

## Compact Size Design with low side lobes of Fractal Linear Array Antenna

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Received on: 12 / 8 /2007

Accepted on: 1/ 9 /2008

### Abstract

In this paper, the fractal concept is used in the linear array antenna design to obtain multiband operation and reduced size. MATLAB programming language version 7.2 (R2006a) is used to simulate the fractal linear array antenna and their radiation pattern. Fractal Cantor linear array antenna of array pattern of 101 has been designed at a frequency of 2700 MHz with uniform and non uniform amplitude distribution. The performance of this array has been simulated. It is found that it operates at frequencies 2700 MHz, 900 MHz, 300 MHz, and 100 MHz. Two proposed models of Cantor linear array antenna having array patterns of 11011 and 1010101 are presented with uniform and non uniform amplitude distribution. The performance of these arrays has been simulated. The resulting frequency behavior is found that it operates at frequencies 2700 MHz and 540 MHz for the first model (11011) and 2700 MHz and 386 MHz for the second model (1010101). The frequencies have been selected in the VHF and UHF bands have been used in many applications in communication systems such as global system mobile (GSM), wireless local area network (WLAN), worldwide interoperability for microwave access (WIMAX) .... etc.

**Keywords:** Fractals, fractal arrays, antenna array, antenna radiation

Patterns, Multi-band antennas, low side lobe antennas.

### الخلاصة

تم في هذا البحث استخدام فكرة الهندسة الجزيئية (Fractal Geometry) في تصميم المصفوفات الخطية للهوائيات وذلك للحصول على هوائيات تعمل على أكثر من تردد وذو ات حجم صغير. استخدمت اللغة البرمجية MATLAB النسخة 7.2 (R2006a) لغرض تمثيل المصفوفة الخطية الجزيئية ورسم شكل الشعاع (Radiation Pattern). تم تصميم المصفوفة الخطية للهوائيات كانتور التي تمتلك النمط 101 عند التردد التصميمي 2700 MHz مع انواع مختلفة من التغذية. حيث تم محاكاة اداء هذه المصفوفة ووجد انها تعمل على الترددات 2700 MHz, 900 MHz, 300 MHz, 100 MHz. تم اقتراح نوعان من مصفوفة كانتور للهوائيات الخطية تمتلك النمط 11011 و 1010101 مع انواع مختلفة من التغذية. حيث تم محاكاة اداء هذه المصفوفات ووجد انها تعمل على الترددات 2700 MHz و 540 MHz للطريقة المقترحة الاولى (11011) و 2700 MHz و 386 MHz للطريقة المقترحة الثانية (1010101). الترددات التي تم استخدامها تقع ضمن الحزم الترددية VHF و UHF التي تستخدم في الكثير من تطبيقات أنظمة الاتصالات مثل GSM, WLAN, WIMAX ..... الخ.

## 1. Introduction

The increasing range of wireless telecommunication services and related applications is driving the attention to the design of multifrequency (multiservice) and small antennas. The telecom operators and equipment manufacturers can produce a variety of communication systems, like cellular communications, global positioning, satellite communications, and others. Each one of these systems operates at several frequency bands. To give service to the users, each system needs to have an antenna that has to work in the frequency band employed for the specific system. The tendency during last years had been to use one antenna for each system, but this solution is inefficient in terms of space usage, and it is very expensive. The variety of communication systems suggests that there is a need for multiband antennas. The use of fractal geometry is a new solution to the design of multiband antennas and arrays.

Fractal was first defined by Benoit Mandelbrot [1] in 1975 as a way of classifying structures whose dimensions are not whole numbers. These geometries have been used previously to characterize unique occurrences in nature that were difficult to define with Euclidean geometries, including the length of coastlines, the density of clouds, and branching of trees.

Fractals can model nature very well. They can be used to generate realistic landscapes or sunsets, wire-frames of mountains, rough terrain, ripples on lakes, coastline, sea floor topography, and plants. Fractals can be divided into many types, as shown in figure (1).

