The Best Fit Equations for the Storage-Elevation and Water Surface Area-Elevation Data of Reservoirs

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Abstract

In this research, the shifted power equation, $y = a\,(x - b)^c$, is proposed to best fit both the storage-elevation and the water surface area-elevation data of reservoirs. This equation is tested through two examples: Bastora reservoir and Al-Adheem reservoir, and compared with other possible best fit equations. It is concluded that the shifted power equation is the best one, and thus it can be adopted to represent the best fit equation for both the storage-elevation and the water surface area-elevation data of reservoirs.

الخلاصة

في هذا البحث تم اقتراح معادلة القوة الزاحفة، $y = a\,(x - b)^c$ ، لتكون افضل معادلة ملائمة لكلا بيانات المخزون . المنسوب والمساحة السطحية للماء . المنسوب للخزانات. تم اختبار هذه المعادلة من خلال مثالين: خزان باستورا وخزان العظيم، وتم مقارنة هذه المعادلة مع المعادلات الاخرى المحتملة الملائمة للبيانات. وتم الاستنتاج بان هذه المعادلة هي افضل معادلة ملائمة للبيانات، وعليه يمكن اعتماد هذه المعادلة لتمثل افضل معادلة ملائمة لبيانات المخزون . المنسوب والمساحة السطحية للماء . المنسوب للخزانات.

1. Introduction

The relationships between storage and elevation and between water surface area and elevation are very important in the design of dams. The best fit equation for the storage and elevation data is used in (***Chow & others 1988, Linsley & Franzini 1979, Linsley & others 1958***):

1. Determining the corresponding elevation of dead storage.
2. Determining the corresponding elevation of live storage.
3. Determining the corresponding elevation of total storage.
4. Determining the normal operation level.
5. Determining the upstream cofferdam height.
6. The hydrological method of flood routing through reservoirs.
7. The dam breaks study.

and the best fit equation for the area and elevation data is used in (***Chow & others 1988, Linsley & Franzini 1979***):

1. Determining the dam reservoir capacity.
2. Determining the annual losses due to evaporation in reservoirs.
3. The hydraulic method of flood routing through reservoirs.

There is no fixed formula is found in literature for the storage-elevation data. Similarly, there is no fixed formula is found in literature for the water surface area-elevation data. In this research, the shifted power equation, $y = a\,(x - b)^c$, is proposed to best fit both the storage-elevation and the water surface area-elevation data of reservoirs. Thus, this research is a trial to fix one equation for the storage-elevation and the water surface area-elevation data of reservoirs instead of searching for the best fit equations for every reservoir.

2. The proposed best fit equations

The scatter diagram of the storage-elevation data of reservoirs is look like that shown in Fig. 1. The scatter diagram of area-elevation data is similar to the scatter diagram of storage-elevation data. The difference is that storage-elevation data contain zero volume.

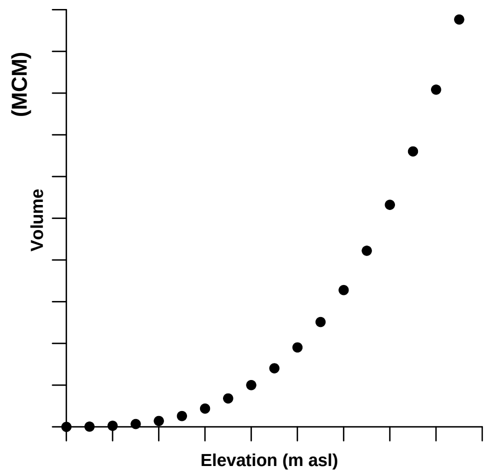

Fig. 1: Scatter diagram of the storage-elevation data

From the scatter diagram, the best fit equation for the storage-elevation data may be polynomial, logarithmic, exponential, or power equation. Polynomial equation of degree higher than 2 is not preferable because of the higher number of coefficients, the high number of digits in every coefficient, and the powers of the independent variable. In this research, the shifted power equation:

$$Volume = a\,(Elv. - b)^c \qquad (1)$$

where : **Elv.** = Water reservoir level (m asl)

Volume = Volume of the reservoir at elevation (Elv.) (MCM)

a, b, c = Parameters

is proposed to best fit the storage-elevation data of reservoirs.

The parameter b in the **Shifted power equation**, **Eq.1**, is proposed equal to the elevation corresponding to zero volume. The parameters a and c are proposed such that Eq.1 gives reasonable residual. These values of a, b, and c are then adjusted through iteration diagram using numerical methods, such as Gauss-Newton method (***Burden & Faires 2001, Chapra & Canale2006***), or optimization methods, such as Nelder-Mead method (***Mathews & Fink 2004***).

Similarly, from the scatter diagram, the best fit equation for the water surface area-elevation data may be polynomial, logarithmic, exponential, or power equation. In this research, the shifted power equation:

$$Area = d\,(Elv. - f)^g \qquad (2)$$

where: **Elv.** = Water reservoir level (m asl)

Area = Reservoir water surface area at elevation (Elv.) (km^2)

d, f, g = Parameters

is proposed to best fit the water surface area- elevation data of reservoirs. The final values of the parameters d, f, and g are determined as mentioned in Eq.1

Eq.1 and Eq.2 will be tested through two examples: Bastora reservoir and Al-Adheem reservoir, and compared with the other equations: polynomial, logarithmic, exponential, and power equations.

2.1 Storage-elevation relationship

Example 1

The Elevation - Volume data of Bastora reservoir are shown in the following table (***El Concorde Consultant Engineers 2006***); find the best fit equation for these data.

Elevation (m asl)	810	820	830	840	850	860	870	880	890
Volume (MCM)	0.00	1.18	5.28	13.71	28.31	51.96	87.63	136.33	200.27

Elevation (m asl)	900	910	920	930	940	950	960	970	980
Volume (MCM)	280.97	380.88	502.74	655.91	844.48	1064.73	1320.65	1616.82	1953.11

Solution

Figures 2 to 11 show the graphical representation of possible best fit equations for the storage-elevation data of Bastora reservoir and Table1 shows the characteristics of these equations.

The linear, 2nd order polynomial, and logarithmic equations cannot be selected because they give negative values for volumes as shown in Figures 2, 3, and 4. The Exponential and Power-1 equations cannot be selected because they have low characteristics. Power-2 equation has the form of Eq.1. From this equation, the parameter b is the elevation corresponding to zero volume. Thus b=810 and Power-2 equation becomes:

$$Volume = a\,(Elv. - 810)^{c} \tag{3}$$

The parameters a and c are found using the least squares method (***Gerald 1984, McCUEN 1985***). Power-2 equation has good characteristics.

