

## Synchronization Capabilities of Heterodyne Optical Solid-State Laser Receiver Based On OPLL

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### **Abstract**

The coherent optical heterodyne receiver for a solid-state laser based on Optical Phase-Locked-Loop (OPLL) is investigated. The phase shift effect of IF-stage at the output laser frequency is proposed under noise terms conditions: shot noise and the components of laser frequency noise. Integral expressions in terms of delay-bandwidth product  $\omega_n \tau_d$  are derived to specify the optimal loop bandwidth required to minimize the phase error variance for various values of carrier-to-noise ratios ( $A^2/N_o$ ) in dB-Hz. The obtained results reveal that the synchronization limits of 0.25 (rad) maintain the standard deviation of phase error to be less than  $10^\circ$  and BER of  $10^{-9}$  for  $\zeta=0.707$ . Further, the normalized degradation factor ( $\Delta_{min}$ ) of the total phase error variance is also determined to be less than 1.6.

### **إمكانات تزامن المستلمة الضوئية الهيتروداينية ذات ليزر صلب المعتمدة OPLL**

#### **الخلاصة**

تم تقصي المستلمة الضوئية نوع هيتروداين ذات ليزر-صلب للاسترجاع التزامني المعتمدة دائرة قفل الطور الضوئية. الدراسة تقتفي تأثير إزاحة الطور لمرحلة IF-stage في تردد إخراج الليزر تحت شروط من الضوضاء: الضوضاء الحراري و ضوضاء shot ومركبات ضوضاء تردد الليزر. تم اشتقاق تعبير تكاملية بدلالة نسبة حاصل ضرب التأخير الزمني-سعة النطاق الترددي  $\omega_n \tau_d$  لتحديد سعة النطاق الترددي المثالية من أجل تقليل تغيرية فرق الطور لحالات مختلفة من نسب إشارة الحامل-إلى-الضوضاء ( $A^2/N_o$ ) المقاسة بـ (dB-Hz). النتائج المستحصلة تبين حدود التزامن 0.25 للمحافظة على بقاء معيارية انحراف فرق الطور أقل من  $10^\circ$  واحتمالية خطأ (BER) بقيمة  $10^{-9}$  (أي جهد العتبة) في حالة  $\zeta=0.707$ ، بالإضافة على تحديد معامل التدهور ( $\Delta_{min}$ ) لتغيرية فرق الطور التي لا تتجاوز 1.6.

## **1- Introduction**

A number of publications on the analysis of coherent optical systems in the presence of noise terms are based on Optical Phase-Locked-Loop (OPLLs) [1-7]. Various types of loops have been analyzed and their corresponding advantages and dis-advantages discussed in detail for hetero- and homodyne receivers. There are two advantages for coherent heterodyne technology over (non-coherent) direct detection systems: increased receiver sensitivity by 10 to 20 dB and better rejection capabilities of noise. These advantages make coherent optical systems attractive for both long-haul transmission and for broadband local networks. Furthermore, OPLLs are extensively used to achieve timing synchronism with the weak incident signal power. For instance, recently by locking together three external-cavity lasers, 33-40.5GHz signals are produced with total phase variance of  $2.5 \times 10^{-4} \text{ rad}^2$  (very low phase noise). To our knowledge, the generated frequencies are twice as high as any generated previously with the semiconductor laser OPLL's of solitary-diode lasers of 18 GHz (variance of  $0.04 \text{ rad}^2$ ) and the phase variance is an order of magnitude lower than the external-cavity laser of 9 GHz with variance of  $4 \times 10^{-3} \text{ rad}^2$  [5-8]. Recently, Yifeng [9], for example, developed a novel phase noise control approach, coined an "Optical Frequency Locked Loop" (OFLL), to achieve low noise operation. Unlike conventional techniques of OPLLs, this scheme utilizes a long fiber delay in place of an external reference oscillator to correct the phase error. Further, the master oscillator is either a single-frequency Nd:YAG solid-state laser or a distributed-feedback fiber laser. The power amplifier is a diode-laser-pumped double-clad Nd doped fiber with

polarization control, 20 dB gain, and about 1.3 W output power [10]. On the other hand, the recent advances in diode-pumped solid-state laser technology encourages to use sub-kilohertz line width frequency stabilized lasers in applications of free-space optical systems to maintain synchronization and retrieving of the received signal phase in the presence of weak incident signal power [3-9].

In the light of these studies, this paper elucidates the effect of IF-stage incorporated with heterodyne optical receiver based on OPLL associated with Light wave model 120-01A frequency stabilized diode-pumped solid-state laser [1,5]. In such a model, the received optical signals detected uses a balance detector to cancel the LO laser intensity noise. The IF signal is filtered through an IF band-pass filter and mixed down to base band using RF mixer and 30 MHz stable frequency reference. The error signal at the mixer output is filtered by a loop filter to estimate the phase error between the received and LO laser signals. The output signal of the loop filter is then fed back into frequency tuning input of LO laser. Nevertheless, the frequency of Light wave model 120-01A lasers is achieved through its two BNC inputs and the continuous frequency tuning is possible over a range of 16 GHz [5] by applying a voltage to the thermal BNC input. In consequence, the excess phase shift of IF-stage (delay-bandwidth product  $\omega_n \tau_d$ ) in the transient time of the error signal and its effect on the phase error variance is investigated to optimize the loop performance and to specify the normalized degradation in the phase error variance for several values of carrier-to-noise ratios ( $A^2/N_o$ ) in dB-Hz. In addition, the model under consideration assumes the frequency noise spectral density of solid-

state laser as a white noise, a  $1/f$  and a strong  $1/f^2$  components at the laser output.

