Interfacial Friction Factor in Horizontal and Inclined Annular Two-Phase Flow in Pipes

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Abstract

In the present work, a simple model to predict the interfacial friction factor in annular two-phase flow is suggested. The experimental data conducted are by two different sources for same operation and design system. The comparison procedure has achieved using the RMS function which is based on the average error between the experimental readings and the theoretical results for the whole used methods included the proposed model. The results have displayed in tabular form and graphically. The comparison reveals that the best performance of the suggested model.

الخلاصة:

في العمل الحالي: تم اقتراح موديل بسيط لتخمين معامل الاحتكاك التداخلي لجريان ثنائي الطور للجريان الاسطواني. البيانات العملية استحصلت من مصدرين مختلفين لنفس موائع الجريان ولظروف مختلفة. تمت المقارنة باستخدام دالة (RMS) والتي تعتمد على معدل الخطأ بين القراءات العملية النتائج النظرية لكل الطرق المستخدمة بضمنها الطريقة المقترحة .تم عرض النتائج بأستخدام اسلوب الجداول والرسومات. بينت المقارنة أن الأداء الأفضل للموديل المقترح.

Nomenclatures:

A	Cross Section Area	m^2
f	Friction Factor	less
Pav	Average Pressure	N/m^2
Tav	Average Temperature	^oC
s	Perimeter	m
V	Velocity	m/s
h	Liquid Leveling	m
Q	Flow rate	m^3/s
fi_{cal}	Calculated friction factor	
fi_{meas}	Measured friction factor	

The Dimensionless Groups:

N_{vg}	Gas velocity number
N_{Lv}	Liquid velocity number
N_L	Liquid viscosity number
N_d	Diameter number
Re	Reynolds number

Greek Symbols:

δ	Liquid Film Thickness	m
τ	Shear Stress	N/m^2
μ	Viscosity	$N.s/m^2$
ρ	Density	N/m^3
σ	Surface Tension	N/m
φ	inclination angle	
ε	roughness	

Superscripts

* modified ≈ dimensionless

Subscripts

L	liquid film
sg	superficial Gas core
sL	superficial liquid
i	interfacial
t	translation
wg	wall-gas
wL	wall liquid
c	gas-core

Introduction:

The interfacial friction factor represents one of the most significant concepts affect on gas-liquid, two-phase flow in pipe. This factor was treated in numerous studies. When the gas phase flows in contact to the liquid phase there are several ripples or waves will forming which will achieve type of resistance to the flow, this resistance is more similar to the resistance where rigid bodies move on each other. Due to this resistance some of pressure will loosed. In annular flow pattern the gas will flow near the center of the pipe while the liquid will be near to the inside walls, because of the roughness of the wall, the liquid will flow slower than the gas which will be in high velocity. Now, between the two surfaces of the liquid and the gas there is an interfacial shear stress will occur. This stress will try to prevent the gas to flow fast than the liquid. This process will loose the pressure force of the two-phase flow. Therefore the study of this interfacial surface is important to overcome the happened shear stress.

In the literature, there are more than tens of investigators have developed correlations to predict the interfacial friction factor empirically or semi- empirically as: Kowalski (1987), Laurinat et al. (1985), Crowley and Rothe (1986), Lee and Bankoff (1983), Tsiklauri et al. (1979), Eck (1973), Xiao et al. (1990), Paras et al (1994), Spedding and Hand (1997), Ben Asante (2000), Petolaz and Aziz (1998), Vlachos et al (1997), Taitel-Dukler (1976)… etc.

Some investigators used the interfacial friction factor which is developed in stratified flow to operate in annular flow and vice versa such as; Naji, (2004) and (2006). To this time, no one approached to mechanistic model to predict it, all these correlations are developed empirically used experimental tests and the accuracy of any method is related with the volume of tests. The aim of this work is to develop a mechanistic model to predict it.

Measured Friction Factor:

In Annular flow shown in figure (1), it is possible to summarize the configuration of the flow geometrically. The treating of such flow will be considered as two phases flowing together in form of two-cylindrical shape.