The **Shifted power-1** equation has the form of Eq.1. The parameter b is proposed equal to the elevation corresponding to zero volume, b=810, and a and c are proposed such that Eq.1 gives reasonable residual. These values of a, b, and c are then adjusted through iteration diagram. The following equation shows the final values of these parameters:

$$Volume = 0.00003\,(Elv. - 794.8802)^{3.446761} \tag{4}$$

This equation has the best characteristics compared with the all equations mentioned before. For this reason, the **Shifted power equation** is proposed in this research to best fit the storage-elevation data of reservoirs.

For the dam break analysis, it is required to find equation that gives zero volume at certain elevation. Eq.4 does not give zero volume at elevation 810m asl. Table 2 shows the observed and predicted values. Thus, a linear equation is used for the first two points:

$$Volume = 0.118\,Elv. - 95.58 \tag{5}$$

For points greater than point 2 to the end of points, either Eq.4 is used and is applied for elevations greater than 820m asl or another equation is found. Power-3 equation is for 17 points, all points except the first point, and has good characteristics. Power-4 equation is for 17 points, all points except the first point, and has the form of Eq.3. It has good characteristics. The **Shifted power-2** equation is same as **Shifted power-1** equation but for 17 points, all points except the first one. This equation is:

$$Volume = 0.00003\,(Elv. - 794.8716)^{3.446962} \tag{6}$$

and has the better characteristics than Power-3 and Power-4 equations.

As a summary, the best fit equation for the storage-elevation data of Bastora reservoir is Eq.4. For dam break analysis, Eq.5 is used for the first two points and for the other points Eq.4 or Eq.6 is used.

Example 2

The Elevation - Volume data of Al-Adheem reservoir are shown in the following table (***Binnie and Partners 1988***); find the best fit equation for these data.

Elevation (m asl)	100	110	115	118	120	125	130	131.5	135	140	143
Volume (MCM)	70	160	310	450	520	980	1400	1600	2150	3130	3750

Solution

As it is mentioned before, the parameter b is proposed equal to the elevation corresponding to zero volume and a and c are proposed such that Eq.1 gives reasonable residual. In this example, the data of elevation corresponding to zero volume is missing. Therefore, a best fit curve is extended to cut the elevation axis as shown in Fig.12. From this Figure, the elevation corresponding to zero volume is about 91m asl.

Figures 13 to 22 show the graphical representation of possible best fit equations for the storage-elevation data of Al-Adheem reservoir and Table3 shows the characteristics of these equations.

The linear, 2nd order polynomial, and logarithmic equations cannot be selected because they give negative volumes as shown in Figures 13, 14, and 15. The Exponential and Power-1 equations cannot be selected because they have weak correlation coefficients. Power-2 equation has the form of Eq.1. Thus b=91 and Power-2 equation becomes:

$$Volume = a\,(Elv. - 91)^{c} \tag{7}$$

The parameters a and c are found using the least squares method. Power-2 equation has good characteristics.

The **Shifted power-1** equation has the form of Eq.1. The parameter b is proposed equal to the elevation corresponding to zero volume, b=91, and a and c are proposed such that Eq.1 gives reasonable residual. These values of a, b, and c are then adjusted through iteration diagram. The following equation shows the final values of these parameters:

$$Volume = 0.000248(Elv. - 82.1987)^{4.026552} \tag{8}$$

This equation has the best characteristics comparing with the all equations mentioned before. Table 4 shows the observed and predicted values.

Eq.8 does not give zero volume at elevation 91m asl, therefore for the dam break analysis, a linear equation is used for the first two points:

$$Volume = \frac{70}{9}(Elv. - 91) \tag{9}$$

For points greater than point 2 to the end of points, either Eq.8 is used and is applied for elevations greater than 100m asl or another equation is found. Power-3 equation is for 11 points, all points except the first point, and has good characteristics. Power-4 equation is for 11 points and has the form of Eq.7. It has good characteristics. The **Shifted power-2** equation is same as **Shifted power-1** equation but for 11 points. The following equation shows the final values of the parameters a, b, and c:

$$Volume = 0.000255(Elv. - 82.27192 \tag{10}$$

and has the better characteristics than Power-3 and Power-4 equations.

As a summary, the best fit equation for the storage-elevation data of Al-Adheem reservoir is Eq.8. For dam break analysis, Eq.9 is used for the first two points and for the other points Eq.8 or Eq.10 is used.

2.2 Water surface area -elevation relationship

Example 3

The Elevation-Area data of Bastora reservoir are shown in the following table (***El Concorde Consultant Engineers 2006***); find the best fit equation for these data.

Elevation (m asl)	810	820	830	840	850	860	870	880	890
Area (km^2)	0.039	0.222	0.633	1.073	1.884	2.881	4.299	5.464	7.371

Elevation (m asl)	900	910	920	930	940	950	960	970	980
Area (km^2)	8.791	11.241	13.155	17.587	20.155	23.949	27.271	32.026	35.259

Solution

Figures 23 to 30 show the graphical representation of possible best fit equations for the water surface area-elevation data of Bastora reservoir and Table 5 shows the characteristics of these equations.

The linear and logarithmic equations cannot be selected because they give negative areas as shown in Figures 23 and 25. The 2nd order polynomial has good characteristics. The Exponential and Power-1 equations do not have high characteristics. Power-2 equation has the form of Eq.3 and has weak correlation coefficient. Power-3 equation has the form of Eq.1, but b is from Eq.4. That is

$$Area = a\,(Elv. - 794.8802)^c \tag{11}$$

The parameters a and c are found using the least squares method. Power-3 equation has good characteristics.

The **Shifted power** equation has the form of Eq.1. The following equation shows the final values of the parameters a, b, and c:

$$Area = 0.000252(Elv. - 801.3651)^{2.28873} \tag{12}$$

This equation has the best characteristics compared with the all equations mentioned before.

As a summary, the shifted power equation, Eq.12, can be selected to represent the best fit equation for the water surface area-elevation data of Bastora reservoir because it has the best characteristics.

Example 4

The Elevation-Area data of Al-Adheem reservoir are shown in the following table (***Binnie and Partners 1988***); find the best fit equation for these data.

Elevation (m asl)	100	110	115	118	120	125	130	131.5	135	140	143
Area (km^2)	3	28	41	52	60	85	122	135	170	233	270

Solution

Figures 31 to 38 show the graphical representation of possible best fit equations for the water surface area-elevation data of Al-Adheem reservoir and Table 6 shows the characteristics of these equations.

