Filters Topology in light of Fuzzy Logic

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Abstract

This paper is a try to promote the principles of filters on a classical sets into a fuzzy sets under definitions of mathematical operations such intersections(disjunctions) and union(conjunction),…,etc .Some definitions on classical sets is traditionally true on the fuzzy sets , also some theorems and remarks were executed .Several examples occurred to demonstrate these views. Eventually this work is represent a restricts to the general view for fuzzy into some points(members) . Fuzzy filter base is an other term edited under several definitions supported by some theorems holds for the crisp sets under these principles .This research is one from a series of researches try to prove and then improve a vision to combine a view(s) of fuzzy logic for many directions for subjects from mathematics ,geometry and sciences computers and calculations ,this work one for topology .

Key words: Fuzzy logic, Fuzzy set, Filter, Fuzzy Filter, Fuzzy Filter base .

الخلاصة

هذا البحث هو محاولة للترويج عن مبادئ الفلاتر على المجموعات الكلاسيكية (الاعتيادية) إلى المجموعات الضبابية تحت تعاريفِ العملياتِ الرياضيةِ مثل التقاطعات والإتحاد ,...، الخ. بعض التعاريف على المجموعات الكلاسيكية هي بشكل تقليدي(بصورة تقليدية) حقيقية على المجموعات الضبابية، كذلك نفذت بعض النظريات والملاحظات. وردت عِدّة أمثلة لعَرْض وجهات النظر هذه. في النهاية يمثل هذا العملِ تقييد لوجهة النظر العامّة للضبابيةِ إلى بَعْض النقاط (عناصر). قاعدة المرشح الضبابية هي تعبير آخرُ حرّرَ تحت عِدّة تعاريف مدعومة من قبل بعْض النظريات التي تَحْملُ للمجموعاتِ الهشة(الاعتيادية) تحت هذه المبادئِ .هذا البحث واحد مِنْ سلسلة من البحوث تحاول إثبات و من ثم تحسين رؤية لجَمْع وجهة(وجهات) نظر المنطقِ الضبابيِ للعديد من الاتجاهات لمواضيع من الرياضيات ,الهندسة والعلوم الحاسبات و الحسابات، هذا العمل للهندسة اللاكمية.

1.Introduction

Going off from the importance of combining fuzzy set theory and the fuzzy logic in many ways with several models of artificial intelligent which named as "*Intelligent modeling*", fuzzy logic like other paradigms such as ;Expert Systems ,Neural Networks,…,etc ,which were used as intelligent techniques that offer some advantages in real world applications(Jain L.& Jain R.,1997).These techniques are fusing together by Fusion Technology. These fusing be susceptible with mistakes and risks .To avoid these all in this work we need to study the topology of fuzzy logic which is based fuzzy set theory.

Most ways that in majority exploit fuzzy logic such in domain of control ,domains of science from biology to particle physics(Thuillard,2000).Since a fuzzy logic extend values from two values to many values, there are a subtle questions to be answered : (i)What is(are) result(s) gotten on if we invert a view of fuzzy logic from multiplicity by filtering sets of points(members) ,or in other words ;Are the results for filtering results of fuzzy logic(sets) by executables be acceptably?(ii)Is the fuzzy version image of image is equal original?(iii)Is filtering fuzzy get on the original(restricted solves)? (iv)Do the operations on fuzzy sets that satisfies for classical sets be true under theorems of filters topology? .

Such questions are needed in practical applications such signal processing .

In that context the theorems in topology of filters were proven for crisp(classical) proves of topology and several proves occurred for some theorems by fuzzy logic , fuzzy set theory that to explain what caused the need for this all .

2. Preview Structures for Fuzzy Set Theory, Filters Topology and their Representations

In this section ,we aim to lighting on basic needed definitions for filters structure and fuzzy sets to prove many concepts on it depending on features for both, and then reset these all in view of fuzzy logic like define filter as fuzzy filter, and then try to design a conditions and properties for each concept .Eventually exploiting this in prove theorems .

