

## ELASTIC - PLASTIC FRACTURE MECHANICS OF DUCTILE CAST IRON PIPES

HAIDER HADI JASIM

Chemical Engineering Department, College of Engineering,  
Basrah University, Basrah, Iraq  
e-mail: raidhani73@yahoo.com

### **Abstract**

In this paper, depends on the finite element method, the J-Integral program is developed for a stationary circumferential crack problem in elastic – plastic fracture mechanics in pipes under static loading and pure bending moment condition. The program developed is applied to ductile cast iron pipes (DCIP) to analyse the integrity assessment, i.e., the significance of crack growth by drawing both failure assessment diagram (FAD) and crack driving force diagram (CDF). A numerical procedure is used for elastic-plastic analysis depending on special equation to predict J-values taking account of the crack geometry and load condition. It is cleared that the results obtained from failure assessment diagram and crack driving force diagram are identical and J-integral method can be used to the onset of crack growth in (DCIP) under bending moment conditions.

**ميكانيكية الانكسار المرنة\_اللدنة ل الأنابيب المصنوعة من حديد الزهر المطيلي**

حيدر هادي جاسم

قسم الهندسة الكيميائية، كلية الهندسة

جامعة البصرة

**الملخص**

في هذا البحث، اعتماداً على طريقة العناصر المحددة، تم تطوير برنامج لطريقة تكامل (J) لدراسة وتحليل ميكانيكية الانكسار المرنة-اللدنة لمسائل الشفوق المحيطية في الأنابيب تحت شروط الأحمال السكونية وعزم الانحناء. البرنامج المطور طبق على أنابيب مصنوعة من حديد الزهر المطيلي لتحليل تقييمها الكامل، اي تحديد بداية نمو الشق والعزم المناظر لبداية نمو الشق من خلال رسم مخطط القوة المسيرة للشق ومخطط تقييم الفشل معاً. تم استخدام طريقة عدبية للتحليل المرن - اللدن بالاعتماد على معادلات خاصة لحساب قيمة (J)، اخذين بنظر الاعتبار شكل الشق وشروط التحميل. لقد اظهرت النتائج تطابق القيم المستحصلة من رسم مخطط القوة المسيرة للشق ومخطط تقييم الفشل كما تم إثبات ان طريقة تكامل (J) تكون جداً مفيدة لتحديد بداية ونمو الشق في الأنابيب المصنوعة من حديد الزهر المطيلي وتحت شروط عزم الانحناء.

### **Introduction**

Cast iron exists in two forms, grey and ductile. In 1955 the ductile cast iron was introduced, when alloying element were added to make iron less brittle, it was also called nodular cast irons since the shape of the graphite inclusion in the iron is changed to a nodular form. The nodular shape of the graphite inclusions decreases the risk of getting crackes in the material. In general cast iron is the most commonly used material for water main pipes since it is economical to produce and able to withstand the pull stress from the internal pressure [1].

Most ductile cast irons pipes (DCIP) are manufactured in large dimensions because they are less complicated to manufacture and are less expensive.

There are two types of (DCIP) according to uses. Aboveground and underground pipes.

The causes of (DCIP) failures can be classified into two categories as determined from government statistics. These are outside force and corosions [2]. Homagoun [3] study the combining fuzzy infloreena and evidential reasoning to quantify the corrosion rate of cast irons and ductile irons pipes. Zencker [4] study the stress intesity factor of dynamically loaded sample of ductile cast iron

containers having crack-like material defects by using numerically calculated method and then verified the result by a comparision with analytical solution.

In this paper, depends on the finite element method, a J-Integral program is developed for a stationary crack under elastic-plastic condition and applied for (DCIP) under bending moment load. Two approaches are used to check integrity assessment, the failure assessment diagram (FAD) and crack driving force (CDF). It is cleared that both approaches are complemetary and give identical result.

### **Fracture Mechanics Procedure**

Fracture mechanics problems can be classified into three types according to the dominant opering deformation modes in the cracked bodies [5]:

1- Linear -- elastic fractuer mechanics (LEFM). In this type the stress-strain behavior and load deformation behavior are linaer . The relevant crack tip parameter is the stress intensity factor ( K), in this type the plastic zone is small.

