

# The Fundamental Natural Frequency and Critical Flow Velocity Evaluation of a Simply Supported Stepped Pipe Conveying Fluid

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## Abstract

This work investigated the fundamental natural frame frequency and critical inlet flow velocity of the simply supported stepped pipe conveying fluid. The influence of some outline parameters, similar pipe diameter ratio with different cross-sectional areas (sudden enlargement or sudden contraction), length ratio, and thickness were studied. A theoretical model for the dynamic behavior of the pipe was investigated. The dynamic behavior of the stepped pipe carrying fluid was described by means of finite element technique. A computer program has been promoted utilizing Matlab language in order to predict the vibration characteristics and to embrace the theoretical work. It was found that the fundamental natural frequency of the stepped pipe decreased in different percentages with increasing inlet fluid flow velocity. The outer diameter ratio, length ratio, and thickness of the stepped pipe conveying fluid had a great effect on the dynamic behavior of stepped pipe. The fundamental natural frame frequency increased as the length ratios ( $L_2/L_1$ ) and the outer diameter ratios ( $Od_2/Od_1$ ) decreased. When the length ratio of the pipe being less than 1, the effect of the diameter ratio on the critical inlet flow velocity was obvious. As the outer diameter ratios ( $Od_2/Od_1$ ) increased, the inlet critical flow velocity increased, until being reached to the maximum value, then it dropped smoothly. The fundamental frame frequency of the stepped pipe increased as the thickness of the stepped pipe increased until being reached to the maximum value, then it dropped smoothly. For each inlet fluid flow velocity, length ratios, and diameter ratios there was an optimum pipe thickness that gave the best dynamic characteristics.

**Keywords:** Natural frequency, critical flow velocity, finite element analysis, stepped pipe conveying fluid.

## الخلاصة

في هذا العمل، تم التحقيق في مبادئ التردد الطبيعي الاساسي و السرعة الحرجة للجريان الداخلي للانبوب المتدرج و الناقل للمائع ذو المساند البسيطة. و تم دراسة تأثير بعض العوامل التصميمية مثل نسبة الاقطار باختلاف مساحة المقطع العرضي (الانفراج الفجائي او التقلص الفجائي)، و كذلك تم دراسة نسبة الطول و السمك للانبوب. و تم دراسة السلوك الديناميكي للانبوب المتدرج الناقل للمائع نظريا. و قد وصف هذا السلوك بواسطة تقنية العناصر المحددة. و طور برنامج حسابي باستخدام لغة Matlab من اجل التنبؤ بخواص الاهتزاز لتغطية العمل النظري.

في هذا البحث وجد ان التردد الطبيعي الاساسي للانبوب المتدرج يقل بنسب مختلفة مع زيادة سرعة جريان المائع الداخل للانبوب. ان نسبة الاقطار الخارجية للانبوب المتدرج، نسبة الطول، و نسبة السمك لها تأثير كبير على السلوك الديناميكي. ان التردد الطبيعي الاساسي يزداد بنقصان نسبة الطول و نسبة الاقطار الخارجية.

عندما تكون نسبة الطول للانبوب اقل من واحد فان تأثير نسبة الاقطار على سرعة الجريان الحرجة الداخلة تكون واضحة. و عندما تزداد نسبة الاقطار الخارجية للانبوب فان سرعة الجريان الحرجة الداخلة تزداد حتى تصل الى اعظم مقدار ثم تتخفف تدريجيا. ان التردد الطبيعي الاساسي للانبوب المتدرج تزداد عندما يزداد سمك الانبوب حتى تصل الى اعظم مقدار ثم تبدأ بالانخفاض تدريجيا. ان لكل سرعة جريان داخلة للمائع، نسبة اطوال، و نسبة اقطار، هناك مقدار مثالي لسمك الانبوب و الذي يعطي افضل خواص ديناميكية.

