Simulation and Analysis of Adaptive Beamforming Algorithms for Phased Array Antennas

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Abstract

Adaptive phased antennas, known as SMART ANTENNAS attract so much attention with the increase of wireless communications implementation. The smart antennas can change their shape of transmission placing nulls in the direction of interference, and steer their main lobe to the direction of interest. This process leads to maximizing Signal to Interference Ratio (SIR) maximizing the throughput of the network. They can mitigate channel fading by searching for the best alternative path. This paper investigates the principles and the algorithms used to steer the main lobe and shape the radiation pattern to optimize the performance. Only analogue techniques are considered.

Keywords: Smart Antennas, Phased Array Antenna, Warless Communication, Analog Beamforming.

الخلاصة

مع ازدياد استخدام الشبكات اللاسلكية، اجتذبت الهوائيات المتكيفة والمعروفة أيضاً بـ (الهوائيات الذكية) اهتمام الباحثين. الهوائيات الذكية قادرة على تغيير نمط بثها ووضع نقاط استلام صفرية باتجاه الإشارات غير المرغوب بها وتحريك كتلة الاستلام الأساسية باتجاه المصدر المرغوب به. هذا يؤدي إلى زيادة نسبة قدرة الإشارة إلى التداخل مما يؤدي إلى زيادة في خرج الشبكة. تستطيع هذه المنظومات أيضا أن تخفف من تأثير التوهين في القناة عن طريق البحث الأوتوماتيكي لمسار آخر للإشارة. هذا البحث يتناول اللوغارتميات المستخدمة لتكييف شكل الإرسال للهوائي.

1. Introduction

The exponential growth of wireless communications systems and the limited bandwidth available for those systems has created problems which all wireless providers are working to solve. One potential solution to the bandwidth limitation is the use of smart antenna systems [Okamoto, 2002]. The demand for increased capacity in wireless networks motivated recent research toward wireless systems that exploit space selectivity. As a result, there are many efforts devoted to the design of "smart antenna arrays".[Garg, and Huntington, 1997, Bellofiore *et. al.*, 2002].

The term *smart* implies the use of signal processing in order to shape the beam pattern according to certain conditions. For an array to be smart implies sophistication beyond merely steering the beam to a direction of interest. Smart essentially means computer control of the antenna performance. Smart antennas hold the promise for improved radar systems, improved system capacities with mobile wireless, and improved wireless communications through the implementation of space division multiple access (SDMA) [Godara 2004, Gross 2005]. The adaptation algorithms can be, generally, categorized into three methods: 1. Estimating the Angle Of Arrival (AOA) then steering, 2. Non-blind adaptation, and 3. Blind adaptation.

2. Beamsteered linear array

For any phased array antenna, the radiation pattern is the multiplication of two main parts: the element radiation pattern and Array Factor (AF). For *N* elements array, AF is given by[Godara 2004, Gross 2005, and Mailloux 2005]:

$$AF=\sum_{n=1}^{N} e^{j(n-1)(kd\sin\theta+\delta)} = \sum_{n=1}^{N} e^{j(n-1)\psi} \tag{1}$$

A beamsteered linear array is an array where the phase shift (δ) is variable thus allowing the main lobe to be directed toward any Direction Of Arrival (DOA) [Gross 2005]. The phase shift can be written as $\delta = -kd\sin\theta_0$ (where θ_0 is the DOA). The array factor can be written in terms of beamsteering such that [Gross 2005, Visser 2005, and Sun *et*. *al*. 2009]

$$\mathrm{AF}_n = \frac{1}{N}\frac{\sin\left(\frac{Nkd}{2}\left(\sin\theta - \sin\theta_0\right)\right)}{\sin\left(\frac{kd}{2}\left(\sin\theta - \sin\theta_0\right)\right)} \tag{2}$$

Figure (1) shows polar plots for the beamsteered 8-element array for the $d/\lambda = 0.5$ (where d is the inter-elements spacing and λ is the wavelength), and θ_0 = 20, 40, and 60°. Major lobes exist above and below the horizontal because of array is symmetry.

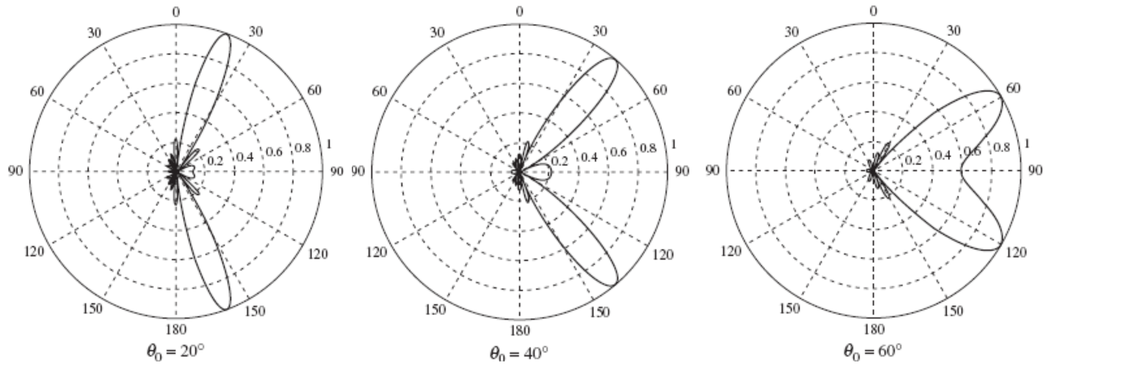


Figure (1) Beamsteered linear array

3. Estimating the Angle Of Arrival (AOA) Then Steering (EAOATS)

The smart antenna needs to estimate, at first, the angle of arrival so as to steer the main beam towards it. *Angle-of-arrival* (AOA) estimation has also been known as *spectral* estimation, *direction of arrival* (DOA) estimation, or *bearing* estimation.

3.1 Array Correlation Matrix

Many of the AOA algorithms rely on the array correlation matrix. In order to understand the array correlation matrix, let us begin with a description of the array, the received signal, and the additive noise. Figure (2) depicts a receive array with incident plane waves from various directions. It also shows D signals arriving from D directions. They are received by an array of M elements with M potential weights w_m.

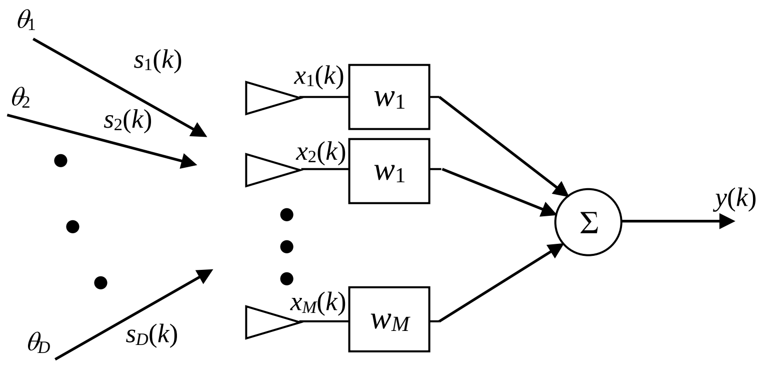


Figure (2) *M*-element array with arriving signals.