2- Elastic-plastic fracture mechanics (EPFM). This type is used for large scale of plasticity and with J-Integral as the relevant crack tip parameters.

3-Time-dependent fracture mechanics (TDFM).This type is used when stress-

strain and load-displacement behavior are time dependent due to either dynamic loading or due to creep.

In ductile material, failure may be governed by growth and fracture which is ( J-based ) by yielding and plastic collapse or by an interaction of the two.

### Plastic Collapse Load and Plastic Collapse Bending Moment

There are two method of integrity assessment, the first is carried out without postulation of crack and the second is carried out with postulation of cracks. If the piping material is extremely ductile or tough , one need not adopt the fracture mechanics principle at all, elastic – plastic fracture mechanics and limit load concept is sufficient for it's integrity assessment [5]

The plastic collapse load (limit load) and plastic collapse moment ( limit moment) of a pipe with circumferential crack under the bending load is given by the following equations [6]:

$$P_l = \frac{16R^2t\sigma_f}{Z-L} [\cos(\frac{\theta}{2}) - 0.5\sin(\theta)] \quad \dots(1)$$

$$M = 4R^2t\sigma_f [\cos(\frac{\theta}{2}) - 0.5\sin(\theta)] \quad \dots(2)$$

where,

R: outer pipe radius  
t: pipe wall thickness.

$$\sigma_f = \frac{\sigma_y + \sigma_u}{2}$$

$\sigma_y$ : yeild strength.

$\sigma_u$ : tensile strength.

Z: outer span.

L: inner span.

$\theta$  : half crack angle.

### J-Integral Approaches

Under elastic-plastic condition and deformation theory of plasticity, the crack driving force,  $J$  , can be obtained by adding the elastic component,  $J_e$  , and plastic component ,  $J_p$  , [5] i.e.

$$J = J_e + J_p \quad \dots(3)$$

For a circumferential crack pipe under bending , closed form expression can be developed for both  $J_e$  and  $J_p$  .

They are discussed as follows:

#### A-Plastic solutions

The plastic component ,  $J_p$  can be found by using two approaches:

1- The plastic compoenent  $J_p$  can be defined as [7]:

$$J_p = \int_0^M \frac{\partial \phi_p}{\partial A} dm = \frac{\partial}{\partial A} \int_0^M \phi_p dm \quad \dots(4)$$

where,

$$\varphi_p = \left(\frac{t}{t_e}\right)^{n-1} \left(\frac{\pi}{4\hat{K}}\right)^n \left(\frac{m}{m_0}\right)^{n-1} * \left(\frac{2m}{\pi R_M^4 t^2 E}\right) I\left(\frac{\theta}{\pi}\right) * \alpha$$

$$\hat{K} = \frac{\sqrt{\pi}}{2} \frac{\Gamma\left(1 + \frac{1}{2n}\right)}{\Gamma\left(\frac{3}{2} + \frac{1}{2n}\right)}$$

$$I\left(\frac{\theta}{\pi}\right) = (2\theta)\left[\left(R_M - \frac{t}{2}\right) \int a F_B^2\left(\frac{\theta}{\pi}\right) da + \int a^2 F_B^2\left(\frac{\theta}{\pi}\right) da\right]$$

$$F_B\left(\frac{\theta}{\pi}\right) = 1.1 + [-0.09967 + 5.0057\left(\frac{\theta}{\pi}\right)^{0.565} - 2.8329\left(\frac{\theta}{\pi}\right)]$$

and,

$$m_0 = \pi R_m^2 t \sigma_0 \text{ (reference moment).}$$

$R_m$ : mean radius of pipe.

$t_e$ : equivalent thickness of uncrack .

$\alpha$  : model parameter.

$n$ : strain hardining exponent.

$\sigma_0$ : reference stress .

$a$  : distance of crack from inner radius.