The linear and logarithmic equations cannot be selected because they give negative areas as shown in Figures 31 and 33. The 2nd order polynomial, Exponential and Power-1 equations have good characteristics. Power-2 equation has the form of Eq.7 and has good characteristics. Power-3 equation has the form of Eq.1, but b is from Eq.8, that is

$$Area = a\,(Elv. - 82.1987)^c \tag{13}$$

The parameters a and c are found using the least squares method. Power-3 equation has good characteristics.

The **Shifted power** equation has the form of Eq.1. The following equation shows the final values of the parameters a, b, and c:

$$Area = 0.000003(Elv. - 66.62782)^{4.234478} \quad (14)$$

This equation has the best characteristics compared with the all equations mentioned before.

As a summary, the shifted power equation, Eq.14, can be selected to represent the best fit equation for the water surface area-elevation data of Al-Adheem reservoir because it has the best characteristics.

3. Conclusions

The shifted power equation, $y = a\,(x - b)^c$, is the most suitable equation for both the storage-elevation and the water surface area-elevation data of Bastora and Al-Adheem reservoirs compared with other possible best fit equations. Therefore, it can be concluded that the shifted power equation can be adopted to represent the best fit equation for both the storage-elevation and the water surface area- elevation data of reservoirs.

4. Recommendations

Application of the shifted power equation, proposed in this research, for other reservoirs, especially other Iraqi reservoirs, is required to show the validity of this type of equation.

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Appendix 1: Tables

Table 1: Bastora reservoir, Volume – Elevation Equations

Equation type	**Residual sum of squares**	**Coef. of determination**	**Coef. of correlation**
Linear	1.13253E+006	0.82167	0.90646
2^{nd} order polynomial	46774.7	0.99264	0.99631
Logarithmic	1.25748E+006	0.80199	0.89554
Exponential	191.484	0.53402	0.73077
Power-1	184.658	0.55063	0.74204
Power-2	26.6462	0.93516	0.96704
Shifted power-1	**87.7621**	**0.99998**	**0.99999**
Power-3	5.37164	0.92669	0.96265
Power-4	0.328611	0.99552	0.99776
Shifted power-2	**87.6409**	**0.99998**	**0.99999**

Table 2: Observed and predicted values of Bastora reservoir (MCM)
($Pridicted\ Volume = 0.000030\,(Elv. - 794.8802)^{3.446761}$)

Elevation (m asl)	**810**	**820**	830	840	850	860	870	880	890	900
Observed volumes	**0.00**	**1.18**	5.28	13.71	28.31	51.96	87.63	136.33	200.27	280.97
Predicted volumes	**.35**	**2.00**	6.35	15.07	30.04	53.37	87.32	134.34	197.01	278.05

Elevation (m asl)	910	920	930	940	950	960	970	980
Observed volumes	380.88	502.74	655.91	844.48	1064.73	1320.65	1616.82	1953.11
Predicted volumes	380.32	506.80	660.59	844.90	1063.06	1318.49	1614.70	1955.33

Table 3: Al- Adheem reservoir, Volume – Elevation Equations

Equation Type	**Residual sum of squares**	**Coef. of determination**	**Coef. of correlation**
Linear	3.80023E+006	0.77636	0.88111
2^{nd} order polynomial	210433	0.98762	0.99379
Logarithmic	4.77804E+006	0.71882	0.84783
Exponential	172.248	0.56829	0.75385
Power-1	150.852	0.62192	0.78862

Power-2	4.75893	0.98807	0.99402
Shifted power-1	**11951.5**	**0.99930**	**0.99965**
Power-3	15.8846	0.99514	0.99757
Power-4	0.903147	0.94342	0.97130
Shifted power-2	**11949.3**	**0.99922**	**0.99961**

Table 4: Observed and predicted values of Al- Adheem reservoir (MCM)

($Pridicted\ Volume = 0.000248(Elv. - 82.1987)^{4.026552}$)

Elevation (m asl)	**91**	**100**	110	115	118	120	125	130	132	135	140	143
Observed volumes	**0**	**70**	160	310	450	520	980	1400	1600	2150	3130	3750
Predicted volumes	**2**	**27**	162	315	448	558	919	1435	1625	2141	3083	3779

Table 5: Bastora reservoir, Area – Elevation Equations

Equation Type	Residual sum of squares	Coef. of determination	Coef. of correlation
Linear	194.321	0.91428	0.95618
2^{nd} order polynomial	2.92665	0.99871	0.99935
Logarithmic	228.086	0.89938	0.94836
Exponential	8.04242	0.86447	0.92977
Power-1	7.14172	0.87965	0.93790
Power-2	24.2688	0.59102	0.76878
Power-3	0.277952	0.99532	0.99766
Shifted power	**2.51209**	**0.99890**	**0.99945**

Table 6: Al- Adheem reservoir, Area – Elevation Equations

Equation Type	Residual sum of squares	Coef. of determination	Coef. of correlation
Linear	7399.61	0.90069	0.94905
2^{nd} order polynomial	355.731	0.99523	0.99761
Logarithmic	9953.96	0.86641	0.93081
Exponential	1.45424	0.90905	0.95344
Power-1	1.03633	0.93519	0.96705
Power-2	0.0984267	0.99384	0.99692
Power-3	0.29925	0.98128	0.99060
Shifted power	**65.7897**	**0.99912**	**0.99956**

