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Some Methods for Estimating the Parameters of A New Generalized Gamma- Weibull Distribution

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Abstract

In this paper, four different estimation methods are presented to estimate distribution parameters of A New Generalized Gamma-Weibull Distribution (NGGW) it represented Maximum Likelihood Method (ML), Weighted Least Square Method (WLS) , Power Density Method (PD), and Mean Standard Deviation Method (MSD). In order to obtain the best results, the study compared the estimation methods by applying the Monte Carlo simulation method using the Wolfram Mathematica 13 program. Several experiments were conducted with different sample sizes (n=20,50,100 and 250), and the results showed the superiority of the maximum potential method in finding estimates of distribution parameters compared to the rest of the methods used in this paper.

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Introduction

Researchers today face difficulty in determining the appropriate probability distribution model for the data of the studied phenomenon, as most of the traditional probability distributions have become inadequate in representing the phenomena data as a result of the tremendous technological development that the world has achieved today, which in turn has made the spread of phenomena data suffer from skewing right and left or oscillating upwards and down. Which prompted researchers to strive and try to build new probability distributions from traditional distributions that are characterized by their flexibility in representing data phenomena. The new two-parameter generalized Gamma-Weibull distribution is one of these distributions, which was proposed by (Aleshinloye and et all, 2023) compounding Weibull and generalized Gamma distribution [4]. This distribution is characterized by its superiority and flexibility in representing data, especially data for studying the survival function and reliability. In this paper, we discuss the four methods; Maximum Likelihood Method (ML), Weighted Least Square Method (WLS) , Power Density Method (PD), and Mean Standard Deviation Method (MSD to estimate distribution parameters. Moreover, using simulation study, these methods are compared using the mean square error (MSE) and bias.

Aims of paper

This paper aims to study the new generalized Gamma-Weibull distribution and estimate its parameters using four methods (Maximum Likelihood, Weighted least squares, Mean-Standard Deviation, and Power Density) and use the simulation study to compare between these methods according to the mean square error criterion and bias.

A New Generalized Weibull-Gamma distribution (NGGW)

The New Generalized Weibull-Gamma distribution (NGGW) developed by compounding Weibull and generalized gamma distribution [2].The probability density and distribution functions of the New generalized Weibull-gamma distribution (NGGW) are defined mathematically by the following equations:

$$f[x,\lambda,\theta] = \frac{\lambda\theta^3}{12(\theta^2+10)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda} \tag{1}$$

$$F[x,\lambda,\theta] = 1-e^{-\theta x}\left(1+\frac{\theta^5x^{5\lambda}+5\theta^3x^{3\lambda}\left(4+x^\lambda\right)+60\theta x^\lambda\left(\theta x^\lambda+2\right)}{12(\theta^2+10)}\right) \tag{2}$$

Or

$$F[x,\lambda,\theta] = \frac{120+12\left(1-e^{-x^\lambda\theta}\right)\theta^2-\Gamma\left[6,x^\lambda\theta\right]}{12(10+\theta^2)}$$

The reliability analysis of a distribution is determined by the survival and hazard rate function of the distribution. These are obtained as follows.
The Survival, S(x) function of the NGGW distribution is:

$$S[x,\lambda,\theta] = 1-F[x,\lambda,\theta]$$

$$S[x,\lambda,\theta] = 1-\frac{120+12\left(1-e^{-x^\lambda\theta}\right)\theta^2-\Gamma\left[6,x^\lambda\theta\right]}{12(10+\theta^2)}$$

Similarly, the hazard rate function, h(x), of the NGGW distribution is obtained as:

$$h[x,\lambda,\theta] = \frac{f[x,\lambda,\theta]}{S[x,\lambda,\theta]}$$

$$h[x,\lambda,\theta] = \frac{\frac{\lambda\theta^3}{12(\theta^2+10)}\left(\theta^3x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}}{1-\frac{120+12\left(1-e^{-x^\lambda\theta}\right)\theta^2-\Gamma[6,x^\lambda\theta]}{12(10+\theta^2)}}$$

$$h[x,\lambda,\theta] = \frac{e^{(x-x^\lambda)\theta}x^{-1+\lambda}\theta^3(12+x^{5\lambda}\theta^3)\lambda}{120+\theta(12\theta+x^\lambda(120+x^\lambda\theta(60+x^\lambda\theta(20+x^\lambda(5+x^\lambda\theta^2)))))}$$