**الكلمات المفتاحية:** التردد الطبيعي، سرعة التدفق الحرجة، تحليل العناصر المحددة، الانبوب المتدرج الناقل للمائع.

## 1. Introduction

The pipe conveying fluid becomes sensitive to resonance or fatigue failure if its natural frequency falls underneath specific points of confinement. At high velocities of fluid, the pipe may end up noticeably insecure. Because of the fluid flow through the pipe, a force introduced into the pipe causes the pipe to vibrate at a vast amplitude when an inlet velocity of the flow goes beyond the critical flow velocity. This force

conforms the fluid to the pipe at all times. The vibration of the pipe conveying fluid is coupled by the forces exerted on the structure by the fluid (Kadhim, 2005).

A considerable research of analytical work has been recently done to look into the vibration effect on pipes structures carrying fluid. The most extensively studied structure is the circular cylinder, largely due to its convenient shape. The vibration of rods and tubes in a parallel flow is a common problem, more importantly, many engineering applications like encourage lines to imaginative elite rockets and aircrafts, reactors system components, water turbines, and heat exchangers, have been used in modern power generation, chemical industries operate on either single cylinders-tubes in cross flow, or on one row and multi-row banks, high energy plants. These flow induced vibrations can cause fretting and fatigue, and can degrade heat transfer performance and reactor core physics. The majority of vibration of rods in parallel flow induced by turbulence in the flow, is the most common practical problem (Kadhim, 2005).

Piping systems are flexible and affected by many kinds of vibration sources. These sources create problems such as fatigue damage, vibration and noise. Supports are applying at the principle purposes of the piping system to solve these problems, (Mahdi, 2001).

In specific points of applications including high velocity flows through flexible pipes combined with vibration such as (encourage lines to rockets and water turbines), the pipe may become sensitive to resonance and fatigue failure if its natural frequency falls beneath specific points of limits (Blevins, 1979).

The inflow within the pipe at a sudden enlargement or contraction take place at a numerous mechanical implementation which have been portrayed with increasing pressure amissions brought about by flow breakaway near adjustment in the cross sectional zone. This increase in pressure loss increases the corrosion rates and heat in the zones where breakaway flow take place (Mahdi, 2001).

(Al-Saffar,1989), studied the out-of-plane vibration characteristics of an intermediately supported curved-straight tube system conveying fluid, where theoretical and experimental works being achieved. The effect of constant thermal force, fluid flow, support location and straight segment length to curve segment radius ratio on the vibration characteristics of this type of tubes, were accounted for. (Sugiyama *et.al.*,1996), covered in their study the balancing out component of the vibration concealment because of an inner fluid flow practically implemented. The technique depended on the way that an inward flowing fluid can balance out the structure by scattering vibrational energy.

(Qing *et.al.*,2002), in this study, the author considered large amplitude oscillation and the case of the clamor for a structure of the nuclear pipe analysis experimentally and theoretically. (Lolov and Markova , 2006), investigated the dynamic behavior of bended pipe, which conveying fluid, and its stability analysis to obtain the flow velocity and the frequency as a non dimensional parameters. (Ismael *et.al.*,2007), investigated a simply support annulus pipe conveying fluid experimentally with free and forced vibration as well as its effect on the forced convection heat transfer coefficient which consequence a relation betwixt the heat flux and natural frequency. A pipe conveying fluid combined with vibration on these pipes was studied by (Salim, 2008). The effect of support type (flexible, simply and rigid) on the natural frequency and corresponding mode shape of a straight pipe conveying laminar flowing fluid was contemplated. Additionally, the effect of some outline parameters like pipe diameter, wall thickness, pipe material, and the effect of fluid velocity were investigated for Reynolds number ranging between (250 to 1500).

(Mediano and Garcia,2014), modeled the dynamic performance of conveying pipe with pinned-fixed supported. In this model, the materials of the pipe played an important role in the performance of the whole piping structures. (Veerapandi *et.al.*, 2014), offered an practical study and analysis of the vibration produced by a flow, by studying the influence of the gas flow in turbulence case of system through sub cooling of static flight of flame test or trial of flow. The critical velocity of a fluid was computed by an analytical approach. Also, the analysis of the modal in the flow of the pipe conformation was studied.