2- The plastic component can also be determine by using the following equation [8]:

$$J_p = J_{p_0} + \int_{\theta_0}^{\theta} \gamma J_{p_0} d\theta \quad \dots (5)$$

where,

$$\gamma = \frac{[0.5 \cos(0.5\theta) - \sin\theta]}{\sin(0.5\theta) + \cos\theta}$$

$$J_{p_0} = \int_0^{\Delta P_i} \eta_{p_0} d(\Delta P_i)$$

$$\eta_{p_0} = \frac{0.5 [\sin(0.5\theta) + \cos\theta]}{2Rt [\cos(0.5\theta) - 0.5\sin\theta]}$$

$$\Delta P_i = \alpha \varepsilon_0 ab(W_r, n) \left[ \frac{f}{f_y} \right]^n$$

$$\varepsilon_0 = \frac{\sigma_0}{E}$$

and,

$\theta$  : semi-circumferential crack angle.

P: total applied load (plastic load).

$\alpha$  : dimensions parameter.

F and  $F_y$  : applied and limit load .

$\Delta P_i$  : load – line displacement

$b(W_r, n)$  : factor depending on strain hardining (n) and crack to width ratio ( $W_r$ )

The functions  $I\left(\frac{\theta}{\pi}\right)$  and  $J_{p_0}$  can be

computed by using the finite element method. The program is developed by using four – noded shell element with three displacement and two rotational degree of freedom at each node. Then  $J_p$  can be culculated using eq.4 and eq.5.

#### B- Elastic solution

The value of J-Integral in elastic portion for a pipe having circumferential crack for plain strain conditions is given by [6]:

$$J_e = \frac{1 - \nu^2}{E} K_I^2 \quad \dots (6)$$

The stress intensity factor ( $K_I$ ) for a pipe subjected to bending moment is :

$$K_I = \sigma_b \sqrt{\pi R \theta F_b}$$

where,

$$\sigma_b = \frac{M}{\pi R^2 t}$$

$$F_b = 1 + A \left[ 4.5967 \left( \frac{\theta}{\pi} \right)^{1.5} + 2.6422 \left( \frac{\theta}{\pi} \right)^{4.24} \right]$$

$$A = \left( 0.4 \frac{R}{t} - 3.0 \right)^{0.25} \text{ for } \frac{R}{t} \geq 10$$

and,

M: bending moment.

$\nu$ : poisson's ratio.

E: Young modulus.

$= 12.08''$  (318.356 mm);  $r_o = 12.4''$  (327.5 mm); and  $t = 0.36''$  (9.144 mm)), and having the following elastic properties [10]:

$$E = 169 \text{ GPa}; \rho = 7100 \text{ kg/m}^3$$

$$\sigma_y = 276 \text{ MPa}; \nu = 0.29$$

$$\sigma_u = 414 \text{ MPa};$$

Also for DCIP material the following values of constant are given [7]:

$$\alpha = 1, n = 0.106, t_e = 0.5 t$$



Fig.1 Ductile iron pipe on support

### Ductile Iron Pipe Dimensions and Properties

Ductile iron pipe is normally manufactured in (216"- 240"), nominal length depends on the pipe manufactured and pipe size Fig.1 [9].

Fig.2 show DCIP and finite element mesh for the following dimensions ( $r_i$

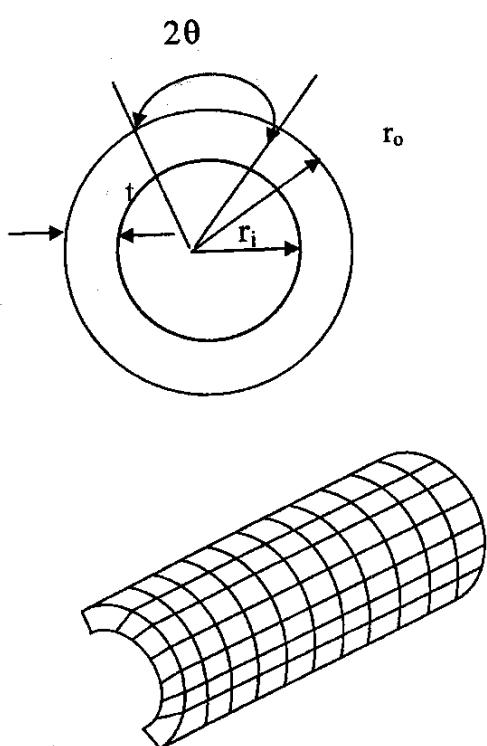


Fig.2 Details of iron pipe and finite element mesh for half pipe.