Fuzzy Logic is a departure from classical two-valued sets and logic, that uses "soft" linguistic (e.g. large, hot, tall) system variables and a continuous range of truth values in the interval [0,1], rather than strict binary (True or False) decisions and assignments(Bonde,2000).Formally, fuzzy logic is a structured, model-free estimator that approximates a function through linguistic input/output associations.

Definitions for fuzzy set, empty fuzzy set, fuzzy subset, intersections ,union and complemented sets that needed here depended as in references(Babuška & et al 1996; Bilgiç & et al,1995).To define a filters on a fuzzy sets depending on definitions of crisp sets in topology books. Cite by references(Weisstein,1999;Sharma,1977; MacIver,2004):

Definition 1: Let X be any non empty universal set and A is nonempty fuzzy set on X by membership function μ as:

$$A = \{(u, \mu(u)) : u \in X\} \qquad \text{….(1)}$$

A ***Fuzzy Filter*** on A is a non_empty family $\mathcal{F}$ of fuzzy subsets for A having the following axioms(let us name it properties or conditions):

1. $\phi \notin \mathcal{F}$,that is $\mathcal{F} = \{A : \mu_A(x) \neq 0 \ \forall x \in X\}$.

2. if $F \in \mathcal{F}$ and $A \supset H \supset F$,that is $F \subset H \Rightarrow \mu_F \leq \mu_H$ $,\forall x \in X$, then $H \in \mathcal{F}$.

3. if $F \in \mathcal{F}$ and $H \in \mathcal{F}$,then $F \cap H \in \mathcal{F}$,that is $\mu_{F \cap H} = \min\{\mu_F, \mu_H\}, \forall x \in X$.

☼ It is clear that the filter of fuzzy set does not had an empty fuzzy set $\ni \mu(x) \neq 0$ $,\forall x \in X$.

☼ The nonempty means that family has at least one fuzzy subset say F of A and since for: $A \supset F$ or $F \subset A \Rightarrow A \in \mathcal{F}$

☼ In Third axiom ; if $F_1, F_2, ..., F_n$ are members in $\mathcal{F}$,then $F_1 \cap F_2 \cap ... \cap F_n$ is also a member of $\mathcal{F}$,Then by axiom(1); $F_1 \cap F_2 \cap ... \cap F_n \neq \phi$ .Then $\mathcal{F}$ has a ***Finite Intersection Property***(***FIP***),for more see(Sharma,1977);So suppose a fuzzy set A ,and the fuzzy subsets from it as $\{B_\lambda : \lambda \in \Lambda, \Lambda = \phi\}$ the intersections:

$$B = \cap\{B_\lambda : \lambda \in \Lambda, \Lambda = \phi\} \qquad \text{….(2)}$$

or as more simply $\quad B = \cap\{B_\lambda : \lambda \in \phi\}$ ….(3)

and $\quad \mu_{F_1 \cap F_2 \cap .. \cap F_n} = \min\{\mu_{F_1}, \mu_{F_2}, ..., \mu_{F_n}\} \neq 0 \ ,\forall x \in X$ ….(4)

Since B is the intersection of the empty subfamily of $\mathcal{F}$,So by **1** and **2**, $\mathcal{F}$ is closed for finite intersection .

☼ The ***Power Set*** $P(A)$ is not a fuzzy filter on a fuzzy set A .Since(By definition of power sets) $\phi \in P(A)$,Thus any fuzzy filter on A must be a proper subset of $P(A)$.

☼ If $F \in \mathcal{F}$,then $A - F \notin \mathcal{F}$ (not necessary if that $\mathcal{F}$ is a fuzzy point and crossover_point see reference(Bilgiç &et al,1995) $\mu_{A-F} = 1 - \mu_F$

If F and $A - F \in \mathcal{F}$,then by axiom(3); $\Rightarrow F \cap A - F$

$\Rightarrow \mu_{F\cap A-F} = \min\{\mu_F, 1-\mu_F\}$ either $\Rightarrow \neq \phi$ or $\Rightarrow = \phi$
that contradict with axiom(1);

In this case $\mu_F = \mu_{A-F}$ this condition not necessary satisfies .