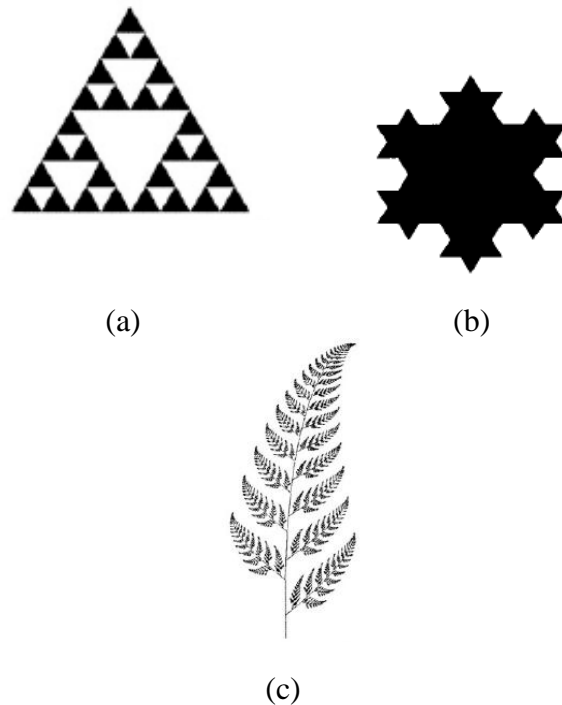


Figure (1) Three fractal examples (a) Sierpinski gasket (b) Koch snowflake (c) Tree [1].

Fractal applications have appeared in many branches of engineering and science. One such area is fractal electrodynamics in which fractal geometry is combined with electromagnetics. An introduction to the subject of fractal electrodynamics may be found in the excellent review by Jaggard [2].

This paper focuses on the application of fractal geometric concepts for the analysis and design of fractal linear array antennas.

## 2. Fractal Antenna Arrays

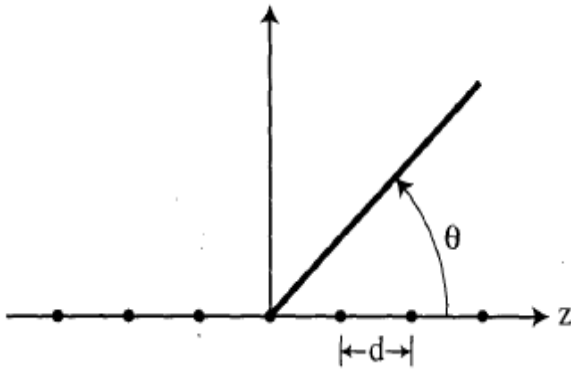
Fast recursive algorithms for calculating the radiation patterns of fractal arrays have recently been developed in [3-5]. These algorithms are based on the fact that fractal arrays can be formed recursively through the repetitive application of a generating

array. A generating array is a small array at level one ( $P=1$ ) used to recursively construct larger arrays at higher levels (i.e.  $P>1$ ). In many cases the generating sub array has elements that are turned on and off in a certain pattern. A set formula for copying, scaling, and translating the generating array is then followed in order to produce a family of higher order arrays.

The array factor for a fractal antenna array may be expressed in the general form [4-5]

$$AF_P = \prod_{p=1}^P GA(\delta^{p-1}\psi) \quad (1)$$

where  $GA(\psi)$  represents the array factor associated with the generating array. The parameter  $\delta$  is a scaling or expansion factor that governs how large the array grows with each successive application of the generating array and  $P$  is a level of iteration. The expression for the fractal array factor given in eq. (1) is simply the product of a scaled version of a generating sub array factor. Therefore, they may regard eq. (1) as representing a formal statement of the pattern



multiplication theorem for fractal arrays. The analysis and design of fractal linear arrays will be considered in the following section.

## 2. Fractal Linear Array Antennas

A linear array of isotropic elements, Uniformly spaced a distance  $d$  apart along the  $z$ -axis, is shown in figure (2).

Figure (2) The linear array geometry of uniformly spaced isotropic sources [6].

The array factor corresponding to this linear array may be expressed in the form [7-8]

$$AF(\psi) = \begin{cases} I_0 + 2 \sum_{n=1}^N I_n \cos(n\psi) & \text{for } (2N+1) \text{ elements} \\ 2 \sum_{n=1}^N I_n \cos\left(\left(\frac{2n-1}{2}\right)\psi\right) & \text{for } (2N) \text{ elements} \end{cases} \quad (2)$$

Where

$I_n$ : Excitation coefficients of the array elements

$$\psi = kd \cos \theta \quad (3)$$

and

$$k = \frac{2\pi}{\lambda} \quad (4)$$

$k$ : The wave number

This array becomes fractal-like when appropriate elements are turned off or removed, so that

$$I_n = \begin{cases} 1 & \text{if element } n \text{ is turned on} \\ 0 & \text{if element } n \text{ is turned off} \end{cases} \quad (5)$$