It is worth noting here that the analysis given in this paper assumes linear approximation model to obtain optimal straight forward design for an OPLL with laser frequency noise model. The obtained results reveal that the theoretical predictions of the optimal synchronization limit is about 0.5 (rad) in the case of  $\zeta=1/\sqrt{2}$ .

This paper is organized as follows: In Section 2, a description of the system is given together with the signal processing analysis for calculating loop performance in the presence of excess phase shift due to IF-stage. The behavior of OPLL is discussed in **Section 3** and the synchronization limit is also obtained to maintain the loop in tracking mode. Moreover, optimizing the parameters of OPLL such as the optimal loop bandwidth will help to reduce the effect of phase shift by calculating the total minimum standard deviation of phase error ( $\sigma_{\phi_e}$ ) as a function of a carrier signal-to-noise ratio ( $A^2/N_o$ ). The discussion concerning the effect of loop delay on OPLL performance is presented in **Section 4**. Further, a degradation of the loop performance is normalized for shot noise, white noise,  $1/f$  and  $1/f^2$  components at the laser output. **Section 5** involves the simulated results for two cases of damping factor. Finally, **Section 6** contains the conclusion of this paper.

## 2- Noisy OPLL Model

A block diagram of Optical Phase-Locked Loop (OPLL) and its equivalent model is shown in **Figure 1**. A second order linearized OPLL is described as [5],

$$\frac{d\phi}{dt} = F_N(t) - K_p \int_{-\infty}^{+\infty} [A \phi_e(\tau) + n(\tau)] F_P(t-\tau) d\tau \delta(t-\tau_d) \quad (1)$$

where,

$\phi_e$  = the phase error,

$A^2$  = the average IF signal power,

$N(t)$  = the additive noise,

$K_p$  = tuning constant of the local oscillator (LO) laser,

$F_P(t)$  = impulsive response of the loop filter,

$F_N(t)$  = frequency noise process due to both the received signal and LO laser,

$\delta(t-\tau_d)$  = Dirac delta function, and  $\tau_d$  is the overall delay incorporated within the loop due to IF-stage.

The loop filter  $F_P(t)$  is taken to be first order active filter  $F_P(s) = (1+s\tau_2)/s\tau_1$ , with  $\tau_1$  and  $\tau_2$  being arbitrary time constants. This allows the loop natural frequency  $\omega_n$  and damping factor  $\zeta$  to be independently defined as,

$$\omega_n = \left( \frac{AK_p}{\tau_1} \right)^2 \quad (2-a) \text{ and}$$

$$\zeta = \frac{\tau_2 \omega_n}{2} \quad (2-b)$$

Here, in terms of frequency acquisition, a perfect second-order loop is chosen with that loop filter to provide an infinite pull-in range as compared with first-order and imperfect second-order loops. Therefore, the closed loop transfer function is defined as,

$$H_1(s) = \frac{AK_p F_p(s) e^{-s\tau_d}}{s + AK_p F_p(s) e^{-s\tau_d}} \quad (3)$$

Further, the transfer function  $H_2(s)$  can be obtained from  $H_1(s)$  to define the phase-error by,

$$H_2 = [1 - H_1(s)] \frac{1}{s} \quad (4)$$

On the other hand, the overall performance is affected by shot noise and frequency noise at the input signal. When the loop is operating in linear region, the effect of each noise can be determined separately and combined to obtain the desired result. Thus the total phase-error variance can be obtained as,

$$\sigma_{\phi_e}^2 = \sigma_{\phi_n}^2 + \sigma_{\phi_f}^2 \quad (5)$$

where  $\sigma_{\phi_n}^2$  and  $\sigma_{\phi_f}^2$  being phase-error variances due to additive noise and frequency noise respectively.

### 3- OPLL Design and Performance

This section includes the derivation of the total phase-error variance  $\sigma_{\phi_e}^2$ . As a fact, the ability of a phase-locked receiver to retrieve an incoming signal with minimum error is directly proportional to its ability to track the phase of that signal. If there is any error in that tracking, then a degradation of the loop performance will occur. When the phase error is assumed to be Gaussian, then the standard derivation of the phase error should be less or equal  $10^\circ$  to maintain 0.5 dB power penalty over the shot noise limit at a bit error rate (BER) of  $10^{-9}$ . Also, if  $\sigma_{\phi_e} > 12.5^\circ$ , then a BER of  $10^{-9}$  cannot be achieved [3,5]. Therefore, the addition noise  $n(t)$  due to the LO shot noise can be modeled as an addition WG noise with PSD of  $N_o$ . Then

$$\sigma_{\phi_n}^2 = \frac{N_o}{A^2} B_L \quad (6)$$

$B_L$  represents the one sided-loop bandwidth defined by,

$$B_L = \int_0^\infty |H_1(j2\pi f)|^2 df \quad (7)$$

Equation 6 takes the phase shift of IF-stage into account as a delay-bandwidth product

and then the phase error variance is contributed the frequency noise terms with Power Spectral Densities (PSD)s of the three components: a  $1/f$ ,  $1/f^2$  components at low frequencies and a White frequency noise. The mathematical model of this PSD frequency noise can be defined as [5],

$$S_f(f) = K_o + \frac{K_1}{f} + \frac{K_2}{f^2} \quad 0 < f < \infty \quad (8)$$