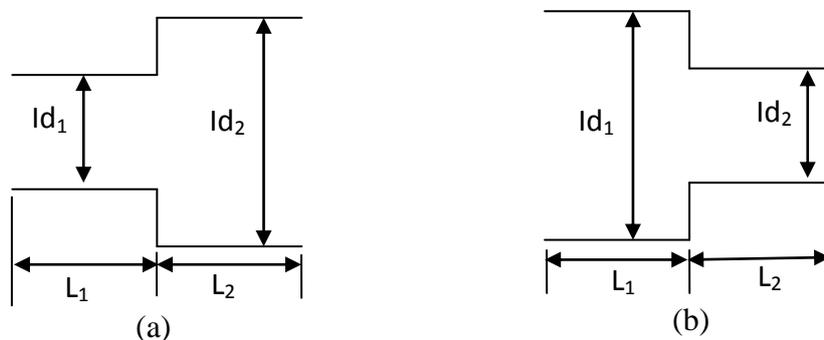
Going through these investigations that dealt with this field, it is possible to say that a few of last investigations studied the effects of the simply supported on the behavior of pipes conveying fluid with different cross-sectional areas, but they dealt with straight pipes and pipes contained orifice plate and exposed to vibration. As a result of that, it is important to investigate the vibration characteristic on a pipe conveying fluid with various cross-sectional areas for example (sudden enlargement and sudden contraction) and different length ratio.

The supported continuous pipes have many practical applications; one of these is their use in conveying petroleum products for a very long distance. The goal of this restriction is to minimize the vibrational influences and additionally to oppose the influence of pipe thermal expansion. Hence, this work is concerned with the effect of vibration on sudden augmentation and constriction pipes carrying fluid to find the natural frequencies and critical flow velocity (as the velocity for which the system loses stability) for a simply end stepped pipe supports conveying fluid for different pipe diameter ratios (different cross-sectional areas such as sudden enlargement or sudden contraction), length ratios, and thickness.

A numerical analysis adopted finite element approach is employed to form the basic elemental matrices and a Matlab program will be developed to solve the equation of motion developed for this purpose. The damping (Coriolis), mass and stiffness matrices are resulting and analysis of the eigenvalue are evaluated.

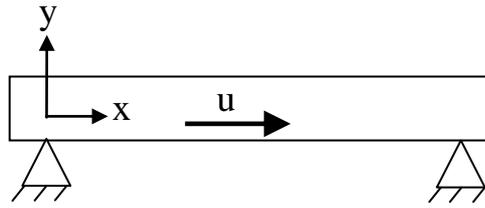
## 2. Derivation of Equation of Motion

Considering a stepped pipe conveys uniform internal flow as shown in fig.1. The stepped pipe, simply supported at both ends, has two regime 1 and 2 (for a sudden contraction or enlargement), with dimensions given by the length ( $L_1$ ) and ( $L_2$ ), the outer diameters are ( $Od_1$ ) and ( $Od_2$ ), and ( $t$ ) is thickness. The pipe is assumed to be sufficiently slender,  $(D/L) \ll 0.1$ . The fluid considered to be incompressible, so that the velocity inside the pipe is uniform and the flow is laminar, where negligible effects of the secondary flow.



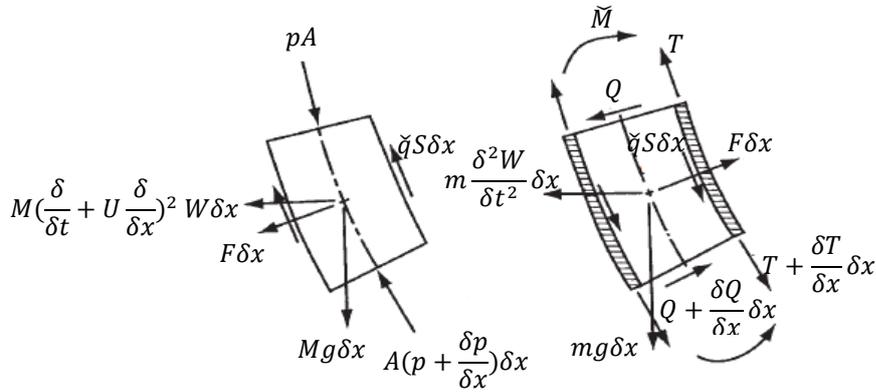
**Fig. 1 A simply supported pipe system (a) sudden enlargement (b)sudden contraction.**

Fig. 2 exemplifies a straight pipe convey fluid with a simply supported ends, with inlet fluid velocity of  $U$ .