One of the simplest schemes for constructing a fractal linear array follows the recipe for the cantor set [9]. Some other aspects of Cantor arrays have been investigated more recently in [4,6,10,11]. Cantor arrays also own multiband properties, so it has multi frequencies ( $F_n$ ):

$$F_n = \frac{F_0}{\delta^n} \quad n = 0, 1, 2, \dots, P-1 \quad (6)$$

Where  $F_0$  is the design frequency

The basic Cantor array may be created by starting with a three element generating sub array, and then applying it repeatedly over  $P$  scales of growth. The generating sub array in this case has three uniformly spaced elements, with the center element turned off or removed, i.e., 101. The Cantor array is generated recursively by replacing 1 by 101 and 0 by 000 at each level of the construction. Table (1) provides the array pattern for the first four levels of the Cantor array.

The array factor of the three element generating sub array with the representation 101 is

$$GA(\psi) = 2I_1 \cos(\psi) \quad (7)$$

Which may be derived from eq. (2) by setting  $N=1, I_0 = 0$ . Substituting eq. (7) into eq. (1) and choosing an expansion factor of three ( $\delta=3$ ), the results in an expression for the Cantor array factor is given by

$$AF_P(\psi) = \prod_{p=1}^P GA(3^{p-1}\psi) = 2 \prod_{p=1}^P I_1 \cos(3^{p-1}\psi) \quad (8)$$

#### **4. A New Approache of Cantor**

##### **Linear Array**

The proposed methods of Cantor linear array may be created by starting with a five or seven elements generating subarray, and then applying it repeatedly over  $P$  scales of growth. The generating subarray in this case has five or seven uniformly spaced elements, with the center element turned off or removed, i.e., 11011 and 1010101 respectively. The Cantor array is generated

recursively by replacing 1 by 11011 or 1010101 and 0 by 00000 or 0000000 at each level of the construction. Table (2) and (3) provides the array pattern for the first two levels of the Cantor array for the array pattern 11011 and 1010101. The array factor of five elements generating sub array with 11011 is

$$GA(\psi) = 2I_1 \cos(\psi) + 2I_2 \cos(2\psi) \quad (9)$$

While, the array factor of seven elements generating sub array with 1010101 is

$$GA(\psi) = 2I_1 \cos(\psi) + 2I_3 \cos(3\psi) \quad (10)$$

Substituting eqs. (9) and (10) into eq. (1) and choosing an expansion factor of five ( $\delta=5$ ) and seven ( $\delta=7$ ) respectively results in an expression for the two cases above for the Cantor array factor given by

Model one, for 11011

$$AF_P(\psi) = \prod_{p=1}^P GA(5^{p-1}\psi) = 2 \prod_{p=1}^P (I_1 \cos(5^{p-1}\psi) + I_2 \cos(5^{p-1}2\psi)) \quad (11)$$

Model two, for 1010101

$$AF_P(\psi) = \prod_{p=1}^P GA(7^{p-1}\psi) = 2 \prod_{p=1}^P (I_1 \cos(7^{p-1}\psi) + I_3 \cos(7^{p-1}3\psi)) \quad (12)$$

#### **5. Computer Simulation Tests And**

##### **Results**

Let, a linear array will be designed and simulate at a frequency ( $F_0$ ) equal to 2700MHz, (then the wavelength  $\lambda_0 = 0.1111$  m), with quarter-wavelength ( $d = \lambda/4$ ) spacing between the adjusted

array elements and 16 active elements in the array. The level four of Cantor linear array (101) have the number of active elements of 16, the total elements number of 81 and the total array length ( $L$ ) of  $20\lambda = 2.222$  m. The Cantor linear array will operate at four frequencies depending on eq. (6). These frequencies are 2700MHz, 900MHz, 300MHz, and 100MHz.

In the proposed model one for 11011, level two has the number of active elements of 16, the total elements number of 25 and the total array length ( $L$ ) of  $6\lambda = 0.6667$  m. Model one will operate at two frequencies depending on eq. (6). These frequencies are 2700MHz, and 540MHz.

The proposed model two for 1010101, in level two has the number of active elements of 16, the number of total elements of 49 and the total array length ( $L$ ) of  $12\lambda = 1.3333$  m. Model two will operate at two frequencies depending on eq. (6). These frequencies are 2700MHz, and 386MHz.