$K_o$ ,  $K_1$  and  $K_2$  are the parameters associated with the frequency noise model to represent the frequency noise floor of Light wave no planar ring oscillator laser. As a result, the phase error variance contributed by the frequency noise can be expressed as,

$$\sigma_{\phi_f}^2 = \int_0^\infty |H_2(j2\pi f)|^2 S_f df \quad (9)$$

Finally, using equation 5, 6 and 9 the total error variance can be written as

$$\sigma_{\phi_e}^2 = \frac{N_o}{A^2} B_L + \omega_{\phi_o}^2 + \sigma_{\phi_1}^2 + \sigma_{\phi_2}^2 \quad (10)$$

where  $\sigma_{\phi_o}^2$ ,  $\sigma_{\phi_1}^2$  and  $\sigma_{\phi_2}^2$  are the phase error variances due to the three components of frequency noise at the laser output. These terms can be defined as follows

$$\begin{aligned} \sigma_{\phi_o}^2 &= \frac{N_o}{2\pi} \int_0^\infty |H_2(j\omega)|^2 d\omega \\ \sigma_{\phi_1}^2 &= K_1 \int_0^\infty |H_2(j\omega)|^2 \frac{d\omega}{\omega} \\ \sigma_{\phi_2}^2 &= 2\pi K_2 \int_0^\infty |H_2(j2\pi f)|^2 \frac{df}{f^2} \end{aligned} \quad (11)$$

Equation 11 can be rewritten in another simplified integral expression to compute  $\sigma_{\phi_e}^2$ ,

$$\sigma_{\phi_e}^2 = \left( \frac{N_o}{A^2} \right) R_d B_{LP} + \frac{K_o'}{B_{LP}} G_o(x) + \frac{K_1'}{B_{LP}^2} G_1(x) + \frac{K_2'}{B_{LP}^3} G_2(x)$$

(12) where,

$$\begin{aligned} K_o' &= K_o (1 + 4\zeta^2) / 8\pi\zeta \\ K_1' &= K_1 (1 + 4\zeta^2)^2 / 32\zeta^2 \\ K_2' &= \pi (1 + 4\zeta^2)^3 / 128\zeta^3 \end{aligned}$$

(13)

From equation 12, the loop parameters are written in terms of phase-shift error,  $R_d = B_L / B_{LP}$ , where  $2B_L / \omega_n = (1 + 4\zeta^2) / 4\zeta$  represents the loop-bandwidth of standard OPLL, and all  $G_o(x)$ ,  $G_1(x)$  and  $G_2(x)$  are also functions of phase-shift  $\omega_n \tau_d$  (See Appendix A).

However, the optimal loop performance can also be determined including the optimal loop bandwidth  $B_{LP}$  (represented as  $B_{opt}$ ) to estimate the minimum phase error variance. By differentiating eqn. 12 with respect to  $B_{LP}$ , the optimal loop bandwidth ( $B_{opt}$ ) can be numerically computed as a function of  $A^2/N_o$ ,  $\zeta$  and  $\omega_n \tau_d$ . With substituting the  $B_{opt}$  in equation 12 a minimum phase error variance  $\sigma_{\phi_e(\min)}^2$  is obtained as a function  $B_{opt}$ ,  $A^2/N_o$  and  $\omega_n \tau_d$ .

#### **4-Degradation of the loop performance**

It is clear that the laser diodes either semiconductors or solid-state lasers with external grating cavities [1-7] are the most suitable sources for coherent optical systems, being continuously tunable and having line widths of the order of 10-50kHz. Therefore, they don't suffer from effects of propagation delays whilst coherent systems with line widths of the order of 10MHz are likely to experiences

propagation delay problems. In this paper, if  $\tau_d$  is allowed to take on some non negligible value, then its effects on the loop performance can be evaluated in terms of delay bandwidth product  $\omega_n \tau_d$  to estimate the minimum phase error variance.

Nevertheless, the normalized variance degradation factors for the contribution of shot noise and the frequency noise model components in the solid-state lasers are also obtained with respect to the negligible model ( $\tau_d=0$ ). From equations 12 and 13 these factors for the shot noise and the frequency noise model are obtained as,

$$\Delta_n = B_L / B_{LP} \quad (14-a)$$

$$\Delta_o = \frac{8\zeta}{\pi} G_o(x) \quad (14-b)$$

$$\zeta < 1 \quad \left\{ \begin{aligned} & \frac{16\zeta\sqrt{1-\zeta^2}}{\pi} \left[ \pi/2 - \tan^{-1} \frac{\zeta - 1/2}{\zeta\sqrt{1-\zeta^2}} \right] G_1(x) \\ & \Delta_1 = \left( \frac{128}{25} \right)^2 G_1(x) \end{aligned} \right.$$

$$\zeta = 1 \quad \left\{ \begin{aligned} & \frac{16\zeta\sqrt{\zeta^2-1}}{\ln \left[ \frac{\zeta^2 - 0.5 + \zeta\sqrt{\zeta^2-1}}{\zeta^2 - 0.5 - \zeta\sqrt{\zeta^2-1}} \right]} G_1(x) \end{aligned} \right.$$

$$\zeta > 1 \quad (14-c)$$

c)  
and,

$$\Delta_2 = \frac{8\zeta}{\pi} G_2(x) \quad (14-d)$$

d)

and for  $\zeta = 1/\sqrt{2}$  the  $\Delta_1$  can be expressed as  $\Delta_1 = 8\zeta G_1(x)/\pi$  and in consequence the normalized degradation factor  $\Delta_{\min}$  of the total phase error variance is determined as a ratio of  $\sigma_{\min}$  at  $\tau_d \neq 0$  to  $\sigma_{\min}$  of negligible  $\tau_d$  model. This factor is used to illustrate the actual loop performance as compared with that of semiconductor lasers proposed in [5].