**Fig. 2 Simply supported ends pipe system.**

In fig. 3, the reaction forces and moments of a fluid and pipe elements are appeared. The system involve of uniform pipe of length ( $L$ ), mass of a pipe for each length unity ( $m$ ), flexural rigidity ( $EI$ ), mass per length unity of conveyance fluid ( $M$ ), and fluid velocity flowing pivotally ( $U$ ). The cross-sectional flow zone is indicated by ( $A$ ) and the pressure of the fluid is indicated by ( $p$ ).  $F\delta x$  indicates the pipe reaction forces of the fluid, it's normal on element.  $\delta x$  deems elements length of fluid and pipe.



**Fig.3 Elements of pipe, fluid and reaction of moments and forces, (Paidoussi, 1998).**

The equation of motion of a system appears in fig.3, (Mustafa. N. H, 2014) :

$$EI \frac{\partial^4 W}{\partial x^4} + MU^2 \frac{\partial^2 W}{\partial x^2} + 2MU \frac{\partial^2 W}{\partial x \partial t} + (m + M) \frac{\partial^2 W}{\partial t^2} = 0 \quad \dots\dots\dots (1)$$

The phrase ( $EI \frac{\partial^4 W}{\partial x^4}$ ) describe a component of influencing force on the pipe as an outcome of the bending pipe. The term ( $MU^2 \frac{\partial^2 W}{\partial x^2}$ ) exemplifies a component of the force performing on the pipe resulted from flow in the region of a deflected pipe (curvature in pipe). This term greatly affects the pipe stability which makes a pipe unstable. The expression ( $2MU \frac{\partial^2 W}{\partial x \partial t}$ ) indicates the inertial force that correlated by Coriolis acceleration caused by flow of fluid velocity  $U$  according to the pipe. The effecting force on the pipe, because of the pipe inertia and the fluid flowing through it, is referred in terms of ( $(m + M) \frac{\partial^2 W}{\partial t^2}$ ). Finally, it is observed that the dynamic behavior of the framework greatly relies on a pipe stiffness, velocity of flow, and lateral displacement (boundary conditions). Thus, varying elastic structure flexibility should alter the dynamic behavior. The boundary conditions of a simply supported pipe are:

$$w|_{x=0,L_2} = 0 \quad EI \frac{\partial^2 W}{\partial x^2} |_{x=0,L_2} \quad \dots\dots(2)$$

$$u = u_1|_{x=0} \quad \text{inlet velocity to the first regime} \quad \dots(3)$$

$$u = u_2|_{x=L_2} \quad \text{inlet velocity to the second regime} \quad \dots(4)$$

Where:  $u_2 = u_1 * (Id_1/Id_2)$

### 3. Finite Element Discretization of the Governing Equation

The equation of element deflection for straight two dimensional beam element could have the form, (Rao, 2004):

$$W(x) = \sum_{i=1}^n N_i(x)q_i \quad \dots(5)$$

Where  $q_i$  is the generalized coordinates ( displacement and rotation, see fig.4).  $N_i$  is the bending shape functions and  $W(x)$  is the deformation polynomial cubic function which defines the displacements and rotations at the nodes.

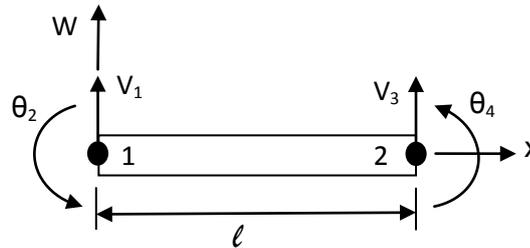


Fig. 4 coordinate system of beam element.

The shape functions  $N_i$  are given by (Rao, 2004):

$$N_1 = \frac{1}{l^3}(2x^3 - 3lx^2 + l^3) \quad \dots(6-i)$$

$$N_2 = \frac{1}{l^2}(x^3 - 2lx^2 + l^2x) \quad \dots(6-ii)$$

$$N_3 = \frac{1}{l^3}(3lx^2 - 2x^3) \quad \dots(6-iii)$$

$$N_4 = \frac{1}{l^2}(x^3 - lx^2) \quad \dots(6-iv)$$

Where:  $l$ , element length.