Each received signal $x_M(k)$ includes additive white Gaussian zero with mean noise. Time is represented by the kth time sample. Thus, the array output y can be given in the following form:

$$y(k) = \bar{w}^T \cdot \bar{x}(k) \tag{3}$$

where

$$\bar{x} = \bar{A} \cdot \bar{s}(k) + \bar{n}(k) \tag{4}$$

and

$\overline{w} = \begin{bmatrix} w_1 & w_2 & \cdots & w_M \end{bmatrix}^T$ = array weights

$\overline{s}(k)$ = vector of incident complex monochromatic signals at time k

$\overline{n}(k)$ = noise vector at each array element m, zero mean, variance σ_n^2

$\overline{a}(\theta_i)$ = M-element array steering vector for the θ_i direction of arrival

$\overline{A} = \begin{bmatrix} \overline{a}(\theta_1) & \overline{a}(\theta_2) & \cdots & \overline{a}(\theta_D) \end{bmatrix}$ is an $M \times D$ matrix of steering vectors.

The D-complex signals arrive at angles θ_i and are intercepted by the M antenna elements. It is initially assumed that the arriving signals are monochromatic and the number of arriving signals $D < M$. It is understood that the arriving signals are time varying and thus our calculations are based upon time snapshots of the incoming signal. Obviously if the transmitters are moving, the matrix of steering vectors is changing with time and the corresponding arrival angles are changing, unless otherwise stated, the time dependence will be suppressed in Eqs. (3) and (4). In order to simplify the notation let us define the $M \times M$ array correlation matrix $\overline{R}_{xx}$ as

$$\overline{R}_{xx} = E\left[\overline{x} \cdot \overline{x}^H\right] = \overline{A}\,\overline{R}_{ss}\overline{A}^H + \overline{R}_{nn} \tag{5}$$

where $E[\cdot]$: the expected value

$\overline{R}_{ss} = D \times D$ source correlation matrix

$\overline{R}_{nn} = \sigma_n^2 \bar{I} = M \times M$ noise correlation matrix

$\bar{I} = N \times N$ identity matrix

^{H}: superscript is the Hermitian operator (transpose complex conjugate)

The exact statistics for the noise and signals are unknown, but we can assume that the process is ergodic. Hence, the correlation can be approximated by the use of a time-averaged correlation. In that case the correlation matrices are defined by

$$\hat{R}_{xx} \approx \frac{1}{K}\sum_{k=1}^{K}\overline{x}(k)\overline{x}^H(k),\ \hat{R}_{ss} \approx \frac{1}{K}\sum_{k=1}^{K}\overline{s}(k)\overline{s}^H(k),\ \hat{R}_{nn} \approx \frac{1}{K}\sum_{k=1}^{K}\overline{n}(k)\overline{n}^H(k) \tag{6}$$

where K is the number of snapshots.

The goal of AOA estimation techniques is to define a function that gives an indication of the angles of arrival based upon maxima vs. angle. This function is traditionally called the pseudospectrum $P(\theta)$ and the units can be in energy or in watts (or at times energy or watts squared).

3.2 AOA Estimation Methods

The core operation of any smart antenna relies on the estimation of AOA, This principles lead to formulate many algorithms to find the AOA. The following are the most used for AOA estimation. All algorithms are simulated with MATLAB. The proposed **scenario 1** is M=8, uncorrelated equal amplitude sources, (s_1, s_2), $d = \lambda/2$, and σ_n^2 = 0.1, and the two different pairs of arrival angles given by ± 10° and ±5°, assuming ergodicity.

3.2.1 Bartlett AOA estimate

If the array is uniformly weighted, we can define the Bartlett AOA estimate as [Gross 2005, and El Zooghby 2005]

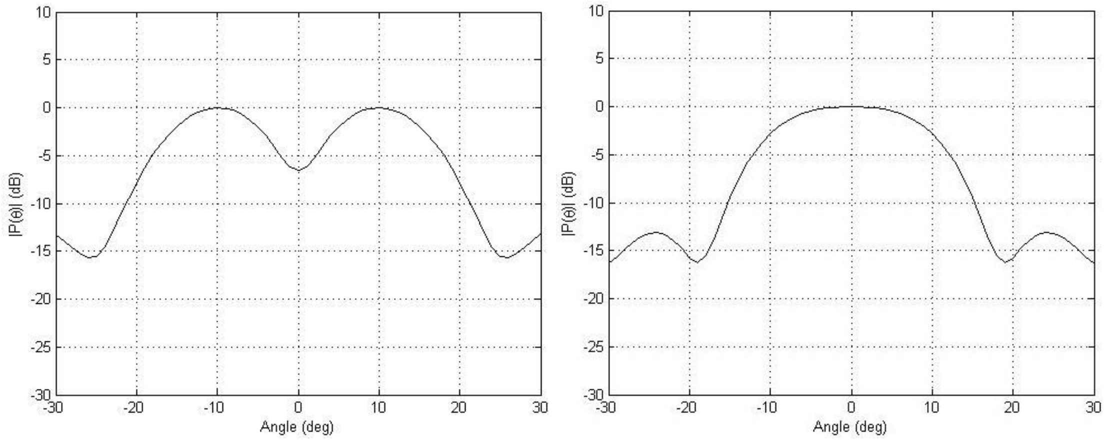
$$P_B(\theta) = \overline{a}^H(\theta)\overline{R}_{xx}\overline{a}(\theta) \tag{7}$$

The Bartlett AOA estimate is the spatial version of an averaged periodogram and is a beamforming AOA estimate. Under the conditions where $\overline{s}$ represents uncorrelated monochromatic signals and there is no system noise, Eq. (7) is equivalent to the following long-hand expression [Blaunstein and Christodoulou 2007, Gross 2005]:

$$P_B(\theta) = \left| \sum_{i=1}^{D} \sum_{m=1}^{M} e^{j(m-1)kd(\sin\theta - \sin\theta_i)} \right|^2 \quad (8)$$

The periodogram is thus equivalent to the spatial finite Fourier transform of all arriving signals. This is also equivalent to adding all beamsteered array factors for each angle of arrival and finding the absolute value squared.

Figure (3) shows the simulation results for Bartlett AOA estimate for the proposed scenario.



a *b*

Figure (3) *a*. ± 10° spacing angle, *b*. ±5° spacing angle

From Figure (3) it can be seen that the Bartlett algorithm fails to resolve the ±5° spacing angle. Thus despite its simplicity it requires more array elements to achieve the required as its resolution is approximately 1/*M*. This is the resolution limit of Bartlett method.

3.2.2 Capon AOA estimate

The Capon AOA estimate [Gross 2005, El Zooghby 2005] is known as a *minimum variance distortionless response* (MVDR). Its goal is to maximize the signal-to-interference ratio (SIR) while passing the signal of interest undistorted in phase and amplitude. The source correlation matrix $\overline{R}_{ss}$ is assumed to be diagonal. Maximized SIR is accomplished with a set of array weights $\overline{w} = [w_1 \quad w_2 \quad \cdots \quad w_M]$ as shown in Figure (2), where the array weights are given by

$$\overline{w} = \frac{\overline{R}_{xx}^{-1}\overline{a}(\theta)}{\overline{a}^H(\theta)\overline{R}_{xx}^{-1}\overline{a}(\theta)} \quad (9)$$

The periodogram is thus

$$P_C(\theta) = \frac{1}{\overline{a}^H(\theta)\overline{R}_{xx}^{-1}\overline{a}(\theta)} \quad (10)$$

Apply the scenario with angle spacing ±5°, the result is shown in Figure (4).