The array factors for all above types are plotted by using MATLAB version 7.2(R2006a) program with uniformly and non uniformly amplitude distribution which they are feeding to active elements. The field patterns are illustrating as shown in figures (3-5), While the values of the side lobe level (SLL), half power beamwidth (HPBW), and directivity (D) are illustrating in tables (4-6).

## 6. Conclusions

It can be concluded that;

1. The field pattern for the different models of fractal array has higher directivity and lower side lobes when feeding with Dolph-Tschebyscheff amplitude distribution type.
2. The model one with 11011 has lower side lobe level ( $SLL$ ) of  $-7.8939$  dB and less number of side lobes, while

$SLL = -5.1361$  dB for model two and  $SLL = -5.934$  dB for conventional Cantor linear array.

3. Model one with 11011 has the total length ( $L$ ) which is the shortest and equal to  $6\lambda_0$ , while  $L = 12\lambda_0$  for model two and  $L = 20\lambda_0$  for conventional Cantor linear array.
4. The drawback of the proposed models is lower directivity than the conventional Cantor linear array.

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**Table (1) First four levels of the conventional Cantor fractal linear array**

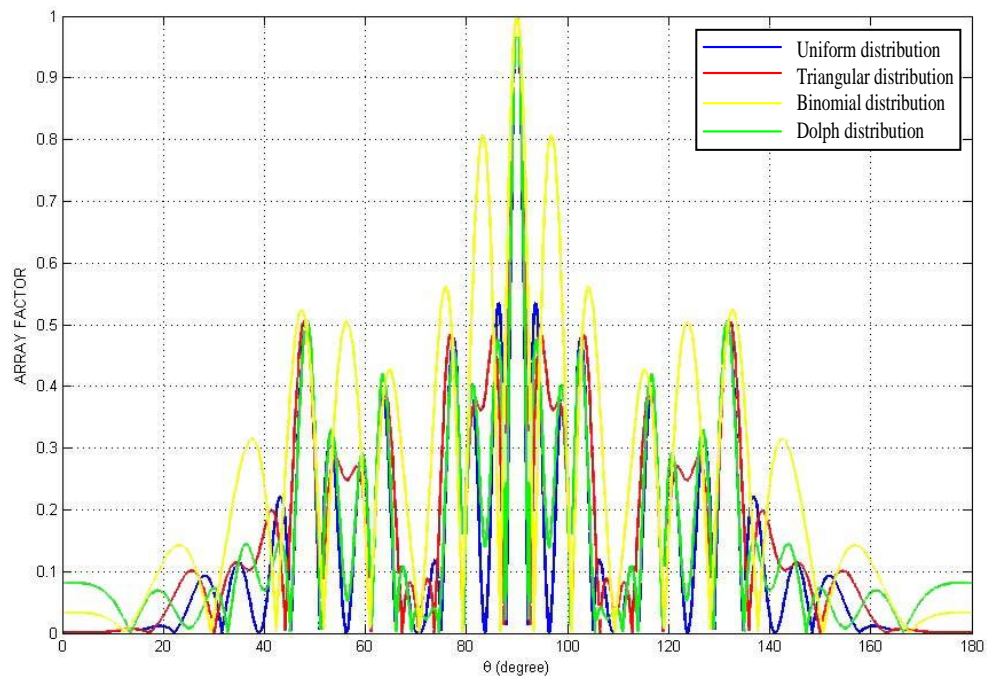
P	Elements array Pattern	Active Elements	Total Elements
1	101	2	3
2	101000101	4	9
3	101000101000000000101000101	8	27
4	1010001010000000001010001010000000000000000000000 0101000101000000000101000101	16	81

**Table (2) First two levels for proposed model one of the fractal linear array**

P	Elements array Pattern	Active Elements	Total Elements
1	11011	4	5
2	1101111011000001101111011	16	25