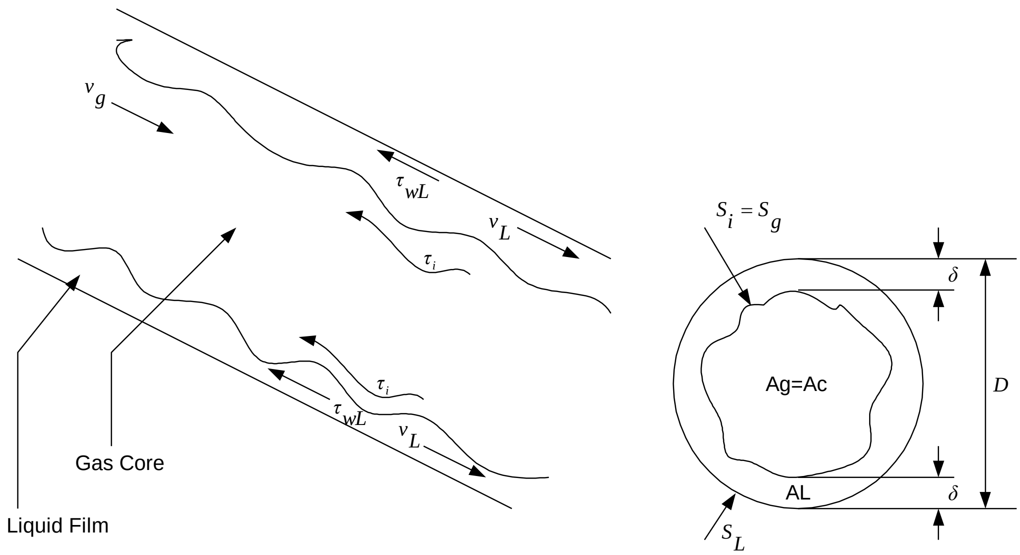

Figure (1): Annular Flow Pattern Configuration

For the gas core stream the force balance:

$$-A_g\left(\frac{dP}{dL}\right)-\tau_i s_i - gA_g \rho_g \sin\varphi = 0 \qquad \text{---------------------- (1)}$$

For the liquid film stream the force balance:

$$-A_L\left(\frac{dP}{dL}\right)+\tau_i s_i-\tau_{wL} s_L-gA_L\rho_L\sin\varphi=0 \qquad \text{---------------------- (2)}$$

Taitel-Dukler, (1976) proposed that the pressure gradient $\left(\frac{dP}{dL}\right)$ will be same in each phase, the combination of equations (1) and (2) will result:

$$\tau_{wL}\frac{s_L}{A_L}-\tau_i s_i\left(\frac{1}{A_L}+\frac{1}{A_g}\right)+\left(\rho_L-\rho_g\right)g\sin\varphi=0 \qquad \text{------------------- (3)}$$

The equation (3) is called the momentum equation of the annular flow pattern and (φ**)** is the inclination angle of the pipe flow.

Based on experimental information, the interfacial shear stress could be calculated from equation (3) and the friction factor is consequently calculated from the following equation:

$$f_i=\frac{2\tau_i}{\rho_g V_g{}^2} \qquad \text{-- (4)}$$

The resulted magnitude of equation (4) will be considered as the measured interfacial friction factor (Xiao et al, 1990)

<u>The Used Methods:</u>

The semi-empirical methods which are used in the comparison procedure are outlined in table (1):

Table (1): The available models in the literature

1.	Andritsos and Hanratty [1987]	$\frac{f_i}{f_{wg}}=1$ if $Vsg \le Vsgt$ $\frac{f_i}{f_{wg}}=1+15\sqrt{\tilde{\delta}_L}\left[\frac{Vsg}{Vsgt}-1\right]$ if $Vsg > Vsgt$ $Vsgt=\sqrt{\frac{Patm}{Pav}}$
2.	Taitel and Dukler [1976]	$\frac{f_i}{f_{wg}}=1$
3**.**	Andrussi and Persen [1987]	$\frac{f_i}{f_{wg}}=1+29.7\left(F_1-0.36\right)^{0.67}\left(\tilde{\delta}_L\right)$ $F_1=V_g\sqrt{\frac{\rho_g}{\rho_L-\rho_g}\frac{1}{A_g}\frac{1}{g\cos(\theta)}\frac{dA_L}{dh_L}}$

4.	Baker et al [1988]	$\varepsilon_i = \frac{34\sigma}{\rho_g V_L^2}$ if $N_{we} N_\mu \le 0.005$ $\varepsilon_i = \frac{170\,\sigma (N_{we} N_\mu)^{0.5}}{\rho_g V_L^2}$ if $N_{we} N_\mu > 0.005$
5.	Cheremisinoff and Davis [1979]	$f_i = 0.008 + 0.00005\,Re_L$ $Re_L = \frac{\rho_L Q_L}{\mu_L (s_L + s_i)}$
6.	Hamersma and Hart [1987]	$\varepsilon_i = 2.3\,\delta_L$ and Re_g.