Capon AOA estimate has better resolution than the Bartlett AOA estimate. When sources are highly correlated, the Capon resolution worsens. The derivation of the Capon weights was conditioned upon considering that all other sources are interferers.

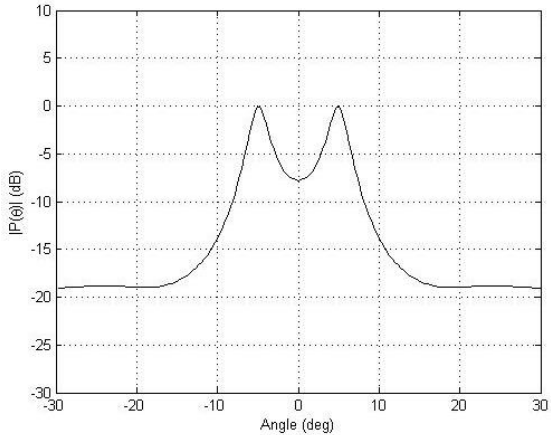


Figure (4) Capon pseudospectrum for $\theta_1 = -5°$, $\theta_2 = 5°$.

3.2.3 Linear Prediction AOA Estimate

The goal of the linear prediction method is to minimize the prediction error between the output of the *m*th sensor and the actual output. In a similar vein as Eq. (9), the solution for the array weights is given as [Blaunstein and Christodoulou 2007, and Gross 2005]

$$\bar{w}_m = \frac{\bar{R}_{xx}^{-1}\bar{u}_m}{\bar{u}_m^T\bar{R}_{xx}^{-1}\bar{u}_m} \tag{11}$$

$\bar{u}_m$ is the Cartesian basis vector which for the *m*th column of the $M \times M$ identity matrix.

The pseudo-spectrum can be shown that

$$P_{LP_m} = \frac{\bar{u}_m^T\bar{R}_{xx}^{-1}\bar{u}_m}{\left|\bar{u}_m^T\bar{R}_{xx}^{-1}\bar{a}(\theta)\right|^2} \tag{12}$$

The choice for which *m*th element output for prediction is random. The choice made can dramatically affect the final resolution. If the array center element is chosen, the linear combination of the remaining sensor elements might provide a better estimate because the other array elements are spaced about the phase center of the array. This would suggest that odd array lengths might provide better results than even arrays because the center element is precisely at the array phase center [Kaiser *et. al*. 2005, Gross 2005, El Zooghby 2005]. The AOA estimation for the proposed scenario is shown in Figure (5).

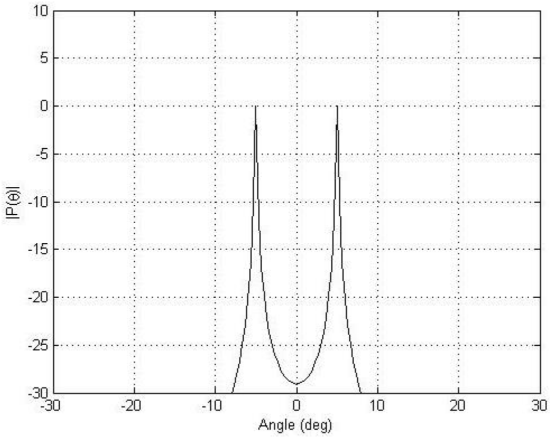


Figure (5) Linear predictive pseudospectrum for $\theta_1 = -5°$, $\theta_2 = 5°$.

3.2.4 Pisarenko Harmonic Decomposition AOA Estimate

The goal of this algorithm is to minimize the mean-squared error of the array output under the constraint that the norm of the weight vector be equal to unity. The eigenvector that minimizes the mean-squared error corresponds to the smallest eigenvalue. For an M = 6 element array, with two arriving signals, there will be two eigenvectors associated with the signal and four eigenvectors associated with the noise. The corresponding PHD pseudospectrum is given by [Kaiser *et. al*. 2005, Gross 2005, El Zooghby 2005]

$$P_{PHD}(\theta) = \frac{1}{\left|\bar{a}^{T}(\theta)\bar{e}_{1}\right|^{2}} \tag{13}$$

where $\bar{e}_1$ is the eigenvector associated with the smallest eigenvalue λ_1.

The performance of PHD algorithm is shown in Figure (6). The Pisarenko peaks are not an indication of the signal amplitudes. These peaks are the roots of the polynomial in the denominator of Eq. (13). It is clear that for this example, the Pisarenko solution has the best resolution.

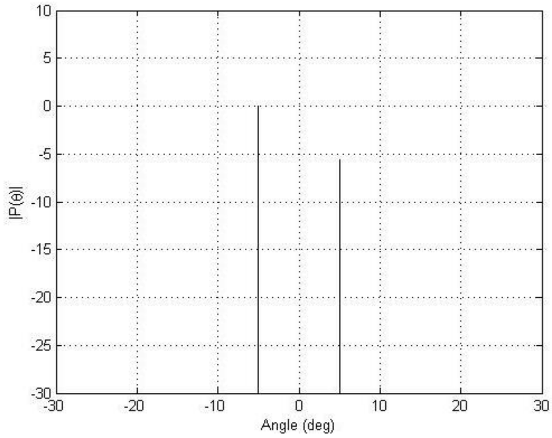


Figure (6) PHD pseudospectrum for $\theta_1 = -5°$, $\theta_2 = 5°$.

3.2.5 MUSIC AOA Estimate

MUSIC is an acronym which stands for the term which is (MUltiple SIgnal Classification) [Shahbazpanahi *et. al*. 2001, Gross 2005, and Dandekar, *et. al*. 2002]. MUSIC promises to provide unbiased estimates of the number of signals, the angles of arrival, and the strengths of the waveforms. MUSIC makes the assumption that the noise in each channel is uncorrelated making the noise correlation matrix diagonal. The incident signals may be somewhat correlated creating a nondiagonal signal correlation matrix. However, under high signal correlation the traditional MUSIC algorithm breaks down and other methods must be implemented to correct this weakness. If the number of signals is D, the number of signal eigenvalues and eigenvectors is D too, and the number of noise eigenvalues and eigenvectors is $M–D$ (M is the number of array elements). The array correlation matrix assuming uncorrelated noise with equal variances is.

$$\bar{R}_{xx} = \bar{A}\bar{R}_{ss}\bar{A}^{H} + \sigma_n^2 \bar{I} \tag{14}$$

We next find the eigenvalues and eigenvectors for $\bar{R}_{xx}$. We then produce D eigenvectors associated with the signals and $M–D$ eigenvectors associated with the noise. We choose the eigenvectors associated with the smallest eigenvalues. For uncorrelated signals, the smallest eigenvalues are equal to the variance of the noise. We can then construct the $M×(M–D)$ dimensional subspace spanned by the noise eigenvectors such that

$$\bar{E}_N = \begin{bmatrix} \bar{e}_1 & \bar{e}_2 & \cdots & \bar{e}_{M-D} \end{bmatrix} \tag{15}$$