Where $\Gamma[6,x^\lambda\theta]$ represents the higher gamma function.
Figure (1) below shows the behavior of the new generalized Gamma-Weibull distribution functions with different values of the two distribution parameters.

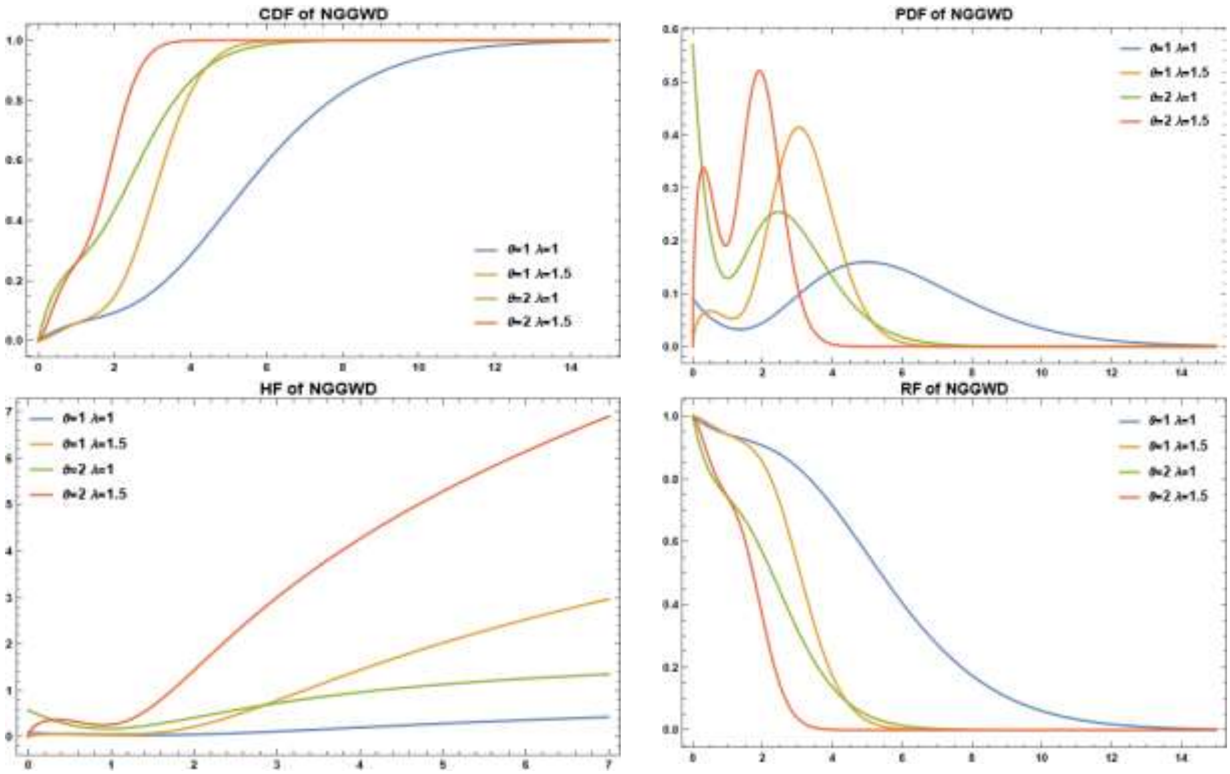

Figure (1): (PDF , CDF, HF and RF of NGGW Distribution)