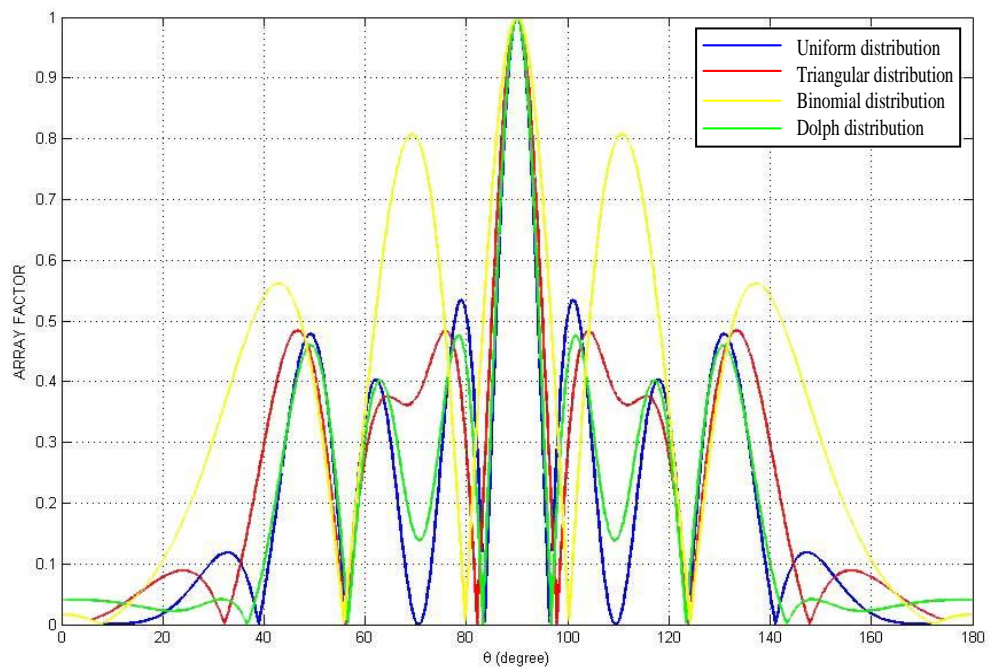
**Table (3) First two levels for proposed model two of the fractal linear array**

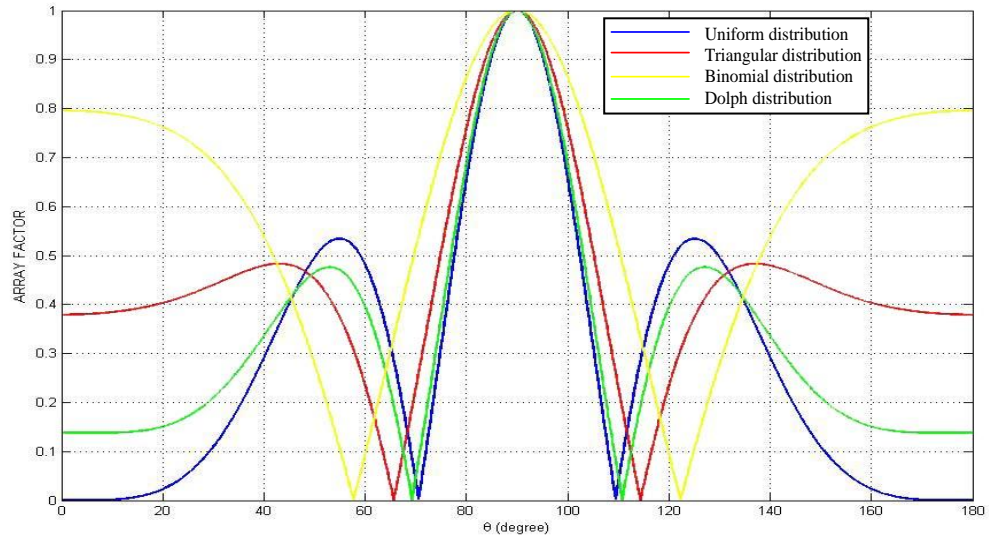
P	Elements array Pattern	Active Elements	Total Elements
1	1010101	4	7
2	1010101000000010101010000000101010100000001010101	16	49



