Attitude estimation of satellite using the extended Kalman filter

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Abstract

Computer simulation (Simulink – Matlab 7.4.4.a) have been developed to estimate the angular velocity and the attitude in terms of the quaternions of a proposed low earth orbit satellite from noisy measurements using the extended Kalman filter . The dynamic and Kinematic models of the rigid spacecraft were distorted by external torques (atmospheric drag, solar radiation , Earth magnetic field interaction with the dipole moment of the satellite ,…etc.) .These torques were represented by white noise .The extended Kalman filter was modeled inside the embedded Matlab function . Linearization of the state space transition matrix of the spacecraft was performed by using the Jacobian matrix . Simulation results showed that the estimated attitude parameters were approximately close to the calculated values .

تخمين الوضع الزاوي للقمر الصناعي باستخدام مرشح كالمن الموسع

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الخلاصة :

طور برنامج حاسوبي (السيمولينك – ماتلاب 7.44a) لتخمين مركبات السرعة الزاوية والوضع الزاوي بدلالة الرباعيات لمركبة فضائية مفترضة تدور في مدار واطئ وايجاد القيم الحقيقية من القياسات المشوشة . الديناميكية والكاينماتيكية للمركبة تعرضت الى تشويشات بسبب العزوم الخارجية (الكبح الايروديناميكي و ضغط الاشعاع الشمسي وتفاعل المجال المغناطيسي الارضي مع عزم ثنائي القطب للقمر الاصطناعي) . اذ مثلت هذه العزوم بالضوضاء الابيض . وقد نمذج مرشح كالمن الموسع داخل دالة ضمن الماتلاب . وخطية مصفوفة الحالة الانتقالية تمت باستخدام مصفوفة جاكوب . وقد أظهرت النتائج تقارب تقريبي بين القيم المخمنة والقيم المحسوبة لمعلمات التوجيه الزاوي للمركبة .

1-Introduction

The Kalman filter is a set of equations that provides a method to estimate the state of the process . This series of equations consist of two steps , predict and correct . These two steps were used recursively . In real time ,the raw data would be added to the filter during the correct step . After the current data points were received , the correct step was used to estimate the state and it's variances . The Kalman filter was frequently used for linear systems. However the extended Kalman filter (EKF) was developed to help account for the nonlinear systems such as spacecraft's attitude dynamics and kinematics in free space .

Three axis attitude determination via Kalman filtering of magnetometer data was presented by M .L. Psiaki , 1990 , [1].]. Attitude determination of NCUBE satellite was investigated by K. Svartveit ,2003 [2]. Non-linear attitude control of micro-satellite ESEO was introduced by M. P. Topland ,2004 [3] . The attitude estimation of rigid body in space using low cost low-pass sensors such as accelerometers , and magnetometers was introduced by A. Tayebi et. al.,2007 ,[4]. The estimation algorithm was coupled with quaternion based attitude stabilization scheme. The Kalman filter for spinning spacecraft attitude estimation was presented by F. L. Markely and J. E. Sedlak ,2008 [5]. Attitude determination for spinning nanosatellite using geomagnetic field data and solar panels as sun sensors was investigated by P. S. Hur et. al. , 2008 [6] . Kalman filter for attitude determination of student satellite was presented by J. Rohde ,2007 [7]. Passive attitude techniques of small satellite was introduced by S. A. Rawashdeh, 2009 ,[8].

In this paper the extended Kalman filter algorithm was used for estimation the attitude dynamics and kinematics of a proposed octagonal spinning spacecraft under the influences of external disturbing torques which was represented by white noise.

2- The coordinate systems

The coordinate system used to describe the spacecraft's attitude dynamics and kinematics were as follows :

2.1 body fixed coordinate system (x,y,z)

Right handed coordinates with origin at the center of mass of the spacecraft . These axes represent the principal axes of inertia of the spacecraft that were rotated with respect to inertial frame fixed at the center of mass of the earth through Euler's angles.

2.2 Inertial coordinate system (xi,yi,zi)

It is right handed coordinate system with origin at the earth’s center.The x_axis towards vernal equinox . the z-axis points toward the north pole . the y-axis is orthonormal to x and z axes

3-Mathmatical model of the system

The attitude dynamics of free rigid body in space is the Euler's equation of rotational motion about the center of mass given by : .

$$\dot{\vec{H}} = \vec{N} - \vec{\omega} \times \vec{H} = I\dot{\vec{\omega}}$$
(3.1)

Where $\vec{H}$ is the total angular momentum in inertial coordinate system, and $\vec{N}$ is the total external torques (which includes , e.g., control torques , areodynamic drag torques , solar radiation pressure torques , etc,) , w and I are the angular velocity and inertia matrix of the spacecraft respectively.