Appendix 2: Figures

1. Bastora reservoir, Volume - Elevation

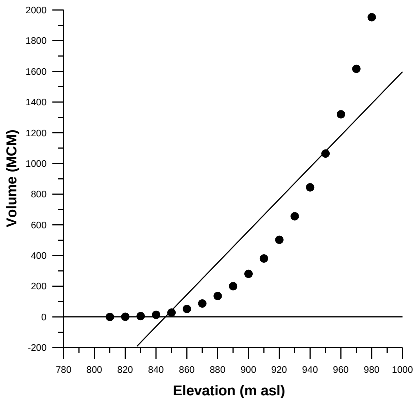


Fig. 2: Linear

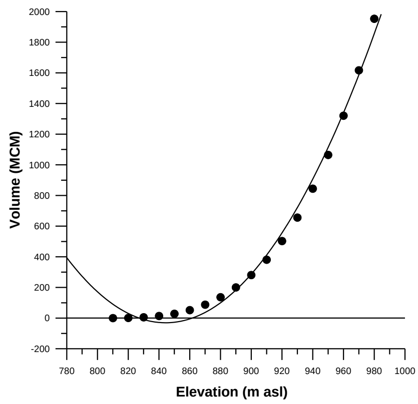


Fig. 3: 2^{nd} order polynomial

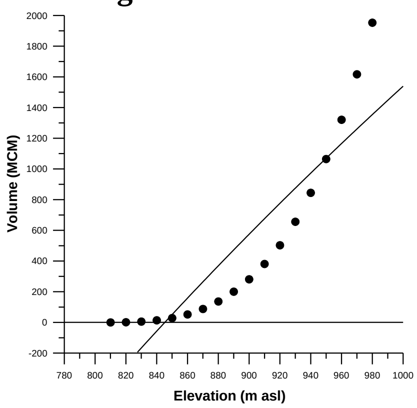


Fig. 4: Logarithmic

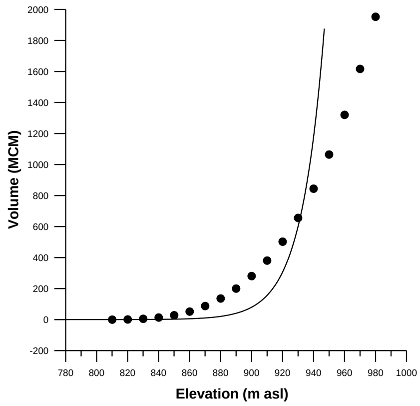


Fig. 5: Exponential

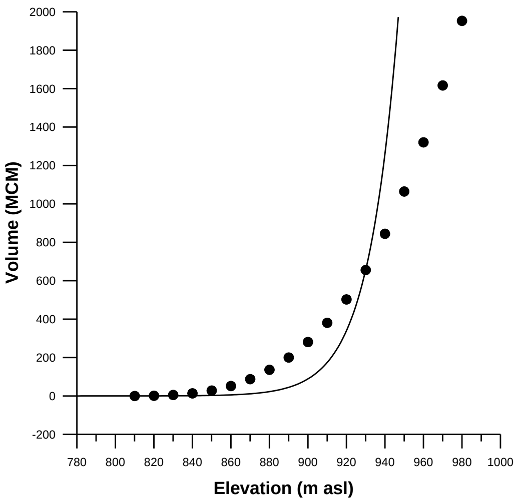


Fig. 6: Power-1

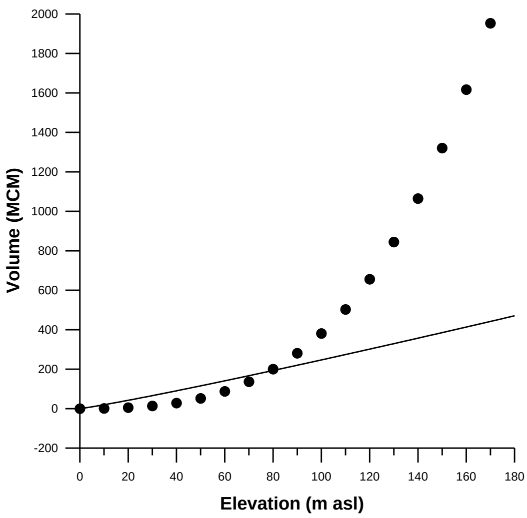


Fig. 7: Power-2

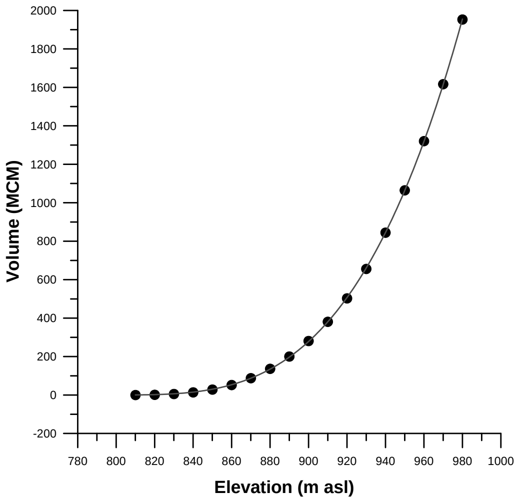


Fig. 8: Shifted power-1

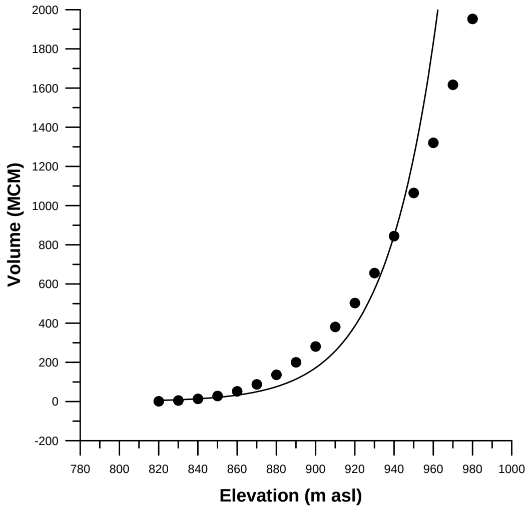


Fig. 9: power-3

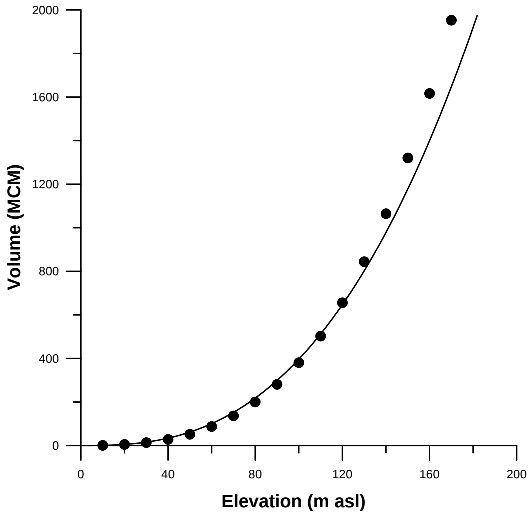


Fig. 10: power 4

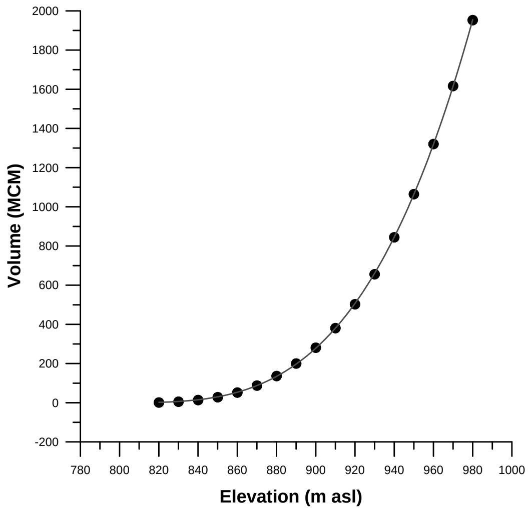
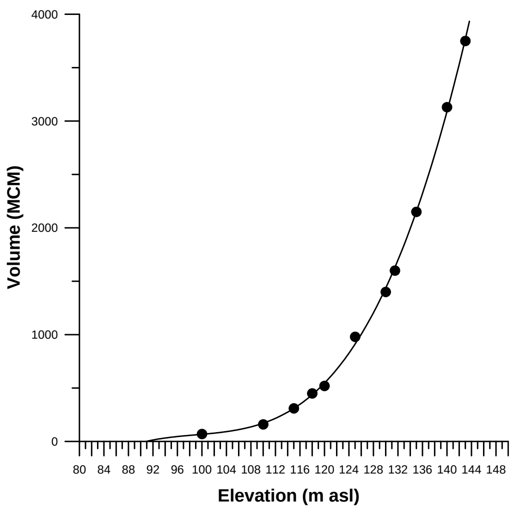