The noise subspace eigenvectors are orthogonal to the array steering vectors at the angles of arrival θ_1, θ_2, … , θ_D. Because of this orthogonality condition, the Euclidean distance $d^2 = \bar{a}(\theta)^H \bar{E}_N \bar{E}_N^H a(\theta) = 0$ for each and every arrival angle θ_1, θ_2, … , θ_D. Placing this distance expression in the denominator creates sharp peaks at the angles of arrival. The MUSIC pseudospectrum is now given as:

$$P_{MU}(\theta) = \frac{1}{\left|\bar{a}(\theta)^H \bar{E}_N \bar{E}_N^H \bar{a}(\theta)\right|^2} \quad (16)$$

The performance of MUSIC for the proposed scenario is given in Figure (7)

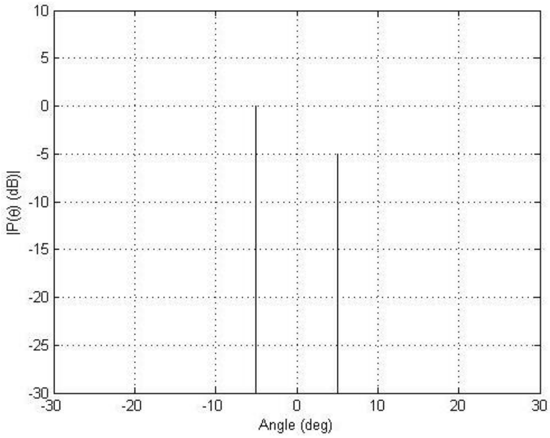


Figure (7) MUSIC pseudospectrum for $\theta_1 = -5°$, $\theta_2 = 5°$.

3.2.6 ESPRIT AOA Estimate

ESPRIT stands for *Estimation of Signal Parameters via Rotational Invariance Techniques* [Jeon, *et. al.* 2005, Gross 2005, Dandekar, *et. al.* 2002]. The goal of the ESPRIT technique is to exploit the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure. ESPRIT inherently assumes narrowband. As with MUSIC, ESPRIT assumes that there are $D < M$ narrow-band sources centered at the center frequency f_0. These signal sources are assumed to be of a sufficient range so that the incident propagating field is approximately planar. The sources can be either random or deterministic and the noise is assumed to be random with zero-mean. ESPRIT assumes multiple identical arrays called *doublets*. These can be separated arrays or can be composed of subarrays of one larger array. It is important that these arrays are displaced translationally but not rotationally. An example is shown in Figure (8) where a four element linear array is composed of two identical three-element subarrays or two doublets. These two subarrays are translationally displaced by the distance *d*. Let us label these arrays as array 1 and array 2.

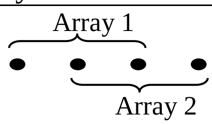


Figure (8) Doublet composed of two identical displaced arrays.

The signals induced on each of the arrays are given by

$$\bar{x}_1 = \bar{A} \cdot \bar{s}(k) + n_1(k) \quad (17)$$

and

$$\bar{x}_2 = \bar{A} \cdot \bar{s}(k) + n_2(k) = \bar{A} \cdot \bar{\Phi} + n_2(k) \quad (18)$$

where $\bar{\Phi} = \text{diag}\left\{e^{jkd\sin\theta_1} \quad e^{jkd\sin\theta_2} \quad \cdots \quad e^{jkd\sin\theta_D}\right\}$=$D\times D$ diagonal unitary matrix with phase shifts between the doublets for each AOA.

$\overline{A}_i$ = Vandermonde matrix of steering vectors for subarrays i = 1, 2

The total received signal considering the contributions of both subarrays is given as

$$\overline{x}(k) = \begin{bmatrix} \overline{x}_1(k) \\ \overline{x}_2(k) \end{bmatrix} = \begin{bmatrix} \overline{A}_1 \\ \overline{A}_1 \cdot \overline{\Phi} \end{bmatrix} \cdot \overline{s}(k) + \begin{bmatrix} n_1(k) \\ n_2(k) \end{bmatrix} \tag{19}$$

The correlation matrix for the complete array is given by

$$\overline{R}_{xx} = E\left[\overline{x} \cdot \overline{x}^H\right] = \overline{A}\overline{R}_{ss}\overline{A}^H + \sigma_n^2 \overline{I} \tag{20}$$

where the correlation matrices for the two subarrays are given by

$$\overline{R}_{11} = E\left[\overline{x}_1 \cdot \overline{x}_1^{\ H}\right] = \overline{A}\overline{R}_{ss}\overline{A}^H + \sigma_n^2 \overline{I} \tag{21}$$

and

$$\overline{R}_{22} = E\left[\overline{x}_2 \cdot \overline{x}_2^{\ H}\right] = \overline{A}\overline{\Phi}\overline{R}_{ss}\overline{\Phi}^H\overline{A}^H + \sigma_n^2 \overline{I} \tag{22}$$

Each of the full rank correlation matrices given in Eq. (21) and (22) has a set of eigenvectors corresponding to the D signals present. Creating the signal subspace for the two subarrays results in the two matrices $\overline{E}_1$ and $\overline{E}_2$. Creating the signal subspace for the entire array results in one signal subspace given by $\overline{E}_x$. Both $\overline{E}_1$ and $\overline{E}_2$ are $M \times D$ matrices whose columns are composed of the D eigenvectors corresponding to the largest eigenvalues of $\overline{R}_{11}$ and $\overline{R}_{22}$. Since the arrays are translationally related, the subspaces of eigenvectors are related by a unique non-singular transformation matrix Ψ such that

$$\overline{E}_2 = \Psi \overline{E}_1 \tag{23}$$

There must also exist a unique non-singular transformation matrix $\overline{T}$ such that

$$\overline{E}_1 = \overline{A}\overline{T} \tag{24}$$

and

$$\overline{E}_2 = \overline{A}\overline{\Phi}\overline{T} \tag{25}$$

By substituting Eqs. (23) and (24) into Eq. (25) and assuming that $\overline{A}$ is of full-rank, we can derive the relationship

$$\overline{T}\overline{\Psi}\overline{T}^{-1} = \overline{\Phi} \tag{26}$$

Thus, the eigenvalues of $\overline{\Psi}$ must be equal to the diagonal elements of $\overline{\Phi}$ such that $\lambda_1 = e^{jkd\sin\theta_1}$, $\lambda_2 = e^{jkd\sin\theta_2}$, …, $\lambda_D = e^{jkd\sin\theta_D}$, and the columns of $\overline{T}$ must be the eigenvectors of $\overline{\Psi}$. $\overline{\Psi}$ is a rotation operator that maps the signal subspace $\overline{E}_1$ into the signal subspace $\overline{E}_2$. If we are restricted to a finite number of measurements and we also assume that the subspaces $\overline{E}_1$ and $\overline{E}_2$ are equally noisy, we can estimate the rotation operator $\overline{\Psi}$ using the *total least-squares* (TLS) criterion. This procedure is outlined as follows.