7.	Hart et al [1989]	$\frac{f_i}{f_{wL}} = 0.00926\ Re_{sL}^{0.726}$
8.	Kim et al. [1985]	$f_i = 0.021 + 0.14\ 10^{-5} Re_L$
9.	Linehan [1968]	$f_i = 0.021 + 0.23\ 10^{-5} Re_L$
10.	Shoham and Taitel [1984]	$f_i = 0.0142$
11.	Kowalski [1987] correlation	$f_i = 7.5\times10^{-5} H_L^{-0.25} Re_g^{-0.3} Re_L^{0.83}$ $Re_g = \frac{\rho_g v_g d}{\mu_g}$ and $Re_L = \frac{\rho_L v_L d}{\mu_L}$
12.	Laurinat et al. [1984]	$\frac{f_i}{f_{wg}} = \left[2 + \frac{2.5\times10^{-5}\ Re_{sL}}{d}\right](1-H_L)^{5/2}$
13.	Crowley and Rothe [1986]	$\frac{f_i}{f_{wg}} = 1$ if $\frac{f_i}{f_{wg}} < 1$ $\frac{f_i}{f_{wg}} = (1 + 75\,H_L)$ if $\frac{f_i}{f_{wg}} > (1 + 75\,H_L)$
14.	Bendiksen et al. [1989]	$\Delta h_w = h_x + \sqrt{h_x^2 - h_y}$ $h_x = \frac{\rho_g (V_g - V_L)^2}{4g(\rho_L - \rho_g)\cos\Theta}$ and $h_y = \frac{\sigma}{g(\rho_L - \rho_g)\cos\Theta}$
15.	Lee and Bankoff (1983)	$f_i = 0.012 + 5.179\times10^{-7}\times(Re_g - Re_g^*)$ if $Re_g \le Re_g^*$ $f_i = 0.012 + 2.694\times10^{-7}\times(Re_g - Re_g^*)\left(\frac{Re_L}{1000}\right)^{1.534}$ if $Re_g > Re_g^*$ where: $Re_g^* = 1.837\times10^5\,Re_L^{-0.184}$

16.	Tsiklauri et al. (1979)	$f_i = 0.0055 + 2.6 \times 10^{-5} Re_L$ where: $Re_L = \frac{\rho_L Q_L}{\mu_L (s_L + s_i)}$
17.	Xiao et al. (1990)	$f_i = 0.053\ N_{gv}^{0.23}\ N_{Lv}^{0.202}\ N_d^{-0.46}\ N_L^{0.076}$ $N_{gv} = v_g \sqrt[4]{\frac{\rho_L}{g\sigma}}$, $N_{Lv} = v_L \sqrt[4]{\frac{\rho_L}{g\sigma}}$ $N_d = d\sqrt{\frac{\rho_L g}{\sigma}}$ and $N_L = \mu_L \sqrt[4]{\frac{g}{\rho_L \sigma^3}}$
18.	Eck [1973]	$f_i = \frac{0.0625}{\left(\frac{2.3 S_g}{3.715 d} + \frac{15}{Re_g}\right)^2}$
19.	Ellis and Gay [1954]	$f_i = 1.29 Re_G^{-0.57}$
20.	Ben Asante [2000]	$f_i = 0.61\left(\varepsilon^{0.35} \times \tilde{\delta}^{0.61} \times Re_g^{-0.52} \times Re_L^{0.6}\right) + 0.32$
21.	Wallis [1969]	$f_i = 0.005 + 1.5\left(\frac{\delta}{d}\right)$
22.	Petolas and Aziz [1998]	$\frac{f_i}{f_c} = 0.24\left(\frac{\sigma}{\rho_c V_c^2 d_c}\right)^{0.085} Re_f^{0.305}$
23.	Vlachos et al [1997]	$f_i = 0.024 (1 - H_L)^{0.35} Re_{sL}^{0.18}$

Present Model:

In the present work, a simplification to the equation (3) has done using the geometrical configuration in figure (1) by using the relations between each term of equation (3) and the thickness of the film zone in the annular flow pattern. From the literature, it is deduced that the inclination angle has a negligible affect on the estimation of the interfacial friction factor, equation (3) has solved yielded for (f_i), and it is found that:

$$\frac{f_i}{f_L} = \left(\frac{\rho_L V_L^2}{\rho_g V_g^2}\right)\left(\frac{d_c}{S_L}\right) \quad \text{--} \ (5)$$

Experimental Tests:

No experimental apparatus has done in the present work, but all the tests are conducted from tests published in the literature from two different sources as explained in table (2), the used system used the air as the gas phase while the kerosene as the liquid phase with the ranges shown in table (3). The properties of both fluids could be predicted by using the facilities correlations as cited in Abdul-Majeed (1996) in the following:

$$\rho_L = 832.34 - 0.8333\ Tav$$
$$\rho_g = Pav/[0.287\ (273 + Tav)]$$
$$\mu_L = 0.001\ \exp(0.0664 - 0.0207\ Tav)$$
$$\mu_g = 0.00001(1.7044 + 0.00613\ Tav - 0.0000314\ Tav^2)$$
$$\sigma = 27.6 - 0.09\ Tav$$

Table (2): The used Data

	The Source	No of tests	Inclination Angle
1	Abdul-Majeed (1996)	20	0^o
2	Mukherjee-Brill (1979)	75	$0^o,-5^o,-20^o.-30^o$

Table (3): Flow Conditions Ranges

R	The Property	Minimum	Maximum
1	Superficial gas velocity m/sec	24.06	48.908
2	Superficial liquid velocity m/sec	0.634	6.3
3	Average Pressure KPa	377.9	603.4
4	Average Temperature ^oC	21.9	47.8
5	Liquid Holdup dimensionless	0.0621	0.28

The statistical Tools:

To investigate which model or method has accurate prediction of the interfacial friction factor, RMS tool was used for this purpose. This tool measures the error (e) with respect to the reference line with zero error. Moreover, this line could be represented by line inclined with 45^o, hence, the accurate prediction must be the nearest to this inclined line.