Fig. 11: Shifted power-2

2. Al- Adheem reservoir, Volume – Elevation



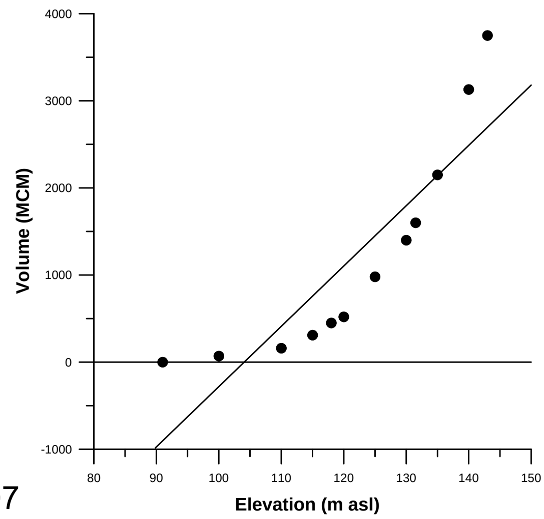


Fig.12: Elevation of zero volume

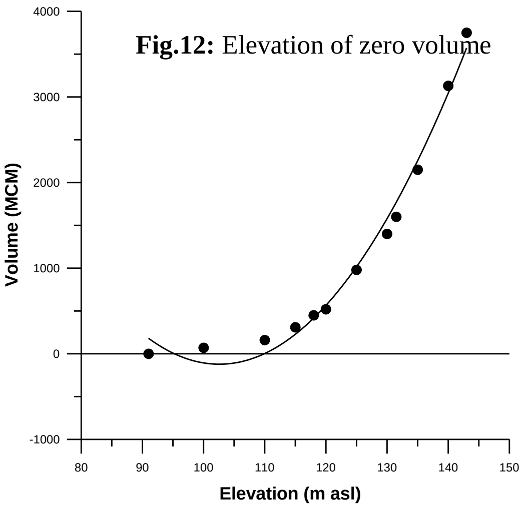


Fig.13: Linear

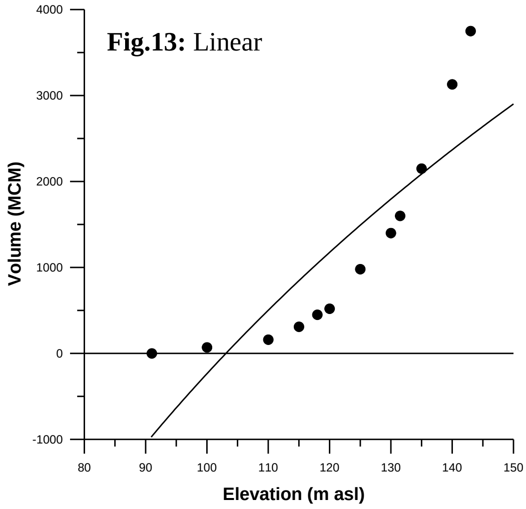


Fig. 14: 2nd order polynomial

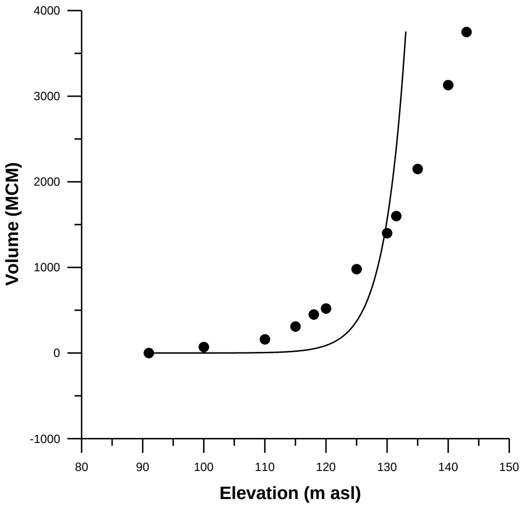


Fig. 15: Logarithmic

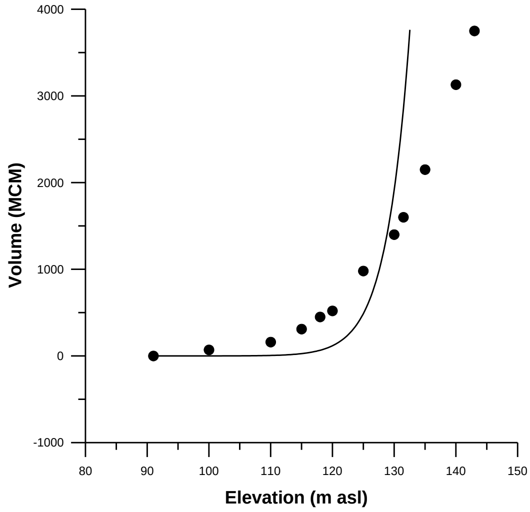


Fig. 16: Exponential

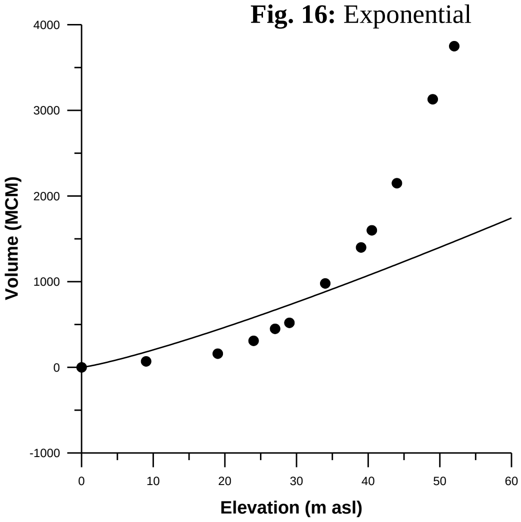
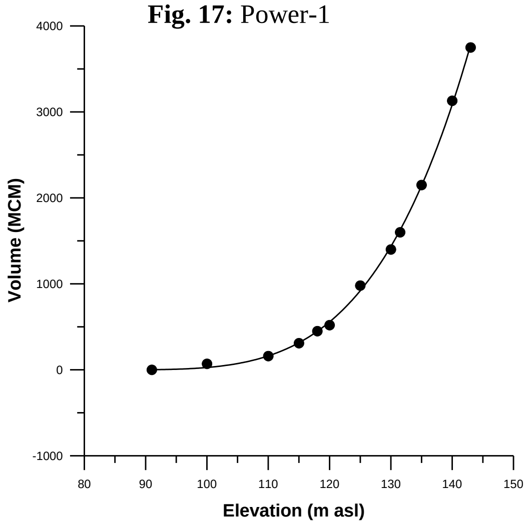