If the material properties are taken as above, the fracture toughness ( $K_{IC}$ ) in specific design are depending in extensive investigation of ferritic in ductile iron over a board range of microstructure and temperature [10].

Elsewhere, used  $K_{IC} \geq 50 \text{ MPa}\sqrt{\text{m}}$

[4].

### Result and Discussions

Fig.3, show J- values for plastic portion goted from Eq.4 and Eq.5. As shown there is a very good agreement between these values.

Fig.4 show failure assessment diagram (FAD) and loading curve. The

(FAD) getting from drawing the values of ( $K_r$ ) i.e., the stress intensity factor divided by fracture toughness of material and ( $L_r$ ) i.e., applied load divided by limit load. From this figure it can be seen two regions: stable region (safe) which defined as the conditions of crack growth during which the applied value of ( $K$ ) for linear elastic conditions equal the resistance of material to fracture, and unstable region (unsafe), this shown fracture condtion to propagate untile complete fracture occurs.

The point where intersects between failure assessment diagram and loading curve give the value of ( $L_r$ ) for crack intiation and from this crack intiation moment can be calculated. In Fig.4  $L_r = 0.39$  and the crack intiation moment =  $471 \text{ kN.m}$

To check this value of crack intiation moment we plot the intiation value of ( $J_i$ ) and applied J- value against applied moment in Fig.5, and the point of intersection of J- values and  $J_i$  - line give applied moment  $1.2 \text{ MN.m}$  and the crack initiation moment is equal to  $(1.2 \times 10^3) * 0.39 = 468 \text{ kN.m}$ .

To obtain failure bending moment, the J-values calculated for elastic -

plastic using given procedure and for increasing values of applied moment (by take any some of values of applied moment above crack initiation moment) are plotted against varies values of crack size as shown in Fig.6. The J-material curve (J-values calculated from  $K_m$ ) is also plotted in the same figure. The point where the slope of J-against half crack angle and J-material curve are the same will give failure moment. Failure bending moment = 503 kN.m.