As a result, the limits on the loop stability can be established as in [2] using standard Nyquist technique. In particular, for  $\zeta = 1/\sqrt{2}$  the limit on the stability occurs when the product  $\omega_n \tau_d$  is less than 0.736 (42.2°) to maintain the loop in-lock. However, in our model this limit will vary according to the BER of  $10^{-9}$  to maintain the standard deviation of the phase error  $\sigma_{\min}$  to be less than  $10^\circ$ . The results are illustrated in **Figures 2-6**.

### 5- The results and Discussion

This section is devoted to present illustrative results showing the performance of the model under consideration using linearized OPLL disturbed by shot noise and frequency noise model. The parameters associated with the noise terms are considered as  $K_0=0.2\text{Hz}$ ,  $K_1=1.5 \times 10^4 \text{Hz}^2$  and  $K_2=1 \times 10^7 \text{Hz}^3$  for White, a  $1/f$  and  $1/f^2$  components, respectively. The results are computed emphasizing the optimization of the loop performance for two cases of  $\zeta = 0.707$  and 1.5.

**Figure 2** depicts the minimum standard deviation of the phase error  $\sigma_{\min}$  for various values of delay-bandwidth product  $\omega_n \tau_d$  and carrier-to-noise ratios ( $A^2/N_0$ ) in dB-Hz, respectively. **Figure 3** displays optimal loop bandwidth required  $B_{\text{opt}}$  (in Hz) for two cases of damping factor ( $\zeta$ ). The normalized degradation variances of

the noise terms contributions: shot noise ( $\Delta_{\phi_n}$ ), White noise ( $\Delta_o$ ), a  $1/f$  ( $\Delta_1$ ) and  $1/f^2$  ( $\Delta_2$ ) components are also illustrated in **Figure 4**. Further, a comparison of the total minimum normalized phase error variances ( $\Delta_{\min}$ ) is established between 50dB-Hz and 100dB-Hz in **Figure 5**. Finally, **Figure 6** introduces an example for the phase error variance that can be accepted when the lower-values of  $A^2/N_0$  are considered for instance 50 and 60 dB-Hz in both cases of negligible and non negligible  $\tau_d$  models with loop bandwidth range 400Hz-20kHz.

Investigating the figures highlights the following significant facts:

- 1) The propagation delay should be not exceeded a certain limit to maintain the loop in-lock, for instance when  $\zeta = 0.707$  the delay-bandwidth product is less than 0.736 (42.2°) and for 1.5 it is 0.4.
- 2) The optimal loop bandwidth ( $B_{\text{opt}}$ ) increases as  $\zeta > 1$  (over damping) and a good tracking is achieved by 3 widening loop bandwidth with minimum required value (**Figure 3**).
- 3) Increasing delay-bandwidth product factor will increase  $\sigma_{\min}$  as  $A^2/N_0$  decreases,. Whilst  $\sigma_{\min}$  can be less than  $10^\circ$  for BER of  $10^{-9}$  when  $A^2/N_0$  equals 50 dB-Hz and then the synchronization limit should be less than 0.25 (rad) to keep the loop in-lock. As a result, for more safety design  $A^2/N_0$  can be greater than the value of 60 dB-Hz without exceeding the stability condition of 0.736 in the case of  $\zeta = 0.707$  as shown in **Figure 2**.
- 4) For high values of  $A^2/N_0$ , the loop bandwidth required ( in kHz) should be

widened to maintain the phase error tracking but the excess delay will increase it rapidly that the loop may tends to loss its lock. For this reason, the optimization is employed to specify the minimum loop bandwidth required.

- 5) It is found that either semiconductors or solid-state lasers with external grating cavities are the most suitable sources for coherent optical systems being continuously tunable and having line widths of the order or 10-50 kHz. If  $\tau_d$  is taken into account as a phase shift of the IF-stage in heterodyne (Light wave model 120-01A diode-pumped solid-state laser) optical receiver then the delay-bandwidth product ( $\omega_n \tau_d$ ) is introduced as a serious parameter for low values of  $A^2/N_o$  less or equal to 50 dB-Hz (very weak incident signals power).
- 6) The lower-values of degradation  $\Delta_{\phi_n}$ ,  $\Delta_o$ ,  $\Delta_1$  and  $\Delta_2$  can be obtained in the case of  $\zeta = 0.707$  as compared with 1.5. The result of  $\Delta_{\phi_n}$  and  $\Delta_o$  are matched with that of [5] for shot noise and phase noise in semiconductor laser of homodyne optical receiver (**Figure 4**).
- 7) The ( $\Delta_{\min}$ )s should be not exceeded 1.6 to maintain BER of  $10^{-9}$  so that for  $\zeta = 1.5$  the range of synchronization limit becomes 0.25 while for  $\zeta = 0.707$  it achieves 0.5 (rad) to keep the loop in stable-state.