The moment of inertia matrix of the proposed spacecraft assumed symmetric and is given by

$$I = \begin{bmatrix} 1.4 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.0 \end{bmatrix}$$

The time rate of change of angular momentum in inertial space is related with that in body centered frame by the relation [2],[3]:

$$\left.\frac{dH}{dt}\right|_I = \left.\frac{dh}{dt}\right|_b + \omega \times h$$
(3.2)

The orientation of the rigid body in inertial space can be expressed in terms of the three Euler's angles or quaternions . The quaternions are preferred for non- singularity (i.e. division by zero) and the absence of trigonometric functions .

The quaternion consists of three vectors and one scalar defined as follows [2]

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$
(3.3)

With vector components given by

$$q_i = n_i \sin\frac{\Phi}{2}$$
(3.4a)

and a scalar part is given by

(3.4b)
$$q_4 = \cos\left(\frac{\Phi}{2}\right)$$

Where $\hat{n}$ is the direction cosine of the axis of rotation with respect to inertial coordinates and Φ is the angle of rotation about that axis..

The normalization of quaternions can be written as follows [8]

$$n_1^{\ 2} + n_2^{\ 2} + n_3^{\ 2} + n_4^{\ 2} = 1 \qquad (3.5)$$

The quaternion Kinematic equations of motion can be written in terms of the spacecraft's angular velocity components as follows[2]

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}_i = \tfrac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

(3.6)

4. The extended Kalman filter model

The Kalman filter is a set of equations that provides a method to estimate the state of the process in discrete time. This series of equations can be arranged into two steps , predict or/and corrector , that are used recursively . The raw data (w1 ,w2 ,w3 ,q1 ,q2 ,q3 ,q4) would be added to the filter after passing through the zero order hold which converts these data into discrete values .

State space representation of the dynamic and kinematic system of the spacecraft without control is shown below :

State space equation of the dynamic and kinematic system

$$\dot{X} = AX$$

(4.1)

Where the state vector X consists of seven elements as follows

$$X = [\omega_1, \omega_2, \omega_3, q_{1,}, q_2, q_3, q_4]^T$$

(4.2)

The state space dynamic and kinematic matrix A is given by

$$A = \begin{bmatrix} 0 & -\frac{I_3}{I_1}X_3 & \frac{I_2}{I_1}X_2 & 0 & 0 & 0 & 0 \\ \frac{I_1}{I_2}X_3 & 0 & \frac{I_3}{I_2}X_1 & 0 & 0 & 0 & 0 \\ \frac{I_2}{I_3}X_2 & \frac{I_1}{I_3}X_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5X_3 & -0.5X_2 & 0.5X_1 \\ 0 & 0 & 0 & -0.5X_3 & 0 & 0.5X_1 & 0.5X_2 \\ 0 & 0 & 0 & 0.5X_2 & -0.5X_1 & 0 & 0.5X_3 \\ 0 & 0 & 0 & -0.5X_1 & -0.5X_2 & -0.5X_3 & 0 \end{bmatrix}$$

Where

$$X_1 = \omega_1, X_2 = \omega_2, X_3 = \omega_3, X_4 = q_1, X_5 = q_2, X_6 = q_3, X_7 = q_4$$

The extended Kalman filter algorithm can be summarized in the following steps

1. Initialization of the state vector $\tilde{x}$

$\tilde{x} = [\omega_1, \omega_2, \omega_3, q_1, q_2, q_{3,}, q_4]^T$

Where the superscript T denote the transpose of the vector

And the covariance matrix P[7 x 7]. each element is zero

P=zeros(7)

The state transition matrix for the dynamic system of the spacecraft in terms of moments of inertia could be obtained by the Jacobian maxtrix and the result was given by

$$\Phi = \begin{bmatrix} 0 & -\frac{I_3}{I_1} & \frac{I_2}{I_1} \\ -\frac{I_1}{I_2} & 0 & \frac{I_3}{I_2} \\ -\frac{I_2}{I_3} & \frac{I_1}{I_3} & 0 \end{bmatrix}$$

And that of the quaternions was given by

$$\Phi = \begin{bmatrix} 0 & 0.5 & -0.5 & 0.5 \\ -0.5 & 0 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0 & 0.5 \\ -0.5 & -0.5 & -0.5 & 0 \end{bmatrix}$$