In previous case the two fuzzy sets F and $A-F$ may be equals that is $\mu_F = \mu_{A-F} = 0.5$ special case of fuzzy sets that named crossover point the definition of free and fixed filter is not different on a crisp sets .

☼ In signal processing, a filter is a function or procedure which removes unwanted parts of a signal. The concept of filtering and filter functions is particularly useful in engineering. One particularly elegant method of filtering ***Fourier transforms*** a signal into frequency space, performs the filtering operation there, then transforms back into the original space (Weisstein,1999).

Definition 2: A fuzzy filter $\mathcal{F}$ on A is ***free fuzzy filter*** iff

$$\cap\{F : F \in \mathcal{F}\} = \phi \ \cap\{F_i : F_i \in \mathcal{F}, i = 1,...,n\} = \phi$$

that is $\mu_{\overset{n}{\underset{i=1}{\cap}} F_i} = \min\{\mu_{Fi} : i = 1,...,n\} = 0$, $\forall u \in F_i$,and ***fixed fuzzy filter*** iff

$$\cap\{F_i : F_i \in \mathcal{F}, i = 1,...,n\} \neq \phi \text{ that is } \mu_{\overset{n}{\underset{i=1}{\cap}} F_i} = \min\{\mu_{F_i} : i = 1,...,n\}$$

Here we give an example on a fuzzy filter:

Example 1: Suppose $X = [0,10]$ define a fuzzy set $A = \{u : u \text{ is about } 5\}$,for $u \in X$ as:

$$A = \{(5,1),(0,0),(2,0.5),(8,0.5),(6,0.9),(4,0.9),(7,0.7),(10,0),(1,0.2),(3,0.3),(9,0.1)\}$$

$$F_1 = about\ 5(from\ 3\ to\ 7), F_2 = about\ 5(from\ 0\ to\ 5),\ F_3 = about\ 5(at\ 5)$$

Defined as:

$F_1 = \{(5,1),(3,0),(7,0),(6,0.5),(4,0.5)\}$,

$F_2 = \{(5,1),(0,0),(3,0.3),(4,0.9),(2,0.5),(1,0.2)\}$,

$F_3 = \{(5,1)\}$,respectively.

A fuzzy filter $\mathcal{F}_f = \{F_1, F_2, F_3,\}, F_1, F_2, F_3 \neq \phi$.

$F_3 \subset F_1 \subset F_2$ and $F_1, F_2 \in \mathcal{F}_f$

At $F_1 \cap F_3 \Rightarrow \mu_{F_1\cap F_3} = \min\{\mu_{F_1}, \mu_{F_3} : \forall\ x\}$

$F_1 \cap F_3 = \{(5,1),(3,0),(7,0),(6.0),(4,0)\}$

$= \{(5,1)\} = F_3 \in \mathcal{F}_f$

Since when a membership function value equal to zero that which means a member is not belongs to that set(Reznik,1997),so it be excusable to not write it within the set.

So $\mathcal{F}_f$ is a fuzzy filter on A .

Definition 3: The ***indirect fuzzy filter*** on a non_empty fuzzy set say A is a filter that has only A ,that is $\mathcal{F} = \{A\}$.

Or in other apt word a filter on fuzzy set that has only that set is ***indirect fuzzy filter*** .

Definition 4: Let A be a non_empty fuzzy set and let $(u_0, \mu_F(u_0)) \in A$,then the family $\mathcal{F} = \{F : (u_0, \mu_F(u_0)) \in F\}$ is a filter called ***The discrete fuzzy filter.***

To prove at less this definition and his causes, $\mathcal{F}$ is non_empty since

$$(u_0, \mu_F(u_0)) \in \{(u_0, \mu_F(u_0))\} \Rightarrow \{(u_0, \mu_F(u_0))\} \in \mathcal{F}$$

◙Since $(u_0, \mu_F(u_0)) \in F$, $\forall F \in \mathcal{F}$,then no member of $\mathcal{F}$ is empty and so $\phi \notin \mathcal{F}$.