$A^2/N_o$ (dB-Hz)	$\sigma_{\phi_e}$ (deg.) for $B_{LP}=3\text{kHz}$	
	No delay	Delay-bandwidth product (0.2 rad)
50	$10^\circ$	$12^\circ$
60	$3.5^\circ$	$4^\circ$

Finally, low-values of  $A^2/N_o$  introduces that the approximation mode throughout

the analysis becomes invalid and these values are undesirable in practical systems. For instance, a few kilo-Hertz of loop bandwidth such as 3 kHz allows the beat signals to be generated with total phase error variances as shown in **Table (1a)**. Further, **Table (1b)** illustrates the resultant  $\sigma_{\phi_e}$  may exceed  $12.5^\circ$  ( $0.047 \text{ rad}^2$ ) and then the BER of  $10^{-9}$  cannot be achieved.

## 6- Conclusions

The paper emphasizes three problems: shot noise, frequency noise contributions (a White, a  $1/f$  and a  $1/f^2$  components) and delay-product factor of IF-stage in heterodyne optical receiver for diode-pumped solid-state lasers based on linearized OPLL. The excess phase shift of non negligible  $\tau_d$  model has a serious role when low-values of  $A^2/N_o$  below 60 dB-Hz are considered. As a result, OPLL can be designed successfully to operate under weak incident signal power with optimal tracking bandwidth. Integral expressions in terms of delay-product factor are derived and numerically computed to specify the optimal loop bandwidth required to minimize the phase error variance for various values of carrier-to-noise ratios ( $A^2/N_o$ ). In general, it can be concluded that for all  $A^2/N_o$  ratios (50-100 dB-Hz), the synchronization limit is 0.25 to maintain  $\sigma_{\min}$  to be less than  $10^\circ$  with BER of  $10^{-9}$  and  $\Delta_{\min}$  of 1.6. Furthermore, for more safety design, the theoretical reveal that the OPLL can operate with  $A^2/N_o > 60$  dB-Hz in spite of the presence of delay. It means that  $\omega_n \tau_d$  has no important effect on the operating range of  $A^2/N_o > 60$  dB-Hz until the loop reaches the stability condition of 0.736 (rad) in the case of  $\zeta = 1/\sqrt{2}$ .

**Table (1a)**

**Table (1b)**

$A^2/N_0$ (dB-Hz)	$\sigma_{\phi_e}$ (deg.) for $B_{LP}=1.5\text{kHz}$	
	No delay	Delay-bandwidth product (0.2 rad)
50	$8^\circ$	$9^\circ$
60	$4.75^\circ$	$5^\circ$

**Table (1):** The resultant comparison for low-values of carrier-to-noise ratios.

### (Appendix A)

#### Derivation of $G_0(x)$ , $G_1(x)$ , and $G_2(x)$

Referring to expression of  $B_L$  of equation 7, the integral is simplified corresponding to the OPLL associated IF-stage as a ratio,

$$R_d = B_L / B_{LP} \quad (\text{A-1})$$

and in terms of  $H_1(j\omega)$ , the function  $H_2(j\omega)$  can be expressed as,

$$|H_2(j\omega)|^2 = \frac{1}{\omega^2} \left| \frac{Y_1^2 + Y_2^2}{Z^2} \right| \quad (\text{A-2})$$

where,

$$\begin{aligned} Y_1 &= x^4 - x^2 \cos(x.D) - 2\zeta x^3 \sin(x.D) \\ Y_2 &= x^2 \sin(x.D) - 2\zeta x^3 \cos(x.D), \text{ and} \\ Z &= 1 + x^4 - 2x^2 \cos(x.D) + 4\zeta^2 x^2 \\ &\quad - 4\zeta x^3 \sin(x.D) \end{aligned} \quad (\text{A-3})$$

with  $x = \omega / \omega_n$  and  $D = \omega \tau_d$  is the phase shift term, and in consequence the  $G(x)$ 's function are expressed as,

$$G_0(x) = \int_0^\infty \left| \frac{Y_1^2 + Y_2^2}{Z^2} \right| \frac{dx}{x^2}$$

$$\begin{aligned} G_1(x) &= \int_0^\infty \left| \frac{Y_1^2 + Y_2^2}{Z^2} \right| \frac{dx}{x^3} \\ G_2(x) &= \int_0^\infty \left| \frac{Y_1^2 + Y_2^2}{Z^2} \right| \frac{dx}{x^4} \end{aligned} \quad (\text{A-4})$$