The above shape functions,  $N_i$ , represent the conventional customary two-dimensional beam elements which have two degrees of freedom at each node: one lateral displacement and the other rotational.

The kinetic and potential energies of a pipe element are described as:

$$K.E = \frac{1}{2} \int_0^l (M+m) \left( \frac{\partial W}{\partial t} \right)^2 dx = \frac{1}{2} \sum_e q^T (M+m) \int_0^l N^T N dx q \quad \dots(7)$$

$$P.E_1 = \frac{1}{2} \int_0^l EI \left( \frac{\partial^2 W}{\partial x^2} \right)^2 dx = \frac{1}{2} \sum_e q^T EI \int_0^l N''^T N'' dx q \quad \dots(8)$$

Accordingly, mass ( $\hat{m}$ ) and stiffness ( $\hat{k}_1$ ) matrices are given as, (Rao, 2004):

$$[\hat{m}] = \frac{(m+M)l}{420} \begin{bmatrix} 156 & 22l & -54 & -13l \\ 22l & 4l^2 & -13l & -3l^2 \\ 54 & 13l & -156 & -22l \\ -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \quad \dots(9)$$

$$[\hat{k}_1] = \frac{2EI}{l^3} \begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix} \quad \dots(10)$$

The idiom ( $MU^2 \frac{\partial^2 W}{\partial x^2}$ ) has a potential energy which is described in the expression of displacement shape function derivative for a pipe as, (Rao, S. S., 2004):

$$P.E_2 = \frac{1}{2} \int_0^l M U^2 \left( \frac{\partial W}{\partial x} \right) \left( \frac{\partial W}{\partial x} \right) dx = \frac{1}{2} \sum_e q^T M U^2 \int_0^l N'^T N' dx q \quad \dots(11)$$

The stiffness matrix occurs from flow about the deviated pipe is, (Rao, 2004):

$$[\hat{k}_2] = \frac{MU^2}{30l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \dots(12)$$

Significantly, it obvious that, the stiffness matrix  $[\hat{k}_2]$  tends to debilitate the stiffness of the pipe structure (Paidoussi, 1998).

The dispersal energy is described in idiom of the Coriolis force expression ( $2MU \frac{\partial^2 w}{\partial x \partial t}$ ) as, (Rao, 2004):

$$D.E. = \frac{1}{2} \int_0^l 2MU \left(\frac{\partial w}{\partial x}\right) \left(\frac{\partial w}{\partial t}\right) dx = \frac{1}{2} \sum_e q^T 2MU \int_0^l N'^T N dx \dot{q} \dots(13)$$

It produces the unsymmetrical damping matrix (Paidoussi, 1998):

$$[\hat{C}] = \frac{MU}{30} \begin{bmatrix} -30 & -6l & -30 & 6l \\ 6l & 0 & -6l & l^2 \\ 30 & 6l & 30 & -6l \\ -6l & -l^2 & 6l & 0 \end{bmatrix} \dots(14)$$

Generally, the stiffness matrices mentioned above are assorted as per their class. At that point, the whole pipe length model is described according to the every class of the assembled element matrices. The dynamic behavior of the system is obtained by regulated these overall matrices in an appropriate appearance.

#### 4. Solution of an Eigenvalue

The equation of motion in finite element formulation is, (Meirovitch, L, 1980):

$$[\hat{m}]\{\ddot{q}\} + [\hat{C}]\{\dot{q}\} + [k_{total}]\{q\} = \{0\} \dots(15)$$

Where:  $k_{total} = \hat{k}_1 - \hat{k}_2$ . Here, we name the hereinbefore matrix  $[k_{total}]$  as a inconsistent matrix because it include two contrary component effects. Since, the equation mentioned above has damping phrase with skew-symmetric attribute, consequently the explanation of eigenvalues problem ought to be performed to the characteristic matrix  $[\Omega]$ , that is equivalent to, (Meirovitch, 1980):

$$[\Omega] = \begin{bmatrix} [0] & [I] \\ -[m+M]^{-1}[k_{total}] & -[m+M]^{-1}[C] \end{bmatrix} \dots(16)$$

Results obtained from the eigenvalue problem produce a complex roots. The fundamental natural frequencies of the damping structure are described by a non-real part of these roots, i. e. once the fluid flows within the pipe, whilst the real part denominates to the ratio of declining of the free vibration, (Mohamed J. A., 2013).