Fig. 17: Power-1



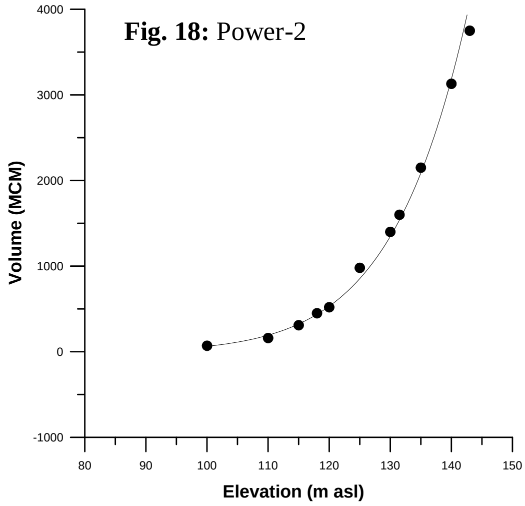


Fig. 18: Power-2

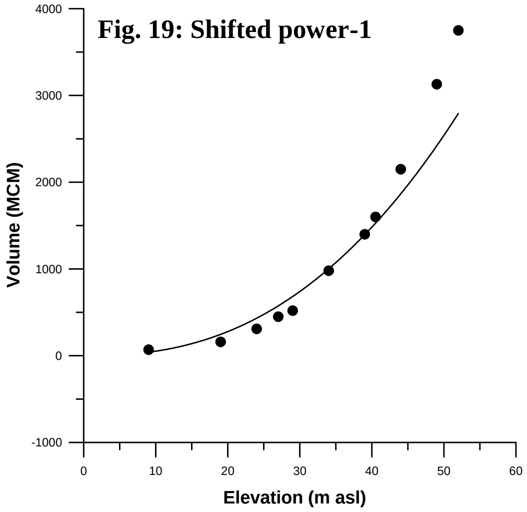


Fig. 19: Shifted power-1

Fig. 20: power-3

Fig. 21: power-4

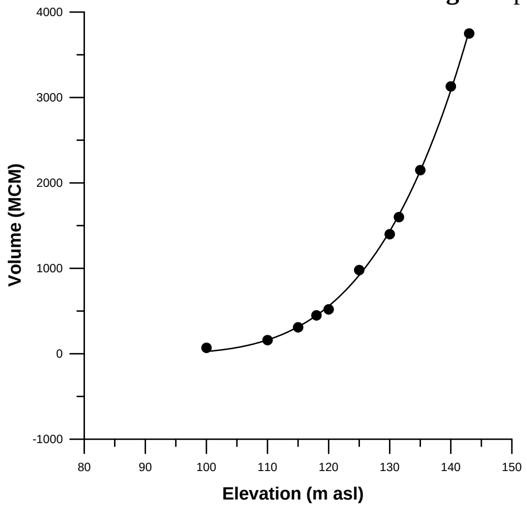


Fig. 22: Shifted power-2

3. Bastora reservoir, Area – Elevation

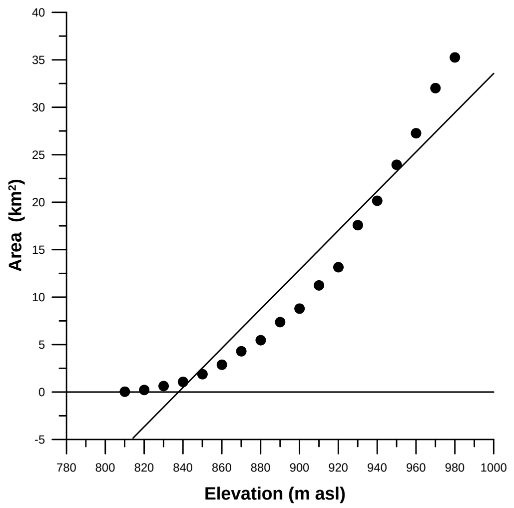


Fig. 23: Linear

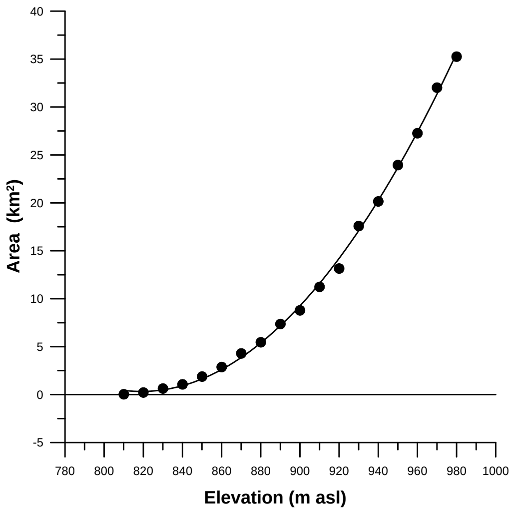