1. Average Error:

---------------------------- (6) $$AE = \frac{1}{n}\sum_{i=1}^{n} e_i$$

2. Absolute Average Error:

--------------------------- (7) $$AAE = \frac{1}{n}\sum_{i=1}^{n} |e_i|$$

3. Root Mean Square based on Error:

------------------------(8) $$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (e_i)^2}$$

$$e_i = f_{i\ cal} - f_{i\ meas}$$ Where

Results and Discussion:

In the present work, the twenty-three methods in table (1) and the new model has programmed to predict the interfacial friction factor and tested with actual magnitude by using the experimental data shown in tables (2) and (3). The results presented by using the Root Mean Square (RMS) which is given by equation (8) and displayed in tables (4) through table (8). The predicted and measured magnitudes of the interfacial friction factor are presented also graphically into figures (2) through (6).

The tables and figures are displaying the results for each data with single inclination angle except the table (8) which represent the results of the using of the whole data.

Table (4) represents the performance for the whole methods at horizontal data. It is clear that the new model has good results than the others while the correlation of Eck (1973) was the second best method. There methods designed to predict the interfacial roughness as Baker et al. (1988), Hamersma and Hart (1987) and Bendiksen et al. (1989). These methods depend on the equation of Colebrook and White to calculate the friction factor; therefore, they get the same results as shown in all tables of the results.

Figure (2) shows that the prediction of the present model is satisfied the measured interfacial friction factor.

Table (5) displayed the results of the whole methods where using the data with -5^{o} inclination angle. The table reveals that the accuracy of the new model is the best while the correlation of the Ellis and Gay (1954) was the second best method, while figure (3) presents the distribution of the output results of the present model for data with angle of inclination is -5^{o}.

Table (6) shows the results of the whole methods by the using of data with -20^{o} inclination angle; this table shows that the best performance is by the new model while the correlation of Kowalski (1987) was the second best, also, figure (4) displays the excellent estimation of the present model.

Table (7) displays the results of the whole methods where using the data of -30^{o} inclination angle only. It is appear that the performance of the Ellis and Gay (1954) is the best among the others except the new model which is give the best results absolutely; as well as figure (5) shows that the prediction by the present model is more reliable one.

Table (8) displays the results for the whole method and by using the whole data in the testing procedure. The table shows that the best performance is by the new model while the second one is the correlation of Ellis and Gay (1954). The worst results are still by the correlation of Andrussi and Persen (1987), while, figure (6) displays the behavior of the present model estimation, this grope used the whole data (95) and regardless the specialization of the inclination angle.

Conclusions

1. All models gave overestimation results to predict of the friction factor except the new model which seems to be underestimation
2. The possibility of using the models those developed to operate in stratified flow to operate in annular flow, because of the results of Taitel and Dukler (1976) and Kowalski (1987) which gave best results than the correlation of Laurnat et al (1985) in spite of the firsts had designed for the estimation in stratified flow while the last is designed for estimating in annular flow.
3. It is clear that the correlation of Ellis and Gay (1954) is valid for annular flow in 0^{o}, -5^{o} and -30^{o} inclination angles, while the correlation of Kowalski (1987) is valid only for annular flow in -20^{o} inclination angle.
4. Due to the best accuracy of the new model, it is recommended to be valid for estimating in horizontal and downwardly inclined flow.

5. By looking to all figures and tables, it is clear that the inclination angle has no significant affect on the estimation of interfacial friction factor, this conclusion supports the assumption of the present model.

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Table (4):The results when using Horizontal Data Only (40 points)

R	The model	AE 10^{-4}	AAE 10^{-4}	RMS 10^{-4}
1	Andritsos-Hanratty [1987]	190	190	1202
2	Taitel-Dukler [1976]	1.89	1.89	11.9
3	Andrussi-Persen [1987]	2294	2294	14513
4	Baker et al [1988]	8.29	8.29	52.4
5	Cheremisinoff-Davis [1979]	38	38	240
6	Hamersma-Hart [1987]	8.29	8.29	52.4
7	Hart et al [1989]	70.8	70.8	448
8	Kim et al. [1985]	7.77	7.77	49.1
9	Linehan [1968]	418	418	2645
10	Shoham-Taitel (1984)	3.54	3.54	22.4
11	Kowalski(1987)	2.86	2.86	18.0
12	Laurinat et al. (1984)	33.2	33.2	210
13	Crowley- Rothe (1986)	10.8	10.8	68.3
14	Bendiksen et al. [1989]	8.29	8.29	52.4
15	Lee and Bankoff (1983)	644	644	4075
16	Tsiklauri et al. (1979)	48.2	48.2	305
17	Xiao et al. (1990)	11.6	11.6	73.6
18	Eck (1973)	1.53	1.53	9.69
19	Ellis and Gay [1954]	1.87	1.87	11.8
20	Ben Asante (2000)	18.1	18.1	114
21	Wallis (1969)	184	184	1169
22	Petolas and Aziz (1998)	1339	1339	8470
23	Vlachos et al (1997)	2.2	2.2	13.9
24	New Model (present)	$-1.5x10^{-4}$	$1.5x10^{-4}$	$9.7x10^{-4}$