- Estimate the array correlation matrices $\overline{R}_{11}$, $\overline{R}_{22}$ from the data samples.
- Knowing the array correlation matrices for both subarrays, the total number of sources equals to the number of large eigenvalues in either $\overline{R}_{11}$ or $\overline{R}_{22}$.
- Calculate the signal subspaces $\overline{E}_1$ and $\overline{E}_2$ based upon the signal eigenvectors of $\overline{R}_{11}$ and $\overline{R}_{22}$. $\overline{E}_1$ can be constructed by selecting the first M/2 + 1 rows ((M + 1)/2 + 1 for odd arrays) of $\overline{E}_x$. $\overline{E}_2$ can be constructed by selecting the last M/2+1 rows ((M+ 1)/2 + 1 for odd arrays) of $\overline{E}_x$.
- Next form a 2D × 2D matrix using the signal subspaces such that

$$\overline{C} = \begin{bmatrix} \overline{E}_1^H \\ \overline{E}_2^H \end{bmatrix} \begin{bmatrix} \overline{E}_1 & \overline{E}_2 \end{bmatrix} = \overline{E}_C \overline{\Lambda} \overline{E}_C^H \quad (27)$$

where the matrix $\overline{E}_C$ is from the *eigenvalue decomposition* (EVD) of $\overline{C}$ such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{2D}$ and $\overline{\Lambda}$ = diag $\{\lambda_1, \lambda_2, \ldots, \lambda_{2D}\}$

- Partition $\overline{E}_C$ into four $D \times D$ submatrices such that

$$\overline{C} = \begin{bmatrix} \overline{E}_{11} & \overline{E}_{12} \\ \overline{E}_{21} & \overline{E}_{22} \end{bmatrix} \quad (28)$$

- Estimate the rotation operator $\overline{\Psi}$ by

$$\overline{\Psi} = -\overline{E}_{12}\overline{E}_{22}^{-1} \quad (29)$$

- Calculate the eigenvalues of $\overline{\Psi}$, $\lambda_1, \lambda_2, \ldots, \lambda_D$
- Now estimate the angles of arrival, given that $\lambda_i = |\lambda_i| e^{j \arg(\lambda_i)}$

$$\theta_i = \sin^{-1}\left(\frac{|\lambda_i|}{kd}\right) \qquad i=1, 2, \ldots, D \quad (30)$$

If so desired, one can estimate the matrix of steering vectors from the signal subspace $\overline{E}_s$ and the eigenvectors of $\overline{\Psi}$ given by $\overline{E}_\Psi$ such that $\hat{A} = \overline{E}_s \overline{E}_\Psi$.

4. Non-Blind Adaptive Beamforming Algorithms

These algorithms depend on a stores reference signal at the receiver. This signal is predefined before the transmission. The task of the algorithm is to minimize the error between the received signal and the reference signal. The proposed scenario for tracking algorithms. **Scenario 2** is M= 8, d= 0.5 λ, AOA θ_0=0˚, interference θ_0=-60˚, the traced function $s(k) = \cos\left(2\pi \, t(k)/T\right)$, T=1 msec, $t = (1 \rightarrow 100)T/100$

4.1 Least Mean Squares

The least mean squares algorithm is a gradient based approach [Gross 2005]. It is established quadratic performance surface. When the performance surface is a quadratic function of the array weights, the performance surface $J(\overline{w})$ is in the shape of an elliptic paraboloid having one minimum. We can establish the performance surface (cost function) by again finding the *Mean Square Error* (MSE). The error, as shown in Figure (9), is

$$\varepsilon(k) = d(k) - \overline{w}^H(k)\overline{x}(k) \quad (31)$$

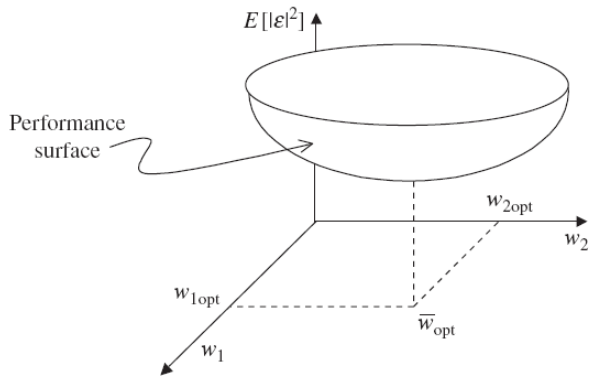


Figure (9) Quadratic surface for MSE.

The squared error is given as

$$\left|\varepsilon(k)\right|^2 = \left|d(k) - \overline{w}^H(k)\overline{x}(k)\right|^2 \quad (32)$$

Momentarily, we will suppress the time dependence. The cost function is given as

$$J(\overline{w}) = D - 2\overline{w}^H\overline{r} + \overline{w}^H R_{xx}\overline{w} \quad (33)$$

Where: $D= E[|d|^2]$

To find the optimum weight vector $\overline{w}$ we can differentiate Eqn. (33) with respect to *w* and equating it to zero. This yields:

$$\overline{w}_{opt} = \overline{R}_{xx}^{-1}\overline{r} \quad (34)$$

Because we don't know signal statistics we must resort to estimating the array correlation matrix ($\overline{R}_{xx}$) and the signal correlation vector ($\overline{r}$) over a range of snapshots or for each instant in time. The instantaneous estimates are given as

$$\hat{R}_{xx}(k) \approx \overline{x}(k)\overline{x}^H(k) \quad (35)$$

and

$$\hat{r}(k) \approx d^*(k)\overline{x}(k) \quad (36)$$

We can employ an iterative technique called the method of *steepest descent* to approximate the gradient of the cost function. The method of steepest descent can be approximated in terms of the weights using the LMS method advocated by Widrow [Gross 2005]. The steepest descent iterative approximation is given as

$$\overline{w}(k+1) = \overline{w}(k) - \frac{1}{2}\mu\nabla_{\overline{w}}\left(J(\overline{w})\right) \quad (37)$$

where, μ is the step-size parameter and $\nabla_{\overline{w}}$ is the gradient of the performance surface.

Substituting the instantaneous correlation approximations, we have the *Least Mean Square* (LMS) solution.

$$\overline{w}(k+1) = \overline{w}(k) + \mu e^*(k)\overline{x}(k) \quad (38)$$

where $e(k) = d(k) - \overline{w}^H(k)\overline{x}(k) =$ error signal

The convergence of the LMS algorithm is directly related to the *step-size parameter* μ. If the step-size is too small, the convergence is slow and we will have the *overdamped* case. If the convergence is slower than the changing angles of arrival, it is possible that the adaptive array cannot acquire the signal of interest fast enough to track the changing signal. If the step-size is too large, the LMS algorithm will overshoot the optimum weights of interest. This is called the *underdamped case*. If attempted convergence is too fast, the weights will oscillate about the optimum weights but will not accurately track the solution desired. It is therefore imperative to choose a step-size in a range that insures convergence. It can be shown that stability is insured provided that the following condition is met

$$0 \le \mu \le \frac{1}{\lambda_{\max}} \quad (39)$$

where $\lambda_{\max}$ is the largest eigenvalue of $\hat{R}_{xx}$.