The process noise of the dynamic system is given by the following diagonal matrix

$$Q = diag(0.05 \quad 0.04 \quad 0.08)$$

And the corresponding measurement error covariance R is given to each of the element of the state space matrix

$$R = diag(0.3\text{^}2 \quad 0.02\text{^}2 \quad 0.04\text{^}2)$$

The process noise and the corresponding measurement error covariance matrices that was applied to the quaternions were given by

$$Q = diag(0.05 \quad 0.06 \quad 0.02 \quad 0.1)$$

$$R = diag(.4\text{^}2 \quad .02\text{^}2 \quad .03\text{^}2 \quad 0.02\text{^}2)$$

2. The propagation of the covariance matrix in terms of the state transition matrix and process noise was given by the following matrix operations

$$P = \Phi . P . \Phi^T + Q$$

Where the superscript T denotes the transpose matrix

3. Propagation of the state space vector estimation is given by

$$\tilde{x} = \Phi . \tilde{x}$$

4a. computation of the observation estimates

$$\tilde{\omega}_{11} = \tilde{x}(1); \tilde{\omega}_2 = \tilde{x}(2); \tilde{\omega}_3 = \tilde{x}(3); \tilde{q}_1 = \tilde{x}(4); \tilde{q}_2 = \tilde{x}(5); \tilde{q}_3 = \tilde{x}(6); \tilde{q}_4 = \tilde{x}(7)$$

4b. computation of observation vector as a transpose of the following vector

$$\tilde{y} = [\tilde{\omega}_1 \quad \tilde{\omega}_2 \quad \tilde{\omega}_3 \quad \tilde{q}_1 \quad \tilde{q}_2 \quad \tilde{q}_3 \quad \tilde{q}_4]^T$$

4c. The measurement matrix M for the dynamic system is the direction cosine of rotation of the orbit plane about y-axis , given by[7]

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Where θ is the angle between the resultant angular velocity of the spacecraft and the transverse component in y- direction .

And the rotation matrix using Euler's parameters

(quaternions) was given by[8]

$$\begin{bmatrix} q_4^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_4q_3) & 2(q_1q_3 + q_4q_2) \\ 2(q_1q_2 + q_4q_3) & q_4^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_4q_1) \\ 2(q_1q_3 - q_4q_2) & 2(q_2q_3 + q_4q_1) & q_4^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

4.d The residual estimation error was the difference between the measured and the observation vectors

Residual= meas - $\tilde{y}$

5. The Kalman gain matrix was defined as follows :

$$K = PM^T.(MPM^T + R)^{-1}$$

6. Updating the estimates

$$\tilde{x} = \tilde{x} + K.residual$$

7. Updating the covariance matrix P

$$P = (I - K.M).P.(I - K.M)^T + K.R.W^T$$

Where I[7x7] was the square identity matrix

8. Finaly the output state estimator was related to the updated state estimator $\tilde{x}$ by

$$\tilde{x}_{out} = \tilde{x}$$

5-Results and discussion

The dynamics and kinematics of the spinning satellite have been simulated without control in simulink-matlab . The dynamic equations of the spacecraft were defined in terms of Euler's equation of motion of rigid body . the orientation of the spacecraft with respect to inertial frame were expressed in terms of quaternions .(Euler symmetric parameters). These equations were corrupted by external noises . The extended Kalman filter was used to extract the actual parameters from noisy data . The nonlinear set of equations of motion were linearized by implementing the Jacobian matrix. . Mathlab simulink program were developed to represent spacecraft's rotational motion under the effects of external disturbances which are represented by white noise. The results were shown in the following figures .

The transverse components of the angular velocity about the x and y-axes , w1 w2 were shown in figures 1 a and b ,the calculated , measured and estimated by the EKF . It can be seen from these figures that the w1 and w2 behavior were sinusoidal due to the nutational motion of the spacraft without control . The calculated one was the sinusoidal , the measured was the one corrupted by noise and estimated values by the EKF were approximately close to the calculated values . The nutation motion of the spacecraft about the spin axis was shown in fig(1c). It can be seen from this figure that during the early times of simulation the calculated values of w3 were compatible with the predicted values after entering the Kalman filter .The residual of these components which was the difference between measured and estimated values were shown in fig(1d) . The orientations of the spacecraft with respect to inertial frame in terms of quaternions were shown in fig(2).it can be seen from this figure that the quaternions q1,q2,q3 and q4 that represent the attitude of the spacecraft were in oscillation state and the estimated ones were approximately close to the calculated values . The residuals of the four attitude parameters were shown in fig(3). Which shows the difference between the measured and observed values .