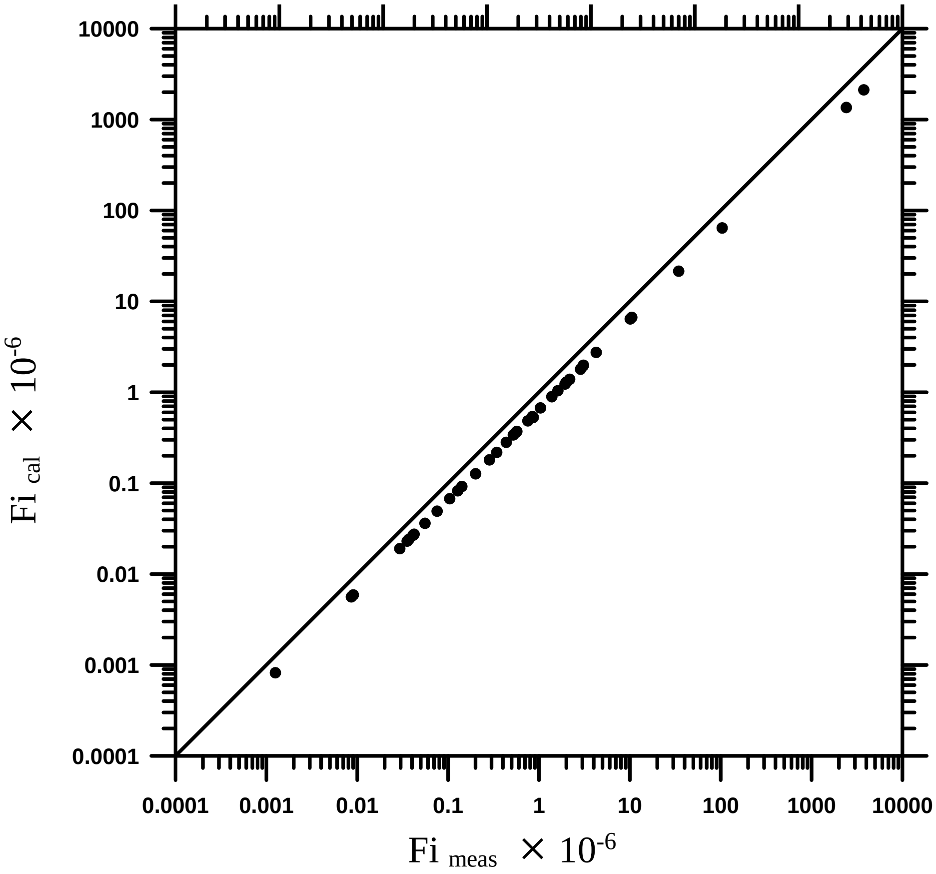

Figure (2): The Prediction of the Present Model for Horizontal Data Only

Table (5):The results when using Inclined Data with -5º Only (17 points)

R	The model	AE 10^{-4}	AAE 10^{-4}	RMS 10^{-4}
1	**Andritsos-Hanratty [1987]**	113	113	859
2	Taitel-Dukler [1976]	1.21	1.21	9.17
3	Andrussi-Persen [1987]	1191	1191	8992
4	Baker et al [1988]	5.19	5.19	39.1
5	Cheremisinoff-Davis [1979]	71.8	71.8	542
6	Hamersma-Hart [1987]	5.19	5.19	39.1
7	Hart et al [1989]	84.5	84.5	638
8	Kim et al. [1985]	8.61	8.61	65.0
9	Linehan [1968]	812	812	6137
10	Shoham-Taitel (1984)	2.48	2.48	18.7
11	Kowalski(1987)	5.55	5.55	41.9
12	Laurinat et al. (1984)	61.4	61.4	464
13	Crowley- Rothe (1986)	7.78	7.78	58.7
14	Bendiksen et al. [1989]	5.19	5.19	39.1

15	Lee and Bankoff (1983)	130	130	981
16	Tsiklauri et al. (1979)	92.5	92.5	699
17	Xiao et al. (1990)	9.49	9.49	71.7
18	Eck (1973)	1.90	1.9	14.3
19	Ellis and Gay [1954]	1.03	1.03	7.77
20	Ben Asante (2000)	17.4	17.4	131
21	Wallis (1969)	126	126	954
22	Petolas and Aziz (1998)	1227	1227	9266
23	Vlachos et al (1997)	3.34	3.34	25.2
24	New Model (present)	$-1.0x10^{-3}$	$1.0x10^{-3}$	$7.6x10^{-3}$