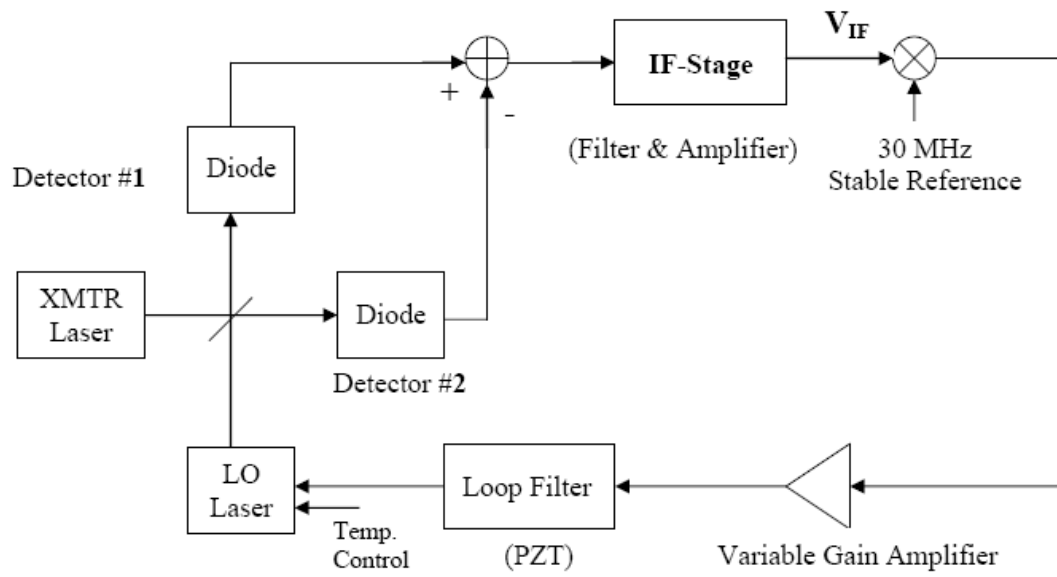
Using numerical integration method, the values of these three functions of  $G(x)$ 's can be computed with integral limit of  $x$  to be around  $10^{-6}$  to 300. The results are in match with one in the case of  $\tau_d = 0$  for  $G_0(\zeta)$ ,  $G_1(\zeta)$  and  $G_2(\zeta)$  of model in [5].

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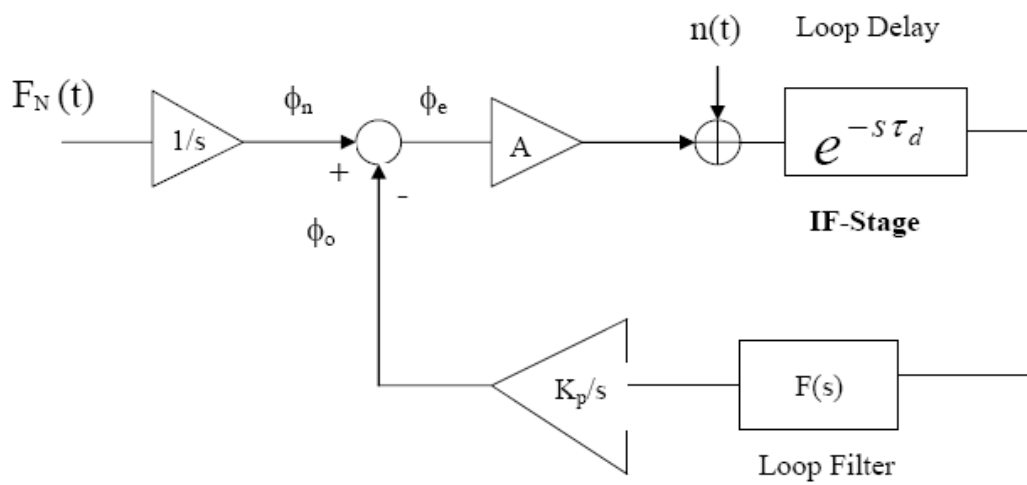
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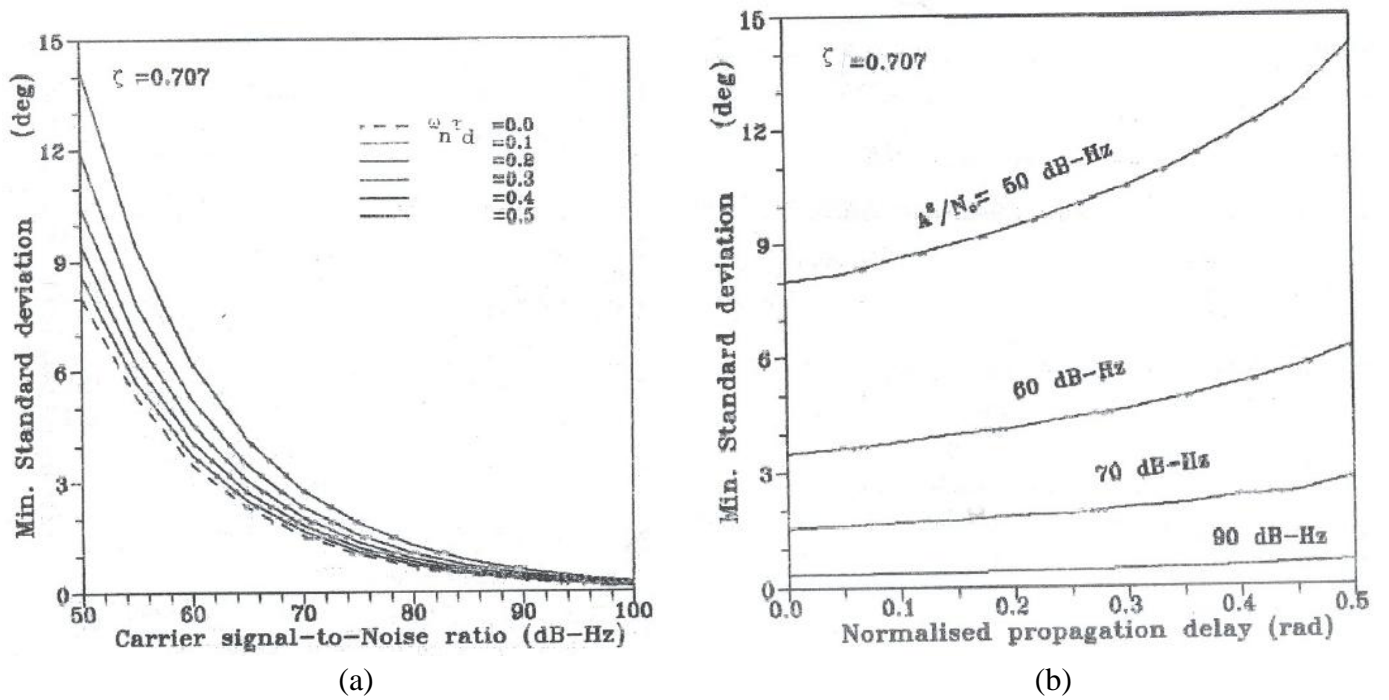