The rth moment of NGGW distribution

The rth moment of NGGW can be found as follow:

$$E(x^r) = \int_0^\infty x^r f(x)\, dx$$

$$E(x^r) = \int_0^\infty x^r \frac{\lambda\theta^3}{12(\theta^2+10)}\left(\theta^3 x^{5\lambda}+12\right)x^{\lambda-1}e^{-\theta x^\lambda}\, dx$$

$$E(x^r) = \frac{\theta^{-\frac{r}{\lambda}}\left(\Gamma[6+\frac{r}{\lambda}] + 12\theta^2\, \Gamma\left[\frac{r+\lambda}{\lambda}\right]\right)}{12(10+\theta^2)} \tag{3}$$

Mean value of NGGW distribution

By using equation (3) can be found mean value of New BXII-E distribution by Replacement r with 1:

$$E(x) = \mu_1' = \frac{\theta^{-\frac{1}{\lambda}}\left(\Gamma[6+\frac{1}{\lambda}] + 12\theta^2\, \Gamma\left[\frac{1+\lambda}{\lambda}\right]\right)}{12(10+\theta^2)} \tag{4}$$

Variance of NGGW distribution

We can found variance value of NGGW distribution by the following equation:

$$V(x) = E(x^2) - \left(E(x)\right)^2 \tag{5}$$

By using equation (3) can be found $E(x^2)$ by Replacement r = 2:

$$E(x^2) = \mu_2' = \frac{\theta^{-\frac{2}{\lambda}}\left(\Gamma[6+\frac{2}{\lambda}] + 12\theta^2\, \Gamma\left[\frac{2+\lambda}{\lambda}\right]\right)}{12(10+\theta^2)} \tag{6}$$

Then

$$V(x) = \frac{\theta^{-\frac{2}{\lambda}}(\Gamma[6+\frac{2}{\lambda}] + 12\theta^2\,\Gamma\left[\frac{2+\lambda}{\lambda}\right])}{12(10+\theta^2)} - \left(\frac{\theta^{-\frac{1}{\lambda}}(\Gamma[6+\frac{1}{\lambda}] + 12\theta^2\,\Gamma\left[\frac{1+\lambda}{\lambda}\right])}{12(10+\theta^2)}\right)^2 \quad (7)$$

Skewness and kurtosis

From Equation (3), we can express the Third and fourth moments as follows:

$$E(x^3) = \mu'_3 = \frac{\theta^{-\frac{3}{\lambda}}(\Gamma[6+\frac{3}{\lambda}] + 12\theta^2\,\Gamma\left[\frac{3+\lambda}{\lambda}\right])}{12(10+\theta^2)} \quad (8)$$

$$E(x^4) = \mu'_4 = \frac{\theta^{-\frac{4}{\lambda}}(\Gamma[6+\frac{4}{\lambda}] + 12\theta^2\,\Gamma\left[\frac{4+\lambda}{\lambda}\right])}{12(10+\theta^2)} \quad (9)$$

The measure of symmetry is the skewness which describes the symmetry of the distribution and defined as:

$$SK = \frac{\mu'_3 - 3\mu'_2\mu'_1 + 2{\mu'_1}^3}{(\mu'_2 - {\mu'_1}^2)^{\frac{3}{2}}} \quad (10)$$

The skewness of the NGGW can be found by using equations (4,6,8).

Figure (2) below shows the skewness of the New generalized Gamma-Weibull distribution