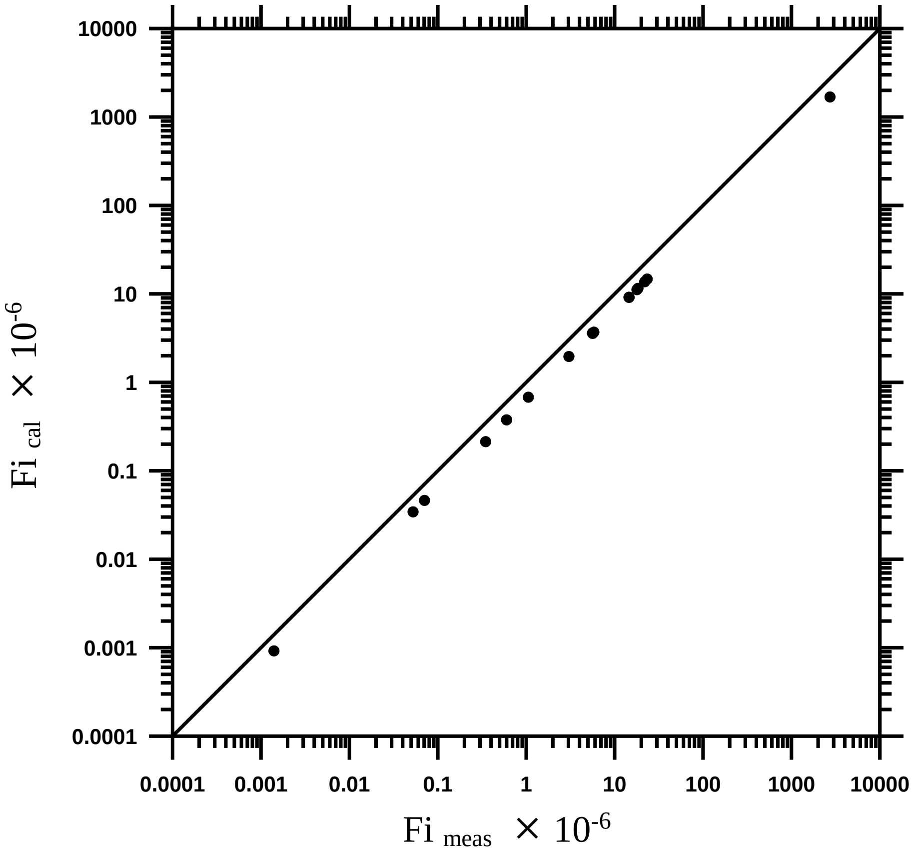

Figure (3): The Prediction of the Present Model for Data with -5º.

Table (6):The results when using Inclined Data with -20º (25 points)

R	The model	AE 10^{-4}	AAE 10^{-4}	RMS 10^{-4}
1	**Andritsos-Hanratty [1987]**	23.0	23.0	209
2	Taitel-Dukler [1976]	1.17	1.17	10.6
3	Andrussi-Persen [1987]	645	645	5841
4	Baker et al [1988]	5.85	5.85	52.9
5	Cheremisinoff-Davis [1979]	7.45	7.45	67.4
6	Hamersma-Hart [1987]	5.85	5.85	52.9
7	Hart et al [1989]	22.2	22.2	201
8	Kim et al. [1985]	3.01	3.01	27.2
9	Linehan [1968]	76.0	76.0	689
10	Shoham-Taitel (1984)	1.73	1.73	15.6
11	Kowalski(1987)	1.07	1.07	9.71
12	Laurinat et al. (1984)	10.6	10.6	96.5
13	Crowley- Rothe (1986)	6.69	6.69	60.6
14	Bendiksen et al. [1989]	5.85	5.85	52.9
15	Lee and Bankoff (1983)	28.0	28.0	254
16	Tsiklauri et al. (1979)	9.09	9.09	82.3
17	Xiao et al. (1990)	3.72	3.72	33.7
18	Eck (1973)	1.14	1.14	10.3
19	Ellis and Gay [1954]	1.84	1.84	16.6
20	Ben Asante (2000)	9.16	9.16	83.0
21	Wallis (1969)	88.8	88.8	804
22	Petolas and Aziz (1998)	580	580	5255
23	Vlachos et al (1997)	1.34	1.34	12.1
24	New Model (present)	-2.9×10^{-4}	2.9×10^{-4}	2.6×10^{-3}