Since the correlation matrix is positive definite, all eigenvalues are positive. If all the interfering signals are noise and there is only one signal of interest, we can approximate the condition in Eqn. (39) as

$$0 \le \mu \le \frac{1}{2\ \mathrm{trace}\left[R_{xx}\right]} \quad (40)$$

For scenario 2, the performance of LMS is given in Figures 10 (a, b, c, and d). It can be seen from Figure (b) that the algorithm tracks the variation function around the 70th iteration. Figure (c) shows that the error degrades to zero at the 70th iteration.

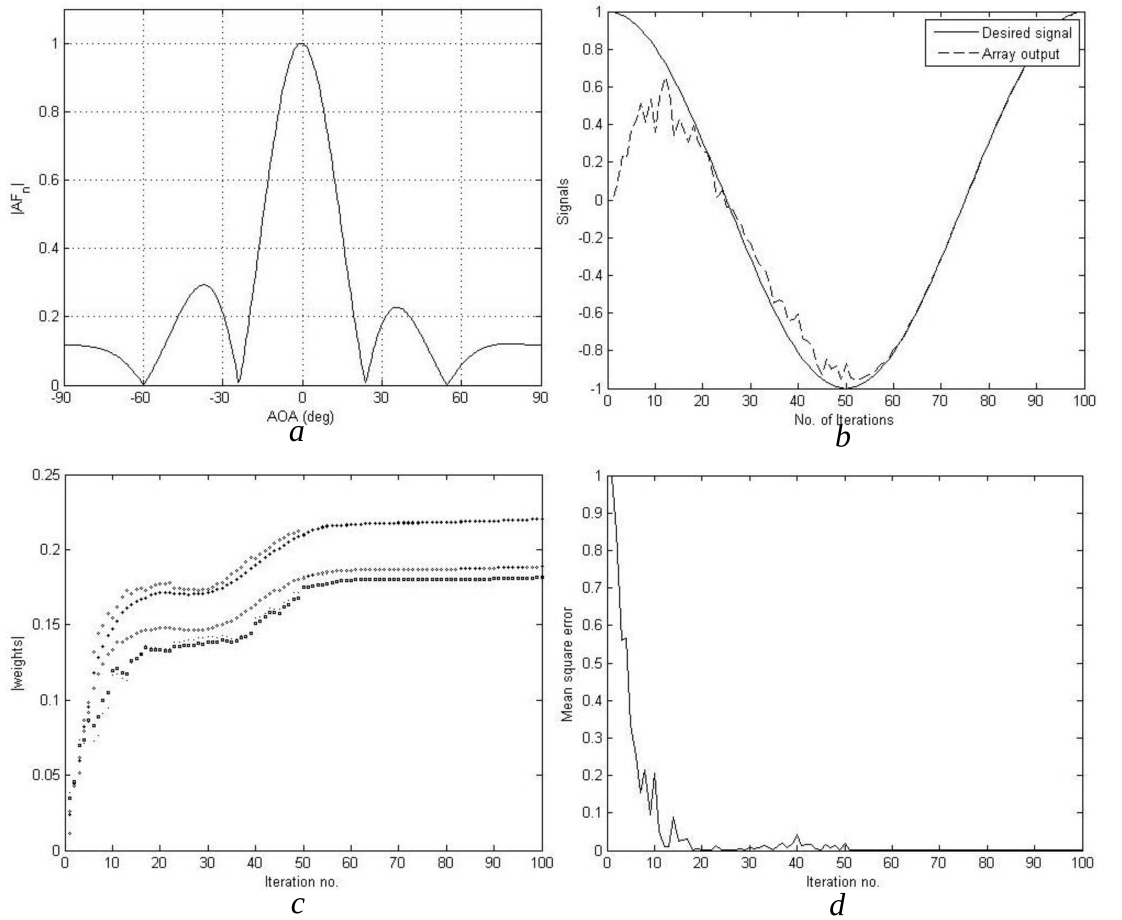


Figure (10) Performance of LMS, *a*. Radiation pattern, *b*. Acquisition and tracking of desired signal, *c*. Magnitude of array weights, *d*. Mean square error.

4.2 Sample Matrix Inversion (SMI)

One of the drawbacks of the LMS adaptive scheme is that the algorithm must go through many iterations before satisfactory convergence is achieved. If the signal characteristics are rapidly changing, the LMS adaptive algorithm may not be able to track of the desired signal. One possible approach to circumventing the relatively slow convergence of the LMS scheme is by use of SMI method [Jeon, *et. al.* 2005, Gross 2005, Dandekar, *et*. *al*. 2002]. This method is also alternatively known as *Direct Matrix Inversion* (DMI). The *sample matrix* is a time average estimate of the array correlation matrix using *K*-time samples. If the random process is ergodic in the correlation, the time average estimate will equal the actual correlation matrix. The optimum array weights are given by the optimum Wiener solution as [Gross 2005]

$$\bar{w}_{\text{opt}} = \bar{R}_{xx}^{-1}\bar{r} \tag{41}$$

where $\bar{r} = E\left[d^* \cdot \bar{x}\right]$

For *K* snapshots, we have

$$\hat{R}_{xx}(k) = \frac{1}{K}\bar{X}_K(k)\bar{X}_K^H(k) \tag{42}$$

and

$$\hat{r}(k) = \frac{1}{K}d^*(k)\bar{X}_K(K) \tag{43}$$

The SMI weights can then be calculated for the *k*th block of length *K* as

$$\bar{w}_{SMI}(k) = \left[\bar{X}_K(k)\bar{X}_K^H(k)\right]^{-1}\bar{d}^*(k)\bar{X}_K(k) \tag{44}$$

The radiation pattern of the algorithm regarding scenario 2 is shown in Figure (11)

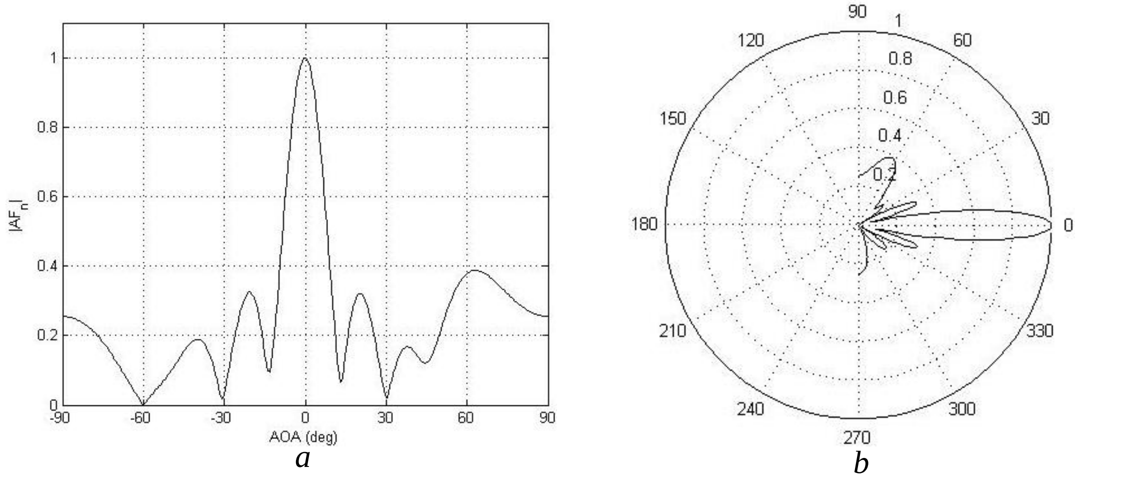


Figure (11) Weighted SMI array pattern, *a*. radiation pattern, *b*. Polar plot

4.3 Recursive Least Squares

The SMI technique has several drawbacks. Even though the SMI method is faster than the LMS algorithm, the computational burden and potential singularities can cause problems [Jeon, *et. al.* 2005, Gross 2005, Dandekar, *et. al.* 2002]. The correlation matrix and the correlation vector omitting *K* (in SMI) as