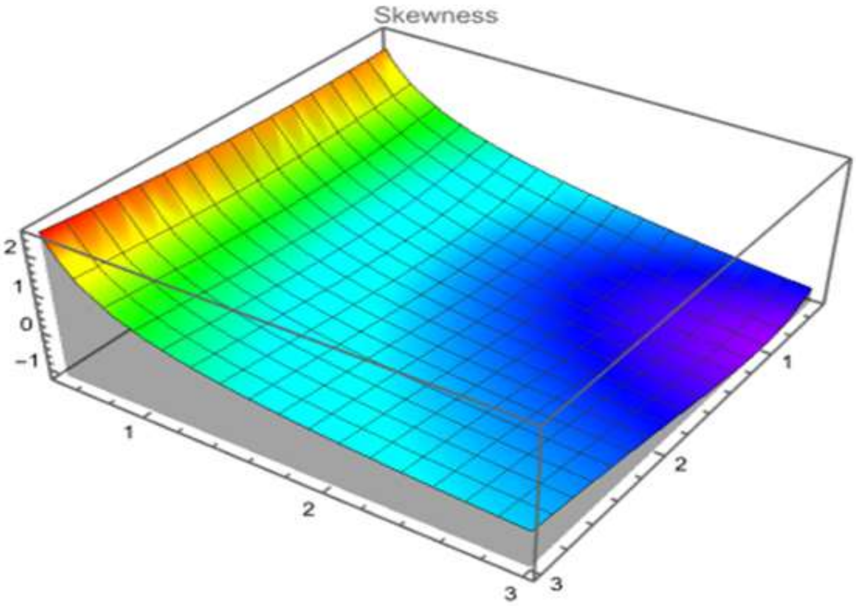

Figure (2): The skewness of NGGW

The kurtosis which describes of the distribution and defined as follows:

$$KU = \frac{\mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2{\mu'_1}^2 - 3{\mu'_1}^4}{\left(\mu'_2 - {\mu'_1}^2\right)^2} \quad (11)$$

The skewness of the NGGW can be found by using equations (4,6,8,9).

Figure (3) below shows the kurtosis of the new generalized Gamma-Weibull distribution

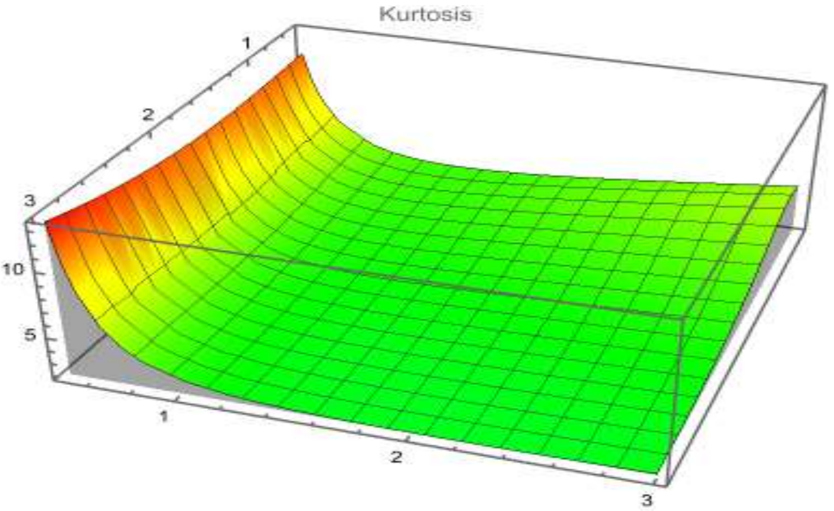

Figure (3): The Kurtosis of NGGW

Estimation parameters of the NGGW distribution

There are many estimation methods for the parameters of probability distributions, and we will use the following methods to estimate the generalized Weibull-kamma distribution of the study, as follows:

Maximum Likelihood Method (ML)

The maximum likelihood estimates (MLE) enjoy desirable properties that can be used when constructing confidence intervals and deliver simple approximations that work well in finite samples.[7]

Let $x_1, x_2, \ldots, x_n$ be independent and identically distributed observed random sample of size n from the NGGW distribution. Then, the log-likelihood function based on observed given by:

$$L\,f[x] \;=\prod_{i=1}^{n}\frac{e^{-\theta x_i^{\lambda}}\theta^3\lambda x_i^{-1+\lambda}(12+\theta^3 x_i^{5\lambda})}{12(10+\theta^2)} \tag{12}$$

$$Log\ L\,f[x] = n(3\mathrm{Log}[\theta]-\mathrm{Log}[12(10+\theta^2)]+\mathrm{Log}[\lambda])+(-1+\lambda)\sum_{i=1}^{n}\mathrm{Log}[x_i]+\sum_{i=1}^{n}\mathrm{Log}\left[12+\theta^3 x_i^{5\lambda}\right] - \theta\sum_{i=1}^{n}x_i^{\lambda} \tag{13}$$