6. Conclusions :

Simulation of the attitude dynamics and kinematics of the rigid spacecraft have been developed under the effects of disturbances represented by white noise . the seven parameters of the attitude dynamics and kinematics were transformed into discrete values by using the zero order hold and then entered into the extended Kalman filter algorithm . the following conclusions could be drawn from these results.

1. The spacecraft sustained detumbling without control
2. Implementing the EKF was to extract the actual values of the attitude parameters of the spacecraft from noisy data .
3. The estimated values of the attitude dynamic and kinematic parameters were approximately close to the calculated values .

7-Refrerences

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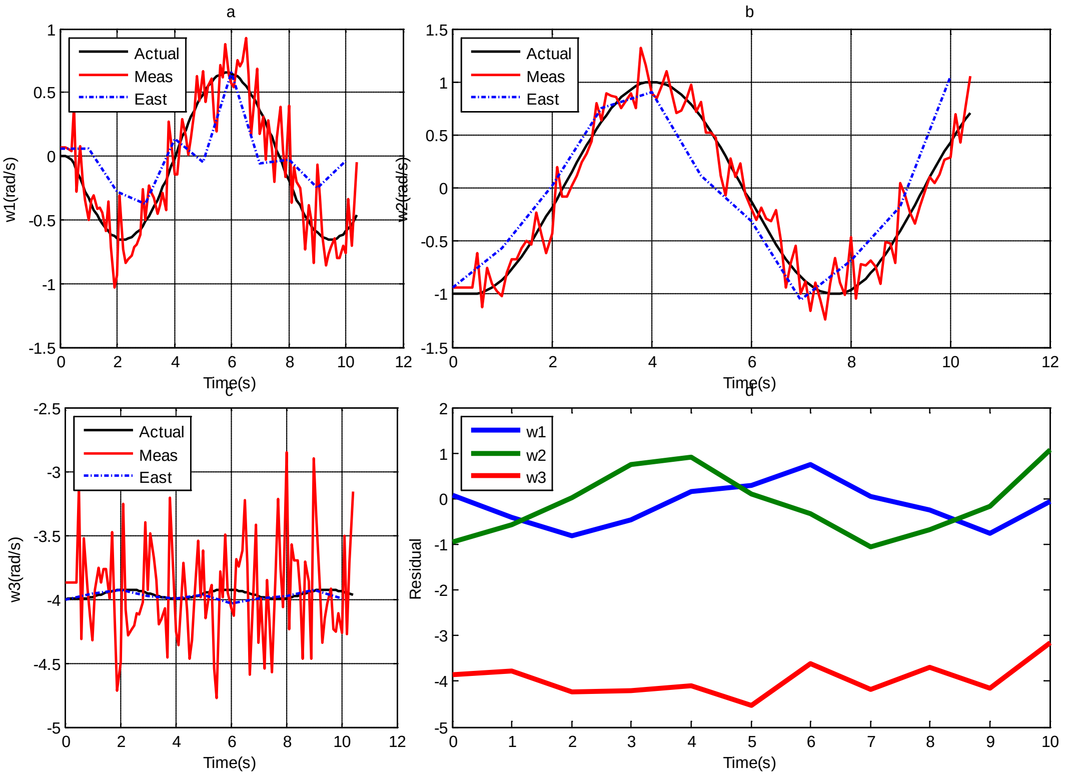


Fig. (1,a,b,c,d):- The actual, measured and estimated anglur velocity components of the spacecraft with respect to body fixed coordinate frame (the red line is the actual , the green line is the measured and the blue lines represent the estimated values after implementing the extended kalman filter(a. w1 b. w2 c. w3 d. Residual)