Fig. 24: 2nd order polynomial

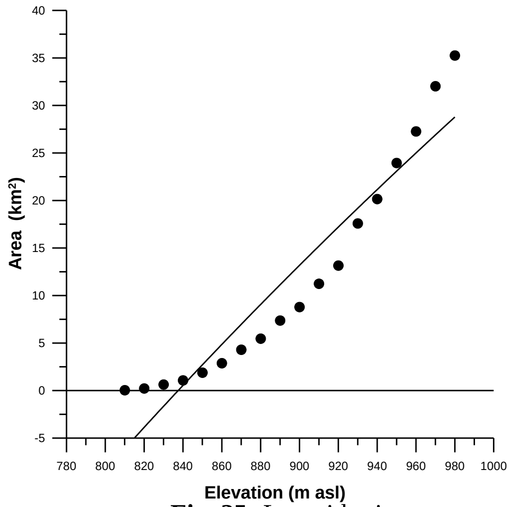


Fig. 25: Logarithmic

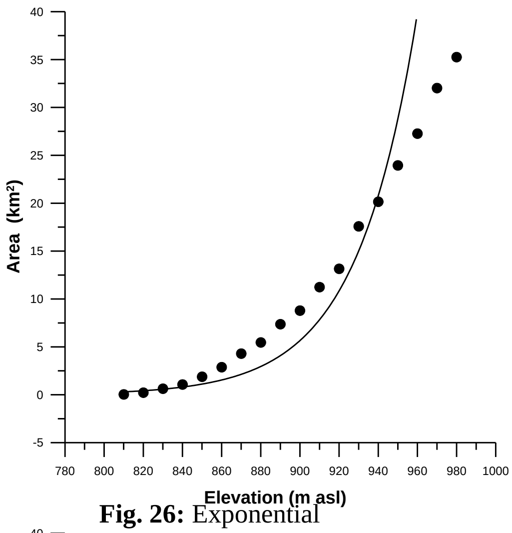


Fig. 26: Exponential

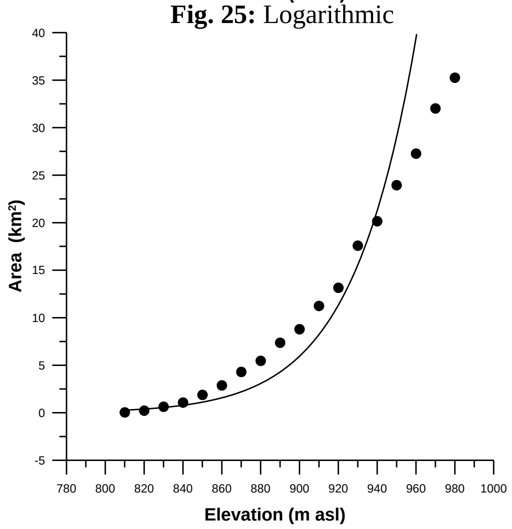


Fig. 27: Power-1

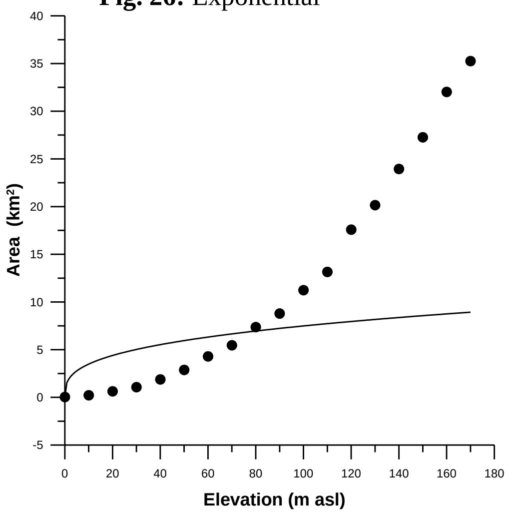


Fig. 28: Power-2

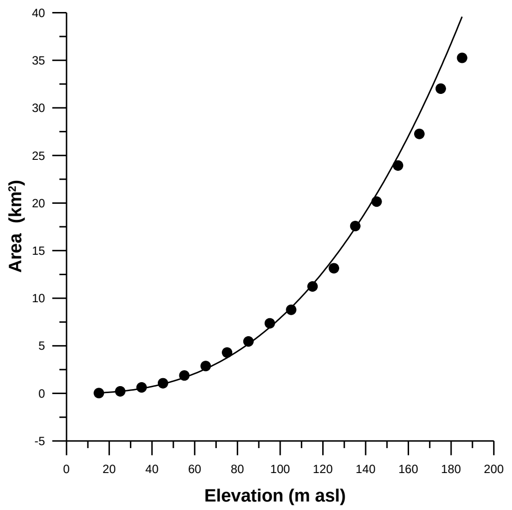


Fig. 29: Power-3

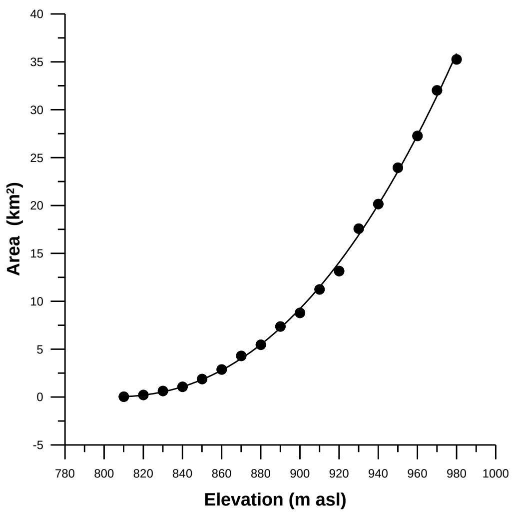
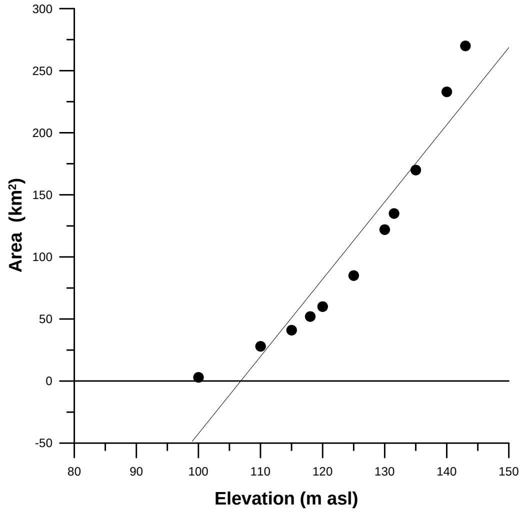