To obtain the potential estimators for the parameters we work to derive the equation (13) for the two parameters (θ, λ) to obtain:

$$\frac{\partial Log\ L\,f[x]}{\partial\theta} = n\left(\frac{3}{\hat\theta}-\frac{2\hat\theta}{10+\hat\theta^2}\right)-\sum_{i=1}^{n}x_i^{\lambda}+\sum_{i=1}^{n}\frac{3\hat\theta^2 x_i^{5\lambda}}{12+\hat\theta^3 x_i^{5\lambda}} \tag{14}$$

$$\frac{\partial Log\ L\,f[x]}{\partial\lambda} = \frac{n}{\hat\lambda}+\sum_{i=1}^{n}Log[x_i]-\theta\sum_{i=1}^{n}\mathrm{Log}[x_i]x_i^{\hat\lambda}+\sum_{i=1}^{n}\frac{5\theta^3\mathrm{Log}[x_i]x_i^{5\hat\lambda}}{12+\theta^3 x_i^{5\hat\lambda}} \tag{15}$$

The Maximum Likelihood Estimates (MLEs), $\hat\theta\ and\ \hat\lambda$ can be obtained by equating (14) and (15) to zero and solving simultaneously. However, analytical expressions for $\hat\theta\ and\ \hat\lambda$ are not feasible. Hence, we computed the (MLEs) numerically using Mathematica software.

Weighted Least-Squares Method (WLS)

To estimate the parameters of various distributions, the weighted least-square (WLS) method are used[5]. Let $x_1 < x_2 < \ldots < x_n$ be a random sample with θ and λ parameters from the NGGW distribution estimators (WLSEs) of the can be obtained by minimizing the following equation:

$$W(\theta,\lambda)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F[x_i]-\frac{i}{n+1}\right]^2$$

$$W(\theta,\lambda)=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[1-\frac{i}{1+n}-e^{-\theta x_i}\left(1+\frac{\theta^5x_i^{5\lambda}+5\theta^3x_i^{3\lambda}(4+x_i^{\lambda})+60\theta x_i^{\lambda}(2+\theta x_i^{\lambda})}{12(10+\theta^2)}\right)\right]^2 \quad (16)$$

derive the equation (16) for the two parameters (θ, λ) to obtain:

$$\frac{\partial W(\theta,\lambda)}{\partial\theta}=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F[x_i]-\frac{i}{n+1}\right]\frac{\partial F[x_i]}{\partial\theta} \quad (17)$$

$$\frac{\partial W(\theta,\lambda)}{\partial\theta}=\sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}\left[F[x_i]-\frac{i}{n+1}\right]\frac{\partial F[x_i]}{\partial\lambda} \quad (18)$$

Where:

$$\frac{\partial F[x_i]}{\partial\theta}=\frac{e^{-x_i{}^{\lambda}\hat{\theta}}\hat{\theta}(-240+x_i{}^{\lambda}\hat{\theta}(10+\hat{\theta}^2)(12+x_i{}^{5\lambda}\hat{\theta}^3)+2e^{x_i{}^{\lambda}\hat{\theta}}\Gamma[6,x_i{}^{\lambda}\hat{\theta}])}{12(10+\hat{\theta}^2)^2} \quad (19)$$

$$\frac{\partial F[x_i]}{\partial\lambda}=\frac{e^{-x_i{}^{\hat{\lambda}}\theta}x_i{}^{\hat{\lambda}}\theta^3(12+x_i{}^{5\hat{\lambda}}\theta^3)\mathrm{Log}[x_i]}{12(10+\theta^2)} \quad (20)$$

The Weighted Least-Squares Estimates (WLSEs), $\hat{\theta}$ *and* $\hat{\lambda}$ can be obtained by equating (17) and (18) to zero and solving simultaneously. However, analytical expressions for $\hat{\theta}$ *and* $\hat{\lambda}$ are not feasible. Hence, we computed the (WLSEs) numerically using Mathematica software.