A mass per unit length of the pipe and fluid is given by:

$$M_t = \rho \frac{\pi}{4} (Od^2 - Id^2) + \rho_f \frac{\pi}{4} Id^2 \dots(17)$$

The expression for the natural frequency and critical velocity of the simply supported pipe conveying fluid is given by;

$$w_n = \left(\frac{\pi^2}{(L_1+L_2)^2}\right) \sqrt{\frac{E*I}{M_t}} \dots(18)$$

$$u_c = \left(\frac{\pi}{(L_1+L_2)}\right) \sqrt{\frac{E*I}{\rho_f*A}} \dots(19)$$

Where: E, is pipe elastic modulus, I is area moment of inertia of the pipe,  $L_1$  and  $L_2$  are the length of the first and second regime of the stepped pipe,  $\rho$  is the pipe Density,  $\rho_f$  is fluid density, and A is the internal pipe area.

#### 5. Results and Discussions

The results obtained from the numerical investigation of the simply supported stepped pipe conveying fluid are discussed in details through this section. The numerical results include results of the inlet fluid flow velocity, fundamental

natural frame frequency of the stepped pipe, and critical inlet fluid flow velocity for different outer diameter ratios ( $Od_2/Od_1$ ) and length ratios ( $L_2/L_1$ )

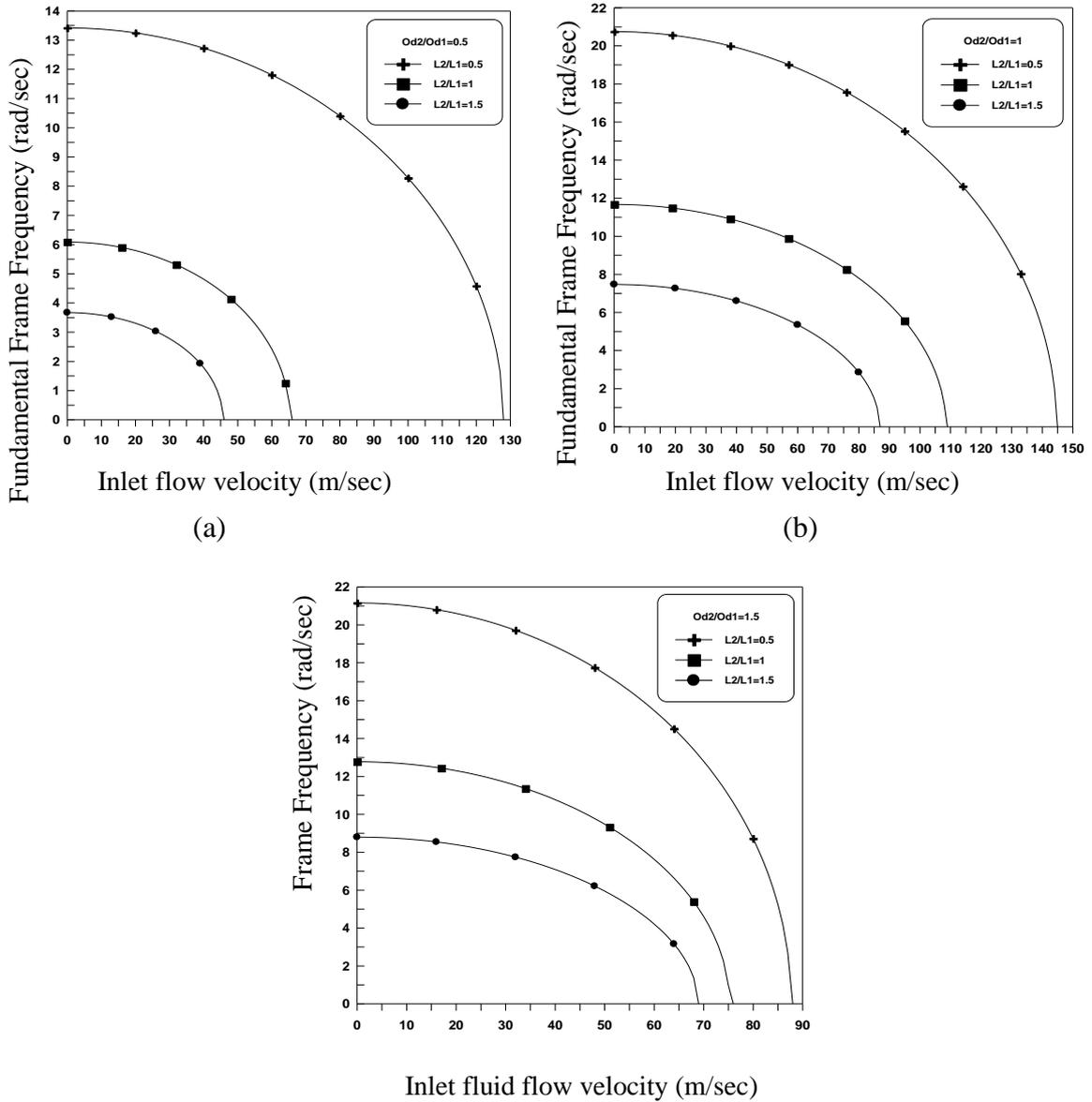
In the first instance of the numerical results, for all the relation between the inlet fluid flow velocity and natural frequency (i.e. all figures), and for all the variation parameters ( $Od_2/Od_1$ ,  $L_2/L_1$ ), the fundamental natural frequency of the pipe leads to a decrease in different percentages with increasing inlet fluid flow velocity, this attribute to that, at a comparatively low inlet fluid flow velocity, the force adapts fluid of the pipe (weaken the effect) that appears to be greater than the axial tension forces (a stiffening effect) produced in the pipe frame. With flow velocity increase, the values of these effects are reversed (stiffening effect becomes larger than weaken effect).