◙Let $F \in \mathcal{F}$ and $H \supset F$, $F \in \mathcal{F}$ impliesthat $\Rightarrow (u_0, \mu_F(u_0)) \in F$ .Since $F \subset H$ then $\mu_F(u) \le \mu_H(u)$, $\forall u \in X \Rightarrow \mu_F(u_0) \le \mu_H(u_0)$ for $u_0 \in X$,viz $F \in \mathcal{F}$ and $F \subset H$

$F \in \mathcal{F} \Rightarrow (u_0, \mu_F(u_0)) \in F$

Since $F \subset H \Rightarrow (u_0, \mu_F(u_0)) \in F \subset H$

$\Rightarrow (u_0, \mu_F(u_0)) \in H$

$\Rightarrow H \in \mathcal{F}$.

◙Let $F \in \mathcal{F}$ and $H \in \mathcal{F}$ to get $F \cap H \in \mathcal{F}$ $\Rightarrow (u_0, \mu_F(u_0)) \in F$ and $(u_0, \mu_F(u_0)) \in H$ then $\Rightarrow (u_0, \mu_F(u_0)) \in F \cap H$,that is $\mu_{F \cap H}(u_0) = \min\{\mu_F(u_0), \mu_H(u_0)\}$ we have three cases:

If $\mu_F(u_0) = \mu_H(u_0) \Rightarrow \mu_{F \cap H}(u_0) = \mu_F(u_0) \Rightarrow (u_0, \mu_F(u_0)) \in F \cap H \Rightarrow F \cap H \in \mathcal{F}$.

If $\mu_F(u_0) < \mu_H(u_0) \Rightarrow \mu_{F \cap H}(u_0) = \mu_F(u_0) \Rightarrow (u_0, \mu_F(u_0)) \in F \cap H \Rightarrow F \cap H \in \mathcal{F}$.

If $\mu_F(u_0) > \mu_H(u_0) \Rightarrow \mu_{F \cap H}(u_0) = \mu_H(u_0)$ since $H \in \mathcal{F} \Rightarrow \mu_H(u_0) = \mu_F(u_0)$

all these cases led to the same solution $\Rightarrow (u_0, \mu_F(u_0)) \in F \cap H \Rightarrow F \cap H \in \mathcal{F}$

◘ This recover a question seems requisite what happen if there is filter at each point in arbitrary set .

Note: The indiscrete filter means a point $(u_0, \mu_F(u_0))$ is within every set in this filter , such the sets named a fuzzy point see((Bilgiç & et al,1995).

This example is satisfied for crisp sets(to be with inform see the reference(Sharma1977)so here the view for fuzzy sets

Example 2: Let X be any non empty set and A be a fuzzy set on empty fuzzy set on it , let F_0 be a non empty fuzzy subset of A ,then the family $\mathcal{F} = \{F : F \supset F_0\}$ is a filter on A **.**

Note: This application is represent the extension of definition of discrete on single point in a set into a set of points ,discrete with large partitions .

Solution: Since $F_0 \in \mathcal{F}$, $\mathcal{F}$ is non_empty.

◙Since $F_0 \ne \phi$, $F_0 \supset F_0 \in \mathcal{F}$ or $F_0 \subset F_0 \in \mathcal{F}$ so $F \ne \phi, \forall F \in \mathcal{F} \Rightarrow \phi \notin \mathcal{F}$

◙Let $F \in \mathcal{F}$ and $H \supset F$ .Now $F \in \mathcal{F}$ implies that $F \supset F_0$ $\mu_{F_0}(u) \le \mu_F(u), u \in X$,

And since $H \supset F \Rightarrow H \supset F \supset F_0 \Rightarrow H \supset F_0$

$\mu_{F_0}(u) \le \mu_F(u) \le \mu_H(u) \Rightarrow \mu_{F_0}(u) \le \mu_H(u)$

◙Let $F \in \mathcal{F}$ and $H \in \mathcal{F}$ then $F \supset F_0$ and $H \supset F_0$ so $\mu_{F_0}(u) \le \mu_F(u)$ and $\mu_{F_0}(u) \le \mu_H(u)$ .Hence $F \cap H \supset F_0$ i.e. $\mu_{F_0}(u) \le \min\{\mu_F(u), \mu_H(u)\}$

it follows that $F \cap H \in \mathcal{F}$.