Mean-standard deviation Method (MSD)

The parameters of the new generalized Gamma-Weibull distribution are estimated using the definition of the experimental and theoretical mean and standard deviation [1][8], as follows:

$$\left[\frac{S}{\bar{x}}\right]^2=\frac{\sigma^2}{\mu^2} \quad (21)$$

Where

$$\sigma^2=V(x)$$

$$\mu^2=\left[\frac{\theta^{-1/\lambda}(\Gamma[6+\frac{1}{\lambda}]+12\theta^2\mathrm{G}\Gamma[\frac{1+\lambda}{\lambda}])}{12(10+\theta^2)}\right]^2$$

Then

$$\left[\frac{S}{\bar{x}}\right]^2=\frac{\frac{\theta^{-2/\lambda}(-(12\theta^2\Gamma[1+\frac{1}{\lambda}]+\Gamma[6+\frac{1}{\lambda}])^2+12(10+\theta^2)(\Gamma[6+\frac{2}{\lambda}]+12\theta^2\Gamma[\frac{2+\lambda}{\lambda}]))}{144(10+\theta^2)^2}}{\left[\frac{\theta^{-1/\lambda}(\Gamma[6+\frac{1}{\lambda}]+12\theta^2\mathrm{G}\Gamma[\frac{1+\lambda}{\lambda}])}{12(10+\theta^2)}\right]^2}$$

$$=\frac{12(10+\theta^2)(\Gamma[6+\frac{2}{\lambda}]+12\theta^2\Gamma[\frac{2+\lambda}{\lambda}])}{(12\theta^2\Gamma[1+\frac{1}{\lambda}]+\Gamma[6+\frac{1}{\lambda}])^2}-1 \quad (22)$$

Where $\bar{x}$ *and* s represent the mean and standard deviation of the sample, respectively, and are calculated as follows:

$$\bar{x}=\frac{\sum x_i}{n} \;\; ; \;\; s^2=\frac{\sum(x_i-\bar{x})^2}{n}$$

To obtain the mean-standard deviation estimates (MSDEs), $\hat{\theta}$ *and* $\hat{\lambda}$ can be obtained by equating (4) and (22) to zero and solving simultaneously. However, analytical expressions for $\hat{\theta}$ *and* $\hat{\lambda}$ are not feasible. Hence, we computed the estimates (MSDEs) numerically using Mathematica software.

Power Density Method (PD)

This method was presented by Akdag and Ali in 2009 [3]. The idea of this method is based on estimating the parameters of the new generalized Gamma-Weibull distribution through the ratio between the third and first moments, experimental and theoretical [1][6], as follows:

$$\bar{x} = \frac{\theta^{-1/\lambda}(\Gamma[6+\frac{1}{\lambda}] + 12\theta^2 \mathrm{G}\Gamma[\frac{1+\lambda}{\lambda}])}{12(10+\theta^2)} \tag{23}$$

To obtain the estimates of parameters with this method, we firstly compute the energy pattern factor. The energy pattern factor which is defined as a ratio between mean of cubic data to cube of mean data.

The cubic mean of data is given in equation (8)

$$\bar{x}_{cub} = \frac{\theta^{-\frac{3}{\lambda}}(\ \Gamma[6+\frac{3}{\lambda}] + 12\theta^2\ \Gamma\left[\frac{3+\lambda}{\lambda}\right])}{12(10+\theta^2)}$$

Then the energy pattern factor (E_{pf}) can define as:

$$E_{pf} = \frac{\bar{x}_{cub}}{(\bar{x})^3} = \frac{144(10+\theta^2)^2(\Gamma[6+\frac{3}{\lambda}] + 12\theta^2\Gamma[\frac{3+\lambda}{\lambda}])}{(12\theta^2\Gamma[1+\frac{1}{\lambda}] + \Gamma[6+\frac{1}{\lambda}])^3} \tag{24}$$

To obtain power density estimates (PDEs), $\hat{\theta}$ *and* $\hat{\lambda}$ can be obtained by equating (4) and (22) to zero and solving simultaneously. However, analytical expressions for $\hat{\theta}$ *and* $\hat{\lambda}$ are not feasible. Hence, we computed the estimates (PDEs) numerically using Mathematica software.

