc S_K, K =0 ,1, 2,… , M For the closed contour $So=S_M$. K-th forming blob is associated with each section $I_K = (S_{K-}, S_K)$

the circulation of the blob is Γ_k. The intersection of Γ_k -th blob is not potential within reach of the range with the same numb- er. Thus, there is:

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$$u = \begin{cases} \frac{\Gamma_k}{2\pi} x(x - x_k\ , y - y_k) & , x\ , y\ \epsilon I_K \\ \frac{\Gamma_k}{2\pi}\ \frac{y - y_k}{r^2} & , x\ , y\ \epsilon I_K \end{cases}$$

$$V = \begin{cases} \frac{\Gamma_k}{2\pi} y(x - x_k\ , y - y_k) & , x\ , y\ \epsilon I_K \\ -\frac{\Gamma_k}{2\pi}\ \frac{x - x_k}{r^2} & , x\ , y\ \epsilon I_K \end{cases} \qquad (31)$$

The components containing brackets vanish outside the K-the range (and are continues in closure of I_k) and the residual terms are potential Utilizing the properties of velocity field described in ps3 we take into account only the pr imary once. Denoting the tangent components (pertaining to not potential interaction) by $\Gamma_k T_K$ (S) (it disappears outsides the range I_k) and similarly, introducing the Symbol Γ_k N_K(s) for the normal component, we obtain 'K

$$V_B^t = V_p^t + \sum_{K=1}^{M} \left(\Gamma_k \,\mathrm{TK} + V_{20}^t + L(V_\beta^n\right) - \sum_{KS}^{M} (\Gamma_k \,\mathrm{L}(N_K) - L\ (V_{20}^n) \qquad (32)$$

The components V_{20}^t and V_{20}^t follow from the existence of the old generation blobs which are at the present moment closer t the line of new blobs formation than i their range of no potential interactions. In turn, I_k disappear outside I obviously, $L(N_K)$ does not possess such property and in general, it is different from ze on the whole considered line in Eq. (32) contains the unknowns ($^{\Gamma}$K=1, 2, ---, M)

the length of the are in an independent component then the calculation of constants $^{\Gamma}$K may be performed after averaging beforehand. (Another method which may be use is the method of collection which consist in satisfying the equation in M chosen points) Two types of averaging are possible: the quadratic or direct, both not presenting the difficulties in calculations. Choosing then will carry out the successive integrations within the ranges of I_k for i=1, 2,…, M. As result we will have:

$$\Gamma_k \int_{S_{K-1}}^{S_K} T_K\, ds - \sum_{P=1}^{\infty} \Gamma_P \int_{S_{K-1}}^{S_K} L\ (N_P) ds = G_k\ , K = 1,2, \dots , M \qquad (33)$$

$$\backslash \Gamma K \int_{S_{K-1}}^{S_K} T_K ds - \sum_{p=1}^{\infty} \Gamma p \int_{S_{k-1}}^{S_k} L\ (N_P) ds = G_K\ , K = 1\ ,2, \dots , M$$

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The term G_K constitutes the Known vector following for the integration of the components and the matrix K with elements

$$K_{ij} = \delta_{ij} \int_{S_{i-1}}^{S_i} T_i(s)ds - \int_{S_{j-1}}^{S_j} L\left(N_j\right)ds \qquad \text{-------------- } 34$$

is determined by the function X and Y eq. (6) as well as by the geometry of region. Let us notice that during the follow around the rigid bodies it is invariable in time. The movement of the old generation blobs alters free terms trough the charge of $\vec{V}_{20}$. In general - $\vec{V}_B$ and $\vec{V}_p$

7. Averaging of the operator L

Integration of the singular integral operator is a kind of regularization and can be carried out in general way Forth case coming within Eq. (19) by the terms of definition the mean value of the integral we write:

$$\int_{S_{k-1}}^{S_k} L\left(N_\varphi\right)ds = -\frac{1}{\pi}\int_{S_{K-1}}^{S_K}\left(\frac{ds}{d(\S_o)}\right)^{-1}\{V.P\int_{-\infty}^{\infty}\frac{N(P)(\frac{ds}{d(\S)})d\xi}{\xi-\xi_o}\, ds$$

$$= -\frac{1}{\pi}\int_{\xi_{k-1}}^{\S_k}\lim_{\in\to 0}\{\int_{-\infty}^{\S-\in}\ldots.+\int_{\S_0+\infty}^{\infty}\ldots.\}d\xi_o$$