Fig. 30: Shifted power

3. Al- Adheem reservoir, Area – Elevation



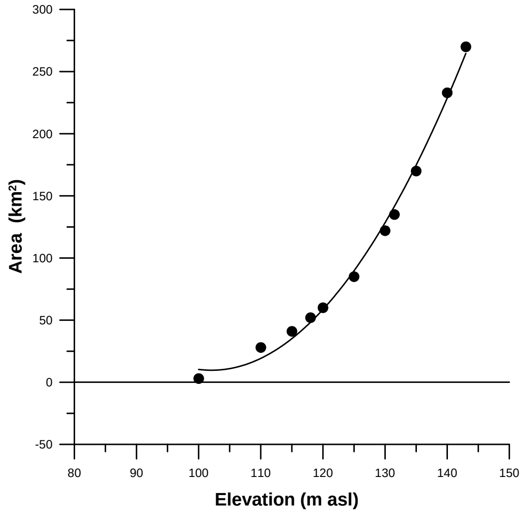
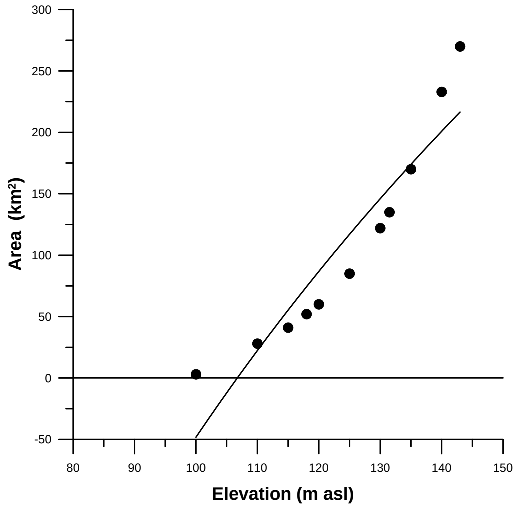


Fig. 31: Linear

Fig. 32: 2[nd] order polynomial



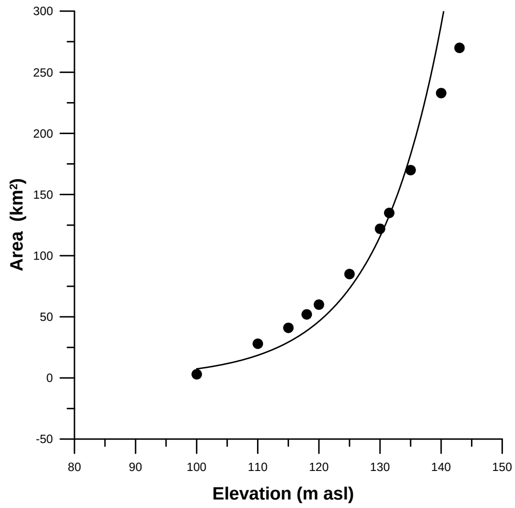
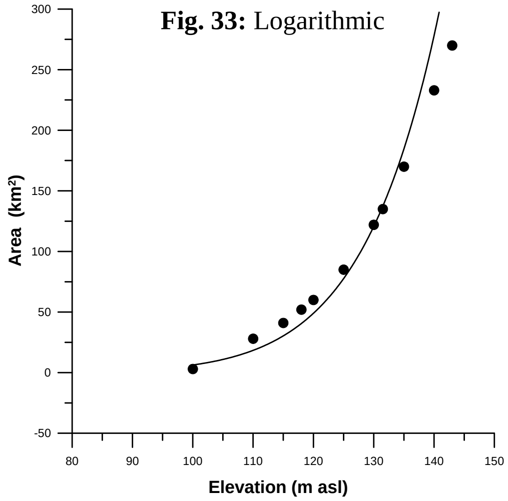


Fig. 33: Logarithmic

Fig. 34: Exponential



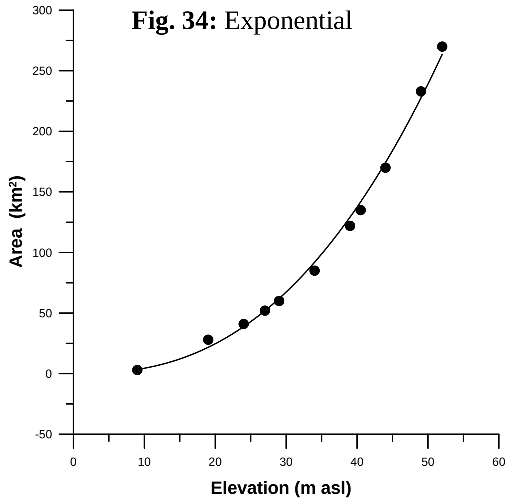


Fig. 35: Power-1

Fig. 36: Power-2

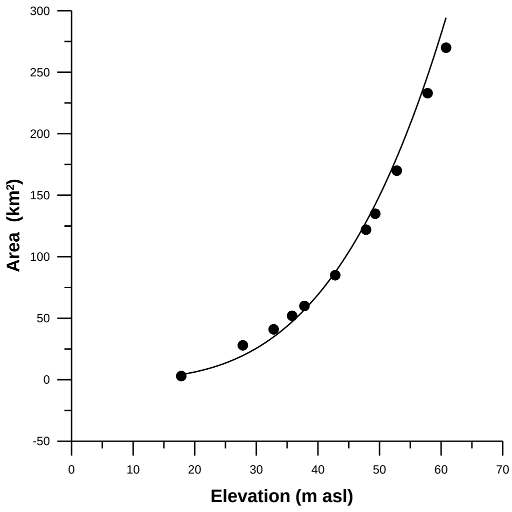


Fig. 37: Power-3

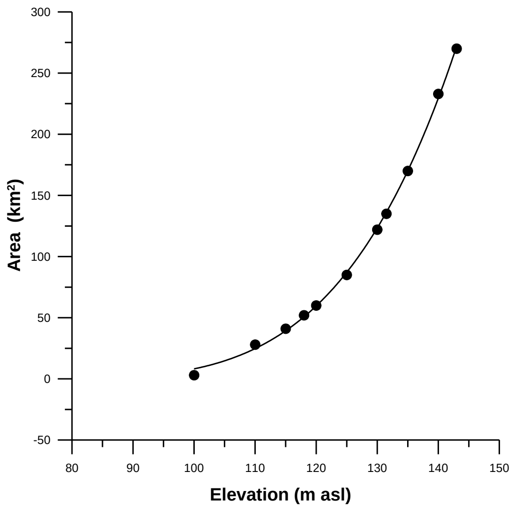


Fig. 38: Shifted power