Simulation study

A simulation study was conducted to explore and compare the behavior of the estimates with respect to their: average of absolute value of average of mean square errors (MSEs), $MSE = \frac{1}{N}\sum_{i=1}^{n}(\widehat{\emptyset} - \emptyset)^2$ and biases $\left|Bias(\widehat{\emptyset})\right| = \frac{1}{N}\sum_{i=1}^{n}\left|\widehat{\emptyset} - \emptyset\right|$.

Table 1: Parameters Estimates

		Case 1		Case 2		Case 3	
Sample size	Method	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\lambda}$
20	ML	0.491358	1.26715	1.55358	1.03014	1.04134	1.00618
	WLS	0.517021	1.25811	1.41618	1.15328	0.98464	0.867382
	PD	0.502998	1.25661	1.55958	1.02812	1.00358	1.03599
	MSD	0.493944	1.26251	1.55697	1.02892	1.03278	1.0175
50	ML	0.471265	1.24118	1.4542	1.03533	0.921912	1.0503
	WLS	0.462529	1.26225	1.44301	1.03856	0.944922	1.03086
	PD	0.457516	1.25904	1.43339	1.05084	0.900723	1.06282
	MSD	0.467742	1.24685	1.44698	1.04156	0.904212	1.0597
100	ML	0.523093	1.1774	1.50285	0.988753	1.00162	1.01143
	WLS	0.520953	1.18479	1.501	0.989255	1.00514	1.00754
	PD	0.517152	1.18335	1.49606	0.991169	0.987398	1.01955
	MSD	0.517203	1.18268	1.49641	0.990942	0.992288	1.01648
250	ML	0.479089	1.22166	1.4739	1.01448	1.03028	0.986686
	WLS	0.486001	1.21467	1.47977	1.00998	1.0298	0.988278
	PD	0.47967	1.22208	1.46986	1.01531	1.04151	0.979661
	MSD	0.47831	1.22294	1.47045	1.0147	1.03491	0.983616

We generate $N = 1000$ random samples of size $n = 20, 50, 100\ and\ 250$ from the NGGW model by using accept-reject method while choosing three cases, case1, θ < λ when (θ = 0.5, λ =1.2) , case2, θ >λ when (θ = 1.5, λ =1) and case3, θ = λ when (θ = 1 , λ = 1) used Mathematica software. For each parameter combination and each sample, we estimated the NGGW parameters θ and λ using four frequentist estimators including ML, WLS, PD, and MSD. Then, the MSEs of the parameter estimates were computed. Simulated outcomes are listed in Tables 1–3.

Table 2: MSE of Parameters Estimates

		Case 1		*Case 2*		*Case 3*	
Sample size	**Method**	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\lambda}$
20	ML	0.04132	0.03661	0.08856	0.01845	0.01960	0.00796
	WLS	0.05224	0.06186	0.23310	0.14308	0.14083	0.14620
	PD	0.04438	0.03873	0.12846	0.02745	0.02716	0.01018
	MSD	0.03867	0.03657	0.11869	0.02562	0.02916	0.00915
50	ML	0.01268	0.01080	0.02921	0.00784	0.04761	0.01900
	WLS	0.01843	0.03488	0.03818	0.01328	0.04070	0.01463
	PD	0.01460	0.01405	0.03448	0.00960	0.05047	0.02236
	MSD	0.01287	0.01163	0.03062	0.00782	0.04849	0.02083
100	ML	0.00666	0.00629	0.01477	0.00399	0.01444	0.00322
	WLS	0.01243	0.01287	0.02129	0.00652	0.01582	0.00474
	PD	0.00745	0.00683	0.01483	0.00408	0.01312	0.00300
	MSD	0.00667	0.00624	0.01544	0.00413	0.01330	0.00293
250	ML	0.00301	0.00319	0.00629	0.00183	0.00736	0.00146
	WLS	0.00256	0.00248	0.00513	0.00131	0.00828	0.00171
	PD	0.00418	0.00451	0.00742	0.00239	0.00866	0.00178
	MSD	0.00371	0.00397	0.00648	0.00196	0.00827	0.00158