In fig. 5 (a, b and c), as the  $L_2/L_1$  ratio decreases, the fundamental natural frequency increases. When  $L_2/L_1$  ratio decreases from 1.5 to 0.5, the fundamental natural frequency of the pipe increases by 72% at  $Od_2/Od_1$  ratio equals to 0.5, while this increase reaches to 64% when  $Od_2/Od_1$  ratio equals to 1.0 and becomes 58% when  $Od_2/Od_1$  ratio equals to 1.5. Likewise, it can be shown from this drawing that the critical inlet fluid flow velocity and the fundamental natural frame frequency increase as the diameter ratio ( $Od_2/Od_1$ ) increases until it reaches 1. Then the critical inlet fluid flow velocity decreases, while the fundamental natural frame frequency still increases. This attribute to the sudden enlargement in the cross-sectional area of the stepped pipe.

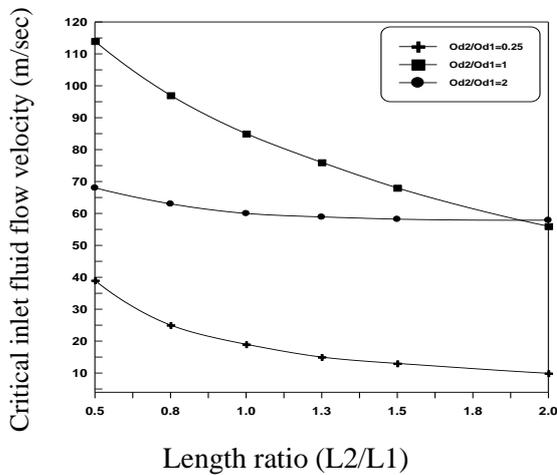
Fig. 6 demonstrates the influence of length ratio on the critical inlet velocity of the flow at different diameter ratio. When the length ratio of the pipe is less than 1, the effect of the diameter ratio on the critical inlet flow velocity is obvious. Then the critical inlet flow velocity drops smoothly as the length ratio increases. This attribute to that the pipe becomes weaker in stiffness with the increase in the length ratio.