(a)

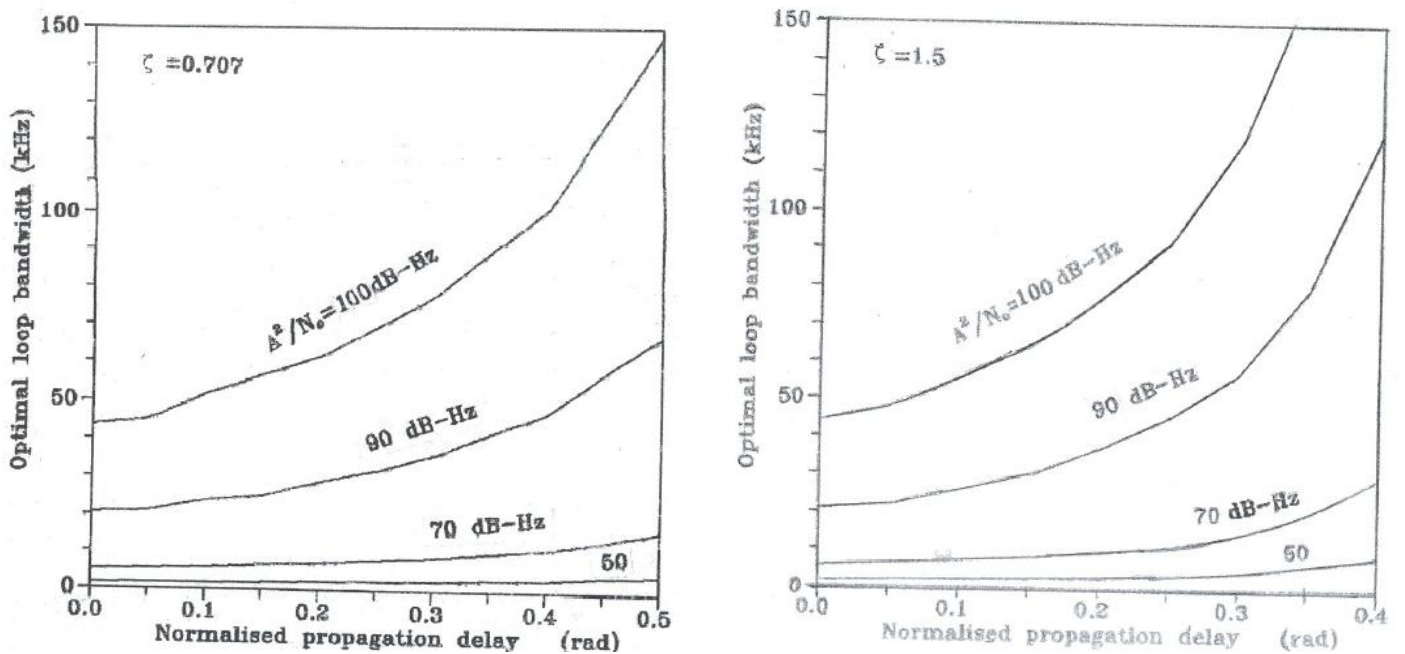


(b)