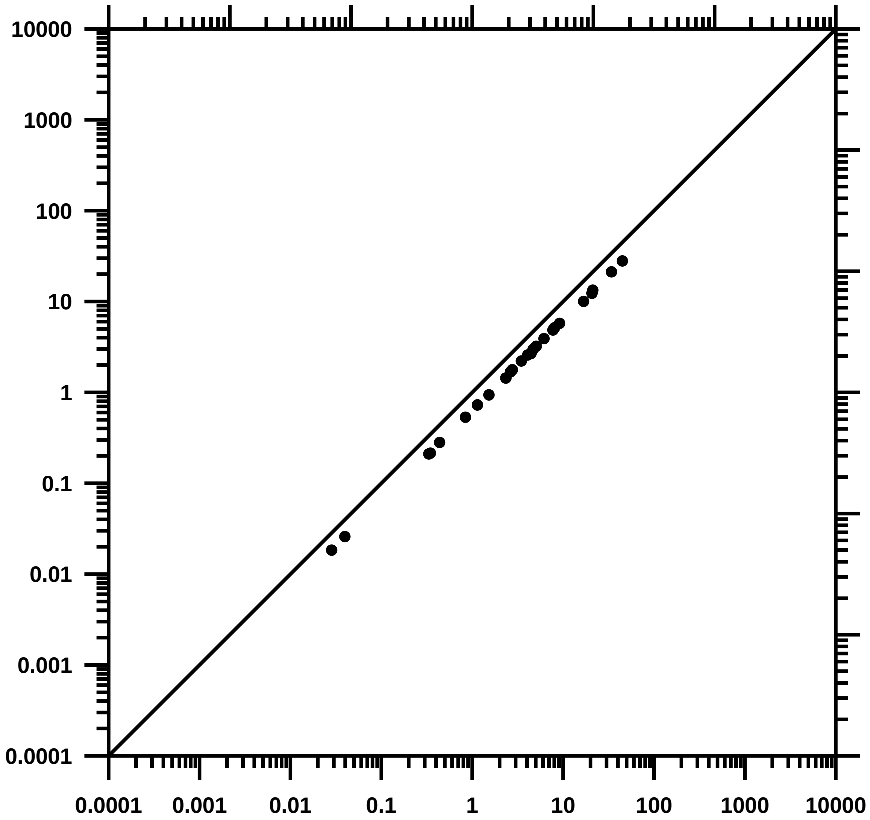

Figure (4): The Prediction of the Present Model for Data with -20º

Table (7):The results when using Inclined Data with -30º (13 points)

R	The model	AE 10^{-4}	AAE 10^{-4}	RMS 10^{-4}
1	**Andritsos-Hanratty [1987]**	56.8	56.8	554
2	Taitel-Dukler [1976]	0.74	0.74	7.26
3	Andrussi-Persen [1987]	682	682	6648
4	Baker et al [1988]	3.19	3.19	31.1
5	Cheremisinoff-Davis [1979]	18.2	18.2	177
6	Hamersma-Hart [1987]	3.19	3.19	31.1
7	Hart et al [1989]	32.7	32.7	319
8	Kim et al. [1985]	3.42	3.42	33.4
9	Linehan [1968]	201	201	1964
10	Shoham-Taitel (1984)	1.49	1.49	14.5
11	Kowalski(1987)	1.71	1.71	16.7
12	Laurinat et al. (1984)	17.6	17.6	171
13	Crowley- Rothe (1986)	4.77	4.77	46.5
14	Bendiksen et al. [1989]	3.19	3.19	31.1

15	Lee and Bankoff (1983)	56.0	56.0	546
16	Tsiklauri et al. (1979)	23.2	23.2	226
17	Xiao et al. (1990)	4.90	4.90	47.8
18	Eck (1973)	1.25	1.25	12.2
19	Ellis and Gay [1954]	0.656	0.656	6.39
20	Ben Asante (2000)	7.26	7.26	70.7
21	Wallis (1969)	75.4	75.4	735
22	Petolas and Aziz (1998)	575	575	5612
23	Vlachos et al (1997)	1.84	1.84	18.0
24	New Model (present)	$-2.8x10^{-4}$	$2.8x10^{-4}$	$2.7x10^{-3}$

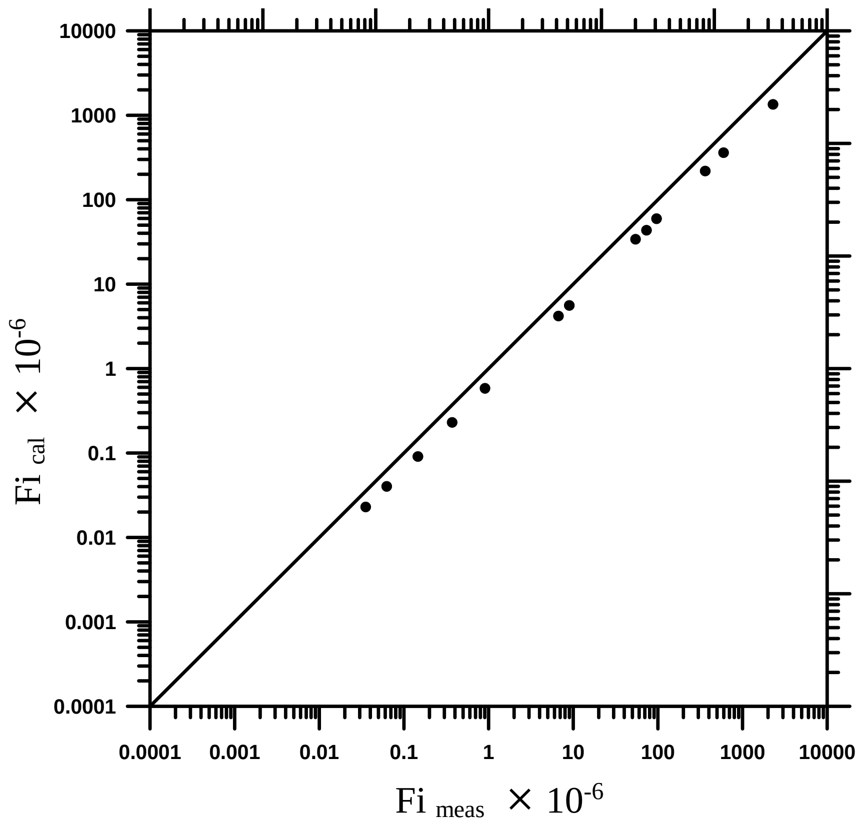

Figure (5): The Prediction of the Present Model for Data with -30º

Table (8): The results when using The overall Data (95 points)