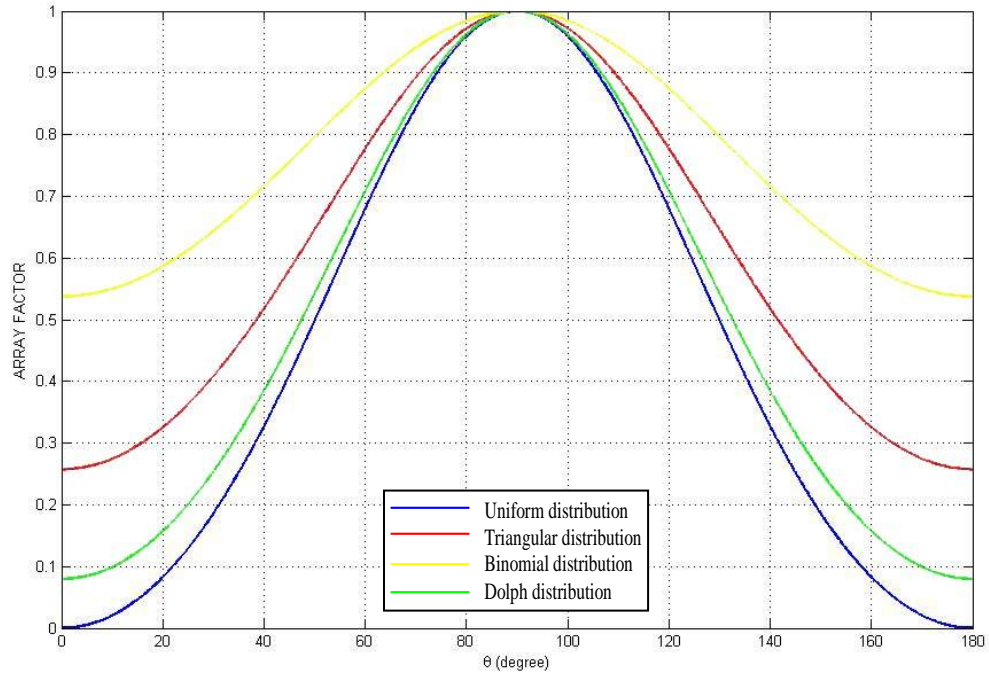
(a)

(b)





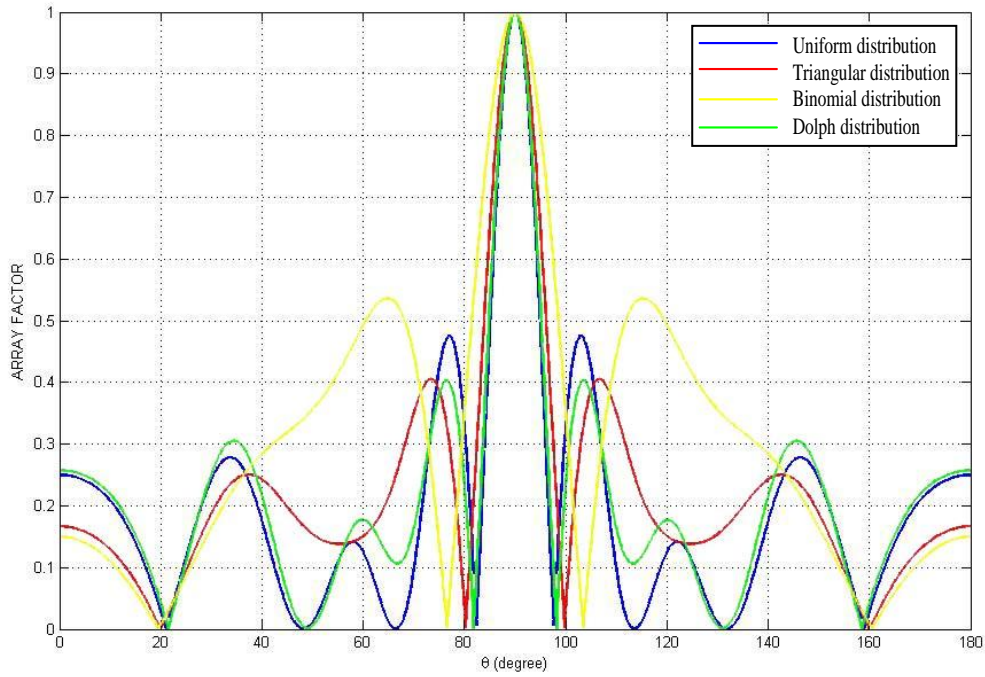
(c)