$\S_{k-1}$ and ξ_k denoted images (conformal) of the points s_{k-1}sand K S_k we will use the equation:

$$\int_{\xi_0+\in}^{\infty}\varphi(\eta)\frac{d\S}{\xi-\xi} = -\frac{d}{d\xi_0} = \int_{\S_{0+\in}}^{\infty}\varphi(\S)in(\S-\S_o)d\S - \varphi(\S_o{+}\in)lne \qquad \text{-------------- } 35$$

and it's equivalent for the second of inner integrals. ds Then assuming the regularity of the function $N_p\frac{ds}{d\xi}$we eliminate the components including Intel. Since Np disappears outside Ip" and the singularity of the logarithmic Karnel is weak, we get:

$$\int_{\xi_{k-1}}^{sk} L(Np)ds\frac{1}{\pi}\int_{-\infty}^{\infty}Np(s)in\left[\frac{\xi(s)-\xi_k}{\xi(s)-\xi_{k-1}}\right]ds \qquad \text{-------------- } 36$$

Similarly, using expression (29) rewritten in the form of

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$$V^t = -\frac{1}{2\pi}\left(\frac{ds}{ad\theta}\right)^{-1} V.P\int_0^{2\pi} V^n \frac{ds}{ad\varphi}\, ctg\frac{\varphi-\theta}{2}\,d\varphi + \frac{1}{\pi}\left(\frac{ds}{ad\theta}\right)^{-1}\int_0^{2\pi} V^n \cos\frac{\varphi}{2} e^{p(\varphi)} d\varphi$$

and denoting the second integral (non-singular) by the symbol γ and $\theta_k = \theta(a_k)$ will have

$$\int_{s_{k-1}}^{s_k} \mathrm{L(Np)}ds = -\frac{1}{2\pi}\int_{\theta_{k-1}}^{\theta_k}\{V.P\int_0^{2\pi} Np\frac{ds}{d\varphi}ctg\frac{\varphi-\theta}{2}d\varphi\}d \in +\left(\frac{\theta_k-\theta_{k-1}}{\pi}\right)a_\gamma$$

Making use of the properties of linear integral we will calculate:

$$\int_{s_{k-1}}^{s_k} \mathrm{L(Np)}ds = -\frac{1}{\pi}\int_0^{2\pi} Np(s)ln\left|\frac{\{\sin(\varphi(s)-\theta_k)\}/2}{\{\sin(\varphi(s)-\theta_{k-1})\}/2}\right|ds + \left(\frac{\theta_k-\theta_{k-1}}{\pi}\right)a_\gamma \qquad (37)$$

8. Final remarks:

The essence of this method consists in following the movement of the blob's structures formed at the previous moment as well as in determining the intensity of circulation blobs originating the present moment of rigid parts of the boundary of motion region. The weakness of this method is the lack of the direct dependence of intensity of formed blobs on the time between samplings. one should then suspect that this process-connected with the runoff-depends evidently on time and Reynolds's number. Introducing this fact into the theory would be a fully satisfactory solution. Another fault (of lesser importance) is not taking into account the evolution of vortices in side blobs-More strictly speaking, considering the finite number of blobs, or should introduce their boundary broadening caused by a Huston (the compact set is into a support of vorticity some terms concerning the introduction of age (analogically to "Fermi age") were carried out by Janien Ko [10]; however he neglected the convection effects. It seems that there exist other possibilities of taking into consideration the evolution of these structures [1]. Finally, there is left an option in determining the functions X and Y (6) defining the local induction what is more: the newly formed blobs not necessarily induce the field described by the mentioned expression [1]. The explanation of these apparent inconsistencies is the following: Eq (6) prevent The catastrophe connected with the unlimited velocity of induction in the meth- od of points singularities and are not of essential importance for sizes of blobs insignificantly small in the relation to the size of the region of motion. In turn, the properties of blobs newly

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originated are considerably different from the one newly originated are consider- ably different from the one formed previously; they model the vortex sheet tied with the flowed-around rigid wall. It is, theoretically, not essential with what shape functions the strength of circulation. applies in the integral sense. It is enough to post- elate the roundedness and continuity following from the acceptance of the functions T_k and N_k

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