Table 3: BIAS of Parameters Estimates

		Case 1		*Case 2*		*Case 3*	
Sample size	**Method**	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\theta}$	$\hat{\lambda}$
20	ML	0.16502	0.16097	0.24552	0.11565	0.11089	0.07205
	WLS	0.16651	0.16018	0.34218	0.20011	0.24231	0.20701
	PD	0.18004	0.17944	0.29920	0.14638	0.12626	0.08018
	MSD	0.16186	0.16633	0.28297	0.14023	0.12970	0.07947
50	ML	0.09609	0.09548	0.13074	0.07397	0.14481	0.09088
	WLS	0.08931	0.10342	0.15059	0.06983	0.14291	0.09163
	PD	0.10494	0.10804	0.15251	0.08112	0.16238	0.09918
	MSD	0.09847	0.09939	0.14506	0.07349	0.15278	0.09405
100	ML	0.06609	0.06525	0.10384	0.05219	0.10110	0.05035
	WLS	0.09909	0.09874	0.13158	0.07073	0.11512	0.06209
	PD	0.07035	0.06983	0.10398	0.05397	0.09702	0.04164
	MSD	0.06545	0.06529	0.11020	0.05126	0.10239	0.04662
250	ML	0.04854	0.04988	0.06812	0.03911	0.06390	0.03005
	WLS	0.04628	0.04499	0.06328	0.03311	0.07176	0.03422
	PD	0.06023	0.06195	0.07611	0.04468	0.06842	0.03215
	MSD	0.05519	0.05685	0.07111	0.04005	0.06737	0.03008

Conclusion

In table 2 and 3 we observe that:

1. The ML method outperforms the rest for both parameters in case 2 and case 3 over other methods, while MSD method was the best in case 1 in sample size 20 according to mean square error and bias.
2. Convergence and diversity of preference estimation methods sample size 50 and 100 for all cases.

3. The ML method outperforms the rest for both parameters in case 1 and case 2 over other methods, in sample size 250 according to mean square error and bias. while MSD method was the best in case 3 according to bias and MSD method was the best in case 3 according to mean square error for θ estimator only and ML method was the best in case 3 according to mean square error for λ estimator.

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بعض الطرق لتقدير معلمات توزيع جاما-ويبل المعمم الجديد

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المستخلص

تم في هذه الورقة تقديم اربع طرائق مختلفة لتقدير معلمات توزيع ويـبل – كـاما المعمم الجديد A New Generalized Gamma-Weibull Distribution (NGGW) تمثلت بطريقة الامكان الاعظم Maximum Likelihood Method (ML) و طريقة المربعات الصغرى الموزونة Weighted Least Square Method (WLS) وطريقة كثافة القوى Power Density Method (PD) وطريقة الانحراف المعياري Mean-Standard Deviation Method (MSD)، ولغرض الحصول على أفضل النتائج فقد تمت المقارنة بين طرائق التقدير انفة الذكر عن طريق تطبيق أسلوب محاكاة مونت كارلو (Monte Carlo Simulation) باستعمال برنامج (Wolfram Mathematica 13) حيث تم إجراء عدة تجارب وبأحجام عينات مختلفة ($n = 20, 50, 100$ and 250) لبيانات تم توليدها من التوزيع ولحالات مختلفة، وقد أظهرت النتائج أفضلية طريقة الإمكان الأعظم (ML) في ايجاد مقدرات معلمات التوزيع مقارنة ببقية الطرائق المستعملة في هذه الورقة.

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