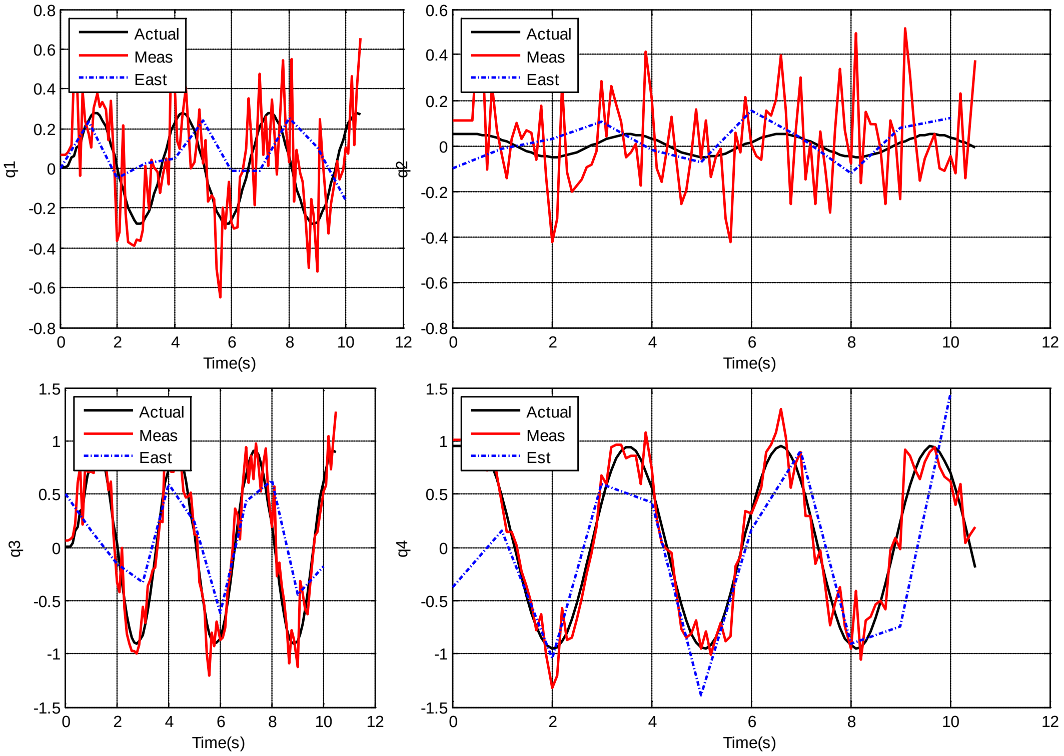
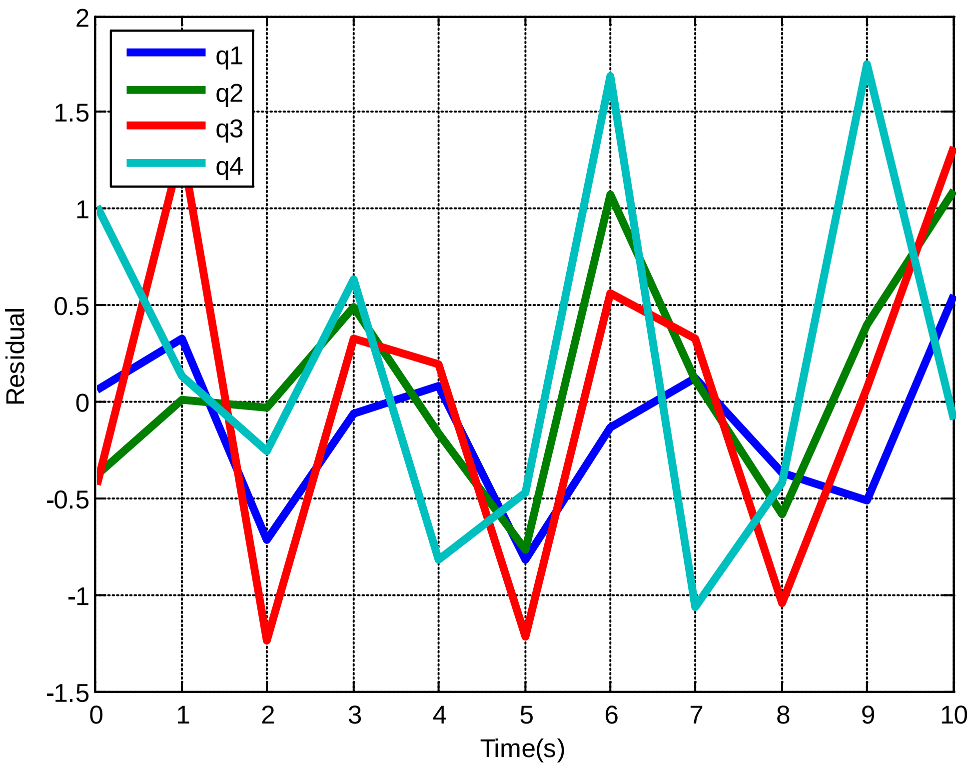


fig. (2)The actual, measured and estimated quaternions (q1,q2,q3,q4). The red curve represents the actual ,the green is the measured and the blue is the estimated .



Fig(3):- The residuals of the quatenions (q1,q2,q3,q4)