R	The model	AE 10^{-4}	AAE 10^{-4}	RMS 10^{-4}
1	**Andritsos-Hanratty [1987]**	56.8	56.8	554
2	Taitel-Dukler [1976]	0.745	0.745	7.26
3	Andrussi-Persen [1987]	682	682	6648
4	Baker et al [1988]	3.19	3.19	31.1
5	Cheremisinoff-Davis [1979]	18.2	18.2	177
6	Hamersma-Hart [1987]	3.19	3.19	31.1
7	Hart et al [1989]	32.7	32.7	319
8	Kim et al. [1985]	3.42	3.42	33.4
9	Linehan [1968]	201	201	1964
10	Shoham-Taitel (1984)	1.49	1.49	14.5
11	Kowalski(1987)	1.71	1.71	16.7
12	Laurinat et al. (1984)	17.6	17.6	171
13	Crowley- Rothe (1986)	4.77	4.77	46.5
14	Bendiksen et al. [1989]	3.19	3.19	31.1
15	Lee and Bankoff (1983)	56.0	56.0	546
16	Tsiklauri et al. (1979)	23.2	23.2	226
17	Xiao et al. (1990)	4.90	4.90	47.8
18	Eck (1973)	1.25	1.25	12.2
19	Ellis and Gay [1954]	0.656	0.656	6.39
20	Ben Asante (2000)	7.26	7.26	70.7
21	Wallis (1969)	75.4	75.4	735
22	Petolas and Aziz (1998)	575	575	5612
23	Vlachos et al (1997)	1.84	1.84	18.0
24	New Model (present)	$-2.8x10^{-4}$	$2.8x10^{-4}$	$2.7x10^{-3}$

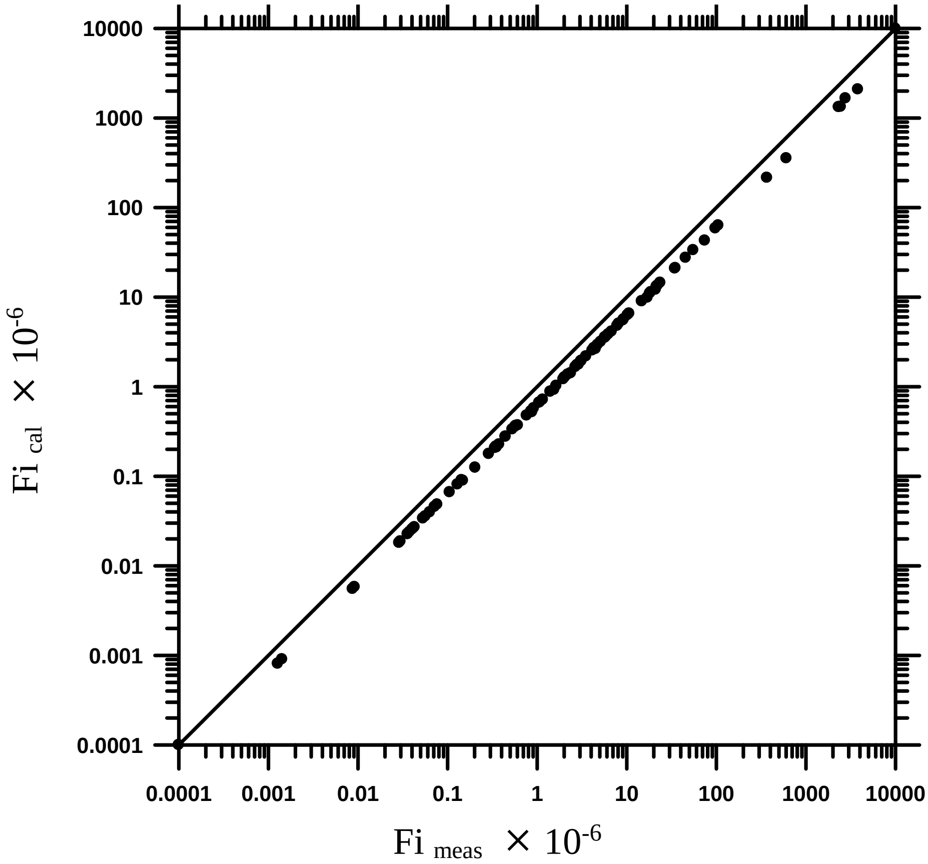

Figure (6): The distribution of the prediction interfacial friction factor by using the overall data (horizontal and inclined tests)