Definition 5: ***Cofinite Filter*** .Let X be any infinite set ,and A be an infinite fuzzy set defined on X .Then $\mathcal{F} = \{F : F \subset A, A - F \text{ is finite}\}$ is a fuzzy filter on A called the cofinite filter(or as named for crisp sets as ***Fréchet filter*** on A) .

That is the complement of an infinite fuzzy set is finite communicated in filter .

◙For relative complement note ;Since $A - A \Rightarrow \mu_A(u) - \mu_A(u) = 0 \Rightarrow A - A = \phi$ is finite, $A \in \mathcal{F}$ and hence $\mathcal{F}$ is non empty ,Further ;

◙Since A is infinite and $A-F$ is finite $\Rightarrow 0 \le \mu_A(u)-\mu_F(u) < 1\ \forall(u,\mu_A(u)) \in A$ it follows that F is an infinite set ,so no member of $\mathcal{F}$ is empty, Hence $\phi \notin \mathcal{F}$.

◙Let $F \in \mathcal{F}$ and $H \supset F$ .Since $A-F$ is finite ,then $A-(F \subset H)$
$H \supset F \Rightarrow A-H \subset A-F \Rightarrow A-H$ is finite $\Rightarrow H \in \mathcal{F}$
$A-F \Rightarrow \mu_A - \mu_F$, $A-H \Rightarrow 0 \le \mu_A - \mu_H < 1$.

◙Let $F \in \mathcal{F}$ and $H \in \mathcal{F}$,then $A-F$ and $A-H$ are both finite .
Now
$A-(F \cap H) = (A-F) \cup (A-H)$ $(\mu_A - \mu_{F\cap H})(u) = \mu_A(u) - \min\{\mu_F(u), \mu_H(u)\}$ that is either $\mu_A(u)-\mu_F(u)$ or $\mu_A(u)-\mu_H(u)$

$$(A-F)\cup(A-H) = \max\{\mu_A(u)-\mu_F(u), \mu_A(u)-\mu_H(u)\}$$
$$= [\mu_A(u)-\mu_F(u)] \cup [\mu_A(u)-\mu_H(u)]$$

Intersection of two finite set is also a finite set ,also a union is a finite set .

¤ ***De_Morgan's laws*** are satisfies for fuzzy logic as one of the most important principles on sets as in references by (Thuillard,2000) see these it so fruitful in this side.

¤ $A \cap B \Rightarrow \mu_{A\cap B} = \min\{\mu_A, \mu_B\}$

$$\sim(A\cap B) \Rightarrow 1-\mu_{A\cap B} = 1-\min\{\mu_A,\mu_B\} = \max\{1-\mu_A, 1-\mu_B\}$$

¤Here $\left.\begin{array}{l} \sim A \Rightarrow 1-\mu_A \\ \sim B \Rightarrow 1-\mu_B \end{array}\right\} \Rightarrow \sim A \cup \sim B = \max\{1-\mu_A, 1-\mu_B\}$

3. Comparison of Fuzzy Filters:

Definition 6: Let $\mathcal{F}$ and $\mathcal{F}'$ be two fuzzy filters on the same fuzzy set A (that were defined on a crisp set X),then $\mathcal{F}$ is said to be finer than $\mathcal{F}'$ or $\mathcal{F}'$ is coarser than $\mathcal{F}$ iff $\mathcal{F}' \subset \mathcal{F}$.

Note: The using of filter on a fuzzy set reduce some of its work in restrict the fuzzy values results .

◙If in addition, $\mathcal{F} \ne \mathcal{F}'$,then $\mathcal{F}$ is said to be strictly finer than $\mathcal{F}'$ or $\mathcal{F}'$ is strictly coarser than $\mathcal{F}$.