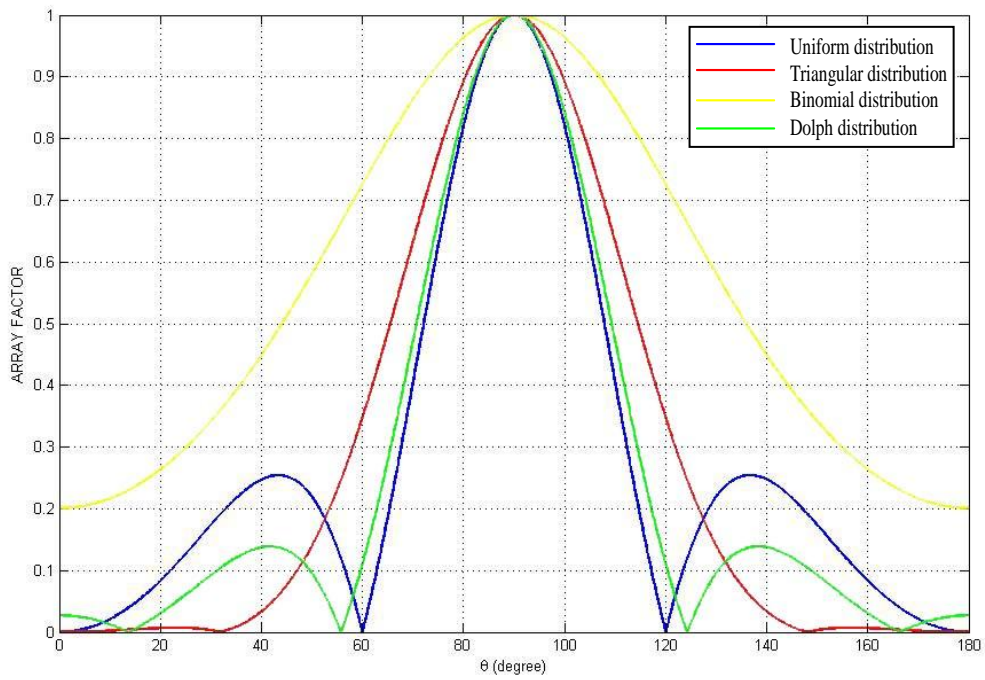
(d)

**Figure (3) Array factor of a linear Cantor array when elements array pattern is  $101$   $F_0 = 2700\text{MHz}$  (b)  $F_1 = 900\text{MHz}$  (c)  $F_2 = 300\text{MHz}$  (d)  $F_3 = 100\text{MHz}$ .**



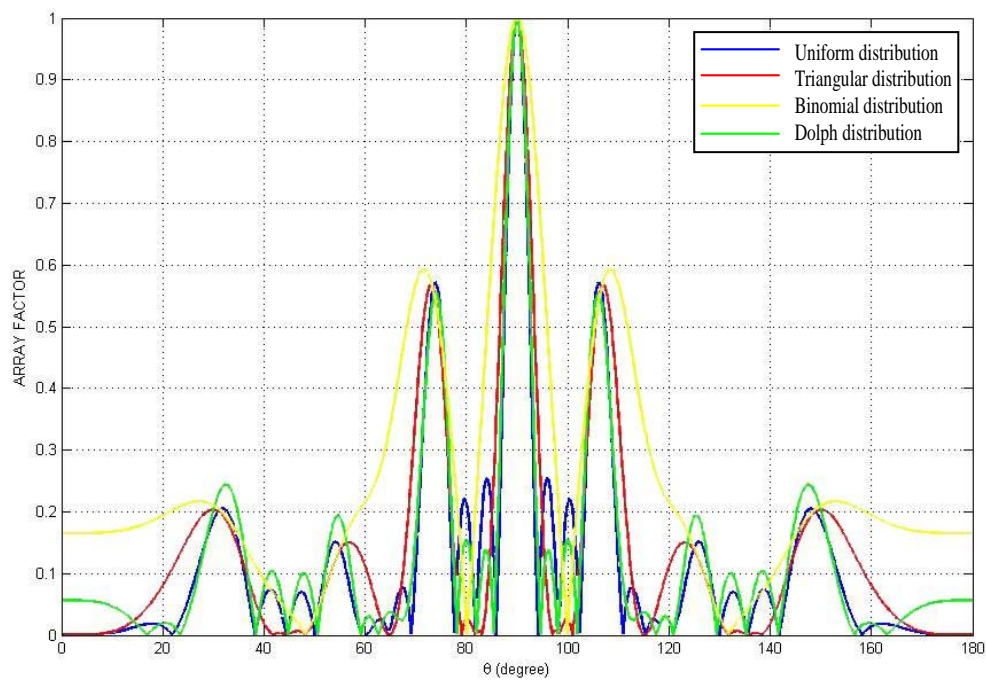


(a)