$$\hat{R}_{xx}(k)=\sum_{i=1}^{k}\bar{x}_K(i)\bar{x}^H(i) \tag{45}$$

$$\hat{r}(k)=\sum_{i=1}^{k}d^*(i)\bar{x}(i) \tag{46}$$

where *k* is the block length and last time sample *k* and $\hat{R}_{xx}(k)$, $\hat{r}(k)$ is the correlation

Both summations (Eqns. (45) and (46)) use rectangular windows, thus they equally consider all previous time samples. Since the signal sources can change or slowly move with time, we might want to deemphasize the earliest data samples and emphasize the most recent ones. This can be accomplished by modifying Eqns. (45) and (46) such that we forget the earliest time samples. This is called a *weighted estimate*. Thus

$$\hat{R}_{xx}(k)=\sum_{i=1}^{k}\alpha^{k-1}\bar{x}_K(i)\bar{x}_K^H(i) \tag{47}$$

$$\hat{r}(k)=\sum_{i=1}^{K}\alpha^{k-1}d^*(i)\bar{x}_K(i) \tag{48}$$

where α is the forgetting factor.

The forgetting factor is also sometimes referred to as the *exponential weighting* factor [Gross 2005]. α is a positive constant such that $0 \le \alpha \le 1$. When $\alpha = 1$, we restore the ordinary least squares algorithm. $\alpha = 1$ also indicates infinite memory. Decomposing the summation in Eqs. (47) and (48) into two terms: the summation for values up to $i = k-1$ and last term for $i = k$.

$$\hat{R}_{xx}(k)=\alpha\hat{R}_{xx}(k-1)+\bar{x}(k)\bar{x}^H(k) \tag{49}$$

$$\hat{r}(k)=\alpha\hat{r}(k-1)+d^*(k)\bar{x}(k) \tag{50}$$

Thus, future values for the array correlation estimate and the vector correlation estimate can be found using previous values. The behavior of the algorithm is show in Figure (12).

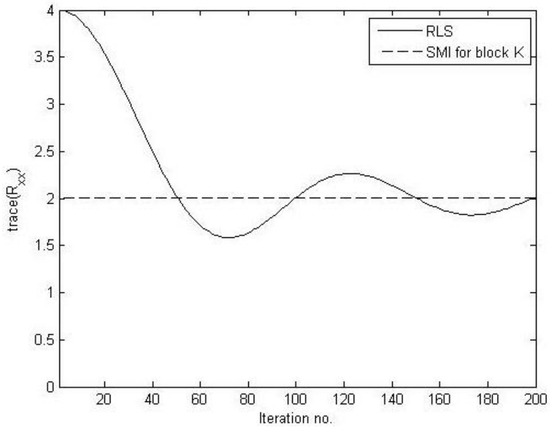
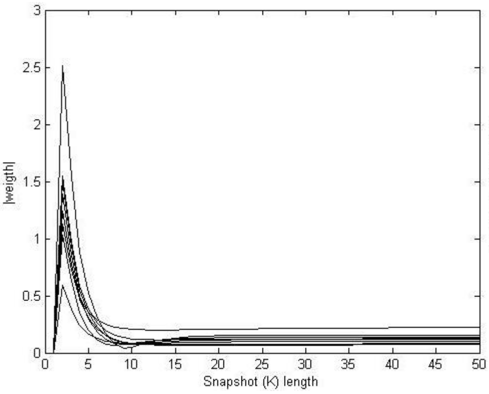
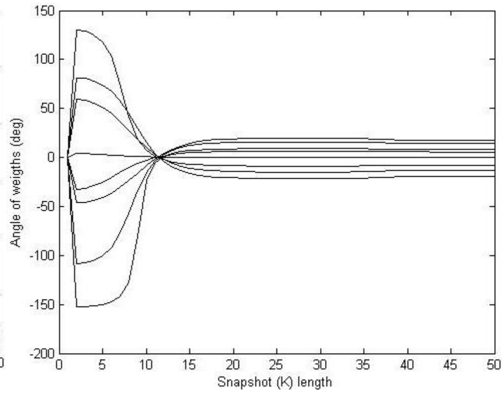


Figure (12) Trace of correlation matrix using SMI and RLS.

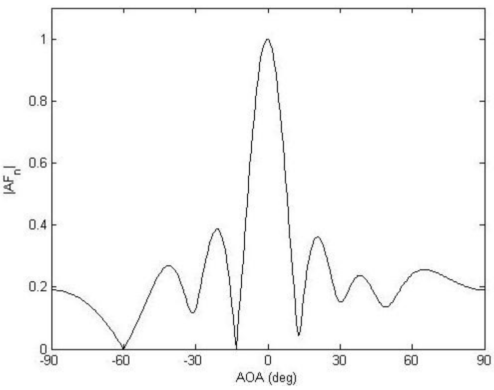
It can be seen that the recursion formula oscillates for different block lengths and that it matches the SMI solution when $k = K$. The recursion formula always gives a correlation matrix estimate for any block length k but only matches SMI when the forgetting factor is 1. The advantage of the recursion approach is that one need not calculate the correlation for an entire block of length K. Rather, update only requires one a block of length 1 and the previous correlation matrix. The performance of the algorithm is shown in Figure 13 (*a*, *b*, and *c*)



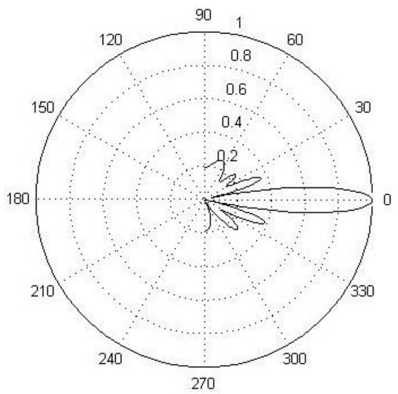
a



b



c



d

Figure (13) *a*. the weight vector values, *b*. the absolute weight vector, *c*. Radiation pattern, *d*. polar plot.

The advantage of the RLS algorithm over SMI is that it is no longer necessary to invert a large correlation matrix. The recursive equations allow for easy updates of the inverse of the correlation matrix. The RLS algorithm also converges much more quickly than the LMS algorithm.