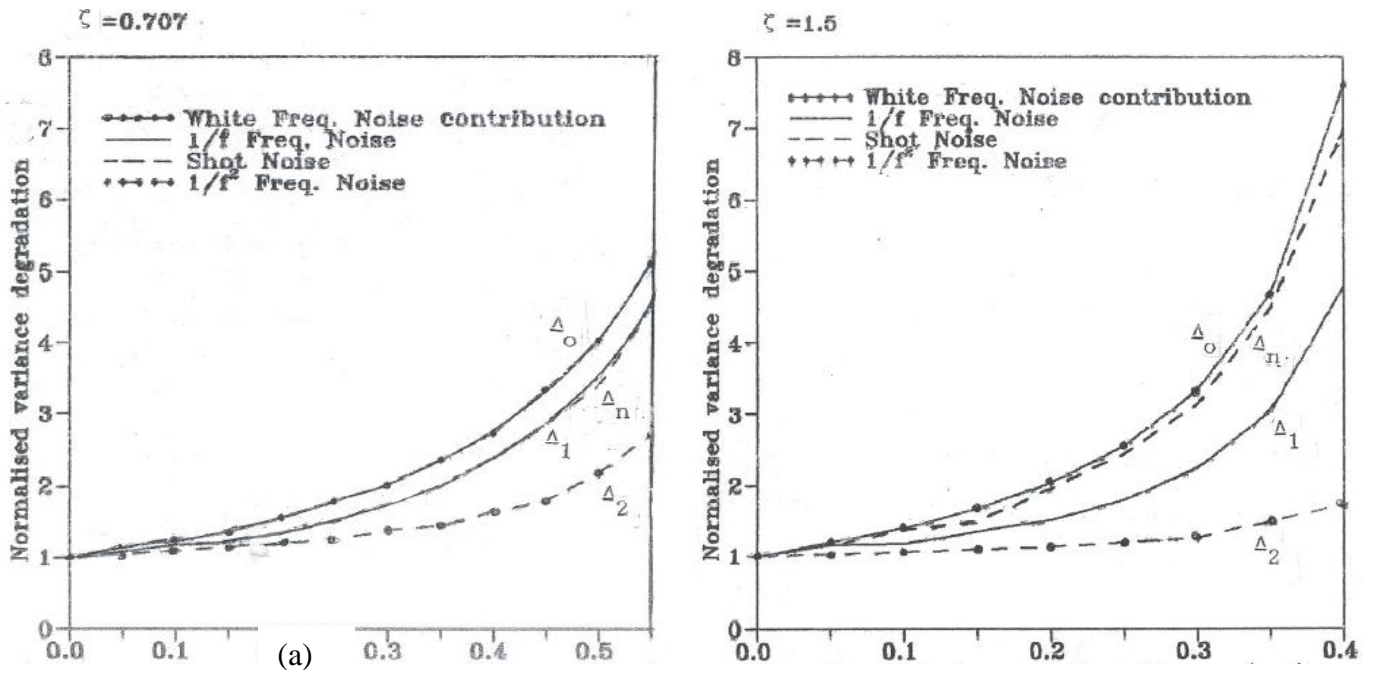
**Fig. 1** Block diagram of optical phase-locked loop (OPLL) and its equivalent model.



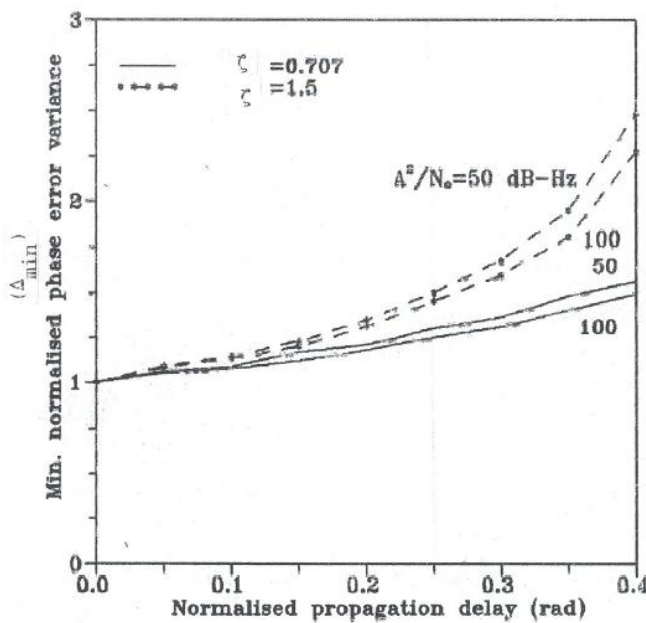
**Fig.2** The minimum standard deviation of the phase error ( $\sigma_{\min}$ ) versus (a) carrier-to-noise ratio ( $A^2/N_0$ ) (b) delay-bandwidth product factor ( $\omega_n \tau_d$ )



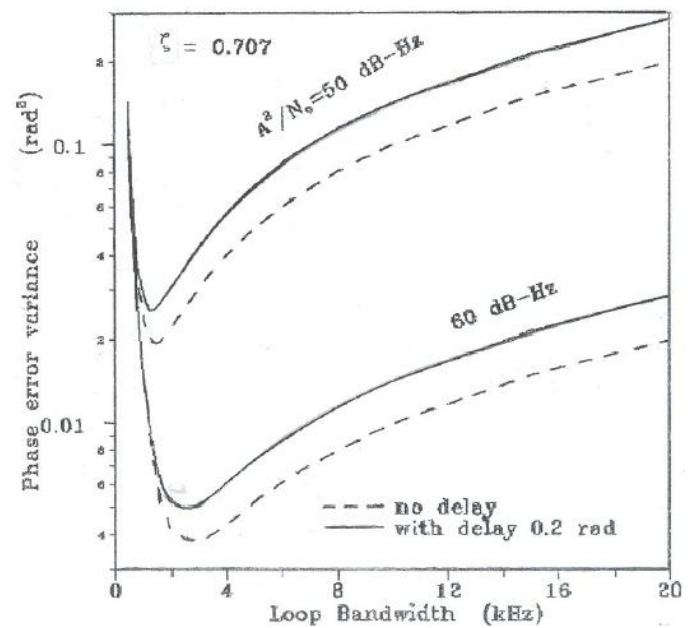
**Fig.3** Optimal loop bandwidth as a function of  $\omega_n \tau_d$  for various values of  $A^2/N_0$  ratios.



**Fig.4** The normalized noise terms contributions versus  $\omega_n \tau_d$ . (a)  $\zeta = 1/\sqrt{2}$  (b)  $\zeta = 1.5$



**Fig.5** The minimum normalized total phase error variance ( $\Delta_{\min}$ ) versus  $\omega_n \tau_d$  for 50 and 100 dB-Hz. (b)



**Fig.6** Total phase error variance as a function of loop bandwidth for lower-values of  $A^2/N_0$ .

----- no delay      ——— with delay of 0.2 rad