◙Two fuzzy filters $\mathcal{F}$ and $\mathcal{F}'$ are said to be comparable iff $\mathcal{F}$ is finer than $\mathcal{F}'$ or $\mathcal{F}'$ is coarser than $\mathcal{F}$.

◙This definition is not different on that one on a crisp sets since here it deals with sets of sets and the features of fuzzy set on the general set properties of fuzzy sets .

3. Fuzzy Filter Bases

Definition 7: Let X be any non_empty set , A is a fuzzy set on X .A filter base on A is a non empty family B of subsets of A satisfying the following axioms :

1. $\phi \notin \mathcal{B}$

2.If $F \in \mathcal{B}$ and $H \in \mathcal{B}$ then there exists a $G \in \mathcal{B}$ such that $G \subset F \cap H$.

$$F = \{(f, \mu_F(f)) : f \in X\}$$
$$H = \{(h, \mu_H(h)) : h \in X\}$$
$$\mu_{F\cap H}(h) = \min\{\mu_F(h), \mu_H(h)\} \text{ if } \mu_{F\cap H}(h) = \mu_H(h) \Rightarrow (h, \mu_H(h)) \in F \cap H$$

◙The intersection of a finite subfamily of B contains a member of $\mathcal{B}$. In other words , $\mathcal{B}$ has ***FIP*** ,which follows from second axiom by finite induction .

◙Every fuzzy filter is a fuzzy filter base .If $\mathcal{F}$ is a fuzzy filter ,the first axiom is arbitrary has been satisfied ,also if $F \in \mathcal{F}$ and $H \in \mathcal{F}$ then $F \cap H \in \mathcal{F}$.
Since $F \cap H \subset F \cap H$ the second once is satisfied .
For $F \cap H \Rightarrow \mu_{F \cap H} = \min\{\mu_F, \mu_H\}$ either $\mu_{F \cap H} = \mu_F$ or $\mu_{F \cap H} = \mu_H$.
If any one from these cases it is satisfy the second condition for fuzzy filter base.
Example 3: The family $B = \{A\}$,where $A \neq \phi$ (non void fuzzy subset)of a non void set X is a fuzzy filter base on X (under fuzzy logic) .
Solution: Since A is the single member in B and since A is non void, $A \neq \phi \Rightarrow \phi \notin B$.
Again since $A \subset A \cap A \Rightarrow A \in B$ .Hence B is a fuzzy filter base on X .
Theorem 2: Let B be a family of subsets on a fuzzy set A .Then B is a fuzzy filter base on A iff the family $\mathcal{F}$ consisting of all those subsets of A which contain a member of B is a fuzzy filter on A (see proof of this theorem for classical sets in(Sharma1977)).
Theorem3: On a fuzzy set A on a set X ,a fuzzy filter $\mathcal{F}$ with base B is finer than a fuzzy filter $\mathcal{F}'$ with base B' iff every member of B contains a member of B' .
Proof: In other word we want to prove this $\forall B \in \mathcal{B}, \exists B' \in \mathcal{B}' \ni B' \subseteq B$.
If Part Let every member of B contains a member of B', hypothesis any $B \in \mathcal{B}$, $\exists B' \in \mathcal{B}' \ni B' \subseteq B$ $\mu_B(a) \leq \mu_{B'}(a)$,and let F be any member of $\mathcal{F}$ ($F \in \mathcal{F}$).
Then by definition in the reference(Sharma,1977,no.6.10 p482) F contains a member of B say B ($B \subset F$)
So by hypothesis($\exists B' \in B \ni B' \subset B \Rightarrow \mu_B(a) \leq \mu_{B'}(a)$, $a \in X$)
Hence F contains B' and consequently by same previous definition, $F \in \mathcal{F}'$ ($B \subset F, B' \subset B \Rightarrow B' \subset F$) $\Rightarrow \mu_B(a) \leq \mu_F(a), \mu_{B'}(a) \leq \mu_B(a)$