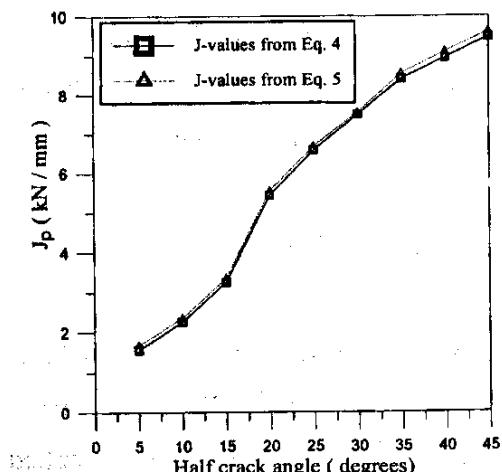


Fig.3: J-values for plastic a portion vs. crack angle

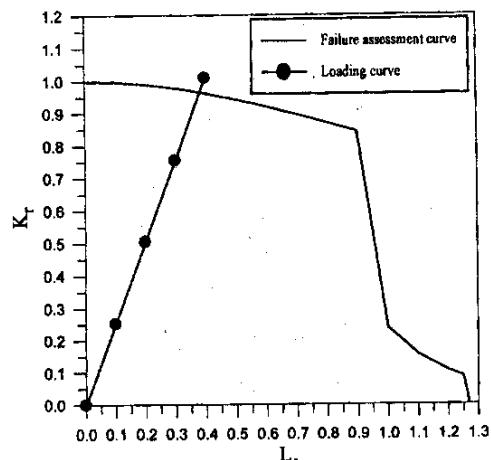


Fig.4 Failure assessment diagram

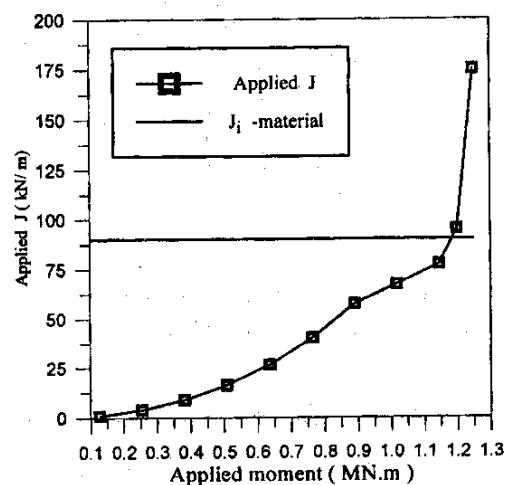


Fig.5 J- values against applied moments

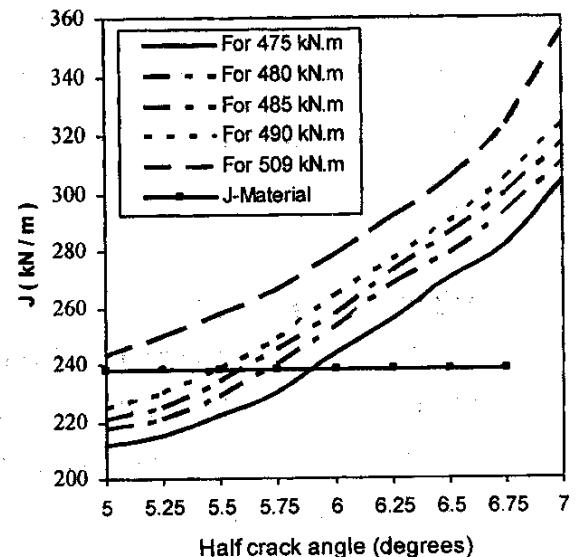


Fig.6 J against crack extension curve

### Conclusions

The J-integral has been analysed for circumferential crack and we may claim that J-integral will be a useful method to evaluate the onset of crack growth under bending moment loading. From discussion before we can conclude the following remarks:

- 1-For plastic problems, J-Integral result presented good agreement between numerical procedure used for analysis.
- 2- An increase in the crack angle will increase the J-values result from increasing stresses in the crack tip.
- 3- For DCIP , there will be significant unstable crack growth before final fracture.

### Acknowledgement

The author express all thanks to all staff in the chemical engineering department, Basrah university for their helps and supporting during different stage in this work.

### References

- [1] Erik Levlin, " Water and waste pipes ", Report of water resource engineering, Royle Institute of Technology, 2004.

[2] Troy F. Stroud , " Design of ductile iron pipe" Report of ductile iron pipe research association, 2003.

[3] Najjarin H., Sadiq R.,and Rajan B. " Condition assessment of water mains using fuzzy evidential reasoning ", 2005 IEEE international conference on system man and cybernetics, pp. 3466 -3471 , Hawaii, USA.

[4] Zencker U., Zeisler P. " Dynamic fracture mechanics assessment for cubic ductile cast iron containers " J. of Ramtrans, vol.11, no.2 , pp. 113-118, 2000.

[5] Araujo T. D., Bittencourt, T. N., Rouhi D., and Martha L. F. "Numerical estimation of fracture parameters in elastic and plastic fracture analysis ", European congress on computational method in applied science and engineering (ECCOMAS), Barcelona 11-14 September 2000.

[6] Ramachandra Murthy D. S. "Assessing the significant of crack in structural components using fracture mechanics", IE(I) Journal-CV, vol. 84, no.1, 2003.

[7] Shaif Rahman " Probabilistic elastic fracture analysis of circumferentially cracked piped with

finite element surface flaws", Nuclear engineering and design journal , no. 195, pp.239-260, 2000.

[8] Chatopadhyay J. " Theoretical and experimental investigation on integrity assessment of pipes and elbows", M.Sc. thesis, Material testing institute university of stuttgart, India, 2004.

[9] Richard W. Bonds, " Design of ductile iron pipe on support ", Report of ductile iron pipe research association, 2001.

[10] Shackelford J. F. " Introduction to material science for engineering ", Fifth edition, 2000.

[11] Chandwani R., Timbress C. M., and Wiehahn M. A. "Crack modelling in power plant component ", International conference on pressure vessel and piping, India, 2006.