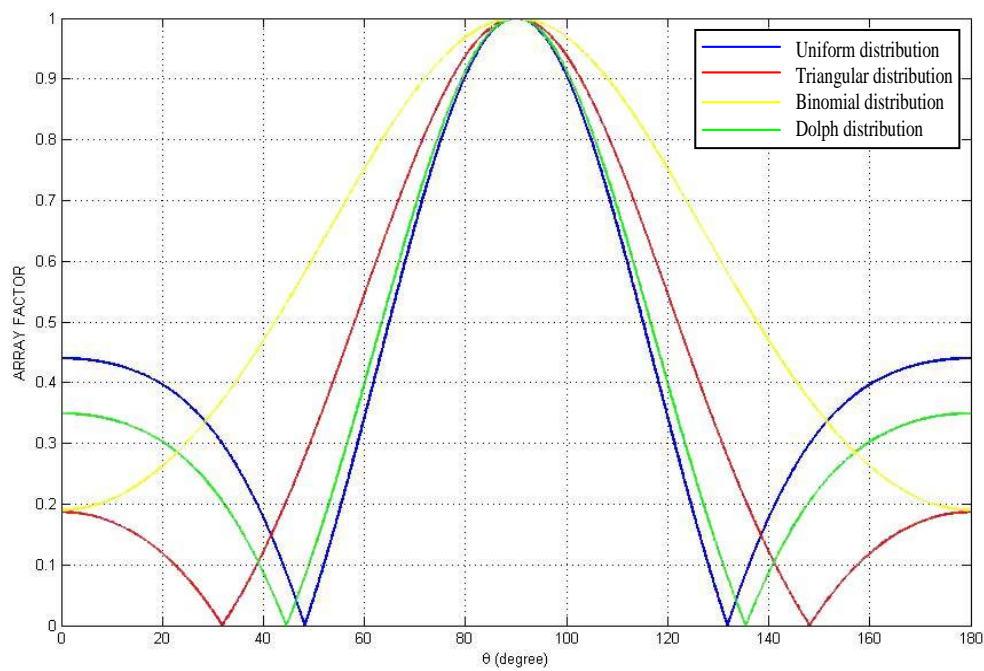


(b)

**Figure (4) Array factor of a linear Cantor array when elements array pattern is 11011: (a)  $F_0 = 2700\text{MHz}$  (b)  $F_1 = 540\text{MHz}$ .**



(a)



(b)

**Figure (5) Array factor of a linear Cantor array when elements array pattern is 1010101:**  
**(a)  $F_0 = 2700\text{MHz}$  (b)  $F_1 = 386\text{MHz}$ .**

Table (4) SLL, D, and HPBW for different types of amplitude distribution fractal Cantor linear array when elements array pattern is 101

## (a) Uniform amplitude distribution

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	16.009	2.0233	0.5339
900	8.3117	6.0721	0.5342
300	4.1725	18.2852	0.5342
100	2.082	56.9372	0

## (b) Binomial amplitude distribution

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	6.926	3.4	0.8072
900	3.6595	10	0.8073
300	2.5017	30.4	0.7958
100	1.38404	104	0

## (c) Triangular amplitude distribution

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	12.7131	2.4555	0.5053
900	6.6422	7.3705	0.4842
300	3.4802	22.2354	0.4831
100	1.7248	70.6869	0

## (d) Dolph amplitude distribution

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	15.3837	2.1285	0.505
900	8.2328	6.388	0.476
300	4.1784	19.2444	0.4761
100	1.9758	60.1923	0

Table (5) SLL, D, and HPBW for different types of amplitude distribution for proposed model one when elements array pattern is 11011

## (a) Uniform amplitude distribution

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	9.9839	7.2069	0.4752
540	2.8979	36.6313	0.44

**(b) Binomial amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	4.8659	12.5643	0.536
540	1.8154	66.3395	0

**(c) Triangular amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	8.2222	8.9619	0.4053
540	2.5138	45.9879	0.1869

**(d) Dolph amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	9.7349	7.5769	0.403
540	2.8664	38.5814	0.3491

**Table (6) SLL, D, and HPBW for different types of amplitude distribution for proposed model two when elements array pattern is 1010101****(a) Uniform amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	16.009	3.6964	0.5714
386	4.0731	26.0741	0.2543

**(b) Binomial amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	6.926	8.5169	0.593
386	1.881	62.5833	0

**(c) Triangular amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	12.7131	4.9022	0.5668
386	3.2213	34.8103	0.00715

**(d) Dolph amplitude distribution**

F(MHz)	D(dimensionless)	HPBW(degree)	SLLmax(dimensionless)
2700	15.3837	3.9575	0.5536
386	3.985	27.9522	0.1388