$$\Rightarrow \mu_{B'}(a) \leq \mu_F(a)$$

Thus we have shown that $F \in \mathcal{F} \Rightarrow F \in \mathcal{F}'$.
Hence $\mathcal{F} \subset \mathcal{F}'$,that is $\mathcal{F}'$ is finer than $\mathcal{F}$.
(Number of sets are larger than that in $\mathcal{F}$)
Only If Part Assume that $\mathcal{F}'$ is finer than $\mathcal{F}$ (i.e. $\mathcal{F} \subset \mathcal{F}'$) and let B be any member of F $\Rightarrow B \in F$,since $B \subset \mathcal{F}$.
Since $\mathcal{F}'$ is finer than $\mathcal{F}$ we have $\Rightarrow B \in \mathcal{F}'$.
Since B' is a fuzzy base of $\mathcal{F}'$ ($B' \subset \mathcal{F}'$) there exists a $B' \in B'$ such that $B' \subset B$ (i.e. $\mu_{B'}(a) \leq \mu_B(a)$)
Thus every member of B contains a member of B' .
Here an important question occurred its solution may be change many principles for fuzzy filters or led to more thoughts on this subject ,let us describe this question through this theorem .
This raise a question here, Did the intersection(conjunction) of fuzzy filters $= \phi$ true or false ?
Theorem 4: Two fuzzy filters bases B and B' on a fuzzy set A on a set X are equivalent iff every member of B contains a member of B' and every member of B' contains a member of B.
There is a proof for this theorem equivalent to that one on crisp sets will occurred later here an other one .
Proof 1: If Part $\forall B \in B, \exists B' \in B' \ni B' \subset B$.
So $\mu_{B'} \leq \mu_B$ ….(1)
Only If $\forall B' \in B', \exists B \in B \ni B \subset B'$.

So $\mu_B \le \mu_{B'}$ ….(2)

From (1)and(2) we get $\mu_{B'} = \mu_B$

So $B' = B$ $\forall B' \in B'$ and $B \in B$ then $B' = B$

This proof is a special case for only true in just supposing B may equal to B'

Proof 2: The proof of this theorem is an immediate corollary of the theorem (satisfied for crisp sets) and will proof it for fuzzy sets .

Proof 3: Assume (the first part of condition) that every member of B contains a member of B' and every member of B' contains a member of B.

$$\forall B \in B, \exists B' \in B' \ni B' \subset B \text{ and } \forall A' \in B', \exists A \in B \ni A \subset A'.$$

Let $\mathcal{F}'$ and $\mathcal{F}$ be the filters generated by B' and B ,respectively.

Let $F \in \mathcal{F}$,then there exists a $B \in B \ni B \subset F$ .By hypothesis, there exists a $B' \in B'$ such that $B' \subset B$

$B' \subset B, \ni B \subset F \Rightarrow$ It follows that $B' \subset F$ .Hence $F \in \mathcal{F}'$ .Thus from $F \in \mathcal{F} \Rightarrow F \in \mathcal{F}'$ and hence $\mathcal{F} \subset \mathcal{F}'$.

This theorem can said to as a two fuzzy filters can be equivalence if they have same fuzzy base or same equivalence fuzzy bases .

Similarly we can show that $\mathcal{F}' \subset \mathcal{F}$,therefore $\mathcal{F} = \mathcal{F}'$ .thus B and B' generate the same filters and so they are equivalent .

Remark: *The fuzzy filter can be generated by two bases ,what about more than two bases (try proving this lifted as new work).*

Conversely ,assume that B and B' are equivalent so that they generate the same filter say $\mathcal{F}$.

Let B be an arbitrary member of B ($B \in B$)

Then $B \in \mathcal{F}$ since B' is also a base of $\mathcal{F}$,so there exists $B' \in B'$ such that $B' \subset B$.

Thus every member of B contain a member of B' .

Similarly it can be shown that every member of B' contains a member of B .

4.Conclusions:

In this study(work) the proposal built would ensure that the crisp set satisfies some property the fuzzy set will also satisfies (even not all) .

This work built into most of the principles for topology filters discussed in most topology books ,It is not as easy to satisfy

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