5. Blind Algorithms

Blind algorithms do not require a reference signal to track the moving source. It depends on the signal properties (such as modulus or phase) to steer the main lob. They are suitable for mobile communications that produces low preambles.

4.4 Conjugate Gradient Method

The problem with the *steepest descent method* is its sensitivity of convergence rates to the eigenvalue spread of the correlation matrix. Greater spreads result in slower convergences. The convergence rate can be accelerated by use of the *conjugate gradient method* (CGM). The goal of CGM is to iteratively search for the optimum solution by choosing conjugate (perpendicular) paths for each new iteration [Godara 2004, Gross 2005]. The method of CGM produces orthogonal search directions resulting in the fastest convergence. Figure (14) depicts a top view of a two-dimensional performance surface where the conjugate steps show convergence toward the optimum solution. Note that the path taken at iteration n + 1 is perpendicular to the path taken at the previous iteration n.

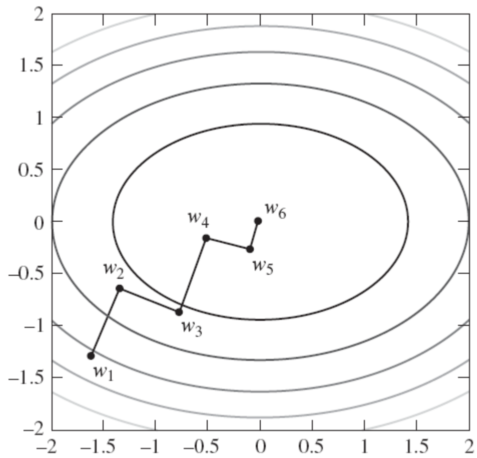


Figure (14) Contours of convergence using conjugate directions.

CGM is an iterative method whose goal is to minimize the quadratic cost function

$$J(\overline{w}) = \frac{1}{2}\overline{w}^H \overline{A}\overline{w} - \overline{d}^H \overline{w} \qquad (51)$$

where

$$\overline{A} = \begin{bmatrix} x_1(1) & x_2(1) & \cdots & x_M(1) \\ x_1(2) & x_2(2) & \cdots & x_M(2) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(K) & x_2(K) & \cdots & x_M(K) \end{bmatrix}$$ $K\times M$ matrix of array snapshots

K = number of snapshots

M = number of array elements

$\overline{w}$ = unknown weight vector

$\overline{d} = [d(1) \quad d(2) \quad \cdots \quad d(K)]^T$ = desired signal vector of K snapshots

We may take the gradient of the cost function and set it to zero in order to find the minimum. It can be shown that

$$\nabla_{\overline{w}} J(\overline{w}) = \overline{A}\overline{w} - \overline{d} \qquad (52)$$

Using the method of steepest descent in order to iterate to minimize Eq. (52). We wish to slide to the bottom of the quadratic cost function choosing the least number of iterations. The general weight update equation is given by

$$\overline{w}(n+1) = \overline{w}(n) - \mu(n)\overline{D}(n) \qquad (53)$$

Where the step size is determined by

$$\mu(n)=\frac{\bar{r}^{H}(n)\bar{A}\bar{A}^{H}r(n)}{\bar{D}^{H}(n)\bar{A}^{H}\bar{A}\bar{D}(n)} \qquad (54)$$

We may now update the residual and the direction vector. We can premultiply Eq. (53) by $-\bar{A}$ and add $\bar{d}$ to derive the updates for the residuals.

$$\bar{r}(n+1)=\bar{r}(n)+\mu(n)\bar{A}\bar{D}(n) \qquad (55)$$

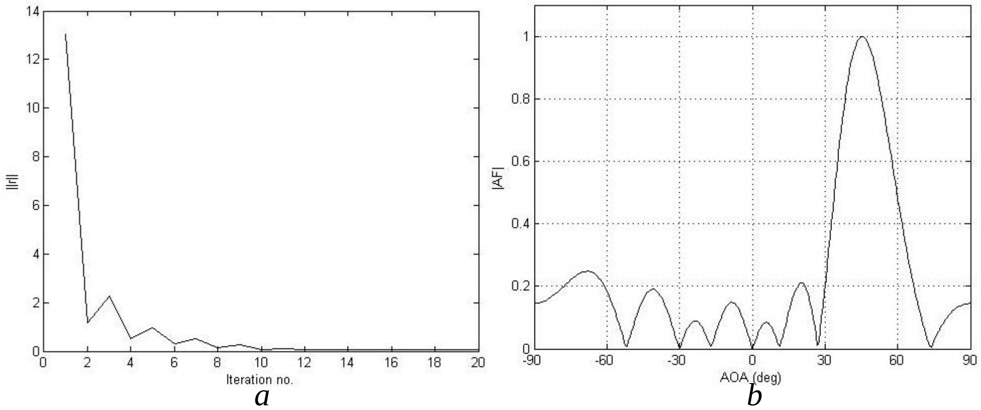
The direction vector update is given by

$$\bar{D}(n+1)=\bar{A}^{H}\bar{r}(n+1)-\alpha(n)\bar{D}(n) \qquad (56)$$

We can use a linear search to determine $\alpha(n)$ which minimizes $\bar{J}(\bar{w}(n))$. Thus

$$\alpha(n)=\frac{\bar{r}^{H}(n+1)\bar{A}\bar{A}^{H}\bar{r}(n+1)}{\bar{r}^{H}(n)\bar{A}\bar{A}^{H}\bar{r}(n)} \qquad (57)$$

Assuming the AOA is 45˚, interference signal at -30˚, 0˚, σ^2 =0.001, K=20; the performance of the algorithm is shown in Figure 15 (*a*, *b*).



a *b*

Figure (15) CGM Algorithm, *a*. Norm of the residuals for each iteration, *b*. Array pattern using CGM.

It can be seen that the residual drops to very small levels after 14 iterations in Figure 15. (*a*). The plot of the resulting pattern is shown in Figure 15.(*b*). It can be seen that two nulls are placed at the two angles of arrival of the interference.

6. Conclusions

Smart antennas have the ability to change its pattern electronically to track the SOI. Hence there is no need for mechanical steering system. The rotation is achieved through the alteration of Array Factor (AF). These algorithms rely heavily on the *correlation matrix R* because of the random nature of the arriving signal. The EAOATS provide very accurate steering algorithms but fails in the environment that constantly changing its behavior. The MUSIC algorithm shows the best accuracy but it fails under highly correlated signals. The ESPRIT shows lesser accuracy but due to its construction it assumes no prior correlation between signals. The non-blind algorithms resolve the weaknesses of EAOATS but need reference signal which might not be available like in mobile stations. The LMS adaptation algorithm is slow, so can’t track fast changing emitter. The SMI is faster but exerts heavy calculation on the processor and suffers from singularities. The RLS proposes a forgetting factor to remove the matrix inversion calculation in every iteration. But its performance is governed by the forgetting factor. For high forgetting factor the algorithm goes unstable, for low forgetting factor its performance reaches the LMS. The blind algorithms such as CGM is a very fast algorithm suitable to track fast changing signals without the need for reference signal but shows higher sidelobes.

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