Fig. 7 demonstrates the influence of the diameter ratio on the critical inlet flow velocity. As the diameter ratio increases, the critical inlet flow velocity increases continuously until it reaches to the maximum value, then it drops smoothly. The increase of the diameter ratio minimizes the fluid velocity through the pipe and the pipe be heavier at the same time. The combination of these two effects gives a complex behavior. Also here, the critical inlet flow velocity decreases as the length ratio of the pipe increases, this is due to the increase of the weight of the pipe and the frame structure becomes heavier.

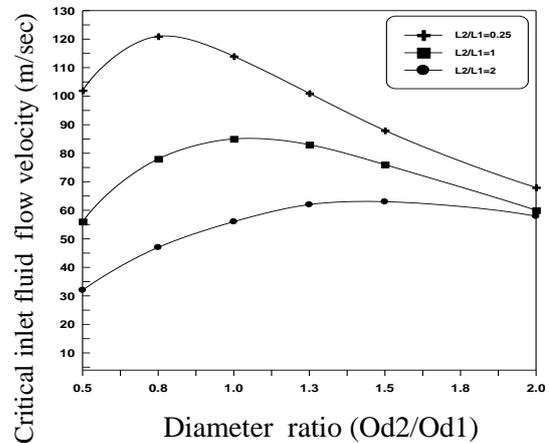
The effect of pipe thickness on the frame frequency at different inlet fluid flow velocities can be shown in fig. 8. With length and diameter ratio equal to 1, the effect of fluid velocity on the frame frequency is obvious when the pipe thickness is relatively small. For each of the inlet fluid flow velocity, as a pipe thickness increases to certain values gives the best ever frame frequency, then the frame frequency decreases, and drops smoothly. This is due to the increase of pipe stiffness and weight with an increase in thickness.



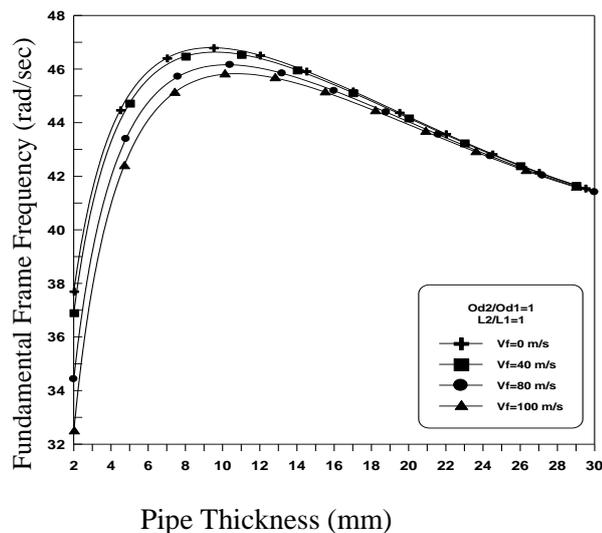
**Fig.5 Influence of inlet fluid flow velocity on the frame frequency with various outer diameter ratios (Od2/Od1) and various length ratios (L2/L1).**



**Fig. 6 Effect of length ratios on the critical inlet fluid velocity at different diameter ratios.**



**Fig. 7 Effect of diameter ratios on the critical inlet fluid velocity at different length ratio.**



**Fig. 8 Effect of pipe thickness on the frame frequency at different inlet fluid flow velocities.**

## 6. Conclusions:

In view of the results obtained, the following main conclusions can be drawn:

1. The fundamental natural frequency of the stepped pipe diminishes at different percentages with increments of inlet fluid flow velocity.
2. The fundamental natural frame frequency increases as the length ratios ( $L_2/L_1$ ) and the diameter ratios ( $Od_2/Od_1$ ) decrease.
3. The inlet fluid flow velocity of the stepped pipe increases as the outer diameter ratios ( $Od_2/Od_1$ ) increase until they reach 1 (straight pipe), then drop smoothly.
4. When the length ratio of the pipe is less than 1, the effect of the diameter ratio on the critical inlet flow velocity is obvious.
5. The fundamental frame frequency of the stepped pipe increases as the thickness of the pipe increases until it reaches the maximum value, then drops smoothly.
6. For each inlet fluid flow velocity, length ratios, and diameter ratios, there is an optimum pipe thickness that gives the best dynamic characteristics.

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